International Journal of Advances in Mathematics

Volume 2018, Number 4, Pages 1-24, 2018

eISSN 2456-6098

Compactness and Continuity On Neutrosophic Soft Metric Space

Tuhin Bera[1] and Nirmal Kumar Mahapatra*

[1] Department of Mathematics, Boror S. S. High School, Bagnan,

Howrah-711312, W.B, India.

* Department of Mathematics, Panskura Banamali College,

Panskura RS-721152, W.B, India.

Email:nirmal_hridoy@yahoo.co.in

ABSTRACT. In this paper, the notion of compact neutrosophic soft metric space is introduced. The concept of neutrosophic soft function and the composition of functions in a neutrosophic soft metric space along with suitable examples also have been brought. The continuity and uniform continuity of a neutrosophic soft function in this space have been defined and verified by proper examples. Several related properties, theorems and structural characteristics of these have been investigated here.

1 Introduction

The theory of Neutrosophic set (NS) introduced by Smarandache [19, 20] is the generalization of many theories e.g., fuzzy set, intuitionistic fuzzy set etc practiced to handle the various uncertainties in many real application over the past many years. The neutrosophic logic includes the information about the percentage of truth, indeterminacy and falsity grade in several real world problem like in law, medicine, engineering, management, industrial, IT sector etc which is not available in fuzzy set theory and intuitionistic fuzzy set theory.

Molodtsov has shown that each of the above topics dealing with uncertainties suffer from inherent difficulties possibly due to inadequacy of their parametrization tool. So, Molodtsov [1] proposed the concept of 'soft set

* Corresponding Author.

Received March 05, 2018; accepted June 17, 2018.

2010 Mathematics Subject Classification: 47H10, 54H25.

Key words and phrases: Compact neutrosophic soft metric space; Neutrosophic soft functions; Composition of neutrosophic soft function; Continuity; Uniform continuity.

theory′ for modeling vagueness and uncertainties. It is completely free from the parametrization inadequacy syndrome. This makes the theory very convenient, efficient and easy to apply in practice. In accordance of this, Maji et al. [2-4] studied the several basic operations in soft sets theory over fuzzy sets and intuitionistic fuzzy sets. The notions of fuzzy metric space were studied in [5-13] from different point of view. Roy and Samanta [14] have defined open and closed sets on fuzzy topological spaces. Park [15] and Alaca et al. [16] defined the concept of intuitionistic fuzzy metric space in term of continuous t-norms and continuous t-conorms as a generalisation of fuzzy metric space. Using all these concepts, Beaula et al. [17,18] proposed the notion of fuzzy soft metric spaces in terms of fuzzy soft points.

After introduction of NS theory, Maji [21] has brought a combined notion Neutrosophic soft set (NSS). In continuation, several mathematicians have presented their research works in different mathematical structures. Deli and Broumi [22], Cetkin and Aygun [24-26], Bera and Mahapatra [27-34] studied some fundamental algebraic structures in NSS theory context. Deli and Broumi [23] have also modified some operations related to indeterministic function of NSSs given by Maji. Broumi et al. [35, 36] have done some consecutive works in graph theory over NSS.

The motivation of the present paper is to extend the concept neutrosophic soft metric space (NSMS) proposed in [32]. The current article presents the notion of compact NSMS, the continuity and uniform continuity of a neutrosophic soft function in an NSMS along with investigation of some related properties and theorems. The content of the present paper is designed as follows :

Section 2 gives some preliminary useful definitions, examples and theorems which will be used through out the paper. In section 3, compactness of NSMS is defined and illustrated by examples. Some related basic properties have been studied here, also. Section 4 deals with the continuity of neutrosophic soft function and the composition of neutrosophic soft functions in an NSMS along with the study of their structural characteristics. The concept of uniform continuity of a neutrosophic soft function in an NSMS has been introduced in section 5. Finally, the conclusion of the present work is stated in section 6.

2 Preliminaries

We recall some basic definitions and theorems related to fuzzy set, soft set, NS, NSS, NSMS for the sake of completeness.

2.1 Definitions related to Fuzzy Set and Soft set

This section gives some important definitions related to Fuzzy set, Soft Set [1, 28] :

1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t - norm if $*$ satisfies the following conditions :

(i) $*$ is commutative and associative.

(ii) $*$ is continuous.

(iii) $a * 1 = 1 * a = a, \ \forall a \in [0,1]$.

(iv) $a * b \leq c * d$ if $a \leq c, \ b \leq d$ with $a, b, c, d \in [0,1]$.

A few examples of continuous t-norm are $a * b = ab, a * b = \min\{a, b\}, a * b = \max\{a + b - 1, 0\}$.

2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t - conorm (s - norm) if $\diamond$ satisfies the following conditions :

(i) $\diamond$ is commutative and associative.

(ii) $\diamond$ is continuous.

(iii) $a \diamond 0 = 0 \diamond a = a, \ \forall a \in [0,1]$.

(iv) $a \diamond b \leq c \diamond d$ if $a \leq c, b \leq d$ with $a, b, c, d \in [0,1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab, a \diamond b = \max\{a, b\}, a \diamond b = \min\{a + b, 1\}$.

3. Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \to P(U)$ is a mapping.

2.2 Definitions related to NS and NSS

Few relevant definitions are given below [19, 21, 23, 33] :

1. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is $T_A, I_A, F_A : X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x)$, $F_A(x)$ and so, $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2. Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F : A \to NS(U)$ is a mapping.

This concept has been modified by Deli and Broumi as given below :

3. Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \to NS(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(U)$ and therefore it can be written as a set of ordered pairs,

$$\begin{aligned} N &= \{(e, f_N(e)) : e \in E\} \\ &= \{(e, \{< x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) >: x \in U\}) : e \in E\} \end{aligned}$$

where $T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) \in [0,1]$ and they are respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_N(e)}(x) + I_{f_N(e)}(x) + F_{f_N(e)}(x) \leq 3$ is obvious.

4. The complement of a neutrosophic soft set N is denoted by N^c and is defined by :

$N^c = \{(e, \{< x, F_{f_N(e)}(x), 1 - I_{f_N(e)}(x), T_{f_N(e)}(x) >: x \in U\}) : e \in E\}$

5. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if $\forall e \in E, \forall x \in U$,

$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x); I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x); F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x)$.

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

6. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined by :

$N_3 = \{(e, \{< x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x) >: x \in U\}) : e \in E\}$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \diamond T_{f_{N_2}(e)}(x)$, $I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x)$ and $F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x)$;

7. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their intersection is denoted by $N_1 \cap N_2 = N_3$ and is defined by :

$N_3 = \{(e, \{< x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x) >: x \in U\}) : e \in E\}$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x)$, $I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x)$ and $F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x)$;

8. A neutrosophic soft set N over (U, E) is said to be null neutrosophic soft set if $T_{f_N(e)}(x) = 0, I_{f_N(e)}(x) = 1, F_{f_N(e)}(x) = 1; \forall e \in E, \forall x \in U$. It is denoted by ϕ_u.

A neutrosophic soft set N over (U, E) is said to be absolute neutrosophic soft set if $T_{f_N(e)}(x) = 1, I_{f_N(e)}(x) = 0, F_{f_N(e)}(x) = 0; \forall e \in E, \forall x \in U$. It is denoted by 1_u.

Clearly, $\phi_u^c = 1_u$ and $1_u^c = \phi_u$.

9. A neutrosophic soft point in an NSS N is defined as an element $(e, f_N(e))$ of N, for $e \in E$ and is denoted by e_N, if $f_N(e) \notin \phi_u$ and $f_N(e') \in \phi_u, \forall e' \in E - \{e\}$.

The complement of a neutrosophic soft point e_N is another neutrosophic soft point e_N^c such that $f_N^c(e) = (f_N(e))^c$.

A neutrosophic soft point $e_N \in M$, M being an NSS if for $e \in E$, $f_N(e) \leq f_M(e)$ i.e., $T_{f_N(e)}(x) \leq T_{f_M(e)}(x)$, $I_{f_N(e)}(x) \geq I_{f_M(e)}(x)$, $F_{f_N(e)}(x) \geq F_{f_M(e)}(x)$, $\forall x \in U$.

Example : Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then,

$e_{1N} = \{< x_1, (0.6, 0.4, 0.8) >, < x_2, (0.8, 0.3, 0.5) >, < x_3, (0.3, 0.7, 0.6) >\}$

is a neutrosophic soft point whose complement is :

$e_{1N}^c = \{< x_1, (0.8, 0.6, 0.6) >, < x_2, (0.5, 0.7, 0.8) >, < x_3, (0.6, 0.3, 0.3) >\}$.

For another NSS M defined on same (U, E), let

$f_M(e_1) = \{< x_1, (0.7, 0.4, 0.7) >, < x_2, (0.8, 0.2, 0.4) >, < x_3, (0.5, 0.6, 0.5) >\}$.

Then $f_N(e_1) \leq f_M(e_1)$ i.e., $e_{1N} \in M$.

2.3 Definitions related to neutrosophic soft metric space

Following necessary definitions are provided here [32]:

1. Let $NS(U_E)$ be the collection of all neutrosophic soft points over (U, E). Then the neutrosophic soft metric in terms of neutrosophic soft points is defined by a mapping $d : NS(U_E) \times NS(U_E) \to [0,3]$ satisfying the following conditions :

$NSM1 : d(e_M, e_N) \geq 0,\ \forall e_M, e_N \in NS(U_E)$.

$NSM2 : d(e_M, e_N) = 0 \Leftrightarrow e_M = e_N$.

$NSM3 : d(e_M, e_N) = d(e_N, e_M)$.

$NSM4 : d(e_M, e_N) \leq d(e_M, e_P) + d(e_P, e_N),\ \forall e_M, e_P, e_N \in NS(U_E)$.

Then $NS(U_E)$ is said to form an NSMS with respect to the neutrosophic soft metric 'd' over (U, E) and is denoted by $(NS(U_E), d)$. Here $e_M = e_N$ in the sense that $T_{e_M}(x_i) = T_{e_N}(x_i)$, $I_{e_M}(x_i) = I_{e_N}(x_i)$, $F_{e_M}(x_i) = F_{e_N}(x_i),\ \forall x_i \in U$.

2. *Example* (i) On $NS(U_E)$ define $d(e_M, e_N) = \min_{x_i}\{(|T_{e_M}(x_i) - T_{e_N}(x_i)|^k + |I_{e_M}(x_i) - I_{e_N}(x_i)|^k + |F_{e_M}(x_i) - F_{e_N}(x_i)|^k)^{\frac{1}{k}}\}, k(\geq 1)$ being any real number. This 'd' satisfies all the metric axioms and so, it is a neutrosophic soft metric over (U, E).

