Article

Dombi Aggregation Operators of Neutrosophic Cubic Sets for Multiple Attribute Decision-Making

Lilian Shi * and **Jun Ye**

Department of Electrical and Information Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing 312000, China; yejun@usx.edu.cn
* Correspondence: cssll@usx.edu.cn; Tel.: +86-575-8834-2802

Abstract: The neutrosophic cubic set can describe complex decision-making problems with its single-valued neutrosophic numbers and interval neutrosophic numbers simultaneously. The Dombi operations have the advantage of good flexibility with the operational parameter. In order to solve decision-making problems with flexible operational parameter under neutrosophic cubic environments, the paper extends the Dombi operations to neutrosophic cubic sets and proposes a neutrosophic cubic Dombi weighted arithmetic average (NCDWAA) operator and a neutrosophic cubic Dombi weighted geometric average (NCDWGA) operator. Then, we propose a multiple attribute decision-making (MADM) method based on the NCDWAA and NCDWGA operators. Finally, we provide two illustrative examples of MADM to demonstrate the application and effectiveness of the established method.

Keywords: neutrosophic cubic sets; neutrosophic cubic Dombi weighted arithmetic average (NCDWAA) operator; Dombi operations; multiple attribute decision-making; neutrosophic cubic Dombi weighted geometric average (NCDWGA) operator

1. Introduction

Fuzzy sets were presented by Zadeh [1] to describe fuzzy problems with the membership function. After Zadeh, some extensions of fuzzy sets have been proposed, including interval-valued fuzzy sets [2], intuitionistic fuzzy sets [3] and interval-valued intuitionistic fuzzy sets [4]. Interval-valued fuzzy sets can be described by the membership degree in an interval value of [0, 1]. Intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets can deal with different types of uncertainties by the non-membership function and membership function. Neutrosophic sets [5] were defined by Smarandache to express fuzzy problems using the truth, indeterminacy and falsity membership functions. Based on the neutrosophic sets, some simplified forms of neutrosophic sets were introduced for engineering applications, including interval neutrosophic sets [6], single valued neutrosophic sets [7] and simplified neutrosophic sets [8] and so on. The simplified forms of neutrosophic sets have been widely applied in multiple attribute decision-making (MADM) problems [9–13] and fault diagnosis [14]. Some extension forms of neutrosophic sets have been proposed by combining neutrosophic sets and other sets, for instance, multi-valued neutrosophic sets [15,16], intuitionistic neutrosophic soft set [17], rough neutrosophic sets [18], single-valued neutrosophic hesitant fuzzy [19], refined single-valued neutrosophic sets [20], neutrosophic soft sets [21], linguistic neutrosophic number [22,23], normal neutrosophic sets [24] and single-valued neutrosophic hesitant fuzzy set [25].

In the real world, the membership function in some fuzzy problems cannot be described completely only by an exact value or an interval-value. Hence, Jun et al. defined cubic sets by the combination of interval-valued fuzzy sets with fuzzy sets [26]. Cubic sets can describe vagueness and uncertainty using an exact value and an interval-value simultaneously. Recently, Ali et al. extended

cubic sets to the neutrosophic sets and introduced the definition of neutrosophic cubic sets (NCSs), containing external NCSs and internal NCSs [27]. Jun et al. discussed the P-union and P-intersection of NCSs [28]. Furthermore, several related studies have been conducted to solve decision-making problems based on NCSs. Zhan et al. presented the concepts of weighted geometric operator (G_W) and the weighted average operator (A_W) on NCSs to solve multi-criteria decision-making problem [29]. Banerjee et al. introduced the grey relational analysis method of NCSs for MADM [30]. Some similarity measures of NCSs were introduced for decision-making problems under neutrosophic cubic set environment [31,32]. Pramanik et al. proposed the NC-TODIM method for solving a multiple attribute group decision-making problem [33].

