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Development of Hybrids of Hypersoft Set with Complex Fuzzy Set, Complex Intuitionistic Fuzzy set and Complex Neutrosophic Set

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Abstract. The complex fuzzy soft set and its generalized hybrids are such effective structures which not only minimize the impediments of all complex fuzzy-like structures for dealing uncertainties but also fulfill all the parametric requirements of soft sets. This feature makes it a completely new mathematical tool for solving problems dealing with uncertainties. Smarandache conceptualized hypersoft set as a generalization of soft set as it transforms the single attribute function into a multi-attribute function. This generalization demands an extension of complex fuzzy soft-like structures to hypersoft structure for more precise results. In this study, hybrids of hypersoft set with complex fuzzy set and its generalized structures i.e. complex intuitionistic fuzzy set and complex neutrosophic set, are developed along with illustrative examples to address the demand of literature. Moreover, some of their fundamentals i.e. subset, equal sets, null set, absolute set etc. and theoretic operations i.e. compliment, union, intersection etc. are discussed.

Keywords: Complex fuzzy sets (CF-Sets), soft set, hypersoft set and complex fuzzy hypersoft set.

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1. Introduction

Zadeh's theory of fuzzy sets [1] is one of those theories which are considered as mathematical means to tackle many complicated problems involving various uncertainties in different fields of mathematical sciences. But these theories are unable to solve these problems successfully due to the inadequacy of the parametrization tool. This shortcoming is addressed by Molodtsov's soft set theory [2] which is free from all such Impediments and appeared as a new parameterized family of subsets of the universe of discourse. Classical complex analysis is useful in algebraic geometry, number theory, analytic combinatorics and many other branches of mathematical sciences. Ramot et al. [3, 4] introduced the concept of complex fuzzy set (CF-set) to tackle the problems of complex analysis under fuzzy environment. This novel concept used complex-valued state for the membership of its elements. Maji et al. [5] developed and conceptualized fuzzy soft set, a new hybrid of fuzzy set with soft set. They also discussed some of its fundamentals terminologies and operations like equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, absolute soft set, AND, OR etc. in their work. Ça̧ğman et al. [6] extended this concept and discussed some other properties and operations. Nadia [7] developed a new hybrid of complex fuzzy set and soft set. Thirunavukarasu et al. [8] established aggregation properties of complex fuzzy soft set and discussed their applications. Atanassov [9] conceptualized intuitionistic fuzzy sets as generalization of fuzzy set. Alkouri et al. [10] extended this concept and developed complex intuitionistic fuzzy soft set and discussed some of its properties. Kumar et al. [11] further discussed its more properties and calculated its distance measures and entropies. Mumtaz et al. [12] extended neutrosophic set [13] to complex neutrosophic set and discussed its fundamentals, theoretic operations and applications. Broumi et al. [14] conceptualized complex neutrosophic soft set and discussed some of its fundamentals.

In 2018, Smarandache [15] introduced the concept of hypersoft set as a generalization of soft set. In 2020, Saeed et al. [16] extended the concept and discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set, aggregation operators along with hypersoft set relation, sub relation,complement relation, function, matrices and operations on hypersoft matrices.

Having motivation from the work in [6], [8]- [16] and [21], novel hybrids of hypersoft set i.e. complex fuzzy hypersoft set, complex intuitionistic fuzzy hypersoft set and complex neutrosophic hypersoft set, are conceptualized along with their some fundamentals and theoretic operations. This is novel and more generalized work as compared to existing related literature for getting more precise results. Moreover, a comparative discussion is presented on particular cases of such hybrids.

The pattern of rest of the paper is: section 2 reviews the notions of soft sets, complex fuzzy set and relevant definitions used in the proposed work. Section 3, presents complex fuzzy hypersoft set and some of its fundamentals. Section 4, presents complex intuitionistic fuzzy hypersoft set and some of its fundamentals. Section 5, presents complex neutrosophic hypersoft set and some of its fundamentals and then concludes the paper.

2. **Preliminaries**

Here some existing fundamental concepts regarding fuzzy set, fuzzy soft set and fuzzy hypersoft set are presented along with their structures with complex fuzzy set from literature.

Throughout the paper, $\mathbb{U}$, $P(\mathbb{U})$, $F(\mathbb{U})$, $C(\mathbb{U})$, $C_{Int}(\mathbb{U})$, $C_{Neu}(\mathbb{U})$, $\coprod$ and $\prod$ will present universe of discourse, power set of $\mathbb{U}$, collection of fuzzy sets, collection of complex fuzzy sets, collection of complex intuitionistic fuzzy sets, collection of complex neutrosophic sets, union and intersection respectively.

Definition 2.1. [1]
Suppose a universal set $\mathbb{U}$ and a *fuzzy set* $X \subseteq \mathbb{U}$. The set X will be written as $X = \{(x, \alpha_X(x))|x \in \mathbb{U}\}$ such that

$$\alpha_X : \mathbb{U} \to [0, 1]$$

where $\alpha_X(x)$ describes the membership percentage of $x \in X$.

Definition 2.2. [3]
A *complex fuzzy set* $\mathbb{C}_f$ is of the form

$$\mathbb{C}_f = \left\{(\epsilon, \mu_{\mathbb{C}_f}(\epsilon)) : \epsilon \in \mathbb{U}\right\} = \left\{(\epsilon, r_{\mathbb{C}_f}(\epsilon)e^{i\omega_{\mathbb{C}_f}(\epsilon)}) : \epsilon \in \mathbb{U}\right\}.$$

where $\mu_{\mathbb{C}_f}(\epsilon)$ is a membership function of $\mathbb{C}_f$ with $r_{\mathbb{C}_f}(\epsilon) \in [0, 1]$ and $\omega_{\mathbb{C}_f}(\epsilon) \in (0, 2\pi]$ as amplitude and phase terms respectively and $i = \sqrt{-1}$.

Zhang et al. [22] and Buckley [23]- [26] presented fuzzy complex number in different way. However, according to [3], [4], both amplitude and phase terms are captured by fuzzy sets.

Definition 2.3. [2]
A *soft set* $\mathfrak{S}$ over $\mathbb{U}$, is defined as

$$\mathfrak{S} = \{(\epsilon, f_{\mathfrak{S}}(\epsilon)) : \epsilon \in E_1\}$$

where $f_{\mathfrak{S}} : E_1 \to P(\mathbb{U})$. and $E_1 \subseteq E$ (set of parameters).

Definition 2.4. [6]
A *fuzzy soft set* (FS-set) Γ_{E_1} on $\mathbb{U}$, is defined as

$$\Gamma_{E_1} = \{(\epsilon, \gamma_{E_1}(\epsilon)) : \epsilon \in E_1, \gamma_{E_1}(\epsilon) \in F(\mathbb{U})\}$$

where $\gamma_{E_1} : E_1 \to F(\mathbb{U})$ such that $\gamma_{E_1}(\epsilon) = \emptyset$ if $\epsilon \notin E_1$, and for all $\epsilon \in E_1$,

$$\gamma_{E_1}(\epsilon) = \left\{\mu_{\gamma_{E_1}(\epsilon)}(v)/v : v \in \mathbb{U}, \mu_{\gamma_{E_1}(\epsilon)}(v) \in [0, 1]\right\}$$

is a fuzzy set over $\mathbb{U}$. Also γ_{E_1} is the approximate function of Γ_{E_1} and the value $\gamma_A(x)$ is a fuzzy set called ϵ-element of FS-set. Note that if $\gamma_{E_1}(\epsilon) = \emptyset$, then $(\epsilon, \gamma_{E_1}(\epsilon)) \notin \Gamma_{E_1}$.

Definition 2.5. [7]
A *complex fuzzy soft set* (CFS-set) χ_{E_1} over $\mathbb{U}$, is defined as

$$\chi_{E_1} = \{(\epsilon, \psi_{E_1}(\epsilon)) : \epsilon \in E_1, \psi_{E_1}(\epsilon) \in C(\mathbb{U})\}.$$

where $\psi_{E_1} : E_1 \to C(\mathbb{U})$ such that $\psi_{E_1}(\epsilon) = \emptyset$ if $\epsilon \notin E_1$ and it is complex fuzzy approximate function of CFS-set χ_{E_1} and its value $\psi_{E_1}(\epsilon)$ is called ϵ-member of CFS-set χ_{E_1} for all $\epsilon \in E_1$. Operations of CFS-sets and CF-sets were defined in [7] and [22] respectively.

Definition 2.6. [27] Let $A = \{(x; \mu_A(x)) : x \in \mathbb{U}\}$ and $B = \{(x; \mu_B(x)) : x \in \mathbb{U}\}$ be two complex fuzzy subsets of $\mathbb{U}$, with membership functions $\mu_A(x) = r_A(x)e^{i\omega_A(x)}$ and $\mu_B(x) = r_B(x)e^{i\omega_B(x)}$, respectively. Then

- A complex fuzzy subset A is said to be a *homogeneous complex fuzzy set* if for all $x, y \in \mathbb{U}$, $r_A(x) \leq r_A(y)$ if and only if $\omega_A(x) \leq \omega_A(y)$
- A complex fuzzy subset A is said to be *homogeneous* with B, if for all $x, y \in \mathbb{U}$, $r_A(x) \leq r_B(y)$ if and only if $\omega_A(x) \leq \omega_B(y)$

Definition 2.7. [10] Let E be a set of attributes with $A \subseteq E$ and $\Psi(a)$ be a CIF-set over $\mathbb{U}$. Then, *complex intuitionistic fuzzy soft set* (CIFS-set) $\xi_A = (\Psi, A)$ over $\mathbb{U}$ is defined as

$$\xi_A = \{(a, \Psi(a)) : a \in A, \Psi(a) \in C_{Int}(\mathbb{U})\}$$

where

$$\Psi : A \to C_{Int}(\mathbb{U}), \qquad \Psi(a) = \emptyset \; if \; a \notin A.$$

is a CIF approximate function of ξ_A and $\Psi(a) = \langle \Psi^T(a), \Psi^F(a) \rangle$.
$\Psi^T(a) = p_T\, e^{i\,\theta_T}$, and $\Psi^F(a) = p_F\, e^{i\,\theta_F}$ are complex-valued membership function, and complex-valued non-membership function of ξ_A respectively and their sum all are lying within the unit circle in the complex plane such that $p_T, p_F \in [0, 1]$ with $0 \leq p_T + p_F \leq 1 (or\ 0 \leq |p_T + p_F| \leq 1)$ and $\theta_T, \theta_F \in (0, 2\pi]$. The value $\Psi(a)$ is called a-member of CIFS-set $\forall a \in A$.