(ii) Let 'd' be a neutrosophic soft metric on $NS(U_E)$. Suppose $d_1(e_M, e_N) = \frac{d(e_M, e_N)}{1+d(e_M, e_N)}$; Then '$d_1$' satisfies all the metric axioms. So, $(NS(U_E), d_1)$ is an NSMS with respect to the neutrosophic soft metric d_1.

3. Let $(NS(U_E), d)$ be a neutrosophic soft metric space and $t \in (0,3]$. An open ball having center at $e_N \in NS(U_E)$ and radius 't' is defined by a set $B(e_N, t) = \{e_{iN} \in NS(U_E) : d(e_N, e_{iN}) < t\}$.

The neutrosophic soft closed ball is defined as : $B[e_N, t] = \{e_{iN} \in NS(U_E) : d(e_N, e_{iN}) \leq t\}$.

A neighbourhood of $e_N \in NS(U_E)$ is defined by an open ball $B(e_N, t)$ with center at e_N and radius $t \in (0,3]$.

4. In an NSMS $(NS(U_E), d)$ over (U, E), a neutrosophic soft point e_N is called an interior point of $NS(U_E)$ if there exist an open ball $B(e_N, t)$ such that $B(e_N, t) \subset NS(U_E)$.

For an NSMS $(NS(U_E), d)$ over (U, E), an NSS M is called open if each of it's points is an interior point.

5. A neutrosophic soft point e_N in an NSMS $(NS(U_E), d)$ is called a limit point/ accumulation point of an NSS $M \subset NS(U_E)$ if for every $t \in (0,3]$, $B(e_N, t)$ contains atleast one neutrosophic soft point of M distinct from e_N.

Collection of all limit points of M is called derived NSS of M and is denoted by $D(M)$. An NSS $M \subset NS(U_E)$ in an NSMS $(NS(U_E), d)$ over (U, E) is closed NSS if $D(M) \subset M$ or M has no limit point.

6. A sequence of neutrosophic soft points $\{e_{nN}\}$ in an NSMS $(NS(U_E), d)$ is said to converge in $(NS(U_E), d)$ if there exists a neutrosophic soft point $e_N \in NS(U_E)$ such that $d(e_{nN}, e_N) \to 0$ as $n \to \infty$ or $e_{nN} \to e_N$ as $n \to \infty$. Analytically, for every $\epsilon > 0$ there exists a natural number n_0 such that $d(e_{nN}, e_N) < \epsilon\ \forall n \geq n_0$.

7. A sequence $\{e_{nN}\}$ of neutrosophic soft point in an NSMS $(NS(U_E), d)$ is said to be a Cauchy sequence if to every $\epsilon > 0$ there exists an $n_0 \in \mathbf{N}$ (set of natural numbers) such that $d(e_{mN}, e_{nN}) < \epsilon\ \forall m, n \geq n_0$ i.e., $d(e_{mN}, e_{nN}) \to 0$ as $m, n \to \infty$.

8. An NSMS $(NS(U_E), d)$ is said to be complete if every Cauchy sequence in $(NS(U_E), d)$ converges to a neutrosophic soft point of $NS(U_E)$.

9. Let $(NS(U_E), d)$ be an NSMS. Then the diameter of $NS(U_E)$ is defined as :

$\delta(NS(U_E)) = \sup\{d(e_{1N}, e_{2N}) : e_{1N}, e_{2N} \in NS(U_E)\}$.

An NSS $M \subset NS(U_E)$ is bounded if it has a finite diameter i.e., if $d(e_{1M}, e_{2M}) \leq r$, for $r \in (0,3]$ and $\forall e_{1M}, e_{2M} \in M$.

2.4 Theorems related to neutrosophic soft metric space

Some necessary theorems are stated for the sake of completeness [32]:

1. In an NSMS $(NS(U_E), d)$, every neutrosophic soft open ball $B(e_N, t)$ is open and every neutrosophic soft closed ball $B[e_N, t]$ is closed.

2. Let $(NS(U_E), d)$ be an NSMS over (U, E). Then,

(i) the intersection of finite number of open NSSs in $(NS(U_E), d)$ is open.

(ii) the intersection of any family of closed NSSs in $(NS(U_E), d)$ is closed.

3. Every finite neutrosophic soft subset of an NSMS is closed.

3 Compactness of NSMS

In this section, the compact NSMS has been defined and illustrated by examples. Some related theorems also have been developed here.

3.1 Definition

An NSMS $(NS(U_E), d)$ is said to be compact if every sequence of neutrosophic soft points $\{e_{nM}\}$ of the space has a subsequence $\{e_{n_k M}\}$ converging to a neutrosophic soft point of $NS(U_E)$.

An NSS $M \subset NS(U_E)$ is said to be compact if every sequence of neutrosophic soft points chosen from M has a subsequence converging to a point of M. If the limit of the subsequence belongs to $NS(U_E)$ and not necessarily to M, then M is said to be compact in $(NS(U_E), d)$.

3.1.1 *Example*

(1) Let $E = \{e\}$ and $U = \{x, y, z\}$. Define a distance function on $NS(U_E)$ as :

$$d(e_M, e_N) = \begin{cases} 1 & \text{if } e_M \neq e_N \\ 0 & \text{if } e_M = e_N. \end{cases}$$

Then 'd' is a neutrosophic soft metric on $NS(U_E)$ and is called discrete neutrosophic soft metric. Thus $(NS(U_E), d)$ is a discrete NSMS. It is a compact NSMS.

(2) Consider the NSMS $(NS(U_E), d)$ where $E = \mathbf{N}$ (the set of natural number) be the parametric set, $U = \mathbf{Z}$ (the set of all integers) be the universal set and d is defined as in (2)(i) of [2.3]; Since T, I, $F \in [0, 1]$, every sequence of neutrosophic soft points of the space has a convergent subsequence and so $(NS(U_E), d)$ is compact.

(3) Take the NSMS $(NS(U_E), d)$ where $E = \mathbf{N}$ (the set of natural number), $U = \mathbf{Z}$ (the set of all integers) and 'd' is defined as in (2)(i) of [2.3]; Consider a sequence of neutrosophic soft points $\{e_{nM}\}$ as, $\forall x \in \mathbf{Z}$:

$T_{e_{nM}}(x) = \frac{1}{2n}$, $I_{e_{nM}}(x) = 1 - \frac{1}{2n}$, $F_{e_{nM}}(x) = \frac{n}{1+2n}$ for $T_{e_{nM}}, I_{e_{nM}}, F_{e_{nM}} \in (0,1)$
Then M is not compact itself but is compact on $NS(U_E)$.

(4) Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ and $U = \mathbf{Z}$. Define 'd' as in (2)(i) of [2.3]; Then $(NS(U_E), d)$ is not compact. We shall verify it by taking a sequence of neutrosophic soft points as given in Table 1.

Table 1 : Tabular form of neutrosophic soft sequence

	e_{1M}	e_{2M}	e_{3M}	e_{4M}	e_{5M}	e_{6M}	e_{7M}	e_{8M}
x_1	$(1,0,0)$	$(0,1,0)$	$(0,0,1)$	$(1,1,0)$	$(1,0,1)$	$(0,1,1)$	$(0,0,0)$	$(1,1,1)$
x_2	$(1,0,0)$	$(0,1,0)$	$(0,0,1)$	$(1,1,0)$	$(1,0,1)$	$(0,1,1)$	$(0,0,0)$	$(1,1,1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then $d(e_{iM}, e_{jM}) \neq 0$ for $i \neq j$. So, neither the sequence nor any of it's subsequence is convergent.

3.2 Theorem

A compact NSMS is complete.

Proof. Let $(NS(U_E), d)$ be a compact NSMS and $\{e_{nM}\}$ be a Cauchy sequence of neutrosophic soft points in $NS(U_E)$. Then to every $\epsilon > 0$ there exists an $n_0 \in \mathbf{N}$ (set of natural numbers) such that $d(e_{mM}, e_{nM}) < \epsilon, \ \forall n > m \geq n_0$.
Since $(NS(U_E), d)$ be compact, $\exists$ a subsequence $\{e_{n_k M}\}$ such that $lim_{n\to\infty} e_{n_k M} = e_P$, say. Then $d(e_{n_k M}, e_P) < \epsilon, \ \forall n_k \geq n_0$. Also $d(e_{mM}, e_{n_k M}) < \epsilon, \ \forall n_k > m \geq n_0$.

Now for $n > m, \ d(e_{nM}, e_P) \leq d(e_{nM}, e_{mM}) + d(e_{mM}, e_{n_k M}) + d(e_{n_k M}, e_P) < 3\epsilon$. Thus $\{e_{nM}\}$ being a Cauchy sequence converges to a point in $NS(U_E)$ and so $(NS(U_E), d)$ be a complete NSMS.

3.3 Theorem

Every compact set in an NSMS is closed and bounded.

Proof. Let M be a compact NSS in an NSMS $(NS(U_E), d)$. Suppose M is not closed. Then there exists a sequence $\{e_{nM}\}$ of neutrosophic soft points in M converging to a point e_M (say) not belong to M. Then every subsequence of $\{e_{nM}\}$ also converges to e_M not belong to M. Thus there is no subsequence of $\{e_{nM}\}$ converging to a point of M which contradicts the compactness of M. Hence M is closed.

Next suppose M is not bounded and e_M be fixed neutrosophic soft point. Then $\exists$ a point $e_{1M} \in M$ such that $d(e_M, e_{1M}) > 3$. By similar argument $\exists$ a point $e_{2M} \in M$ such that $d(e_M, e_{2M}) > d(e_M, e_{1M}) + 3$. Continuing this process, we get a sequence of neutrosophic soft points $e_{1M}, e_{2M}, \cdots, e_{nM}, \cdots \in M$ such that $d(e_M, e_{nM}) > d(e_M, e_{1M}) + d(e_M, e_{2M}) + \cdots + d(e_M, e_{(n-1)M}) + 3$. So, for $n > m, \quad d(e_M, e_{nM}) > d(e_M, e_{mM}) + 3$. Now, $d(e_{nM}, e_M) \leq d(e_{nM}, e_{mM}) + d(e_{mM}, e_M)$ and so $d(e_{nM} e_{mM}) > 3$ whenever $n > m$. This shows that neither the sequence $\{e_{nM}\}$ nor any of it's subsequence can converge, contradicting the fact that M is compact. Hence M is bounded.

3.3.1 *Remark*

Converse of above may not be true. The fact is shown by the example (4) of [3.1.1];

Here, $d(e_{iM}, e_{jM}) < 3$ for all $i \neq j$ and $D(M) = \phi \subset M$. So, M is bounded and closed. But M is not compact.

4 Continuity on NSMS

Here, the concept of neutrosophic soft function, it's continuity on an NSMS, the composition of neutrosophic soft functions have been introduced and illustrated by suitable examples. Several properties, structural characteristics and theorems related to these also have been presented here.