Aggregation operators play an important role in decision making. Hence, many researchers have presented various aggregation operators and their extensions [34–50], such as Harmonic aggregation operators [34,35], weighted Bonferroni mean operators [36–39], Einstein prioritized weighted operators [40], generalized weighted aggregation operators [41], Choquet integral operators [42] and so on. Dombi first developed the Dombi T-norm and T-conorm operations [51]. Recently, Liu et al. presented Dombi Bonferroni mean operators of intuitionistic fuzzy sets and applied them in MADM problems [52]. Chen and Ye also extended the Dombi operations to single valued neutrosophic sets and proposed some single-valued neutrosophic Dombi weighted aggregation operators and applied them in MADM problems [53]. From the above review, the Dombi operations have the advantage of good flexibility with the operational parameter [53] and NCSs contain much more incomplete, inconsistent and indeterminate information to express actual decision-making problems [31]. Hence, the paper extends the Dombi operations to NCSs and proposes a neutrosophic cubic Dombi weighted arithmetic average (NCDWAA) operator and a neutrosophic cubic Dombi weighted geometric average (NCDWGA) operator. In order to solve MADM problems with neutrosophic cubic information, a MADM method based on the NCDWAA and NCDWGA operators is proposed in this paper.

The remainder of the paper is arranged as follows. Section 2 briefly introduces some concepts of NCSs to be used for the following study. Some Dombi operations of NCSs are introduced in Section 3. Section 4 presents the NCDWAA and NCDWGA operators and discusses their properties. In Section 5, we establish a MADM method based on the NCDWAA and NCDWGA operators. Section 6 presents two illustrative examples to demonstrate the effectiveness and feasibility of the proposed method. Conclusions and future research are given in Section 7.

2. Preliminaries

In this section, we firstly present some concepts of interval neutrosophic sets, single-valued neutrosophic sets, cubic sets and NCSs and then introduce some ranking methods of NCSs based on their score, accuracy and certainty functions.

Definition 1 ([6]). *Let Z be a non-empty set. An interval neutrosophic sets G in Z is defined as follows:*

$$G = \{v, <T(v), I(v), F(v)> \mid v \in Z\}, \tag{1}$$

where the intervals $T(v) = [T^-(v), T^+(v)] \subseteq [0, 1]$, $I(v) = [I^-(v), I^+(v)] \subseteq [0, 1]$, *and* $F(v) = [F^-(v), F^+(v)] \subseteq [0, 1]$ *for* $v \in Z$ *represent respectively the degrees of the truth-membership, indeterminacy-membership and falsity-membership.*

Definition 2 ([7]). *Let Z be a universe of discourse. A single-valued neutrosophic sets H in Z is described as follows:*

$$H = \{v, <t(v), i(v), f(v)> \mid v \in Z\}, \tag{2}$$

where the functions $t(v), i(v), f(v) \in [0, 1]$ *with the condition* $0 \leq t(v) + i(v) + f(v) \leq 3$ *for* $v \in Z$, *represent respectively the degrees of the truth-membership, the indeterminacy-membership and falsity-membership.*

Definition 3 ([26]). *Let Z be a non-empty set, then a cubic set C in Z is constructed as the following form:*

$$C = \{v, A(v), \breve{a}(v) \mid v \in Z\}, \tag{3}$$

for $v \in Z$. It can be noted by $C = \{A, \breve{a}\}$. Then, $C = \{A, \breve{a}\}$ in Z is called an internal cubic set if $A^-(v) \le a(v) \le A^+(v)$ for $v \in Z$ and $C = \{A, \breve{a}\}$ in Z is called an external cubic set if $a(v) \notin [A^-(v), A^+(v)]$ for $v \in Z$.

Ali et al. [27] and Jun et al. [28] extended cubic sets to the neutrosophic sets and proposed the concept of a NCS as follows.