Definition 2.8. [14]
Let E be a set of attributes with $A \subseteq E$ and $\Psi(a)$ be a CN-set over $\mathbb{U}$. Then, *complex neutrosophic soft set* (CNS-set) $\xi_A = (\Psi, A)$ over $\mathbb{U}$ is defined as

$$\xi_A = \{(a, \Psi(a)) : a \in A, \Psi(a) \in C_{Neu}(\mathbb{U})\}$$

where

$$\Psi : A \to C_{Neu}(\mathbb{U}), \qquad \Psi(a) = \emptyset \; if \; a \notin A.$$

is a CN approximate function of ξ_A and $\Psi(a) = \langle \Psi^T(a), \Psi^I(a), \Psi^F(a) \rangle$.
$\Psi^T(a) = p_T\, e^{i\,\theta_T}$, $\Psi^I(a) = p_I\, e^{i\,\theta_I}$ and $\Psi^F(a) = p_F\, e^{i\,\theta_F}$ are complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity membership function of ξ_A respectively and their sum all are lying within the unit circle in the complex plane such that $p_T, p_I, p_F \in [0, 1]$ with $^-0 \leq p_T + p_I + p_F \leq 3^+ (or\ 0 \leq |p_T + p_I + p_F| \leq 3)$ and $\theta_T, \theta_I, \theta_F \in (0, 2\pi]$. The value $\Psi(a)$ is called a-member of CNS-set $\forall a \in A$.

For more study about neutrosophic sets see ([28]- [42]).

Definition 2.9. [15]
The pair (H, G) is called a *hypersoft set* over $\mathbb{U}$, where G is the cartesian product of n disjoint sets $H_1, H_2, H_3, ..., H_n$ having attribute values of n distinct attributes $h_1, h_2, h_3, ..., h_n$ respectively and $H : G \to P(\mathbb{U})$.

Definition 2.10. [15]
A hypersoft set over a fuzzy universe of discourse is called *fuzzy hypersoft set.*

For more definitions and operations of hypersoft set, see ([15]- [20]).

3. Complex Hypersoft set(CH-Set) and Complex Fuzzy Hypersoft Set(CFH-Set)

In this section, first we define complex hypersoft set then complex fuzzy hypersoft set is conceptualized with its some fundamentals.

Definition 3.1. Let $\mathbb{C}$ be the set of complex numbers and $P(\mathbb{C})$ be the collection of all non-empty bounded subsets of the set of complex numbers. Let $A_1, A_2, A_3, ..., A_n$ are disjoint sets having attribute values of n distinct attributes $a_1, a_2, a_3, ..., a_n$ respectively for $n \geq 1, A = A_1 \times A_2 \times A_3 \times ..... \times A_n$ then a mapping $\psi : A \to P(\mathbb{C})$ is called a complex hypersoft set. It is denoted by (ψ, A).

Example 3.2. Let $\mathbb{C} = \{2 + 3i, 1 + 2i, 3 + 5i, 4 + 2i, 3 + i\}$ be the set of complex numbers and $E = \{A_1, A_2, A_3\}$ with $A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}$ and $A_3 = \{a_{31}, a_{32}\}$ are disjoint set having attribute values then

$$A = \left\{ \begin{array}{l} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), \\ (a_{21}, a_{21}, a_{31}), (a_{21}, a_{21}, a_{32}), (a_{21}, a_{22}, a_{31}), (a_{21}, a_{22}, a_{32}) \end{array} \right\}$$

$A = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, then (ψ, A) can be considered as a complex hypersoft set where

$$(\psi, A) = \left\{ \begin{array}{l} (x_1, \{2 + 3i, 1 + 2i\}), (x_2, \{2 + 3i, 1 + 2i, 3 + 5i\}), (x_3, \{4 + 2i, 1 + 2i, 3 + 5i\}), \\ (x_4, \{2 + 3i, 4 + 2i, 3 + i\}), (x_5, \{3 + i, 1 + 2i\}), (x_6, \{3 + i, 2 + 3i, 3 + 5i\}), \\ (x_7, \{2 + 3i, 3 + i\}), (x_8, \{4 + 2i, 3 + 5i\}), \end{array} \right\}$$

Definition 3.3. Let $A_1, A_2, A_3, ..., A_n$ are disjoint sets having attribute values of n distinct attributes $a_1, a_2, a_3, ..., a_n$ respectively for $n \geq 1, G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ and $\psi(\underline{\epsilon})$ be a CF-set over $\mathbb{U}$ for all $\underline{\epsilon} = (d_1, d_2, d_3, ..., d_n) \in G$ such that $d_1 \in A_1, d_2 \in A_2, d_3 \in A_3, ..., d_n \in A_n$. Then, *complex fuzzy hypersoft set* (CFH-set) χ_G over $\mathbb{U}$ is defined as

$$\chi_G = \{(\underline{\epsilon}, \psi(\underline{\epsilon})) : \underline{\epsilon} \in G, \psi(\underline{\epsilon}) \in C(\mathbb{U})\}$$

where

$$\psi : G \to C(\mathbb{U}), \qquad \psi(\underline{\epsilon}) = \emptyset \ if \ \underline{\epsilon} \notin G.$$

is a CF-approximate function of χ_G and its value $\psi(\underline{\epsilon})$ is called $\underline{\epsilon}$-member of CFH-set $\forall \underline{\epsilon} \in G$.

Example 3.4. Suppose a Department Promotion Committee (DPC) wants to observe(evaluate) the characteristics of some teachers by some defined indicators for departmental promotion. For this purpose, consider a set of teachers as a universe of discourse $\mathbb{U} = \{t_1, t_2, t_3, t_4\}$. The attributes of the teachers under consideration are the set $E = \{A_1, A_2, A_3\}$, where

$A_1 =$ Total experience in years $= \{3, < 10\} = \{e_{11}, e_{12}\}$

$A_2 =$ Total no. of publications$= \{10, 10 <\} = \{e_{21}, e_{22}\}$

$A_3 =$ Performance Evaluation Report (PER) remarks $= \{eligible, not\, eligible\} = \{e_{31}, e_{32}\}$

and

$$G = A_1 \times A_2 \times A_3 = \left\{ \begin{array}{l} (e_{11}, e_{21}, e_{31}), (e_{11}, e_{21}, e_{32}), (e_{11}, e_{22}, e_{31}), \\ (e_{11}, e_{22}, e_{32}), (e_{12}, e_{21}, e_{31}), (e_{12}, e_{21}, e_{32}), \\ (e_{12}, e_{22}, e_{31}), (e_{12}, e_{22}, e_{32}) \end{array} \right\} = \{e_1, e_2, e_3, ..., e_8\}$$

Complex fuzzy set $\psi_G(e_1), \psi_G(e_2), ..., \psi_G(e_8)$ are defined as,

$$\psi_G(e_1) = \left\{ \frac{0.4e^{i0.5\pi}}{t_1}, \frac{0.8e^{i0.6\pi}}{t_2}, \frac{0.8e^{i0.8\pi}}{t_3}, \frac{1.0e^{i0.75\pi}}{t_4} \right\},$$

$$\psi_G(e_2) = \left\{ \frac{0.6e^{i0.7\pi}}{t_1}, \frac{0.9e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_3) = \left\{ \frac{0.5e^{i0.6\pi}}{t_1}, \frac{0.8e^{i0.9\pi}}{t_2}, \frac{0.6e^{i0.9\pi}}{t_3}, \frac{0.65e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_4) = \left\{ \frac{0.3e^{i0.7\pi}}{t_1}, \frac{0.7e^{i0.9\pi}}{t_2}, \frac{0.5e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.65\pi}}{t_4} \right\},$$

$$\psi_G(e_5) = \left\{ \frac{0.2e^{i0.5\pi}}{t_1}, \frac{0.3e^{i0.8\pi}}{t_2}, \frac{0.8e^{i0.7\pi}}{t_3}, \frac{0.45e^{i0.65\pi}}{t_4} \right\},$$

$$\psi_G(e_6) = \left\{ \frac{0.5e^{i0.9\pi}}{t_1}, \frac{0.3e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.8\pi}}{t_3}, \frac{0.85e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_7) = \left\{ \frac{0.6e^{i0.9\pi}}{t_1}, \frac{0.9e^{i0.6\pi}}{t_2}, \frac{0.5e^{i0.6\pi}}{t_3}, \frac{0.85e^{i0.75\pi}}{t_4} \right\},$$

and

$$\psi_G(e_8) = \left\{ \frac{0.8e^{i0.9\pi}}{t_1}, \frac{0.8e^{i0.8\pi}}{t_2}, \frac{0.6e^{i0.8\pi}}{t_3}, \frac{0.65e^{i0.85\pi}}{t_4} \right\}$$

then CFH-set χ_G is written by,

$$\chi_G = \left\{ \begin{array}{l} (e_1, \frac{0.4e^{i0.5\pi}}{t_1}, \frac{0.8e^{i0.6\pi}}{t_2}, \frac{0.8e^{i0.8\pi}}{t_3}, \frac{1.0e^{i0.75\pi}}{t_4}), (e_2, \frac{0.6e^{i0.7\pi}}{t_1}, \frac{0.9e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.95\pi}}{t_4}), \\ (e_3, \frac{0.5e^{i0.6\pi}}{t_1}, \frac{0.8e^{i0.9\pi}}{t_2}, \frac{0.6e^{i0.9\pi}}{t_3}, \frac{0.65e^{i0.95\pi}}{t_4}), (e_4, \frac{0.3e^{i0.7\pi}}{t_1}, \frac{0.7e^{i0.9\pi}}{t_2}, \frac{0.5e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.65\pi}}{t_4}), \\ (e_5, \frac{0.2e^{i0.5\pi}}{t_1}, \frac{0.3e^{i0.8\pi}}{t_2}, \frac{0.8e^{i0.7\pi}}{t_3}, \frac{0.45e^{i0.65\pi}}{t_4}), (e_6, \frac{0.5e^{i0.9\pi}}{t_1}, \frac{0.3e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.8\pi}}{t_3}, \frac{0.85e^{i0.95\pi}}{t_4}), \\ (e_7, \frac{0.6e^{i0.9\pi}}{t_1}, \frac{0.9e^{i0.6\pi}}{t_2}, \frac{0.5e^{i0.6\pi}}{t_3}, \frac{0.85e^{i0.75\pi}}{t_4}), (e_8, \frac{0.8e^{i0.9\pi}}{t_1}, \frac{0.8e^{i0.8\pi}}{t_2}, \frac{0.6e^{i0.8\pi}}{t_3}, \frac{0.65e^{i0.85\pi}}{t_4}) \end{array} \right\}$$