4.1 Definition

Let $(NS(U_E), d)$ and $(NS(V_{E'}), d')$ be two NSMSs and $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be a neutrosophic soft function where $\varphi : U \to V$ and $\psi : E \to E'$ be two crisp functions. Consider two neutrosophic soft points e_M, e'_N as :

$e_M = \{< x, (T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)) >: x \in U\} \in NS(U_E),\ e \in E$ and

$e'_N = \{< y, (T_{e'_N}(y), I_{e'_N}(y), F_{e'_N}(y)) >: y \in \varphi(U)\} \in NS(V_{E'}),\ e' \in \psi(E)$

(1) Then the image of e_M under (φ, ψ) is denoted by $(\varphi, \psi)(e_M)$. It is also a neutrosophic soft point e'_N (say) $\in NS(V_{E'})$ defined as follows :

$$T_{e'_N}(y) = \begin{cases} \max_{\varphi(x)=y} \max_{\psi(e)=e'} [T_{e_M}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 0 \ , & \text{otherwise.} \end{cases}$$

$$I_{e'_N}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(e)=e'} [I_{e_M}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \ , & \text{otherwise.} \end{cases}$$

$$F_{e'_N}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(e)=e'} [F_{e_M}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \ , & \text{otherwise.} \end{cases}$$

(2) The pre-image of e'_N under (φ, ψ), denoted by $(\varphi, \psi)^{-1}(e'_N)$, is a neutrosophic soft point e_M (say) $\in NS(U_E)$ and is defined as follows, $\forall x \in U$, $\forall e \in \psi^{-1}(E')$:

$$\begin{aligned} T_{e_M}(x) &= T_{[\psi(e)]_N}(\varphi(x)) = T_{e'_N}(\varphi(x)) \\ I_{e_M}(x) &= I_{[\psi(e)]_N}(\varphi(x)) = I_{e'_N}(\varphi(x)) \\ F_{e_M}(x) &= F_{[\psi(e)]_N}(\varphi(x)) = F_{e'_N}(\varphi(x)) \end{aligned}$$

If ψ and φ are injective (surjective), then (φ, ψ) is injective (surjective).

4.1.1 *Example*

Let $E = \mathbf{N}$ (the set of natural numbers) be the parametric set and $U = \mathbf{Z}$ (the set of integers) be the universal set. Consider a neutrosophic soft point $n_M \in NS(\mathbf{Z_N})$ as follows, for any $n \in \mathbf{N}$ and $x \in \mathbf{Z}$:

$$T_{n_M}(x) = \begin{cases} 0 & \text{if } x = 2k-1,\ k \in \mathbf{Z} \\ \frac{1}{n} & \text{if } x = 2k,\ k \in \mathbf{Z}. \end{cases}$$

$$I_{n_M}(x) = \begin{cases} \frac{1}{2n} & \text{if } x = 2k-1,\ k \in \mathbf{Z} \\ 0 & \text{if } x = 2k,\ k \in \mathbf{Z}. \end{cases}$$

$$F_{n_M}(x) = \begin{cases} 1-\frac{1}{n} & \text{if } x = 2k-1,\ k \in \mathbf{Z} \\ 0 & \text{if } x = 2k,\ k \in \mathbf{Z}. \end{cases}$$

Then $(NS(\mathbf{Z_N}), d)$ forms an NSMS where 'd' is defined in (2)(i) of [2.3]. Now, let $\varphi : \mathbf{Z} \to \mathbf{Z}$ and $\psi : \mathbf{N} \to \mathbf{N}$ be two crisp functions defined as $\varphi(x) = 2x + 3 = y$ (say) and $\psi(n) = 2n - 1 = m$ (say), respectively. Then the neutrosophic soft function $(\varphi, \psi) : (NS(\mathbf{Z_N}), d) \to (NS(\mathbf{Z_N}), d)$ is given by $(\varphi, \psi)(n_M) = m_P$, $m \in \mathbf{N}$ and it is defined as :

$$T_{m_P}(y) = \begin{cases} 0 & \text{if } y = 4k+1,\ k \in \mathbf{Z} \\ \frac{2}{1+m} & \text{if } y = 4k+3,\ k \in \mathbf{Z} \\ 0 & \text{if } y = \text{otherwise}. \end{cases}$$

$$I_{m_P}(y) = \begin{cases} \frac{1}{1+m} & \text{if } y = 4k+1,\ k \in \mathbf{Z} \\ 0 & \text{if } y = 4k+3,\ k \in \mathbf{Z} \\ 1 & \text{if } y = \text{otherwise}. \end{cases}$$

$$F_{m_P}(y) = \begin{cases} \frac{m-1}{m+1} & \text{if } y = 4k+1,\ k \in \mathbf{Z} \\ 0 & \text{if } y = 4k+3,\ k \in \mathbf{Z} \\ 1 & \text{if } y = \text{otherwise}. \end{cases}$$

4.2 Proposition

Let $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be a neutrosophic soft function. Then the image set $\{(\phi, \psi)(e_M) : e_M \in NS(U_E)\}$ forms an NSMS with respect to 'd''.

Proof. Let us consider three neutrosophic soft points $e_M, e_N, e_P \in NS(U_E)$. Now,

$$
\begin{aligned}
&(1) && e_M \neq e_N \;\Rightarrow\; d(e_M, e_N) > 0 \\
&\textit{i.e.,} && (T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)) \neq (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x)) \;\Rightarrow \\
& && d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] > 0, \forall x \in U \\
&\textit{i.e.,} && (\max_{\varphi(x)}\max_{\psi(e)}[T_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_M}(x)]) \neq \\
& && (\max_{\varphi(x)}\max_{\psi(e)}[T_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_N}(x)]) \;\Rightarrow \\
& && d'[(\max_{\varphi(x)}\max_{\psi(e)}[T_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_M}(x)]), \\
& && (\max_{\varphi(x)}\max_{\psi(e)}[T_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_N}(x)])] > 0 \\
&\textit{i.e.,} && (\phi, \psi)(e_M) \neq (\phi, \psi)(e_N) \;\Rightarrow\; d'[(\phi, \psi)(e_M), (\phi, \psi)(e_N)] > 0
\end{aligned}
$$

$$
\begin{aligned}
&(2) && e_M = e_N \;\Leftrightarrow\; d(e_M, e_N) = 0 \\
&\textit{i.e.,} && T_{e_M}(x) = T_{e_N}(x), I_{e_M}(x) = I_{e_N}(x), F_{e_M}(x) = F_{e_N}(x), \forall x \in U \;\Leftrightarrow \\
& && d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] = 0, \forall x \in U \\
&\textit{i.e.,} && \max_{\varphi(x)}\max_{\psi(e)}[T_{e_M}(x)] = \max_{\varphi(x)}\max_{\psi(e)}[T_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_M}(x)] = \\
& && \min_{\varphi(x)}\min_{\psi(e)}[I_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_M}(x)] = \min_{\varphi(x)}\min_{\psi(e)}[F_{e_N}(x)] \;\Leftrightarrow \\
& && d'[(\max_{\varphi(x)}\max_{\psi(e)}[T_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_M}(x)]), \\
& && (\max_{\varphi(x)}\max_{\psi(e)}[T_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_N}(x)])] = 0 \\
&\textit{i.e.,} && (\phi, \psi)(e_M) = (\phi, \psi)(e_N) \;\Leftrightarrow\; d'[(\phi, \psi)(e_M), (\phi, \psi)(e_N)] = 0
\end{aligned}
$$

$$
\begin{aligned}
&(3) && d(e_M, e_N) = d(e_N, e_M) \\
&\Rightarrow && d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] \\
& && = d[(T_{e_N}(x), I_{e_N}(x), F_{e_N}(x)), (T_{e_M}(x), I_{e_M}(x), F_{e_M}(x))] \\
&\Rightarrow && d'[(\max_{\varphi(x)}\max_{\psi(e)}[T_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_M}(x)]), \\
& && (\max_{\varphi(x)}\max_{\psi(e)}[T_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_N}(x)])] \\
& && = d'[(\max_{\varphi(x)}\max_{\psi(e)}[T_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_N}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_N}(x)]), \\
& && (\max_{\varphi(x)}\max_{\psi(e)}[T_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[I_{e_M}(x)], \min_{\varphi(x)}\min_{\psi(e)}[F_{e_M}(x)])] \\
&\Rightarrow && d'[(\phi, \psi)(e_M), (\phi, \psi)(e_N)] = d'[(\phi, \psi)(e_N), (\phi, \psi)(e_M)]
\end{aligned}
$$

$$
\begin{aligned}
(4) \quad & d(e_M, e_N) \leq d(e_M, e_P) + d(e_P, e_N) \\
\Rightarrow \quad & d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] \\
& \leq d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_P}(x), I_{e_P}(x), F_{e_P}(x))] + \\
& d[(T_{e_P}(x), I_{e_P}(x), F_{e_P}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))], \forall x \in U \\
\Rightarrow \quad & d'[(\max_{\varphi(x)} \max_{\psi(e)} [T_{e_M}(x)], \min_{\varphi(x)} \min_{\psi(e)} [I_{e_M}(x)], \min_{\varphi(x)} \min_{\psi(e)} [F_{e_M}(x)]), \\
& (\max_{\varphi(x)} \max_{\psi(e)} [T_{e_N}(x)], \min_{\varphi(x)} \min_{\psi(e)} [I_{e_N}(x)], \min_{\varphi(x)} \min_{\psi(e)} [F_{e_N}(x)])] \\
& \leq d'[(\max_{\varphi(x)} \max_{\psi(e)} [T_{e_M}(x)], \min_{\varphi(x)} \min_{\psi(e)} [I_{e_M}(x)], \min_{\varphi(x)} \min_{\psi(e)} [F_{e_M}(x)]), \\
& (\max_{\varphi(x)} \max_{\psi(e)} [T_{e_P}(x)], \min_{\varphi(x)} \min_{\psi(e)} [I_{e_P}(x)], \min_{\varphi(x)} \min_{\psi(e)} [F_{e_P}(x)])] \\
& + d'[(\max_{\varphi(x)} \max_{\psi(e)} [T_{e_P}(x)], \min_{\varphi(x)} \min_{\psi(e)} [I_{e_P}(x)], \min_{\varphi(x)} \min_{\psi(e)} [F_{e_P}(x)]) \\
& (\max_{\varphi(x)} \max_{\psi(e)} [T_{e_N}(x)], \min_{\varphi(x)} \min_{\psi(e)} [I_{e_N}(x)], \min_{\varphi(x)} \min_{\psi(e)} [F_{e_N}(x)])] \\
\Rightarrow \quad & d'[(\phi, \psi)(e_M), (\phi, \psi)(e_N)] \leq d'[(\phi, \psi)(e_M), (\phi, \psi)(e_P)] + d'[(\phi, \psi)(e_P), (\phi, \psi)(e_N)]
\end{aligned}
$$

This completes the proof.