Definition 4 ([27,28]). *Let Z be a universe of discourse, then a neutrosophic cubic set X in Z is denoted as the following form:*

$$X = \{v, <T(v), I(v), F(v)>, <t(v), i(v), f(v)> \mid v \in Z\}, \tag{4}$$

where $<T(v), I(v), F(v)>$ is an interval neutrosophic set [6] in Z and the intervals $T(v) = [T^-(v), T^+(v)] \subseteq [0, 1]$, $I(v) = [I^-(v), I^+(v)] \subseteq [0, 1]$, and $F(v) = [F^-(v), F^+(v)] \subseteq [0, 1]$ for $v \in Z$ represent the truth, indeterminacy and falsity membership degrees, respectively; then $<t(v), i(v), f(v)>$ is a single valued neutrosophic set [5,7] in Z and $t(v), i(v), f(v) \in [0, 1]$ for $v \in Z$ represent the membership degrees of truth, indeterminacy and falsity, respectively.

Then, a neutrosophic cubic sets $X = \{v, <T(v), I(v), F(v)>, <t(v), i(v), f(v)> \mid v \in Z\}$ is called an internal NCS if $T^-(v) \le t(v) \le T^+(v)$, $I^-(v) \le i(v) \le I^+(v)$, and $F^-(v) \le f(v) \le F^+(v)$, for $v \in Z$; and a NCS X is called an external NCS if $t(v) \notin (T^-(v), T^+(v))$, $i(v) \notin (I^-(v), I^+(v))$ and $f(v) \notin (F^-(v), F^+(v))$ for $v \in Z$ [27].

For convenient expression, a basic element $(v, <T(v), I(v), F(v)>, <t(v), i(v), f(v)>)$ in a NCS X is denoted by $x = (< [T^-, T^+], [I^-, I^+], [F^-, F^+] >, < t, i, f >)$, which is called a neutrosophic cubic number (NCN) [31], where $[T^-, T^+], [I^-, I^+], [F^-, F^+] \subseteq [0, 1]$ and $t, i, f \in [0, 1]$ satisfy the condition $0 \le T^+ + I^+ + F^+ \le 3$ and $0 \le t + i + f \le 3$.

For any neutrosophic cubic number, we provide the following score, accuracy and certainty functions.

Definition 5 ([54]). *Let $x = (< [T^-, T^+], [I^-, I^+], [F^-, F^+] >, < t, i, f >)$ be a neutrosophic cubic number. Then, its score, accuracy and certainty functions are defined as follows:*

$$S(x) = [(4 + T^- + T^+ - I^- - I^+ - F^- - F^+) + (2 + t - i - f)]/9 \tag{5}$$

$$A(x) = [(T^- + T^+ - F^- - F^+) + (t - f)]/3 \tag{6}$$

$$C(x) = (T^- + T^+ + t)/3 \tag{7}$$

where, S(x), A(x) and C(x) represent the score, accuracy and certainty functions of the NCNs, respectively.

The score function $S(x)$ is a useful index in ranking NCNs. For a NCN, the bigger the truth-membership is, the greater the NCN is. At the same time, the smaller the memberships of indeterminacy and falsity are, the greater the NCN is. As to the accuracy function $A(x)$, the larger the difference between truth-membership and falsity-membership is, the more effective the statement is. For the certainty function $C(x)$, if the truth membership is bigger, then the NCN is more certainty. Hence, the score, accuracy and certainty functions are defined as shown above.

According to the three functions $S(x)$, $A(x)$ and $C(x)$, the comparison and ranking of two NCNs are defined as following definition.

Definition 6 ([54]). *Let $x_1 = (< [T_1^-, T_1^+], [I_1^-, I_1^+], [F_1^-, F_1^+] >, < t_1, i_1, f_1 >)$ and $x_2 = (< [T_2^-, T_2^+], [I_2^-, I_2^+], [F_2^-, F_2^+] >, < t_2, i_2, f_2 >)$ be two neutrosophic cubic numbers. Then their ranking method is defined as follows:*

(1)　If $S(x_1) > S(x_2)$, then $x_1 \succ x_2$;

(2)　If $S(x_1) = S(x_2)$ and $A(x_1) > A(x_2)$, then $x_1 \succ x_2$;

(3)　If $S(x_1) = S(x_2)$, $A(x_1) = A(x_2)$ and $C(x_1) > C(x_2)$, then $x_1 \succ x_2$;

(4)　If $S(x_1) = S(x_2)$, $A(x_1) = A(x_2)$ and $C(x_1) = C(x_2)$, then $x_1 \sim x_2$.