Definition 3.5. Let $\chi_{G_1} = (\psi_1, G_1)$ and $\chi_{G_2} = (\psi_2, G_2)$ be two CFH-sets over the same $\mathbb{U}$. The set $\chi_{G_1} = (\psi_1, G_1)$ is said to be the *subset* of $\chi_{G_2} = (\psi_2, G_2)$, if

i. $G_1 \subseteq G_2$

ii. $\forall\ \underline{x} \in G_1, \psi_1(\underline{x}) \subseteq \psi_2(\underline{x})$ i.e. $r_{G_1}(\underline{x}) \leq r_{G_2}(\underline{x})$ and $\omega_{G_1}(\underline{x}) \leq \omega_{G_2}(\underline{x})$, where $r_{G_1}(\underline{x})$ and $\omega_{G_1}(\underline{x})$ are amplitude and phase terms of $\psi_1(\underline{x})$, whereas $r_{G_2}(\underline{x})$ and $\omega_{G_2}(\underline{x})$ are amplitude and phase terms of $\psi_2(\underline{x})$.

Definition 3.6. Two CFH-sets $\chi_{G_1} = (\psi_1, G_1)$ and $\chi_{G_2} = (\psi_2, G_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\psi_1, G_1) \subseteq (\psi_2, G_2)$
ii. $(\psi_2, G_2) \subseteq (\psi_1, G_1)$.

Definition 3.7. Let (ψ, G) be a CFH-set over $\mathbb{U}$.Then

i. (ψ, G) is called a *null CFH-set*, denoted by $(\psi, G)_\Phi$ if $r_G(\underline{x}) = 0$ and $\omega_G(\underline{x}) = 0\pi$ for all $\underline{x} \in G$.
ii. (ψ, G) is called a *absolute CFH-set*, denoted by $(\psi, G)_\Delta$ if $r_G(\underline{x}) = 1$ and $\omega_G(\underline{x}) = 2\pi$ for all $\underline{x} \in G$.

Definition 3.8. Let (ψ_1, G_1) and (ψ_2, G_2) are two CFH-sets over the same universe $\mathbb{U}$.Then

i. A CFH-set (ψ_1, G_1) is called a *homogeneous CFH-set*, denoted by $(\psi_1, G_1)_{Hom}$ if and only if $\psi_1(\underline{x})$ is a homogeneous CF-set for all $\underline{x} \in G_1$.

ii. A CFH-set (ψ_1, G_1) is called a *completely homogeneous CFH-set*, denoted by $(\psi_1, G_1)_{CHom}$ if and only if $\psi_1(\underline{x})$ is a homogeneous with $\psi_1(\underline{y})$ for all $\underline{x}, \underline{y} \in G_1$.

iii. A CFH-set (ψ_1, G_1) is said to be a completely homogeneous CFH-set with (ψ_2, G_2) if and only if $\psi_1(\underline{x})$ is a homogeneous with $\psi_2(\underline{x})$ for all $\underline{x} \in G_1 \prod G_2$.

3.1. *Set Theoretic Operations and Laws on CFH-Sets*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CFH-sets.

Definition 3.9. The *complement* of CFH-set (ψ, G), denoted by $(\psi, G)^c$ is defined as

$$(\psi, G)^c = \{(\underline{x}, \psi^c(\underline{x})) : \underline{x} \in G, \psi^c(\underline{x}) \in C(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $\psi^c(\underline{x})$ are given by $r^c_G(\underline{x}) = 1 - r_G(\underline{x})$ and $\omega^c_G(\underline{x}) = 2\pi - \omega_G(\underline{x})$ respectively.

Proposition 3.10. *Let (ψ, G) be a CFH-set over $\mathbb{U}$.Then $((\psi, G)^c)^c = (\psi, G)$.*

Proof. Since $\psi(\underline{x}) \in C(\mathbb{U})$, therefore (ψ, G) can be written in terms of its amplitude and phase terms as

$$(\psi, G) = \left\{ \left(\underline{x}, r_G(\underline{x}) e^{i\omega_G(\underline{x})} \right) : \underline{x} \in G \right\} \tag{1}$$

Now

$$\psi^c(\underline{x}) = \left\{\left(\underline{x}, r^c_G(\underline{x})e^{i\omega^c_G(\underline{x})}\right) : \underline{x} \in G\right\}$$

$$\psi^c(\underline{x}) = \left\{\left(\underline{x}, (1 - r_G(\underline{x}))e^{i(2\pi - \omega_G(\underline{x}))}\right) : \underline{x} \in G\right\}$$

$$((\psi, G)^c)^c = \left\{\left(\underline{x}, (1 - r_G(\underline{x}))^c e^{i(2\pi - \omega_G(\underline{x})^c)}\right) : \underline{x} \in G\right\}$$

$$((\psi, G)^c)^c = \left\{\left(\underline{x}, (1 - (1 - r_G(\underline{x})))e^{i(2\pi - (2\pi - \omega_G(\underline{x})))}\right) : \underline{x} \in G\right\}$$

$$((\psi, G)^c)^c = \left\{\left(\underline{x}, r_G(\underline{x})e^{i\omega_G(\underline{x})}\right) : \underline{x} \in G\right\} \tag{2}$$

from equations (1) and (2), we have $((\psi, G)^c)^c = (\psi, G)$. □

Proposition 3.11. *Let (ψ, G) be a CFH-set over $\mathbb{U}$. Then*

i. $((\psi, G)_\Phi)^c = (\psi, G)_\Delta$
ii. $((\psi, G)_\Delta)^c = (\psi, G)_\Phi$

Definition 3.12. The *intersection* of two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) over the same universe $\mathbb{U}$, denoted by $(\psi_1, G_1) \prod (\psi_2, G_2)$, is the CFH-set (ψ_3, G_3), where $G_3 = G_1 \prod G_2$, and $\psi_3(\underline{x}) = \psi_1(\underline{x}) \prod \psi_2(\underline{x})$ for all $\underline{x} \in G_3$.

Definition 3.13. The *difference* between two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) is defined as

$$(\psi_1, G_1) \setminus (\psi_2, G_2) = (\psi_1, G_1) \prod (\psi_2, G_2)^c$$

Definition 3.14. The *union* of two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) over the same universe $\mathbb{U}$, denoted by $(\psi_1, G_1) \coprod (\psi_2, G_2)$, is the CFH-set (ψ_3, G_3), where $G_3 = G_1 \coprod G_2$, and for all $\underline{x} \in G_3$,

$$\psi_3(\underline{x}) = \begin{cases} \psi_1(\underline{x}) & , if\ \underline{x} \in G_1 \backslash G_2 \\ \psi_2(\underline{x}) & , if\ \underline{x} \in G_2 \backslash G_1 \\ \psi_1(\underline{x}) \coprod \psi_2(\underline{x}) & , if\ \underline{x} \in G_1 \prod G_2 \end{cases}$$

Proposition 3.15. *Let (ψ, G) be a CFH-set over $\mathbb{U}$. Then the following results hold true:*

i. $(\psi, G) \coprod (\psi, G)_\Phi = (\psi, G)$
ii. $(\psi, G) \coprod (\psi, G)_\Delta = (\psi, G)_\Delta$
iii. $(\psi, G) \prod (\psi, G)_\Phi = (\psi, G)_\Phi$
iv. $(\psi, G) \prod (\psi, G)_\Delta = (\psi, G)$
v. $(\psi, G)_\Phi \coprod (\psi, G)_\Delta = (\psi, G)_\Delta$
vi. $(\psi, G)_\Phi \prod (\psi, G)_\Delta = (\psi, G)_\Phi$

Proposition 3.16. *Let (ψ_1, G_1), (ψ_2, G_2) and (ψ_3, G_3) are three CFH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\psi_1, G_1) \prod (\psi_2, G_2) = (\psi_2, G_2) \prod (\psi_1, G_1)$
ii. $(\psi_1, G_1) \coprod (\psi_2, G_2) = (\psi_2, G_2) \coprod (\psi_1, G_1)$
iii. $(\psi_1, G_1) \prod ((\psi_2, G_2) \prod (\psi_3, G_3)) = ((\psi_1, G_1) \prod (\psi_2, G_2)) \prod (\psi_3, G_3)$
iv. $(\psi_1, G_1) \coprod ((\psi_2, G_2) \coprod (\psi_3, G_3)) = ((\psi_1, G_1) \coprod (\psi_2, G_2)) \coprod (\psi_3, G_3)$

Proposition 3.17. *Let (ψ_1, G_1) and (ψ_2, G_2) are two CFH-sets over the same universe $\mathbb{U}$. Then the following De Morganss laws hold true:*

i. $((\psi_1, G_1) \prod (\psi_2, G_2))^c = (\psi_1, G_1)^c \coprod (\psi_2, G_2)^c$
ii. $((\psi_1, G_1) \coprod (\psi_2, G_2))^c = (\psi_1, G_1)^c \prod (\psi_2, G_2)^c$

4. Complex Intuitionistic Fuzzy Hypersoft Set(CIFH-Set)

In this section, fundamental theory of CIFH-set is developed.