4.3 Proposition

Let $(\varphi, \psi) : (NS(U_E), d) \rightarrow (NS(V_{E'}), d')$ be an onto neutrosophic soft function. Then the pre-image set $\{(\varphi, \psi)^{-1}(e'_Q) : e'_Q \in NS(V_{E'})\}$ forms also an NSMS with respect to 'd'. [Note that $(\varphi, \psi)^{-1}$ is the inverse image of $NS(V_{E'})$ under the mapping (φ, ψ). Here $(\varphi, \psi)^{-1}$ may not be a mapping.]

Proof. Let $e_M, e_N, e_P \in NS(U_E)$ and $e'_Q, e'_R, e'_S \in NS(V_{E'})$ such that $(\varphi, \psi)^{-1}(e'_Q) = e_M, (\varphi, \psi)^{-1}(e'_R) = e_N, (\varphi, \psi)^{-1}(e'_S) = e_P$ and $\varphi(x) = y$ for $x \in U, y \in V$. Now,

$$
\begin{aligned}
(1) \quad & e'_Q \neq e'_R \;\Rightarrow\; d'(e'_Q, e'_R) > 0 \\
\textit{i.e.,} \quad & (T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y)) \neq (T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y)) \;\Rightarrow \\
& d'[(T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y)), (T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y))] > 0, \forall y \in V
\end{aligned}
$$

$$
\begin{aligned}
\textit{i.e.,} \quad & (T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)) \neq (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x)) \;\Rightarrow \\
& d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] > 0 \\
\textit{i.e.,} \quad & e_M \neq e_N \;\Rightarrow\; d(e_M, e_N) > 0 \;\; \textit{i.e.,} \\
& (\varphi, \psi)^{-1}(e'_Q) \neq (\varphi, \psi)^{-1}(e'_R) \;\Rightarrow\; d[(\varphi, \psi)^{-1}(e'_Q), (\varphi, \psi)^{-1}(e'_R)] > 0
\end{aligned}
$$

$$\begin{aligned}
&(2) && e'_Q = e'_R \;\Leftrightarrow\; d(e'_Q, e'_R) = 0 \\
&i.e., && T_{e'_Q}(y) = T_{e'_R}(y), I_{e'_Q}(y) = I_{e'_R}(y), F_{e'_Q}(y) = F_{e'_R}(y), \forall y \in V \;\Leftrightarrow \\
& && d[(T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y)), (T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y))] = 0, \forall y \in V \\
&i.e., && T_{e_M}(x) = T_{e_N}(x), I_{e_M}(x) = I_{e_N}(x), F_{e_M}(x) = F_{e_N}(x) \;\Leftrightarrow \\
& && d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] = 0 \\
&i.e., && e_M = e_N \;\Leftrightarrow\; d(e_M, e_N) = 0 \\
&i.e., && (\varphi,\psi)^{-1}(e'_Q) = (\varphi,\psi)^{-1}(e'_R) \;\Leftrightarrow\; d[(\varphi,\psi)^{-1}(e'_Q), (\varphi,\psi)^{-1}(e'_R)] = 0
\end{aligned}$$

$$\begin{aligned}
&(3) && d'(e'_Q, e'_R) = d'(e'_R, e'_Q) \\
&\Rightarrow && d'[(T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y)), (T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y))] = \\
& && d'[(T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y)), (T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y))] \\
&\Rightarrow && d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] = \\
& && d[(T_{e_N}(x), I_{e_N}(x), F_{e_N}(x)), (T_{e_M}(x), I_{e_M}(x), F_{e_M}(x))] \\
&\Rightarrow && d(e_M, e_N) = d(e_N, e_M) \;\; i.e., \\
& && d[(\varphi,\psi)^{-1}(e'_Q), (\varphi,\psi)^{-1}(e'_R)] = d[(\varphi,\psi)^{-1}(e'_R), (\varphi,\psi)^{-1}(e'_Q)]
\end{aligned}$$

$$\begin{aligned}
&(4) && d'(e'_Q, e'_S) \le d'(e'_Q, e'_R) + d'(e'_R, e'_S) \\
&\Rightarrow && d'[(T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y)), (T_{e'_S}(y), I_{e'_S}(y), F_{e'_S}(y))] \\
& && \le d'[(T_{e'_Q}(y), I_{e'_Q}(y), F_{e'_Q}(y)), (T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y))] + \\
& && d'[(T_{e'_R}(y), I_{e'_R}(y), F_{e'_R}(y)), (T_{e'_S}(y), I_{e'_S}(y), F_{e'_S}(y))], \forall y \in V \\
&\Rightarrow && d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_P}(x), I_{e_P}(x), F_{e_P}(x))] \\
& && \le d[(T_{e_M}(x), I_{e_M}(x), F_{e_M}(x)), (T_{e_N}(x), I_{e_N}(x), F_{e_N}(x))] + \\
& && d[(T_{e_N}(x), I_{e_N}(x), F_{e_N}(x)), (T_{e_P}(x), I_{e_P}(x), F_{e_P}(x))] \\
&\Rightarrow && d(e_M, e_P) \le d(e_M, e_N) + d(e_N, e_P) \;\; i.e., \\
& && d[(\varphi,\psi)^{-1}(e'_Q), (\varphi,\psi)^{-1}(e'_S)] \le d[(\varphi,\psi)^{-1}(e'_Q), (\varphi,\psi)^{-1}(e'_R)] \\
& && +d[(\varphi,\psi)^{-1}(e'_R), (\varphi,\psi)^{-1}(e'_S)]
\end{aligned}$$

This completes the proof.

4.4 Proposition

Let $P, Q \subset NS(U_E)$ and $M, N \subset NS(V_{E'})$. Then for a neutrosophic soft function $(\varphi,\psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$, the followings hold.

(1) $(\varphi,\psi)^{-1}(M) \subseteq (\varphi,\psi)^{-1}(N) \;\Leftrightarrow\; M \subseteq N$.

(2) $P \subseteq Q \;\Leftrightarrow\; (\varphi,\psi)(P) \subseteq (\varphi,\psi)(Q)$.

(3) $M \subseteq (\varphi,\psi)(P) \Leftrightarrow (\varphi,\psi)^{-1}(M) \subseteq P$.

(4) $(\varphi,\psi)(Q) \subseteq N \Leftrightarrow Q \subseteq (\varphi,\psi)^{-1}(N)$.

Proof. Let $\varphi(x) = y$ and $\psi(e) = e'$ for $x \in U$, $y \in V$ and $e \in E$, $e' \in E'$. Then,

$$
\begin{aligned}
(1) \quad & M \subseteq N \\
\Leftrightarrow\ & T_{e'_M}(y) \leq T_{e'_N}(y),\ I_{e'_M}(y) \geq I_{e'_N}(y),\ F_{e'_M}(y) \geq F_{e'_N}(y),\ \forall e',\ \forall y; \\
\Leftrightarrow\ & T_{[\psi(e)]_M}(\varphi(x)) \leq T_{[\psi(e)]_N}(\varphi(x)),\ I_{[\psi(e)]_M}(\varphi(x)) \geq I_{[\psi(e)]_N}(\varphi(x)), \\
& F_{[\psi(e)]_M}(\varphi(x)) \geq F_{[\psi(e)]_N}(\varphi(x)),\ \forall e,\ \forall x; \\
\Leftrightarrow\ & T_{e_{(\varphi,\psi)^{-1}(M)}}(x) \leq T_{e_{(\varphi,\psi)^{-1}(N)}}(x),\ I_{e_{(\varphi,\psi)^{-1}(M)}}(x) \geq I_{e_{(\varphi,\psi)^{-1}(N)}}(x), \\
& F_{e_{(\varphi,\psi)^{-1}(M)}}(x) \geq F_{e_{(\varphi,\psi)^{-1}(N)}}(x),\ \forall e,\ \forall x; \\
\Leftrightarrow\ & (\varphi,\psi)^{-1}(M) \subseteq (\varphi,\psi)^{-1}(N)
\end{aligned}
$$

$$
\begin{aligned}
(2) \quad & (\varphi,\psi)(P) \subseteq (\varphi,\psi)(Q) \\
\Leftrightarrow\ & \max_{\varphi(x)} \max_{\psi(e)} [T_{e_P}(x)] \leq \max_{\varphi(x)} \max_{\psi(e)} [T_{e_Q}(x)],\ \min_{\varphi(x)} \min_{\psi(e)} [I_{e_P}(x)] \\
& \geq \min_{\varphi(x)} \min_{\psi(e)} [I_{e_Q}(x)],\ \min_{\varphi(x)} \min_{\psi(e)} [F_{e_P}(x)] \geq \min_{\varphi(x)} \min_{\psi(e)} [F_{e_Q}(x)] \\
\Leftrightarrow\ & T_{e_P}(x) \leq T_{e_Q}(x),\ I_{e_P}(x) \geq I_{e_Q}(x),\ F_{e_P}(x) \geq F_{e_Q}(x),\ \forall e,\ \forall x \\
\Leftrightarrow\ & P \subseteq Q
\end{aligned}
$$

$$
\begin{aligned}
(3) \quad & M \subseteq (\varphi,\psi)(P) \\
\Leftrightarrow\ & T_{e'_M}(y) \leq \max_{\varphi(x)} \max_{\psi(e)} [T_{e_P}(x)],\ I_{e'_M}(y) \geq \min_{\varphi(x)} \min_{\psi(e)} [I_{e_P}(x)], \\
& F_{e'_M}(y) \geq \min_{\varphi(x)} \min_{\psi(e)} [F_{e_P}(x)] \\
\Leftrightarrow\ & T_{[\psi(e)]_M}(\varphi(x)) \leq \max_{\varphi(x)} \max_{\psi(e)} [T_{e_P}(x)],\ I_{[\psi(e)]_M}(\varphi(x)) \geq \\
& \min_{\varphi(x)} \min_{\psi(e)} [I_{e_P}(x)],\ F_{[\psi(e)]_M}(\varphi(x)) \geq \min_{\varphi(x)} \min_{\psi(e)} [F_{e_P}(x)] \\
\Leftrightarrow\ & T_{e_{(\varphi,\psi)^{-1}(M)}}(x) \leq T_{e_P}(x),\ I_{e_{(\varphi,\psi)^{-1}(M)}}(x) \geq I_{e_P}(x),\ F_{e_{(\varphi,\psi)^{-1}(M)}}(x) \geq F_{e_P}(x), \forall e, \forall x \\
\Leftrightarrow\ & (\varphi,\psi)^{-1}(M) \subseteq P
\end{aligned}
$$

$$
\begin{aligned}
(4) \quad & (\varphi,\psi)(Q) \subseteq N \\
\Leftrightarrow\ & \max_{\varphi(x)} \max_{\psi(e)} [T_{e_Q}(x)] \leq T_{e'_N}(y),\ \min_{\varphi(x)} \min_{\psi(e)} [I_{e_P}(x)] \geq I_{e'_N}(y), \\
& \min_{\varphi(x)} \min_{\psi(e)} [F_{e_Q}(x)] \geq F_{e'_N}(y) \\
\Leftrightarrow\ & \max_{\varphi(x)} \max_{\psi(e)} [T_{e_Q}(x)] \leq T_{[\psi(e)]_N}(\varphi(x)),\ \min_{\varphi(x)} \min_{\psi(e)} [I_{e_Q}(x)] \geq \\
& I_{[\psi(e)]_N}(\varphi(x)),\ \min_{\varphi(x)} \min_{\psi(e)} [F_{e_Q}(x)] \geq F_{[\psi(e)]_N}(\varphi(x)) \\
\Leftrightarrow\ & T_{e_Q}(x) \leq T_{e_{(\varphi,\psi)^{-1}(N)}}(x),\ I_{e_Q}(x) \geq I_{e_{(\varphi,\psi)^{-1}(N)}}(x),\ F_{e_Q}(x) \geq F_{e_{(\varphi,\psi)^{-1}(N)}}(x), \forall e, \forall x \\
\Leftrightarrow\ & Q \subseteq (\varphi,\psi)^{-1}(N)
\end{aligned}
$$