Example 1. *Let Ψ_1 and Ψ_2 be two NCNs.*

(1)　*Assume that Ψ_1 = (<[0.8, 0.9], [0.1, 0.2], [0.2, 0.3]>, <0.7, 0.1, 0.2>) and Ψ_2 = (<[0.5, 0.6], [0.3, 0.4], [0.4, 0.5]>, <0.5, 0.3, 0.4>). Referring to Definition 5, $S(\Psi_1)$ = 0.8111, $S(\Psi_2)$ = 0.5889, $A(\Psi_1)$ = 0.5667, $A(\Psi_2)$ = 0.1000, $C(\Psi_1)$ = 0.8000, $C(\Psi_2)$ = 0.5333. According to Definition 6, $S(\Psi_1) > S(\Psi_2)$, therefore, $\Psi_1 \succ \Psi_2$.*

(2)　*Assume that Ψ_1 = (<[0.5, 0.6], [0.2, 0.3], [0.3, 0.4]>, <0.5, 0.2, 0.3>) and Ψ_2 = (<[0.3, 0.4], [0.1, 0.2], [0.2, 0.3]>, <0.3, 0.1, 0.2>). Referring to Definition 5, $S(\Psi_1)$ = 0.6556, $S(\Psi_2)$ = 0.6556, $A(\Psi_1)$ = 0.2000, $A(\Psi_2)$ = 0.1000, $C(\Psi_1)$ = 0.5333, $C(\Psi_2)$ = 0.3333. According to Definition 6, $S(\Psi_1) = S(\Psi_2)$, $A(\Psi_1) > A(\Psi_2)$, therefore, $\Psi_1 \succ \Psi_2$.*

(3)　*Assume that Ψ_1 = (<[0.5, 0.6], [0.2, 0.3], [0.3, 0.4]>, <0.5, 0.2, 0.3>) and Ψ_2 = (<[0.3, 0.4], [0.2, 0.3], [0.1, 0.2]>, <0.3, 0.2, 0.1>). Referring to Definition 5, $S(\Psi_1)$ = 0.6556, $S(\Psi_2)$ = 0.6556, $A(\Psi_1)$ = 0.2000, $A(\Psi_2)$ = 0.2000, $C(\Psi_1)$ = 0.5333, $C(\Psi2)$ = 0.3333. According to Definition 6, $S(\Psi_1) = S(\Psi_2)$, $A(\Psi_1) = A(\Psi_2)$, $C(\Psi_1) > C(\Psi_2)$ therefore, $\Psi_1 \succ \Psi_2$.*

3. Some Dombi Operations of NCNs

Definition 7 ([51]). *Let g and h be two real numbers, then the Dombi T-norm and T-conorm between g and h are defined as follows*:

$$D(g,h) = \frac{1}{1+\{\,(\frac{1-g}{g})^{\rho} + (\frac{1-h}{h})^{\rho}\,\}^{1/\rho}}, \tag{8}$$

$$D^c(g,h) = 1 - \frac{1}{1+\{\,(\frac{g}{1-g})^{\rho} + (\frac{h}{1-h})^{\rho}\,\}^{1/\rho}}, \tag{9}$$

where $(g, h) \in (0, 1) \times (0, 1)$ and if $\rho > 0$ then the operator $D(g, h)$ is conjunctive and $D^c(g, h)$ is disjunctive, satisfying $D(0, 0) = D(0, 1) = D(1, 0) = 0$, $D(1, 1) = 1$, $D^c(0, 1) = D^c(1, 0) = D^c(1, 1) = 1$ and $D^c(0, 0) = 0$ [51] and if $\rho < 0$ then the operator $D(g, h)$ is disjunctive and the operator $D^c(g, h)$ is conjunctive.

According to Equations (8) and (9), some Dombi operations of NCNs are provided as following definition.