Definition 4.1. Let $B_1, B_2, B_3, ..., B_n$ are disjoint sets having attribute values of n distinct attributes $b_1, b_2, b_3, ..., b_n$ respectively for $n \geq 1$, $B = B_1 \times B_2 \times B_3 \times ..... \times B_n$ and $\xi(\underline{\nu})$ be a CIF-set over $\mathbb{U}$ for all $\underline{\nu} = (s_1, s_2, s_3, ..., s_n) \in B$ such that $s_1 \in B_1, s_2 \in B_2, s_3 \in B_3, ..., s_n \in B_n$. Then, *complex intuitionistic fuzzy hypersoft set* (CIFH-set) $\Gamma_B = (\xi, B)$ over $\mathbb{U}$ is defined as

$$\Gamma_B = \{(\underline{\nu}, \xi(\underline{\nu})) : \underline{\nu} \in B, \xi(\underline{\nu}) \in C_{Int}(\mathbb{U})\}$$

where

$$\xi : B \to C_{Int}(\mathbb{U}), \qquad \xi(\underline{\nu}) = \emptyset \ if \ \underline{\nu} \notin B.$$

is a CIF approximate function of Γ_B and $\xi(\underline{\nu}) = \langle \xi^T(\underline{\nu}), \xi^F(\underline{\nu}) \rangle$.
$\xi^T(\underline{\nu}) = \alpha_T \, e^{i\,\beta_T}$ and $\xi^F(\underline{\nu}) = \alpha_F \, e^{i\,\beta_F}$ are complex-valued grade of membership and nonmembership of Γ_B respectively and their sum all are lying within the unit circle in the complex plane such that $\alpha_T, \alpha_F \in [0,1]$ with $0 \leq \alpha_T + \alpha_F \leq 1$ and $\beta_T, \beta_F \in (0, 2\pi]$. The value $\xi(\underline{\nu})$ is called $\underline{\nu}$-member of CIFH-set $\forall \underline{\nu} \in B$.

Example 4.2. Considering example 3.4 with $B = \{e_1, e_2, e_3, ..., e_8\}$, CIF-sets $\xi_B(e_1), \xi_B(e_2), ..., \xi_B(e_8)$ are defined as,

$$\xi_B(e_1) = \left\{ \frac{\langle 0.6, 0.2 \rangle e^{i\langle 0.5, 0.3 \rangle \pi}}{t_1}, \frac{\langle 0.8, 0.1 \rangle e^{i\langle 0.5, 0.3 \rangle \pi}}{t_2}, \frac{\langle 0.6, 0.4 \rangle e^{i\langle 0.7, 0.2 \rangle \pi}}{t_3}, \frac{\langle 0.3, 0.1 \rangle e^{i\langle 0.65, 0.35 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_2) = \left\{ \frac{\langle 0.5, 0.2 \rangle e^{i\langle 0.6, 0.3 \rangle \pi}}{t_1}, \frac{\langle 0.8, 0.01 \rangle e^{i\langle 0.8, 0.02]\pi}}{t_2}, \frac{\langle 0.6, 0.2 \rangle] e^{i\langle 0.8, 0.03 \rangle \pi}}{t_3}, \frac{\langle 0.65, 0.25 \rangle e^{i\langle 0.85, 0.05 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_3) = \left\{ \frac{\langle 0.4, 0.3 \rangle e^{i\langle 0.5, 0.1 \rangle \pi}}{t_1}, \frac{\langle 0.7, 0.o2 \rangle e^{i\langle 0.8, 0.03 \rangle]\pi}}{t_2}, \frac{\langle 0.5, 0.1 \rangle e^{i\langle 0.9, 0.01 \rangle \pi}}{t_3}, \frac{\langle 0.55, 0.25 \rangle e^{i\langle 0.85, 0.05 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_4) = \left\{ \frac{\langle 0.3, 0.1 \rangle e^{i\langle 0.6, 0.1 \rangle \pi}}{t_1}, \frac{\langle 0.6, 0.01 \rangle e^{i\langle 0.8, 0.09 \rangle \pi}}{t_2}, \frac{\langle 0.5, 0.05 \rangle e^{i\langle 0.2, 0.01 \rangle \pi}}{t_3}, \frac{\langle 0.45, 0.25 \rangle e^{i\langle 0.55, 0.15 \rangle \pi}}{t_4} \right\},$$

$$\xi_B(e_5)=\left\{\frac{\langle 0.3,0.2\rangle e^{i\langle 0.4,0.3\rangle\pi}}{t_1},\frac{\langle 0.7,0.1\rangle e^{i\langle 0.7,0.08\rangle\pi}}{t_2},\frac{\langle 0.7,0.01\rangle e^{i\langle 0.6,0.1\rangle\pi}}{t_3},\frac{\langle 0.55,0.05\rangle e^{i\langle 0.45,0.05\rangle\pi}}{t_4}\right\},$$

$$\xi_B(e_6)=\left\{\frac{\langle 0.4,0.01\rangle e^{i\langle 0.5,0.1\rangle\pi}}{t_1},\frac{\langle 0.4,0.1\rangle e^{i\langle 0.8,0.1\rangle\pi}}{t_2},\frac{\langle 0.6,0.070\rangle e^{i\langle 0.7,0.01\rangle\pi}}{t_3},\frac{\langle 0.65,0.05\rangle e^{i\langle 0.85,0.15\rangle\pi}}{t_4}\right\},$$

$$\xi_B(e_7)=\left\{\frac{\langle 0.5,0.09\rangle e^{i\langle 0.8,0.09\rangle\pi}}{t_1},\frac{\langle 0.4,0.09\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_2},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_3},\frac{\langle 0.75,0.15\rangle e^{i\langle 0.65,0.25\rangle\pi}}{t_4}\right\},$$

and

$$\xi_B(e_8)=\left\{\frac{\langle 0.7,0.08\rangle e^{i\langle 0.1,0.09\rangle\pi}}{t_1},\frac{\langle 0.5,0.08\rangle e^{i\langle 0.7,0.02\rangle\pi}}{t_2},\frac{\langle 0.5,0.06\rangle e^{i\langle 0.8,0.03\rangle\pi}}{t_3},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.75,0.15\rangle\pi}}{t_4}\right\}$$

then CIFH-set Γ_B is written by,

$$\Gamma_B=\left\{\begin{array}{l}
\left(e_1,\frac{\langle 0.6,0.2\rangle e^{i\langle 0.5,0.3\rangle\pi}}{t_1},\frac{\langle 0.8,0.1\rangle e^{i\langle 0.5,0.3\rangle\pi}}{t_2},\frac{\langle 0.6,0.4\rangle e^{i\langle 0.7,0.2\rangle\pi}}{t_3},\frac{\langle 0.3,0.1\rangle e^{i\langle 0.65,0.35\rangle\pi}}{t_4}\right),\\
\left(e_2,\frac{\langle 0.5,0.2\rangle e^{i\langle 0.6,0.3\rangle\pi}}{t_1},\frac{\langle 0.8,0.01\rangle e^{i\langle 0.8,0.02\rangle]\pi}}{t_2},\frac{\langle 0.6,0.2\rangle] e^{i\langle 0.8,0.03\rangle\pi}}{t_3},\frac{\langle 0.65,0.25\rangle e^{i\langle 0.85,0.05\rangle\pi}}{t_4}\right),\\
\left(e_3,\frac{\langle 0.4,0.3\rangle e^{i\langle 0.5,0.1\rangle\pi}}{t_1},\frac{\langle 0.7,0.o2\rangle e^{i\langle 0.8,0.03\rangle]\pi}}{t_2},\frac{\langle 0.5,0.1\rangle e^{i\langle 0.9,0.01\rangle\pi}}{t_3},\frac{\langle 0.55,0.25\rangle e^{i\langle 0.85,0.05\rangle\pi}}{t_4}\right),\\
\left(e_4,\frac{\langle 0.3,0.1\rangle e^{i\langle 0.6,0.1\rangle\pi}}{t_1},\frac{\langle 0.6,0.01\rangle e^{i\langle 0.8,0.09\rangle\pi}}{t_2},\frac{\langle 0.5,0.05\rangle e^{i\langle 0.2,0.01\rangle\pi}}{t_3},\frac{\langle 0.45,0.25\rangle e^{i\langle 0.55,0.15\rangle\pi}}{t_4}\right),\\
\left(e_5,\frac{\langle 0.3,0.2\rangle e^{i\langle 0.4,0.3\rangle\pi}}{t_1},\frac{\langle 0.7,0.1\rangle e^{i\langle 0.7,0.08\rangle\pi}}{t_2},\frac{\langle 0.7,0.01\rangle e^{i\langle 0.6,0.1\rangle\pi}}{t_3},\frac{\langle 0.55,0.05\rangle e^{i\langle 0.45,0.05\rangle\pi}}{t_4}\right),\\
\left(e_6,\frac{\langle 0.4,0.01\rangle e^{i\langle 0.5,0.1\rangle\pi}}{t_1},\frac{\langle 0.4,0.1\rangle e^{i\langle 0.8,0.1\rangle\pi}}{t_2},\frac{\langle 0.6,0.070\rangle e^{i\langle 0.7,0.01\rangle\pi}}{t_3},\frac{\langle 0.65,0.05\rangle e^{i\langle 0.85,0.15\rangle\pi}}{t_4}\right),\\
\left(e_7,\frac{\langle 0.5,0.09\rangle e^{i\langle 0.8,0.09\rangle\pi}}{t_1},\frac{\langle 0.4,0.09\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_2},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.5,0.06\rangle\pi}}{t_3},\frac{\langle 0.75,0.15\rangle e^{i\langle 0.65,0.25\rangle\pi}}{t_4}\right),\\
\left(e_8,\frac{\langle 0.7,0.08\rangle e^{i\langle 0.1,0.09\rangle\pi}}{t_1},\frac{\langle 0.5,0.08\rangle e^{i\langle 0.7,0.02\rangle\pi}}{t_2},\frac{\langle 0.5,0.06\rangle e^{i\langle 0.8,0.03\rangle\pi}}{t_3},\frac{\langle 0.4,0.05\rangle e^{i\langle 0.75,0.15\rangle\pi}}{t_4}\right)
\end{array}\right\}$$

Definition 4.3. Let $\Gamma_{B_1}=(\xi_1,B_1)$ and $\Gamma_{B_2}=(\xi_2,B_2)$ be two CIFH-sets over the same $\mathbb{U}$. The set $\Gamma_{B_1}=(\xi_1,B_1)$ is said to be the *subset* of $\Gamma_{B_2}=(\xi_2,B_2)$, if

i. $B_1\subseteq B_2$

ii. $\forall\ \underline{p}\in B_1,\xi_1(\underline{p})\subseteq\xi_2(\underline{p})$ implies ${\xi^T}_1(\underline{p})\subseteq{\xi^T}_2(\underline{p}),{\xi^F}_1(\underline{p})\subseteq{\xi^F}_2(\underline{p})$ i.e.
$\alpha_{TB_1}(\underline{p})\le\alpha_{TB_2}(\underline{p}),\alpha_{FB_1}(\underline{p})\le\alpha_{FB_2}(\underline{p}),\beta_{TB_1}(\underline{p})\le\beta_{TB_2}(\underline{p})$ and $\beta_{FB_1}(\underline{p})\le\beta_{FB_2}(\underline{p})$,
where

$\alpha_{TB_1}(\underline{p})$ and $\beta_{TB_1}(\underline{p})$ are amplitude and phase terms of $\xi_1^T(\underline{p})$,
$\alpha_{FB_1}(\underline{p})$ and $\beta_{FB_1}(\underline{p})$ are amplitude and phase terms of $\xi_1^F(\underline{p})$,
$\alpha_{TB_2}(\underline{p})$ and $\beta_{TB_2}(\underline{p})$ are amplitude and phase terms of $\xi_2^T(\underline{p})$, and
$\alpha_{FB_2}(\underline{p})$ and $\beta_{FB_2}(\underline{p})$ are amplitude and phase terms of $\xi_2^F(\underline{p})$.