4.5 Definition

Let $(NS(U_E), d)$ and $(NS(V_{E'}), d')$ be two NSMSs. Then a neutrosophic soft function $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ is said to be continuous at $e_{0N} \in NS(U_E)$ if for each $\epsilon > 0$ there exists a $\delta > 0$ such that

$$d'[(\varphi,\psi)(e_M), (\varphi,\psi)(e_{0N})] < \epsilon \quad \text{whenever} \quad d(e_M, e_{0N}) < \delta,\ e_M \in NS(U_E).$$

i.e., if $(\varphi,\psi)[B_u(e_{0N},\delta)] \subset B_v((\varphi,\psi)(e_{0N}),\epsilon)$ holds. (φ,ψ) is called neutrosophic soft continuous function if it is continuous at every point in $NS(U_E)$.

4.6 Theorem

Let $(\varphi,\psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be a neutrosophic soft function.

(1) If e_{0N} is a limit point of $NS(U_E)$, then (φ,ψ) is neutrosophic soft continuous at e_{0N} iff $\lim_{e_M \to e_{0N}} (\varphi,\psi)(e_M) = (\varphi,\psi)(e_{0N})$.

(2) (φ,ψ) is continuous at $e_{0N} \in NS(U_E)$ iff for every sequence $\{e_{nN}\}$ of neutrosophic soft points in $NS(U_E)$ converging to e_{0N}, we have $\lim_{n\to\infty} (\varphi,\psi)(e_{nN}) = (\varphi,\psi)(e_{0N})$.

Proof. (1) It is straight forward.

(2) First suppose that (φ,ψ) is continuous at $e_{0N} \in NS(U_E)$ and $\lim_{n\to\infty} e_{nN} = e_{0N}$. Then given $\epsilon > 0$, there exists a $\delta > 0$ such that

$d'[(\varphi,\psi)(e_M), (\varphi,\psi)(e_{0N})] < \epsilon \quad \text{whenever} \quad d(e_M, e_{0N}) < \delta,\ e_M \in NS(U_E)$.

Since $\lim_{n\to\infty} e_{nN} = e_{0N}$, there exists a natural number n_0 such that

$d(e_{nN}, e_{0N}) < \delta,\ \forall n \geq n_0$. Putting $e_M = e_{nN}$, we have

$d'[(\varphi,\psi)(e_{nN}), (\varphi,\psi)(e_{0N})] < \epsilon \quad \text{whenever} \quad d(e_{nN}, e_{0N}) < \delta,\ \forall n \geq n_0$.

Thus $d'[(\varphi,\psi)(e_{nN}), (\varphi,\psi)(e_{0N})] < \epsilon,\ \forall n \geq n_0$ and this completes the 'if' part.

Conversely, let the condition be hold but (φ,ψ) is not continuous at $e_{0N} \in NS(U_E)$. Then given $\epsilon > 0$, there exists a $\delta > 0$ such that

$d'[(\varphi,\psi)(e_M), (\varphi,\psi)(e_{0N})] \geq \epsilon \quad \text{whenever} \quad d(e_M, e_{0N}) < \delta,\ e_M \in NS(U_E); \cdots (1)$

But by hypothesis, there exists a natural number n_0 such that

$d'[(\varphi,\psi)(e_{nN}), (\varphi,\psi)(e_{0N})] < \epsilon \quad \text{whenever} \quad d(e_{nN}, e_{0N}) < \delta,\ \forall n \geq n_0$.

Putting $e_M = e_{nN}$ in (1), we have

$d'[(\varphi,\psi)(e_{nN}), (\varphi,\psi)(e_{0N})] \geq \epsilon \quad \text{whenever} \quad d(e_{nN}, e_{0N}) < \delta$

This contradicts the hypothesis and so (φ,ψ) is continuous at $e_{0N} \in NS(U_E)$.

4.6.1 *Example*

1. Let $E = \mathbf{N}$ (the set of natural numbers), $E' = \mathbf{I}$ (unit interval $[0,1]$) and $U = V = \mathbf{Q}^\star$ (the set of nonzero rational numbers). Consider a neutrosophic soft sequence $\{n_M\}$ in $NS(\mathbf{Q}^\star_{\mathbf{N}})$ as following, for any $n \in \mathbf{N}$:

$T_{n_M}(x) = \frac{n}{n+1},\ I_{n_M}(x) = \frac{1}{2n},\ F_{n_M}(x) = \frac{1}{3n},\ \forall x \in \mathbf{Q}^\star$.

Then $(NS(\mathbf{Q}^\star_{\mathbf{N}}), d)$ forms an NSMS where 'd' is defined in (2)(i) of [2.3]. Now, let $\varphi : \mathbf{Q}^\star \to \mathbf{Q}^\star$ and $\psi : \mathbf{N} \to \mathbf{I}$ be two crisp functions defined as $\varphi(x) = \frac{1}{x} = y$ (say) and $\psi(n) = 1 - \frac{1}{n} = m$ (say), respectively. Then the neutrosophic soft function $(\varphi,\psi) : NS(\mathbf{Q}^\star_\mathbf{N}) \to NS(\mathbf{Q}^\star_\mathbf{I})$ is given by $(\varphi,\psi)(n_M) = m_P,\ m \in \mathbf{I}$ and is defined as :

$T_{m_P}(y) = \frac{1}{2-m},\ I_{m_P}(y) = \frac{1-m}{2},\ F_{m_P}(y) = \frac{1-m}{3},\ \forall y = \frac{1}{x} \in \mathbf{Q}^\star.$

We now define a neutrosophic soft point $a_S \in NS(\mathbf{Q}^\star_\mathbf{N}), a \in \mathbf{N}$ given as :

$T_{a_S}(x) = 1,\ I_{a_S}(x) = 0,\ F_{a_S}(x) = 0,\ \ \forall x \in \mathbf{Q}^\star.$

We shall estimate the distance function 'd' here for $k = 1$ only. Similar conclusion can be drawn for different values of k.

$$\begin{aligned} d(n_M, a_S) &= |T_{n_M}(x) - T_{a_S}(x)| + |I_{n_M}(x) - I_{a_S}(x)| + |F_{n_M}(x) - F_{a_S}(x)| \\ &= |\frac{n}{n+1} - 1| + |\frac{1}{2n} - 0| + |\frac{1}{3n} - 0| \\ &= \frac{1}{n+1} + \frac{1}{2n} + \frac{1}{3n} \\ &= \frac{11n+5}{6n^2+6n} = \frac{11+\frac{5}{n}}{n(6+\frac{6}{n})} \end{aligned}$$

Hence, $d(n_M, a_S) \longrightarrow 0$ as $n \longrightarrow \infty$ i.e., $\{n_M\}$ converges to a_S.

To test the continuity of (φ,ψ) at a_S, we shall use the theorem (2) of [4.6].

Clearly, $(\varphi,\psi)(a_S) = \{< y, (1,0,0) >: y \in \varphi(\mathbf{Q}^\star)\}$; For same '$d$' stated above,

$$\begin{aligned} d[(\varphi,\psi)(n_M), (\varphi,\psi)(a_S)] &= |\frac{1}{2-m} - 1| + |\frac{1-m}{2} - 0| + |\frac{1-m}{3} - 0| \\ &= \frac{1-m}{2-m} + \frac{1-m}{2} + \frac{1-m}{3} \\ &= \frac{(1-m)(16-5m)}{6(2-m)} \end{aligned}$$

This shows $d[(\varphi,\psi)(n_M), (\varphi,\psi)(a_S)] \longrightarrow 0$ as $n \longrightarrow \infty$ (i.e., as $m \to 1$). Hence $\{(\varphi,\psi)(n_M)\}$ converges to $(\varphi,\psi)(a_S)$ and so (φ,ψ) is continuous at (a_S).

2. Consider a neutrosophic soft sequence $\{n_M\}$ in $NS(\mathbf{Z}_\mathbf{N})$ ($\mathbf{Z}$ being the set of integers and $\mathbf{N}$ being the set of natural numbers) as following, for any $n \in \mathbf{N}$:

$T_{n_M}(x) = 1 - \frac{1}{n},\ I_{n_M}(x) = \frac{1}{7n},\ F_{n_M}(x) = \frac{1}{n+1},\ \ \forall x \in \mathbf{Z}.$

Then $(NS(\mathbf{Z}_\mathbf{N}), d)$ forms an NSMS where 'd' is defined in (2)(i) of [2.3]. Now, let a neutrosophic soft function $(\varphi,\psi) : NS(\mathbf{Z}_\mathbf{N}) \to NS(\mathbf{Z}_{\mathbf{N}\cup\{0\}})$ be given by $(\varphi,\psi)(n_M) = m_P,\ m \in \mathbf{N}$ where $\varphi : \mathbf{Z} \to \mathbf{Z}$ and $\psi : \mathbf{N} \to \mathbf{N} \cup \{0\}$ be two crisp functions defined as $\varphi(x) = 3x = y$ (say) and $\psi(n) = n - 1 = m$ (say). Then $(\varphi,\psi)(n_M) = m_P$ is defined as :

$$T_{m_P}(y) = \begin{cases} 1 - \frac{1}{m+1} & \text{if } y = \varphi(x) \\ 0 & \text{otherwise.} \end{cases}$$

$$I_{m_P}(y) = \begin{cases} \frac{1}{7(m+1)} & \text{if } y = \varphi(x) \\ 1 & \text{otherwise.} \end{cases}$$

$$F_{m_P}(y) = \begin{cases} \frac{1}{m+2} & \text{if } y = \varphi(x) \\ 1 & \text{otherwise.} \end{cases}$$

We now define a neutrosophic soft point $a_S \in NS(\mathbf{Z}_\mathbf{N}), a \in \mathbf{N}$ given as :

$T_{a_S}(x) = 1,\ I_{a_S}(x) = 0,\ F_{a_S}(x) = 0,\ \forall x \in \mathbf{Z}$. Then for $k = 1$,

$$\begin{aligned} d(n_M, a_S) &= |T_{n_M}(x) - T_{a_S}(x)| + |I_{n_M}(x) - I_{a_S}(x)| + |F_{n_M}(x) - F_{a_S}(x)| \\ &= |1 - \frac{1}{n} - 1| + |\frac{1}{7n} - 0| + |\frac{1}{n+1} - 0| \\ &= \frac{1}{n} + \frac{1}{7n} + \frac{1}{n+1} \\ &= \frac{15n+8}{7n^2+7n} = \frac{15 + \frac{8}{n}}{n(7 + \frac{7}{n})} \end{aligned}$$

Similar conclusion can be drawn for different choice of k. Hence, $d(n_M, a_S) \longrightarrow 0$ as $n \longrightarrow \infty$ i.e., $\{n_M\}$ converges to a_S. But $\{(\varphi, \psi)(n_M)\}$ does not converge to $(\varphi, \psi)(a_S)$ clearly and hence, (φ, ψ) is not continuous at a_S.