Definition 4.4. Two CIFH-set $\Gamma_{B_1} = (\xi_1, B_1)$ and $\Gamma_{B_2} = (\xi_2, B_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\xi_1, B_1) \subseteq (\xi_2, B_2)$
ii. $(\xi_2, B_2) \subseteq (\xi_1, B_1)$.

Definition 4.5. Let (ξ, B) be a CIFH-set over $\mathbb{U}$.Then

i. (ξ, B) is called a *null CIFH-set*, denoted by $(\xi, B)_\Phi$ if $\alpha_{TB}(\underline{p}) = \alpha_{FB}(\underline{p}) = 0$ and $\beta_{TB}(\underline{p}) = \beta_{FB}(\underline{p}) = 0\pi$ for all $\underline{p} \in B$.
ii. (ξ, B) is called a *absolute CIFH-set*, denoted by $(\xi, B)_\Delta$ if $\alpha_{TB}(\underline{p}) = \alpha_{FB}(\underline{p}) = 1$ and $\beta_{TB}(\underline{p}) = \beta_{FB}(\underline{p}) = 2\pi$ for all $\underline{p} \in B$.

Definition 4.6. Let (ξ_1, B_1) and (ξ_2, B_2) are two CIFH-sets over the same universe $\mathbb{U}$.Then

i. A CIFH-set (ξ_1, B_1) is called a *homogeneous CIFH-set*, denoted by $(\xi_1, B_1)_{Hom}$ if and only if $\xi_1(\underline{p})$ is a homogeneous CIF-set for all $\underline{p} \in B_1$.

ii. A CIFH-set (ξ_1, B_1) is called a *completely homogeneous CIFH-set*, denoted by $(\xi_1, B_1)_{CHom}$ if and only if $\xi_1(\underline{p})$ is a homogeneous with $\xi_1(\underline{q})$ for all $\underline{p}, \underline{q} \in B_1$.

iii. A CIFH-set (ξ_1, B_1) is said to be a completely homogeneous CIFH-set with (ξ_2, B_2) if and only if $\xi_1(\underline{p})$ is a homogeneous with $\xi_2(\underline{p})$ for all $\underline{p} \in B_1 \prod B_2$.

4.1. *Set Theoretic Operations and Laws on CIFH-set*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CFH-set.

Definition 4.7. The *complement* of CIFH-set (ξ, B), denoted by $(\xi, B)^c$ is defined as

$$(\xi, B)^c = \{(\underline{p}, (\xi(\underline{p}))^c) : \underline{p} \in B, (\xi(\underline{p}))^c \in C_{Int}(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $(\xi(\underline{p}))^c$ are given by
$(\alpha_{TB}(\underline{p}))^c = 1 - \alpha_{TB}(\underline{p})$
$(\alpha_{FB}(\underline{p}))^c = 1 - \alpha_{FB}(\underline{p})$
and
$(\beta_{TB}(\underline{p}))^c = 2\pi - \beta_{TB}(\underline{p})$,
$(\beta_{FB}(\underline{p}))^c = 2\pi - \beta_{FB}(\underline{p})$ respectively.

Proposition 4.8. *Let (ξ, B) be a CIFH-set over $\mathbb{U}$.Then* $((\xi, B)^c)^c = (\xi, B)$.

Proof. Since $\xi(\underline{p}) \in C_{Int}(\mathbb{U})$, therefore (ξ, B) can be written in terms of its amplitude and phase terms as

$$(\xi, B) = \left\{ \left(\underline{p}, \left(\alpha_{TB}(\underline{p}) e^{i\beta_{TB}(\underline{p})}, \alpha_{FB}(\underline{p})) e^{i\beta_{FB}(\underline{p})} \right) \right) : \underline{p} \in B \right\} \tag{3}$$

Now

$$(\xi, B)^c(\underline{p}) = \left\{ \left(\underline{p}, \left((\alpha_{TB}(\underline{p}))^c e^{i(\beta_{TB}(\underline{p}))^c}, (\alpha_{FB}(\underline{p}))^c e^{i(\beta_{FB}(\underline{p}))^c} \right) \right) : \underline{p} \in B \right\}$$

$$(\xi, B)^c(\underline{p}) = \left\{ \left(\underline{p}, \left((1 - \alpha_{TB}(\underline{p})) e^{i(2\pi - \beta_{TB}(\underline{p}))}, (1 - \alpha_{FB}(\underline{p})) e^{i(2\pi - \alpha_{FB}(\underline{p}))} \right) \right) : \underline{p} \in B \right\}$$

$$((\xi, B)^c)^c = \left\{ \left(\underline{p}, \left((1 - \alpha_{TB}(\underline{p}))^c e^{i(2\pi - \beta_{TB}(\underline{p}))^c}, (1 - \alpha_{FB}(\underline{p}))^c e^{i(2\pi - \beta_{TB}(\underline{p}))^c} \right) \right) : \underline{p} \in B \right\}$$

$$((\xi, B)^c)^c = \left\{ \left(\underline{p}, \left((1 - (1 - \alpha_{TB}(\underline{p}))) e^{i(2\pi - (2\pi - \beta_{TB}(\underline{p})))}, (1 - (1 - \alpha_{FB}(\underline{p}))) e^{i(2\pi - (2\pi - \beta_{FB}(\underline{p})))} \right) \right) : \underline{p} \in B \right\}$$

$$((\xi, B)^c)^c = \left\{ \left(\underline{p}, \left(\alpha_{TB}(\underline{p}) e^{i\beta_{TB}(\underline{p})}, \alpha_{FB}(\underline{p})) e^{i\beta_{FB}(\underline{p})} \right) \right) : \underline{p} \in B \right\} \tag{4}$$

from equations (3) and (4), we have $((\xi, B)^c)^c = (\xi, B)$. □

Proposition 4.9. *Let* (ξ, B) *be a CIFH-set over* $\mathbb{U}$*. Then*

i. $((\xi, B)_\Phi)^c = (\xi, B)_\Delta$

ii. $((\xi, B)_\Delta)^c = (\xi, B)_\Phi$

Definition 4.10. The *intersection* of two CIFH-set (ξ_1, B_1) and (ξ_2, B_2) over the same universe $\mathbb{U}$, denoted by $(\xi_1, B_1) \prod (\xi_2, B_2)$, is the CIFH-set (ξ_3, B_3), where $B_3 = B_1 \coprod B_2$, and for all $\underline{p} \in B_3$,

$$\xi^T{}_3(\underline{p}) = \begin{cases} \alpha_{TB_1}(\underline{p}) e^{i\beta_{TB_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{TB_2}(\underline{p}) e^{i\beta_{TB_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ min(\alpha_{TB_1}(\underline{p}), \alpha_{TB_2}(\underline{p})) e^{i\,min(\beta_{TB_1}(\underline{p}), \beta_{TB_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

and

$$\xi^F{}_3(\underline{p}) = \begin{cases} \alpha_{FB_1}(\underline{p}) e^{i\beta_{FB_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{FB_2}(\underline{p}) e^{i\beta_{FB_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ min(\alpha_{FB_1}(\underline{p}), \alpha_{FB_2}(\underline{p})) e^{i\,min(\beta_{FB_1}(\underline{p}), \beta_{FB_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

Definition 4.11. The *difference* between two CFH-set (ξ_1, B_1) and (ξ_2, B_2) is defined as

$$(\xi_1, B_1) \setminus (\xi_2, B_2) = (\xi_1, B_1) \prod (\xi_2, B_2)^c$$

Definition 4.12. The *union* of two CFH-set (ξ_1, B_1) and (ξ_2, B_2) over the same universe $\mathbb{U}$, denoted by $(\xi_1, B_1) \coprod (\xi_2, B_2)$, is the CFH-set (ξ_3, B_3), where $B_3 = B_1 \coprod B_2$, and for all $\underline{p} \in B_3$,

$$\xi^T{}_3(\underline{p}) = \begin{cases} \alpha_{TB_1}(\underline{p}) e^{i\beta_{TB_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{TB_2}(\underline{p}) e^{i\beta_{TB_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ max(\alpha_{TB_1}(\underline{p}), \alpha_{TB_2}(\underline{p})) e^{i\,max(\beta_{TB_1}(\underline{p}), \beta_{TB_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

and

$$\xi^F{}_3(\underline{p}) = \begin{cases} \alpha_{F B_1}(\underline{p}) e^{i\beta_{F B_1}(\underline{p})} & , if\ \underline{p} \in B_1 \backslash B_2 \\ \alpha_{F B_2}(\underline{p}) e^{i\beta_{F B_2}(\underline{p})} & , if\ \underline{p} \in B_2 \backslash B_1 \\ max(\alpha_{F B_1}(\underline{p}), \alpha_{F B_2}(\underline{p})) e^{i\, max(\beta_{F B_1}(\underline{p}), \beta_{F B_2}(\underline{p}))} & , if\ \underline{p} \in B_1 \prod B_2 \end{cases}$$

Proposition 4.13. *Let (ξ, B) be a CIFH-set over $\mathbb{U}$.Then the following results hold true:*

i. $(\xi, B) \coprod (\xi, B)_\Phi = (\xi, B)$
ii. $(\xi, B) \coprod (\xi, B)_\Delta = (\xi, B)_\Delta$
iii. $(\xi, B) \prod (\xi, B)_\Phi = (\xi, B)_\Phi$
iv. $(\xi, B) \prod (\xi, B)_\Delta = (\xi, B)$
v. $(\xi, B)_\Phi \coprod (\xi, B)_\Delta = (\xi, B)_\Delta$
vi. $(\xi, B)_\Phi \prod (\xi, B)_\Delta = (\xi, B)_\Phi$

Proposition 4.14. *Let (ξ_1, B_1), (ξ_2, B_2) and (ξ_3, B_3) are three CIFH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\xi_1, B_1) \prod (\xi_2, B_2) = (\xi_2, B_2) \prod (\xi_1, B_1)$
ii. $(\xi_1, B_1) \coprod (\xi_2, B_2) = (\xi_2, B_2) \coprod (\xi_1, B_1)$
iii. $(\xi_1, B_1) \prod ((\xi_2, B_2) \prod (\xi_3, B_3)) = ((\xi_1, B_1) \prod (\xi_2, B_2)) \prod (\xi_3, B_3)$
iv. $(\xi_1, B_1) \coprod ((\xi_2, B_2) \coprod (\xi_3, B_3)) = ((\xi_1, B_1) \coprod (\xi_2, B_2)) \coprod (\xi_3, B_3)$

Proposition 4.15. *Let (ξ_1, B_1) and (ξ_2, B_2) are two CIFH-sets over the same universe $\mathbb{U}$. Then the following De Morganss laws hold true:*

i. $((\xi_1, B_1) \prod (\xi_2, B_2))^c = (\xi_1, B_1)^c \coprod (\xi_2, B_2)^c$
ii. $((\xi_1, B_1) \coprod (\xi_2, B_2))^c = (\xi_1, B_1)^c \prod (\xi_2, B_2)^c$

5. Complex Neutrosophic Hypersoft Set(CNH-Set)

In this section, CNH-set and its some fundamentals are developed.

Definition 5.1. Let $N_1, N_2, N_3, ..., N_n$ are disjoint sets having attribute values of n distinct attributes $n_1, n_2, n_3, ..., n_n$ respectively for $n \geq 1, N = N_1 \times N_2 \times N_3 \times ..... \times N_n$ and $\zeta(\underline{\lambda})$ be a CN-set over $\mathbb{U}$ for all $\underline{\lambda} = (a_1, a_2, a_3, ..., a_n) \in N$ such that $a_1 \in N_1, a_2 \in N_2, a_3 \in N_3, ..., a_n \in N_n$. Then, *complex neutrosophic hypersoft set* (CNH-set) $\Theta_N = (\zeta, N)$ over $\mathbb{U}$ is defined as

$$\Theta_N = \{(\underline{\lambda}, \zeta(\underline{\lambda})) : \underline{\lambda} \in N, \zeta(\underline{\lambda}) \in C_{Neu}(\mathbb{U})\}$$

where

$$\zeta : N \to C_{Neu}(\mathbb{U}), \qquad \zeta(\underline{\lambda}) = \emptyset\ if\ \underline{\lambda} \notin N.$$

is a CN approximate function of Θ_N and $\zeta(\underline{\lambda}) = \langle \zeta^T(\underline{\lambda}), \zeta^I(\underline{\lambda}), \zeta^F(\underline{\lambda}) \rangle$.
$\zeta^T(\underline{\lambda}) = \delta_T\, e^{i\,\eta_T}$, $\zeta^I(\underline{\lambda}) = \delta_I\, e^{i\,\eta_I}$ and $\zeta^F(\underline{\lambda}) = \delta_F\, e^{i\,\eta_F}$ are complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity

membership function of Θ_N respectively and their sum all are lying within the unit circle in the complex plane such that $\delta_T, \delta_I, \delta_F \in [0,1]$ with $^{-}0 \leq \delta_T+\delta_I+\delta_F \leq 3^{+}(or\ 0 \leq |\delta_T+\delta_I+\delta_F| \leq 3)$ and $\eta_T, \eta_I, \eta_F \in (0, 2\pi]$. The value $\zeta(\underline{\lambda})$ is called $\underline{\lambda}$-member of CNH-set $\forall \underline{\lambda} \in N$.

Example 5.2. Considering example 3.4 wit $N = \{e_1, e_2, e_3, ..., e_8\}$, CNF-sets $\zeta_N(e_1), \zeta_N(e_2), ..., \zeta_N(e_8)$ are defined as,

$$\zeta_N(e_1) = \left\{ \frac{\langle 0.6,0.1,0.2\rangle e^{i\langle 0.5,0.2,0.3\rangle\pi}}{t_1}, \frac{\langle 0.8,0.3,0.1\rangle e^{i\langle 0.5,0.4,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.5,0.4\rangle e^{i\langle 0.7,0.6,0.2\rangle\pi}}{t_3}, \frac{\langle 0.3,0.7,0.1\rangle e^{i\langle 0.65,0.55,0.35\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_2) = \left\{ \frac{\langle 0.5,0.2,0.1\rangle e^{i\langle 0.6,0.3,0.2\rangle\pi}}{t_1}, \frac{\langle 0.8,0.01,0.2\rangle e^{i\langle 0.8,0.02,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.2,0.2\rangle e^{i\langle 0.8,0.03,0.4\rangle\pi}}{t_3}, \frac{\langle 0.65,0.25,0.5\rangle e^{i\langle 0.85,0.05,0.5\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_3) = \left\{ \frac{\langle 0.4,0.3,0.3\rangle e^{i\langle 0.5,0.1,0.8\rangle\pi}}{t_1}, \frac{\langle 0.7,0.02,0.3\rangle e^{i\langle 0.8,0.03,0.7\rangle]\pi}}{t_2}, \frac{\langle 0.5,0.1,0.9\rangle e^{i\langle 0.9,0.01,0.7\rangle\pi}}{t_3}, \frac{\langle 0.55,0.25,0.1\rangle e^{i\langle 0.85,0.05,0.4\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_4) = \left\{ \frac{\langle 0.3,0.1,0.9\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_1}, \frac{\langle 0.6,0.01,0.4\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_2}, \frac{\langle 0.5,0.05,0.3\rangle e^{i\langle 0.2,0.01,0.4\rangle\pi}}{t_3}, \frac{\langle 0.45,0.25,0.01\rangle e^{i\langle 0.55,0.15,0.3\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_5) = \left\{ \frac{\langle 0.3,0.2,0.1\rangle e^{i\langle 0.4,0.3,0.4\rangle\pi}}{t_1}, \frac{\langle 0.7,0.1,0.5\rangle e^{i\langle 0.7,0.08,0.4\rangle\pi}}{t_2}, \frac{\langle 0.7,0.01,0.4\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_3}, \frac{\langle 0.55,0.05,0.4\rangle e^{i\langle 0.45,0.05,0.3\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_6) = \left\{ \frac{\langle 0.4,0.01,0.3\rangle e^{i\langle 0.5,0.1,0.4\rangle\pi}}{t_1}, \frac{\langle 0.4,0.1,0.3\rangle e^{i\langle 0.8,0.1,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.070,0.5\rangle e^{i\langle 0.7,0.01,0.1\rangle\pi}}{t_3}, \frac{\langle 0.65,0.05,0.3\rangle e^{i\langle 0.85,0.15,0.4\rangle\pi}}{t_4} \right\},$$

$$\zeta_N(e_7) = \left\{ \frac{\langle 0.5,0.09,0.3\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_1}, \frac{\langle 0.4,0.09,0.4\rangle e^{i\langle 0.5,0.06,0.4\rangle\pi}}{t_2}, \frac{\langle 0.4,0.05,0.1\rangle e^{i\langle 0.5,0.06,0.5\rangle\pi}}{t_3}, \frac{\langle 0.75,0.15,0.4\rangle e^{i\langle 0.65,0.25,0.2\rangle\pi}}{t_4} \right\},$$

and

$$\zeta_N(e_8) = \left\{ \frac{\langle 0.7,0.08,0.3\rangle e^{i\langle 0.1,0.09,0.01\rangle\pi}}{t_1}, \frac{\langle 0.5,0.08,0.3\rangle e^{i\langle 0.7,0.02,0.6\rangle\pi}}{t_2}, \frac{\langle 0.5,0.06,0.5\rangle e^{i\langle 0.8,0.03,0.3\rangle\pi}}{t_3}, \frac{\langle 0.4,0.05,0.35\rangle e^{i\langle 0.75,0.15,0.6\rangle\pi}}{t_4} \right\}$$