4.7 Theorem

Let $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be a neutrosophic soft function. Then (φ, ψ) is continuous on $NS(U_E)$ iff $(\varphi, \psi)^{-1}(P)$ is open in $NS(U_E)$ whenever $P \subset NS(V_{E'})$ is open.

Proof. First suppose (φ, ψ) be continuous on $NS(U_E)$ and $P \subset NS(V_{E'})$ be an open NSS. Let $e_{0M} \in (\varphi, \psi)^{-1}(P)$. Then $(\varphi, \psi)(e_{0M}) \in P$. Since P is open NSS, there exists an open ball $B_v((\varphi, \psi)(e_{0M}), \epsilon) \subset P$. Again as (φ, ψ) is continuous at e_{0M}, there exists $\delta > 0$ such that $d'[(\varphi, \psi)(e_N), (\varphi, \psi)(e_{0M})] < \epsilon$ whenever $d(e_N, e_{0M}) < \delta$ for $e_N \in NS(U_E)$. It implies $(\varphi, \psi)(e_N) \in B_v((\varphi, \psi)(e_{0M}), \epsilon)$, $\forall e_N \in B_u(e_{0M}, \delta)$. But $(\varphi, \psi)(e_N) \in B_v((\varphi, \psi)(e_{0M}), \epsilon) \subset P \Rightarrow e_N \in (\varphi, \psi)^{-1}(P)$. Thus $B_u(e_{0M}, \delta) \subset (\varphi, \psi)^{-1}(P)$ whenever $e_{0M} \in (\varphi, \psi)^{-1}(P)$. Hence e_{0M} is an interior point of $(\varphi, \psi)^{-1}(P)$. Since e_{0M} is arbitrary, $(\varphi, \psi)^{-1}(P)$ is open in $NS(U_E)$.

Conversely, assume that $(\varphi, \psi)^{-1}(P)$ is open in $NS(U_E)$ for every open NSS $P \subset NS(V_{E'})$ and $e_{0M} \in NS(U_E)$ be arbitrary but fixed. Then $(\varphi, \psi)(e_{0M}) \in NS(V_{E'})$ and $B_v((\varphi, \psi)(e_{0M}), \epsilon)$ being an open ball is an open set in $NS(V_{E'})$. So by hypothesis, $(\varphi, \psi)^{-1}[B_v((\varphi, \psi)(e_{0M}), \epsilon)]$ is open in $NS(U_E)$. Now $(\varphi, \psi)(e_{0M}) \in B_v((\varphi, \psi)(e_{0M}), \epsilon)$, clearly and so $e_{0M} \in (\varphi, \psi)^{-1}[B_v((\varphi, \psi)(e_{0M}), \epsilon)]$. Since $(\varphi, \psi)^{-1}[B_v((\varphi, \psi)(e_{0M}), \epsilon)]$ is open in $NS(U_E)$, so $B_u(e_{0M}, \delta) \subset (\varphi, \psi)^{-1}[B_v((\varphi, \psi)(e_{0M}), \epsilon)]$. Let $e_N \in B_u(e_{0M}, \delta) \subset (\varphi, \psi)^{-1}[B_v((\varphi, \psi)(e_{0M}), \epsilon)]$. Then $e_N \in B_u(e_{0M}, \delta)$ and $(\varphi, \psi)(e_N) \in B_v((\varphi, \psi)(e_{0M}), \epsilon)$. This shows that $d'[(\varphi, \psi)(e_N), (\varphi, \psi)(e_{0M})] < \epsilon$ whenever $d(e_N, e_{0M}) < \delta$ i.e., (φ, ψ) is continuous at e_{0M}. Since $e_{0M} \in NS(U_E)$ is arbitrary, so (φ, ψ) is continuous on $NS(U_E)$.

4.8 Theorem

Let $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be an injective and continuous neutrosophic soft function. Then $(\varphi, \psi)^{-1}(Q)$ is closed in $NS(U_E)$ whenever $Q \subset NS(V_{E'})$ is closed.

Proof. Let $e_{0M} \in NS(U_E)$ be a limit point of $(\varphi, \psi)^{-1}(Q) \subset NS(U_E)$ and $e_N \in B_u(e_{0M}, \delta) \cap (\varphi, \psi)^{-1}(Q)$, $e_N \neq e_{0M}$. Then by sense of [2.2] (9), $e_N \in B_u(e_{0M}, \delta)$ and $e_N \in (\varphi, \psi)^{-1}(Q) \Rightarrow e_N \in B_u(e_{0M}, \delta)$ and $(\varphi, \psi)(e_N) \in Q$. Again as (φ, ψ) is continuous at e_{0M}, there exists $\epsilon > 0$ such that $(\varphi, \psi)(e_N) \in B_v((\varphi, \psi)(e_{0M}), \epsilon)$ whenever $e_N \in B_u(e_{0M}, \delta)$ for $e_N \in NS(U_E)$. Thus $(\varphi, \psi)(e_N) \in B_v((\varphi, \psi)(e_{0M}), \epsilon) \cap Q$ with $(\varphi, \psi)(e_N) \neq (\varphi, \psi)(e_{0M})$, as (φ, ψ)

is injective. This shows that $(\varphi,\psi)(e_{0M})$ is a limit point of Q. Since Q is closed in $NS(V_{E'})$, so $(\varphi,\psi)(e_{0M}) \in Q$ i.e., $e_{0M} \in (\varphi,\psi)^{-1}(Q)$. Hence e_{0M} being a limit point of $(\varphi,\psi)^{-1}(Q)$ belongs to $(\varphi,\psi)^{-1}(Q)$. Since e_{0M} is arbitrary, so $(\varphi,\psi)^{-1}(Q)$ is closed in $NS(U_E)$.

4.9 Definition

Let $(NS(U_E), d)$ be an NSMS over (U, E) and $M \subset NS(U_E)$ be an arbitrary NSS. Then the closure of M is denoted by $\overline{M}$ and is defined as follows :

$\overline{M} = \cap\{N \subset NS(U_E) : N \text{ is neutrosophic soft closed and } N \supset M\}$

i.e., it is the intersection of all closed neutrosophic soft supersets of M.

4.9.1 *Example*

Let $(NS(U_E), d)$ be an NSMS with respect to 'd' defined in (2)(i) of [2.3] where $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then every NSS defined over (U, E) is finite. Also every finite NSS on an NSMS is closed by [2.4](3). Now consider four NSSs $M, N, P, 1_u \subset NS(U_E)$ such that $M \subset N, P, 1_u$ only and they are given as following :

$f_M(e_1) = \{< x_1, (0.6, 0.7, 0.8) >, < x_2, (0.5, 0.3, 0.7) >, < x_3, (0.4, 0.4, 0.5) >\}$
$f_M(e_2) = \{< x_1, (0.4, 0.5, 0.7) >, < x_2, (0.3, 0.4, 0.8) >, < x_3, (0.6, 0.4, 0.6) >\}$

$f_N(e_1) = \{< x_1, (0.6, 0.5, 0.8) >, < x_2, (0.6, 0.3, 0.5) >, < x_3, (0.5, 0.3, 0.4) >\}$
$f_N(e_2) = \{< x_1, (0.5, 0.4, 0.7) >, < x_2, (0.4, 0.2, 0.6) >, < x_3, (0.6, 0.2, 0.5) >\}$

$f_P(e_1) = \{< x_1, (0.7, 0.4, 0.6) >, < x_2, (0.8, 0.2, 0.4) >, < x_3, (0.6, 0.2, 0.3) >\}$
$f_P(e_2) = \{< x_1, (0.6, 0.2, 0.5) >, < x_2, (0.5, 0.1, 0.5) >, < x_3, (0.7, 0.1, 0.2) >\}$

$f_{1_u}(e_1) = \{< x_1, (1, 0, 0) >, < x_2, (1, 0, 0) >, < x_3, (1, 0, 0) >\}$
$f_{1_u}(e_2) = \{< x_1, (1, 0, 0) >, < x_2, (1, 0, 0) >, < x_3, (1, 0, 0) >\}$

Then $\overline{M} = 1_u \cap N \cap P = N$. The corresponding t-norm $(*)$ and s-norm $(\diamond)$ are : $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$.

4.9.2 **Proposition**

Let $(NS(U_E), d)$ be an NSMS and $M \subset NS(U_E)$. Then the followings hold.

(1) $\overline{M}$ is the smallest closed NSS containing M.
(2) $\overline{M} = M$ if and only if M is closed.
(3) $M \subset P \Rightarrow \overline{M} \subset \overline{P}$.
(4) $\overline{\overline{M}} = \overline{M}$.
(5) $\overline{M \cup P} = \overline{M} \cup \overline{P}$.
(6) $\overline{M \cap P} \subset \overline{M} \cap \overline{P}$.

Proof. (1) Since intersection of a family of closed NSSs in an NSMS is closed and $\overline{M}$ is the intersection of all closed neutrosophic soft supersets of M, so the proof is completed.

(2) If $\overline{M} = M$, then M is closed by (1).

Conversely, let M be closed. By (1), $M \subset \overline{M}$. Hence, we shall only show $\overline{M} \subset M$.

$$\begin{aligned} \overline{M} &= \cap\{P \subset NS(U_E) : P \text{ is neutrosophic soft closed and } P \supset M\} \\ &\subset \{M \subset NS(U_E) : M \text{ is neutrosophic soft closed and } M \supset M\} = M \end{aligned}$$

(3) $M \subset \overline{M}$ and $P \subset \overline{P} \Rightarrow M \subset P \subset \overline{P} \Rightarrow M \subset \overline{P}$

But $\overline{M}$ is the smallest closed set containing M i.e., $M \subset \overline{M} \subset \overline{P}$. Hence, $\overline{M} \subset \overline{P}$.