then CNH-set Θ_N is written by,

$$\Theta_N = \left\{ \begin{array}{l} \left(e_1, \frac{\langle 0.6,0.1,0.2\rangle e^{i\langle 0.5,0.2,0.3\rangle\pi}}{t_1}, \frac{\langle 0.8,0.3,0.1\rangle e^{i\langle 0.5,0.4,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.5,0.4\rangle e^{i\langle 0.7,0.6,0.2\rangle\pi}}{t_3}, \frac{\langle 0.3,0.7,0.1\rangle e^{i\langle 0.65,0.55,0.35\rangle\pi}}{t_4}\right), \\ \left(e_2, \frac{\langle 0.5,0.2,0.1\rangle e^{i\langle 0.6,0.3,0.2\rangle\pi}}{t_1}, \frac{\langle 0.8,0.01,0.2\rangle e^{i\langle 0.8,0.02,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.2,0.2\rangle] e^{i\langle 0.8,0.03,0.4\rangle\pi}}{t_3}, \frac{\langle 0.65,0.25,0.5\rangle e^{i\langle 0.85,0.05,0.5\rangle\pi}}{t_4}\right), \\ \left(e_3, \frac{\langle 0.4,0.3,0.3\rangle e^{i\langle 0.5,0.1,0.8\rangle\pi}}{t_1}, \frac{\langle 0.7,0.02,0.3\rangle e^{i\langle 0.8,0.03,0.7\rangle]\pi}}{t_2}, \frac{\langle 0.5,0.1,0.9\rangle e^{i\langle 0.9,0.01,0.7\rangle\pi}}{t_3}, \frac{\langle 0.55,0.25,0.1\rangle e^{i\langle 0.85,0.05,0.4\rangle\pi}}{t_4}\right), \\ \left(e_4, \frac{\langle 0.3,0.1,0.9\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_1}, \frac{\langle 0.6,0.01,0.4\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_2}, \frac{\langle 0.5,0.05,0.3\rangle e^{i\langle 0.2,0.01,0.4\rangle\pi}}{t_3}, \frac{\langle 0.45,0.25,0.01\rangle e^{i\langle 0.55,0.15,0.3\rangle\pi}}{t_4}\right), \\ \left(e_5, \frac{\langle 0.3,0.2,0.1\rangle e^{i\langle 0.4,0.3,0.4\rangle\pi}}{t_1}, \frac{\langle 0.7,0.1,0.5\rangle e^{i\langle 0.7,0.08,0.4\rangle\pi}}{t_2}, \frac{\langle 0.7,0.01,0.4\rangle e^{i\langle 0.6,0.1,0.5\rangle\pi}}{t_3}, \frac{\langle 0.55,0.05,0.4\rangle e^{i\langle 0.45,0.05,0.3\rangle\pi}}{t_4}\right), \\ \left(e_6, \frac{\langle 0.4,0.01,0.3\rangle e^{i\langle 0.5,0.1,0.4\rangle\pi}}{t_1}, \frac{\langle 0.4,0.1,0.3\rangle e^{i\langle 0.8,0.1,0.3\rangle\pi}}{t_2}, \frac{\langle 0.6,0.070,0.5\rangle e^{i\langle 0.7,0.01,0.1\rangle\pi}}{t_3}, \frac{\langle 0.65,0.05,0.3\rangle e^{i\langle 0.85,0.15,0.4\rangle\pi}}{t_4}\right), \\ \left(e_7, \frac{\langle 0.5,0.09,0.3\rangle e^{i\langle 0.8,0.09,0.5\rangle\pi}}{t_1}, \frac{\langle 0.4,0.09,0.4\rangle e^{i\langle 0.5,0.06,0.4\rangle\pi}}{t_2}, \frac{\langle 0.4,0.05,0.1\rangle e^{i\langle 0.5,0.06,0.5\rangle\pi}}{t_3}, \frac{\langle 0.75,0.15,0.4\rangle e^{i\langle 0.65,0.25,0.2\rangle\pi}}{t_4}\right), \\ \left(e_8, \frac{\langle 0.7,0.08,0.3\rangle e^{i\langle 0.1,0.09,0.01\rangle\pi}}{t_1}, \frac{\langle 0.5,0.08,0.3\rangle e^{i\langle 0.7,0.02,0.6\rangle\pi}}{t_2}, \frac{\langle 0.5,0.06,0.5\rangle e^{i\langle 0.8,0.03,0.3\rangle\pi}}{t_3}, \frac{\langle 0.4,0.05,0.35\rangle e^{i\langle 0.75,0.15,0.6\rangle\pi}}{t_4}\right) \end{array} \right\}$$

Definition 5.3. Let $\Theta_{N_1} = (\zeta_1, N_1)$ and $\Theta_{N_2} = (\zeta_2, N_2)$ be two CNH-sets over the same $\mathbb{U}$. The set $\Theta_{N_1} = (\zeta_1, N_1)$ is said to be the *subset* of $\Theta_{N_2} = (\zeta_2, N_2)$, if

i. $N_1 \subseteq N_2$

ii. $\forall\ \underline{u} \in N_1, \zeta_1(\underline{u}) \subseteq \zeta_2(\underline{u})$ implies $\zeta_1^T(\underline{u}) \subseteq \zeta_2^T(\underline{u}), \zeta_1^I(\underline{u}) \subseteq \zeta_2^I(\underline{u}), \zeta_1^F(\underline{u}) \subseteq \zeta_2^F(\underline{u})$ i.e.
$\delta_{TN_1}(\underline{u}) \leq \delta_{TN_2}(\underline{u})$,
$\delta_{IN_1}(\underline{u}) \leq \delta_{IN_2}(\underline{u})$,
$\delta_{FN_1}(\underline{u}) \leq \delta_{FN_2}(\underline{u})$,
$\eta_{TN_1}(\underline{u}) \leq \eta_{TN_2}(\underline{u})$,
$\eta_{IN_1}(\underline{u}) \leq \eta_{IN_2}(\underline{u})$ and
$\eta_{FN_1}(\underline{u}) \leq \eta_{FN_2}(\underline{u})$,
where
$\delta_{TN_1}(\underline{u})$ and $\eta_{TN_1}(\underline{u})$ are amplitude and phase terms of $\zeta_1^T(\underline{u})$,
$\delta_{IN_1}(\underline{u})$ and $\eta_{IN_1}(\underline{u})$ are amplitude and phase terms of $\zeta_1^I(\underline{u})$,
$\delta_{FN_1}(\underline{u})$ and $\eta_{FN_1}(\underline{u})$ are amplitude and phase terms of $\zeta_1^F(\underline{u})$,
$\delta_{TN_2}(\underline{u})$ and $\eta_{TN_2}(\underline{u})$ are amplitude and phase terms of $\zeta_2^T(\underline{u})$,
$\delta_{IN_2}(\underline{u})$ and $\eta_{IN_2}(\underline{u})$ are amplitude and phase terms of $\zeta_2^I(\underline{u})$, and
$\delta_{FN_2}(\underline{u})$ and $\eta_{FN_2}(\underline{u})$ are amplitude and phase terms of $\zeta_2^F(\underline{u})$.

Definition 5.4. Two CNH-set $\Theta_{N_1} = (\zeta_1, N_1)$ and $\Theta_{N_2} = (\zeta_2, N_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\zeta_1, N_1) \subseteq (\zeta_2, N_2)$
ii. $(\zeta_2, N_2) \subseteq (\zeta_1, N_1)$.

Definition 5.5. Let (ζ, N) be a CNH-set over $\mathbb{U}$.Then

i. (ζ, N) is called a *null CNH-set*, denoted by $(\zeta, N)_\Phi$ if $\delta_{TN}(\underline{u}) = \delta_{IN}(\underline{u}) = \delta_{FN}(\underline{u}) = 0$ and $\eta_{TN}(\underline{u}) = \eta_{IN}(\underline{u}) = \eta_{FN}(\underline{u}) = 0\pi$ for all $\underline{u} \in B$.
ii. (ζ, N) is called a *absolute CNH-set*, denoted by $(\zeta, N)_\Delta$ if $\delta_{TN}(\underline{u}) = \delta_{IN}(\underline{u}) = \delta_{FN}(\underline{u}) = 1$ and $\eta_{TN}(\underline{u}) = \eta_{IN}(\underline{u}) = \eta_{FN}(\underline{u}) = 2\pi$ for all $\underline{u} \in B$.

Definition 5.6. Let (ζ_1, N_1) and (ζ_2, N_2) are two CNH-sets over the same universe $\mathbb{U}$.Then

i. A CNH-set (ζ_1, N_1) is called a *homogeneous CNH-set*, denoted by $(\zeta_1, N_1)_{Hom}$ if and only if $\zeta_1(\underline{u})$ is a homogeneous CN-set for all $\underline{u} \in N_1$.

ii. A CNH-set (ζ_1, N_1) is called a *completely homogeneous CNH-set*, denoted by $(\zeta_1, N_1)_{CHom}$ if and only if $\zeta_1(\underline{u})$ is a homogeneous with $\zeta_1(\underline{v})$ for all $\underline{u}, \underline{v} \in N_1$.

iii. A CNH-set (ζ_1, N_1) is said to be a completely homogeneous CNH-set with (ζ_2, N_2) if and only if $\zeta_1(\underline{u})$ is a homogeneous with $\zeta_2(\underline{u})$ for all $\underline{u} \in N_1 \prod N_2$.

5.1. *Set Theoretic Operations and Laws on CNH-set*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CNH-set.

Definition 5.7. The *complement* of CNH-set (ζ, N), denoted by $(\zeta, N)^c$ is defined as

$$(\zeta, N)^c = \{(\underline{u}, (\zeta(\underline{u}))^c) : \underline{u} \in B, (\zeta(\underline{u}))^c \in C_{Neu}(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $(\zeta(\underline{u}))^c$ are given by
$(\delta_{TN}(\underline{u}))^c = \delta_{FN}(\underline{u})$,
$(\delta_{IN}(\underline{u}))^c = 1 - \delta_{IN}(\underline{u})$,
$(\delta_{FN}(\underline{u}))^c = \delta_{TN}(\underline{u})$,
and
$(\eta_{TN}(\underline{u}))^c = 2\pi - \eta_{TN}(\underline{u})$,
$(\eta_{IN}(\underline{u}))^c = 2\pi - \eta_{IN}(\underline{u})$,
$(\eta_{FN}(\underline{u}))^c = 2\pi - \eta_{FN}(\underline{u})$ respectively.

Proposition 5.8. *Let* (ζ, N) *be a CNH-set over* $\mathbb{U}$*.Then* $((\zeta, N)^c)^c = (\zeta, N)$.

Proof. Since $\zeta(\underline{u}) \in C_{Neu}(\mathbb{U})$, therefore (ζ, N) can be written in terms of its amplitude and phase terms as

$$(\zeta, N) = \left\{\left(\underline{u}, \left\langle \delta_{TN}(\underline{u})e^{i\eta_{TN}(\underline{u})}, \delta_{IN}(\underline{u})e^{i\eta_{IN}(\underline{u})}, \delta_{FN}(\underline{u}))e^{i\eta_{FN}(\underline{u})}\right\rangle\right) : \underline{u} \in N\right\} \quad (5)$$

Now

$$(\zeta, N)^c(\underline{u}) = \left\{\left(\underline{u}, \left\langle (\delta_{TN}(\underline{u}))^c e^{i(\eta_{TN}(\underline{u}))^c}, (\delta_{IN}(\underline{u}))^c e^{i(\eta_{IN}(\underline{u}))^c}, (\delta_{FN}(\underline{u}))^c e^{i(\eta_{FN}(\underline{u}))^c}\right\rangle\right) : \underline{u} \in N\right\}$$

$$(\zeta, N)^c(\underline{u}) = \left\{\left(\underline{u}, \left\langle (\delta_{FN}(\underline{u}))e^{i(2\pi - \eta_{TN}(\underline{u}))}, (1 - \delta_{IN}(\underline{u}))e^{i(2\pi - \eta_{IN}(\underline{u}))}, (\delta_{TN}(\underline{u}))e^{i(2\pi - \delta_{FN}(\underline{u}))}\right\rangle\right) : \underline{u} \in N\right\}$$

$$((\zeta, N)^c)^c = \left\{\left(\underline{u}, \left\langle (\delta_{FN}(\underline{u}))^c e^{i(2\pi - \eta_{TN}(\underline{u}))^c}, (1 - \delta_{IN}(\underline{u}))^c e^{i(2\pi - \eta_{IN}(\underline{u}))^c}, (\delta_{TN}(\underline{u}))^c e^{i(2\pi - \eta_{TN}(\underline{u}))^c}\right\rangle\right) : \underline{u} \in N\right\}$$

$$((\zeta, N)^c)^c = \left\{\left(\underline{u}, \left\langle \begin{array}{l} \delta_{TN}(\underline{u})e^{i(2\pi - (2\pi - \eta_{TN}(\underline{u}))}, \\ (1 - (1 - \delta_{IN}(\underline{u})))e^{i(2\pi - (2\pi - \eta_{IN}(\underline{u}))}, \\ \delta_{FN}(\underline{u})e^{i(2\pi - (2\pi - \eta_{FN}(\underline{u}))} \end{array} \right\rangle\right) : \underline{u} \in N\right\}$$

$$((\zeta, N)^c)^c = \left\{\left(\underline{u}, \left\langle \delta_{TN}(\underline{u})e^{i\eta_{TN}(\underline{u})}, \delta_{IN}(\underline{u})e^{i\eta_{IN}(\underline{u})}, \delta_{FN}(\underline{u}))e^{i\eta_{FN}(\underline{u})}\right\rangle\right) : \underline{u} \in N\right\} \quad (6)$$

from equations (5) and (6), we have $((\zeta, N)^c)^c = (\zeta, N)$. □

Proposition 5.9. *Let* (ζ, N) *be a CNH-set over* $\mathbb{U}$*.Then*

i. $((\zeta, N)_\Phi)^c = (\zeta, N)_\Delta$

ii. $((\zeta, N)_{\Delta})^c = (\zeta, N)_{\Phi}$

Definition 5.10. The *intersection* of two CNH-set (ζ_1, N_1) and (ζ_2, N_2) over the same universe $\mathbb{U}$, denoted by $(\zeta_1, N_1) \prod (\zeta_2, N_2)$, is the CNH-set (ζ_3, N_3), where $N_3 = N_1 \coprod N_2$, and for all $\underline{u} \in N_3$,

$$\zeta^T{}_3(\underline{u}) = \begin{cases} \delta_{TN_1}(\underline{u})e^{i\eta_{TN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{TN_2}(\underline{u})e^{i\eta_{TN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{TN_1}(\underline{u}) \otimes \delta_{TN_2}(\underline{u})] \,.\, e^{i\,[\eta_{TN_1}(\underline{u}) \otimes \eta_{TN_2}(\underline{u})]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

$$\zeta^I{}_3(\underline{u}) = \begin{cases} \delta_{IN_1}(\underline{u})e^{i\eta_{IN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{IN_2}(\underline{u})e^{i\eta_{IN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{IN_1}(\underline{u}) \otimes \delta_{IN_2}(\underline{u})] \,.\, e^{i\,[\eta_{IN_1}(\underline{u}) \otimes \eta_{IN_2}(\underline{u})]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

and

$$\zeta^F{}_3(\underline{u}) = \begin{cases} \delta_{FN_1}(\underline{u})e^{i\eta_{FN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{FN_2}(\underline{u})e^{i\eta_{FN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{FN_1}(\underline{u}) \otimes \delta_{FN_2}(\underline{u})] \,.\, e^{i\,[\eta_{FN_1}(\underline{u}) \otimes \eta_{FN_2}(\underline{u})]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

where $\otimes$ denotes minimum operator.

Definition 5.11. The *difference* between two CNH-set (ζ_1, N_1) and (ζ_2, N_2) is defined as

$$(\zeta_1, N_1) \setminus (\zeta_2, N_2) = (\zeta_1, N_1) \prod (\zeta_2, N_2)^c$$

Definition 5.12. The *union* of two CNH-set (ζ_1, N_1) and (ζ_2, N_2) over the same universe $\mathbb{U}$, denoted by $(\zeta_1, N_1) \coprod (\zeta_2, N_2)$, is the CNH-set (ζ_3, N_3), where $N_3 = N_1 \coprod N_2$, and for all $\underline{u} \in N_3$,

$$\zeta^T{}_3(\underline{u}) = \begin{cases} \delta_{TN_1}(\underline{u})e^{i\,\eta_{TN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{TN_2}(\underline{u})e^{i\,\eta_{TN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{TN_1}(\underline{u}) \oplus \delta_{TN_2}(\underline{u})] \,.\, e^{i\,[\eta_{TN_1}(\underline{u}) \oplus \eta_{TN_2}(\underline{u})]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

$$\zeta^I{}_3(\underline{u}) = \begin{cases} \delta_{IN_1}(\underline{u})e^{i\,\eta_{IN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{IN_2}(\underline{u})e^{i\,\eta_{IN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{IN_1}(\underline{u}) \oplus \delta_{IN_2}(\underline{u})] \,.\, e^{i\,[\eta_{IN_1}(\underline{u}) \oplus \eta_{IN_2}(\underline{u})]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

and

$$\zeta^F{}_3(\underline{u}) = \begin{cases} \delta_{FN_1}(\underline{u})e^{i\,\eta_{FN_1}(\underline{u})} & , if\ \underline{u} \in N_1 \backslash N_2 \\ \delta_{FN_2}(\underline{u})e^{i\,\eta_{FN_2}(\underline{u})} & , if\ \underline{u} \in N_2 \backslash N_1 \\ [\delta_{FN_1}(\underline{u}) \oplus \delta_{FN_2}(\underline{u})] \,.\, e^{i\,[\eta_{FN_1}(\underline{u}) \oplus \eta_{FN_2}(\underline{u})]} & , if\ \underline{u} \in N_1 \prod N_2 \end{cases}$$

where $\oplus$ denotes maximum operator.

Proposition 5.13. *Let (ζ, N) be a CNH-set over $\mathbb{U}$. Then the following results hold true:*

i. $(\zeta, N) \coprod (\zeta, N)_{\Phi} = (\zeta, N)$
ii. $(\zeta, N) \coprod (\zeta, N)_{\Delta} = (\zeta, N)_{\Delta}$

iii. $(\zeta, N) \prod (\zeta, N)_\Phi = (\zeta, N)_\Phi$
iv. $(\zeta, N) \prod (\zeta, N)_\Delta = (\zeta, N)$
v. $(\zeta, N)_\Phi \coprod (\zeta, N)_\Delta = (\zeta, N)_\Delta$
vi. $(\zeta, N)_\Phi \prod (\zeta, N)_\Delta = (\zeta, N)_\Phi$

Proposition 5.14. *Let (ζ_1, N_1), (ζ_2, N_2) and (ζ_3, N_3) are three CNH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\zeta_1, N_1) \prod (\zeta_2, N_2) = (\zeta_2, N_2) \prod (\zeta_1, N_1)$
ii. $(\zeta_1, N_1) \coprod (\zeta_2, N_2) = (\zeta_2, N_2) \coprod (\zeta_1, N_1)$
iii. $(\zeta_1, N_1) \prod ((\zeta_2, N_2) \prod (\zeta_3, N_3)) = ((\zeta_1, N_1) \prod (\zeta_2, N_2)) \prod (\zeta_3, N_3)$
iv. $(\zeta_1, N_1) \coprod ((\zeta_2, N_2) \coprod (\zeta_3, N_3)) = ((\zeta_1, N_1) \coprod (\zeta_2, N_2)) \coprod (\zeta_3, N_3)$

Proposition 5.15. *Let (ζ_1, N_1) and (ζ_2, N_2) are two CNH-sets over the same universe $\mathbb{U}$. Then the following De Morganss laws hold true:*

i. $((\zeta_1, N_1) \prod (\zeta_2, N_2))^c = (\zeta_1, N_1)^c \coprod (\zeta_2, N_2)^c$
ii. $((\zeta_1, N_1) \coprod (\zeta_2, N_2))^c = (\zeta_1, N_1)^c \prod (\zeta_2, N_2)^c$

Discussion on particular cases of CFH-sets, CIFH-sets and CNH-sets

- If $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}), \zeta^I(\underline{\lambda}), \zeta^F(\underline{\lambda}) \right\rangle, {}^-0 \leq \delta_T + \delta_I + \delta_F \leq 3^+ (or\ 0 \leq |\delta_T + \delta_I + \delta_F| \leq 3)$ is replaced by $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}), \zeta^F(\underline{\lambda}) \right\rangle, 0 \leq \delta_T + \delta_F \leq 1 (or\ 0 \leq |\delta_T + \delta_F| \leq 1)$ with omission of indeterminacy, then complex neutrosophic hypersoft set reduces to complex intuitionistic fuzzy hypersoft set.

- If $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}), \zeta^I(\underline{\lambda}), \zeta^F(\underline{\lambda}) \right\rangle$ is replaced by $\zeta(\underline{\lambda}) = \left\langle \zeta^T(\underline{\lambda}) \right\rangle$ with omission of indeterminacy and falsity, then complex neutrosophic hypersoft set reduces to complex fuzzy hypersoft set.

This concludes that complex fuzzy hypersoft set and complex intuitionistic fuzzy hypersoft set are the particular cases of complex neutrosophic hypersoft set. Since Complex fuzzy hypersoft sets and complex intuitionistic fuzzy hypersoft sets cannot handle imprecise, indeterminate, inconsistent, and incomplete information of periodic nature so to overcome this hurdle, complex neutrosophic hypersoft set is conceptualized.

Conclusion

In this work, new hybrids of hypersoft set i.e. complex fuzzy hypersoft set, complex intuitionistic fuzzy hypersoft set and complex neutrosophic hypersoft set, are conceptualized with their some fundamentals and theoretic operations. Future study may include other hybrids of hypersoft set with interval-valued complex fuzzy set etc., similarity and distance measures, aggregations operators and applications in multi-criteria decision making problems.

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