(4) If N is closed then $N = \overline{N}$. Since $\overline{M}$ is closed, replacing N by $\overline{M}$, we get $\overline{\overline{M}} = \overline{M}$.

(5) $M \subset M \cup P$ and $P \subset M \cup P \Rightarrow \overline{M} \subset \overline{M \cup P}$ and $\overline{P} \subset \overline{M \cup P} \Rightarrow \overline{M} \cup \overline{P} \subset \overline{M \cup P}$.

Also, $M \subset \overline{M}$ and $P \subset \overline{P} \Rightarrow M \cup P \subset \overline{M} \cup \overline{P}$. But we have, $M \cup P \subset \overline{M \cup P} \subset \overline{M} \cup \overline{P}$.

Thus, $\overline{M \cup P} = \overline{M} \cup \overline{P}$.

(6) $M \cap P \subset M$ and $M \cap P \subset P \Rightarrow \overline{M \cap P} \subset \overline{M}$ and $\overline{M \cap P} \subset \overline{P}$

$\Rightarrow \overline{M \cap P} \subset \overline{M} \cap \overline{P}$.

4.10 Theorem

Let $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be an injective as well as continuous neutrosophic soft function. Then,

(1) $(\varphi, \psi)(\overline{N}) \subset \overline{(\varphi, \psi)(N)}$ in $NS(V_{E'})$ for every $N \subset NS(U_E)$.

(2) $\overline{(\varphi, \psi)^{-1}(M)} \subset (\varphi, \psi)^{-1}(\overline{M})$ in $NS(U_E)$ for every $M \subset NS(V_{E'})$.

Proof. (1) Here $(\varphi, \psi)(N) \in NS(V_{E'})$ and so $\overline{(\varphi, \psi)(N)}$ is closed in $NS(V_{E'})$. Since (φ, ψ) is continuous, so $(\varphi, \psi)^{-1}[\overline{(\varphi, \psi)(N)}]$ is closed in $NS(U_E)$ by [4.8]. Then $\overline{(\varphi, \psi)^{-1}[\overline{(\varphi, \psi)(N)}]} = (\varphi, \psi)^{-1}[\overline{(\varphi, \psi)(N)}]$ by [4.9.2](2). Now $\overline{(\varphi, \psi)(N)}$ is the closure of $(\varphi, \psi)(N)$. So, $(\varphi, \psi)(N) \subset \overline{(\varphi, \psi)(N)} \Rightarrow N \subset (\varphi, \psi)^{-1}[\overline{(\varphi, \psi)(N)}] \Rightarrow \overline{N} \subset \overline{(\varphi, \psi)^{-1}[\overline{(\varphi, \psi)(N)}]} = (\varphi, \psi)^{-1}[\overline{(\varphi, \psi)(N)}]$. Thus $(\varphi, \psi)(\overline{N}) \subset \overline{(\varphi, \psi)(N)}$.

(2) Here $\overline{M}$ is closed in $NS(V_E)$ and so is $(\varphi, \psi)^{-1}(\overline{M})$ in $NS(U_E)$ by [4.8]. But $M \subset \overline{M} \Rightarrow (\varphi, \psi)^{-1}(M) \subset (\varphi, \psi)^{-1}(\overline{M}) \Rightarrow \overline{(\varphi, \psi)^{-1}(M)} \subset \overline{(\varphi, \psi)^{-1}(\overline{M})} = (\varphi, \psi)^{-1}(\overline{M})$,

as $(\varphi, \psi)^{-1}(\overline{M})$ is closed. Thus $\overline{(\varphi, \psi)^{-1}(M)} \subset (\varphi, \psi)^{-1}(\overline{M})$.

4.11 Definition

Let $(\varphi_1, \psi_1) : (NS(U_E), d_1) \to (NS(V_{E'}), d_2)$, $(\varphi_2, \psi_2) : (NS(V_{E'}), d_2) \to (NS(W_{E''}), d_3)$ be two neutrosophic soft functions where $(NS(U_E), d_1)$, $(NS(V_{E'}), d_2)$, $(NS(W_{E''}), d_3)$ are three NSMSs. Then the composition of these two functions is given by :

$(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1) : (NS(U_E), d_1) \to (NS(W_{E''}), d_3)$ and is defined as :

$[(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1)](e_M) = (\varphi_2, \psi_2)[(\varphi_1, \psi_1)(e_M)] = (\varphi_2, \psi_2)(e'_N) = e''_R$, where $e_M \in NS(U_E)$, $e'_N \in NS(V_{E'})$, $e''_R \in NS(W_{E''})$ and for $x \in U$, $z \in W$

$$T_{e''_R}(z) = \begin{cases} \max_{(\varphi_2 \circ \varphi_1)(x)=z} \max_{(\psi_2 \circ \psi_1)(e)=e''} [T_{e_M}(x)], & \text{if } x \in (\varphi_2 \circ \varphi_1)^{-1}(z) \\ 0 & \text{otherwise.} \end{cases}$$

$$I_{e''_R}(z) = \begin{cases} \min_{(\varphi_2 \circ \varphi_1)(x)=z} \min_{(\psi_2 \circ \psi_1)(e)=e''} [I_{e_M}(x)], & \text{if } x \in (\varphi_2 \circ \varphi_1)^{-1}(z) \\ 1 & \text{otherwise.} \end{cases}$$

$$F_{e''_R}(z) = \begin{cases} \min_{(\varphi_2 \circ \varphi_1)(x)=z} \min_{(\psi_2 \circ \psi_1)(e)=e''} [F_{e_M}(x)], & \text{if } x \in (\varphi_2 \circ \varphi_1)^{-1}(z) \\ 1 & \text{otherwise.} \end{cases}$$

4.11.1 *Example*

Let $(\varphi_1, \psi_1) : (NS(U_E), d) \to (NS(V_E), d)$, $(\varphi_2, \psi_2) : (NS(V_E), d) \to (NS(W_E), d)$ be two neutrosophic soft functions where d is defined in (2)(i) of [2.3]. Let $U = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. We consider the $NS(U_E)$ as given by the Table 2.

Table 2 : Tabular form of $NS(U_E)$

	e_{1A}	e_{2A}	e_{1B}	e_{2B}	e_{1C}	e_{2C}
x_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)	(0.6,0.2,0.3)	(0.8,0.6,0.2)	(0.7,0.2,0.5)
x_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)	(0.4,0.3,0.5)	(0.5,0.7,0.4)	(0.1,0.5,0.8)

Now let $\varphi_1(x_1) = y_1$, $\varphi_1(x_2) = y_1$ and $\psi_1(e_1) = e_2$, $\psi_1(e_2) = e_1$. Suppose,

$$(\varphi_1, \psi_1)(e_{1A}) = e_{2D}, \ (\varphi_1, \psi_1)(e_{2A}) = e_{1G}, \ (\varphi_1, \psi_1)(e_{1B}) = e_{2G}$$
$$(\varphi_1, \psi_1)(e_{2B}) = e_{1H}, \ (\varphi_1, \psi_1)(e_{1C}) = e_{2H}, \ (\varphi_1, \psi_1)(e_{2C}) = e_{1D}$$

Then the following table (Table 3) represents $(\varphi_1, \psi_1)(NS(U_E))$:

Table 3 : Tabular form of $(\varphi_1, \psi_1)(NS(U_E))$

	e_{1D}	e_{2D}	e_{1G}	e_{2G}	e_{1H}	e_{2H}
y_1	(0.7,0.2,0.5)	(0.5,0.6,0.3)	(0.7,0.3,0.3)	(0.7,0.4,0.2)	(0.6,0.2,0.3)	(0.8,0.6,0.2)
y_2	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)

One calculation is provided here to make out the Table 3.

$$\begin{aligned} T_{e_{1G}}(y_1) &= \max_{\{\varphi_1(x_1), \varphi_1(x_2)\}} \max_{\psi_1(e_2)} [T_{e_{2A}}(x)], \text{ as } x_1, x_2 \in \varphi_1^{-1}(y_1) \\ &= \max(0.6, 0.7) = 0.7 \\ I_{e_{1G}}(y_1) &= \min_{\{\varphi_1(x_1), \varphi_1(x_2)\}} \min_{\psi_1(e_2)} [I_{e_{2A}}(x)], \text{ as } x_1, x_2 \in \varphi_1^{-1}(y_1) \\ &= \min(0.3, 0.4) = 0.3 \\ F_{e_{1G}}(y_1) &= \min_{\{\varphi_1(x_1), \varphi_1(x_2)\}} \min_{\psi_1(e_2)} [F_{e_{2A}}(x)], \text{ as } x_1, x_2 \in \varphi_1^{-1}(y_1) \\ &= \min(0.5, 0.3) = 0.3 \end{aligned}$$

Further $T_{e_{1G}}(y_2) = 0$, $I_{e_{1G}}(y_2) = 1$, $F_{e_{1G}}(y_2) = 1$ as $x_1, x_2 \notin \varphi_1^{-1}(y_2)$.

Now assume $\varphi_2(y_1) = z_2,\ \varphi_2(y_2) = z_1$ and $\psi_2(e_1) = e_1,\ \psi_2(e_2) = e_2$. Suppose,

$$(\varphi_2, \psi_2)(e_{1D}) = e_{1L},\ \ (\varphi_2, \psi_2)(e_{1G}) = e_{1Q},\ \ (\varphi_2, \psi_2)(e_{1H}) = e_{1M}$$
$$(\varphi_2, \psi_2)(e_{2D}) = e_{2M},\ \ (\varphi_2, \psi_2)(e_{2G}) = e_{2L},\ \ (\varphi_2, \psi_2)(e_{2H}) = e_{2Q}$$

Then $(\varphi_2, \psi_2)[(\varphi_1, \psi_1)(NS(U_E))]$ is given by the Table 4.

Table 4 : Tabular form of $(\varphi_2, \psi_2)[(\varphi_1, \psi_1)(NS(U_E))]$

	e_{1L}	e_{2L}	e_{1M}	e_{2M}	e_{1Q}	e_{2Q}
z_1	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)	(0,1,1)
z_2	(0.7,0.2,0.5)	(0.7,0.4,0.2)	(0.6,0.2,0.3)	(0.5,0.6,0.3)	(0.7,0.3,0.3)	(0.8,0.6,0.2)

Thus the Table 4 gives $(\varphi_2, \psi_2)[(\varphi_1, \psi_1)(NS(U_E))] = [(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1)](NS(U_E))$. For convenience, $[(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1)](e_{1A}) = (\varphi_2, \psi_2)[(\varphi_1, \psi_1)(e_{1A})] = (\varphi_2, \psi_2)(e_{2D}) = e_{2M}$ and so on.

4.12 Theorem

Let $(\varphi_1, \psi_1) : (NS(U_E), d_1) \to (NS(V_{E'}), d_2)$, $(\varphi_2, \psi_2) : (NS(V_{E'}), d_2) \to (NS(W_{E''}), d_3)$ be two neutrosophic soft functions where $(NS(U_E), d_1)$, $(NS(V_{E'}), d_2)$, $(NS(W_{E''}), d_3)$ are three NSMSs. If (φ_1, ψ_1) is continuous at $e_{0N} \in NS(U_E)$ and (φ_2, ψ_2) is continuous at the corresponding point $(\varphi_1, \psi_1)(e_{0N}) \in NS(V_{E'})$, then the composite function $(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1) : (NS(U_E), d_1) \to (NS(W_{E''}), d_3)$ is continuous at $e_{0N} \in NS(U_E)$.

Proof. Let $\{e_{nN}\}$ be a sequence of neutrosophic soft points in $NS(U_E)$ such that $\lim_{n\to\infty} e_{nN} = e_{0N} \in NS(U_E)$. Since (φ_1, ψ_1) is continuous at e_{0N}, so $(\varphi_1, \psi_1)(e_{nN}) \to (\varphi_1, \psi_1)(e_{0N}) \in NS(V_{E'})$ as $n \to \infty$. Again since (φ_2, ψ_2) is continuous at $(\varphi_1, \psi_1)(e_{0N})$, so $(\varphi_2, \psi_2)[(\varphi_1, \psi_1)(e_{nN})] \to (\varphi_2, \psi_2)[(\varphi_1, \psi_1)(e_{0N})] \in NS(W_{E''})$ as $n \to \infty$. This implies $[(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1)](e_{nN}) \to [(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1)](e_{0N}) \in NS(W_{E''})$ as $n \to \infty$.
Hence $(\varphi_2, \psi_2) \circ (\varphi_1, \psi_1)$ is continuous at $e_{0N} \in NS(U_E)$.

4.13 Theorem

Continuous image of a compact NSMS is compact.

Proof. Let $(\varphi, \psi) : (NS(U_E), d) \to (NS(V_{E'}), d')$ be a continuous neutrosophic soft function and $NS(U_E)$ be a compact NSMS. We are to show that $(\varphi, \psi)(NS(U_E)) = NS(V'_{E'}) \subseteq NS(V_{E'})$ (say) is compact. Let $\{e'_{nN}\}$ be a soft sequence in $NS(V'_{E'})$. Then for each e'_{nN} there exists $e_{nM} \in NS(U_E)$ such that $(\varphi, \psi)(e_{nM}) = e'_{nN}, n = 1, 2, 3, \cdots$; Since $NS(U_E)$ is compact, the soft sequence $\{e_{nM}\}$ has a subsequence $\{e_{n_kM}\}$ such that $\lim_{k\to\infty} e_{n_kM} = e_{0P} \in NS(U_E)$ (say). Again (φ, ψ) is continuous on $(NS(U_E), d)$, so it is continuous at e_{0P}. Then by (2) of theorem [4.6], $\lim_{k\to\infty}(\varphi, \psi)(e_{n_kM}) = (\varphi, \psi)(e_{0P})$; But, $(\varphi, \psi)(e_{n_kM}) = e'_{n_kN}$ and so $\lim_{k\to\infty} e'_{n_kN} = (\varphi, \psi)(e_{0P})$; Thus a soft sequence $\{e'_{nN}\}$ in $NS(V'_{E'})$ has a subsequence $\{e'_{n_kN}\}$ converging to a soft point in $NS(V'_{E'})$. This follows the theorem.

5 Uniform continuity on NSMS

This section gives the concept of uniform continuity of a neutrosophic soft function on an NSMS and it's characteristics on NSMS.

5.1 Definition

Let $(NS(U_E), d)$ and $(NS(V_{E'}), d')$ be two NSMSs. Then a neutrosophic soft function $(\varphi, \psi) : NS(U_E) \to NS(V_{E'})$ is said to be uniformly continuous on $NS(U_E)$ if for each $\epsilon > 0$ there exists a $\delta > 0$ depending only on ϵ, not on the point such that
$d'[(\varphi, \psi)(e_M), (\varphi, \psi)(e_N)] < \epsilon$ whenever $d(e_M, e_N) < \delta \quad \forall e_M, e_N \in NS(U_E)$.

5.1.1 *Example*

Consider a neutrosophic soft function $(\varphi, \psi) : (NS(\mathbf{Z}_E), d) \to (NS(\mathbf{Z}_E), d)$ where 'd' is defined in (2)(i) of [2.3] and $\mathbf{Z}$ be the set of integers. The function is defined as $(\varphi, \psi)(e_M) = \tilde{\rho}(e_M, P)$ for any NSS $P \subset NS(\mathbf{Z}_E)$ and $e_M \in NS(\mathbf{Z}_E)$, where

$$\begin{aligned}\tilde{\rho}(e_M, P) \quad = \quad & \{< x, \min_{e_P \in P}\{|T_{e_M}(x) - T_{e_P}(x)|\}, \max_{e_P \in P}\{|I_{e_M}(x) - I_{e_P}(x)|\}, \\ & \max_{e_P \in P}\{|F_{e_M}(x) - F_{e_P}(x)|\} >: x \in \mathbf{Z}\}\end{aligned}$$

Now for any two points $e_M, e_N \in NS(\mathbf{Z}_E)$ and for $P \subset NS(\mathbf{Z}_E)$, we have

$$\begin{aligned}
& d[(\varphi, \psi)(e_M), (\varphi, \psi)(e_N)] \\
= \; & d[\tilde{\rho}(e_M, P), \tilde{\rho}(e_N, P)] \\
= \; & \min_x \langle\, |\min_{e_P \in P}\{|T_{e_M}(x) - T_{e_P}(x)|\} - \min_{e_P \in P}\{|T_{e_N}(x) - T_{e_P}(x)|\}| \\
& + |\max_{e_P \in P}\{|I_{e_M}(x) - I_{e_P}(x)|\} - \max_{e_P \in P}\{|I_{e_N}(x) - I_{e_P}(x)|\}| \\
& + |\max_{e_P \in P}\{|F_{e_M}(x) - F_{e_P}(x)|\} - \max_{e_P \in P}\{|F_{e_N}(x) - F_{e_P}(x)|\}|\,\rangle \\
< \; & \min_x \langle\, |\min_{e_P \in P}\{T_{e_M}(x) + T_{e_P}(x)\} - \min_{e_P \in P}\{T_{e_N}(x) + T_{e_P}(x)\}| \\
& + |\max_{e_P \in P}\{I_{e_M}(x) + I_{e_P}(x)\} - \max_{e_P \in P}\{I_{e_N}(x) + I_{e_P}(x)\}| \\
& + |\max_{e_P \in P}\{F_{e_M}(x) + F_{e_P}(x)\} - \max_{e_P \in P}\{F_{e_N}(x) + F_{e_P}(x)\}|\,\rangle \\
= \; & \min_x \langle\, |\min_{e_P \in P}\{T_{e_M}(x) + T_{e_P}(x) - T_{e_N}(x) - T_{e_P}(x)\}| \\
& + |\max_{e_P \in P}\{I_{e_M}(x) + I_{e_P}(x) - I_{e_N}(x) - I_{e_P}(x)\}| \\
& + |\max_{e_P \in P}\{F_{e_M}(x) + F_{e_P}(x) - F_{e_N}(x) - F_{e_P}(x)\}|\,\rangle \\
= \; & \min_x \langle\, |T_{e_M}(x) - T_{e_N}(x)| + |I_{e_M}(x) - I_{e_N}(x)| + |F_{e_M}(x) - F_{e_N}(x)|\,\rangle \\
= \; & d(e_M, e_N) < \delta = \epsilon
\end{aligned}$$

Hence (φ, ψ) is uniformly continuous on $NS(\mathbf{Z}_E)$.

5.2 Theorem

The image of a Cauchy sequence in an NSMS under a uniformly continuous neutrosophic soft function is again a Cauchy sequence.

Proof. Let $(\varphi, \psi) : (NS(U_E), d) \rightarrow (NS(V_{E'}), d')$ be a uniformly continuous neutrosophic soft function. Then for each $\epsilon > 0$ there exists a $\delta > 0$ depending only on ϵ, not on the point such that
$d'[(\varphi, \psi)(e_M), (\varphi, \psi)(e_N)] < \epsilon \quad \text{whenever} \quad d(e_M, e_N) < \delta \ \ \forall e_M, e_N \in NS(U_E)$.
Let $\{e_{nP}\}$ be a Cauchy sequence in $NS(U_E)$. Then to every $\delta > 0$ there exists an $n_0 \in \mathbf{N}$ (set of natural numbers) such that $d(e_{mP}, e_{nP}) < \delta \ \ \forall m, n \geq n_0$.
This shows $d'[(\varphi, \psi)(e_{mP}), (\varphi, \psi)(e_{nP})] < \epsilon \ \ \forall m, n \geq n_0$ and that ends the theorem.

5.3 Theorem

Every uniformly continuous neutrosophic soft function on an NSMS is continuous.

Proof. Replacing e_N by e_{0N}, an arbitrary but fixed neutrosophic soft point, it directly follows from definition [5.1];

5.4 Theorem

Uniform continuous image of a complete NSS in an NSMS is complete.

Proof. Let $(\varphi, \psi) : (NS(U_E), d) \rightarrow (NS(V_{E'}), d')$ be a uniformly continuous neutrosophic soft function and $M \subset NS(U_E)$ be a complete NSS. We are to show that $(\varphi, \psi)(M) = P$ (say) is complete. Let $\{e_{nM}\}$ be a neutrosophic soft Cauchy sequence in M such that $\lim_{n\rightarrow\infty} e_{nM} = e_{0M} \in M$. Then $\{(\varphi, \psi)(e_{nM})\}$ is a Cauchy sequence in P by theorem [5.2]; Again (φ, ψ) being uniformly continuous neutrosophic soft function is continuous by theorem [5.3] and so, $\lim_{n\rightarrow\infty} (\varphi, \psi)(e_{nM}) = (\varphi, \psi)(e_{0M}) \in P$ by theorem (2) of [4.6]; Thus a cauchy sequence $\{(\varphi, \psi)(e_{nM})\}$ in P converges to a point in P and this completes the proof.

5.5 Theorem

Uniform continuous image of a compact NSMS is compact.

Proof. It is the combination of theorem [5.3] and the theorem [4.13];

6 Conclusion

In this paper, the notion of compact NSMS has been introduced and is illustrated by suitable examples. The continuity and uniform continuity of a neutrosophic soft function in an NSMS have been defined and verified by proper examples. Several related properties, theorems and structural characteristics of these in an NSMS have been investigated. Some are justified by suitable examples also. The motivation of the present paper is to put forward the concept introduced in [32]. We expect, these concepts will bring an opportunity of further research work to develop the NSS theory.

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