Definition 8. *Let $X = \{x_1, x_2, \dots, x_n\}$ be a NCS, where $x_j = (< [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] >, < t_j, i_j, f_j >)$ for $j = 1, 2, \dots, n$ is a collection of NCNs and $T_j^-, T_j^+, I_j^-, I_j^+, F_j^-, F_j^+, t_j, i_j, f_j \in (0, 1)$ and $\lambda > 0$ and $\rho > 0$. Then, the Dombi T-norm and T-conorm operations of NCNs are defines as follows:*

$$
(i)\; x_1 \oplus x_2 = \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \frac{1}{1+\{(\frac{T_1^-}{1-T_1^-})^{\rho}+(\frac{T_2^-}{1-T_2^-})^{\rho}\}^{1/\rho}}, 1 - \frac{1}{1+\{(\frac{T_1^+}{1-T_1^+})^{\rho}+(\frac{T_2^+}{1-T_2^+})^{\rho}\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\{(\frac{1-I_1^-}{I_1^-})^{\rho}+(\frac{1-I_2^-}{I_2^-})^{\rho}\}^{1/\rho}}, \\ \frac{1}{1+\{(\frac{1-I_1^+}{I_1^+})^{\rho}+(\frac{1-I_2^+}{I_2^+})^{\rho}\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\{(\frac{1-F_1^-}{F_1^-})^{\rho}+(\frac{1-F_2^-}{F_2^-})^{\rho}\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-F_1^+}{F_1^+})^{\rho}+(\frac{1-F_2^+}{F_2^+})^{\rho}\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{(\frac{t_1}{1-t_1})^{\rho}+(\frac{t_2}{1-t_2})^{\rho}\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-i_1}{i_1})^{\rho}+(\frac{1-i_2}{i_2})^{\rho}\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-f_1}{f_1})^{\rho}+(\frac{1-f_2}{f_2})^{\rho}\}^{1/\rho}} \right\rangle \end{array} \right); \tag{10}
$$

$$(ii) \quad x_1 \otimes x_2 = \left(\begin{array}{c} \left\langle \left[\frac{1}{1+\{ (\frac{1-T_1^-}{T_1^-})^\rho + (\frac{1-T_2^-}{T_2^-})^\rho \}^{1/\rho}}, \frac{1}{1+\{ (\frac{1-T_1^+}{T_1^+})^\rho + (\frac{1-T_2^+}{T_2^+})^\rho \}^{1/\rho}} \right], \left[1 - \frac{1}{1+\{ (\frac{I_1^-}{1-I_1^-})^\rho + (\frac{I_2^-}{1-I_2^-})^\rho \}^{1/\rho}}, \right. \right. \\ \left. 1 - \frac{1}{1+\{ (\frac{I_1^+}{1-I_1^+})^\rho + (\frac{I_2^+}{1-I_2^+})^\rho \}^{1/\rho}} \right], \left[1 - \frac{1}{1+\{ (\frac{F_1^-}{1-F_1^-})^\rho + (\frac{F_2^-}{1-F_2^-})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ (\frac{F_1^+}{1-F_1^+})^\rho + (\frac{F_2^+}{1-F_2^+})^\rho \}^{1/\rho}} \right] \left. \right\rangle, \\ \left\langle \frac{1}{1+\{ (\frac{1-t_1}{t_1})^\rho + (\frac{1-t_2}{t_2})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ (\frac{i_1}{1-i_1})^\rho + (\frac{i_2}{1-i_2})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ (\frac{f_1}{1-f_1})^\rho + (\frac{f_2}{1-f_2})^\rho \}^{1/\rho}} \right\rangle \end{array} \right); \quad (11)$$

$$(iii) \quad \lambda x_1 = \left(\begin{array}{c} \left\langle \left[1 - \frac{1}{1+\{ \lambda(\frac{T_1^-}{1-T_1^-})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ \lambda(\frac{T_1^+}{1-T_1^+})^\rho \}^{1/\rho}} \right], \left[\frac{1}{1+\{ \lambda(\frac{1-I_1^-}{I_1^-})^\rho \}^{1/\rho}}, \right. \right. \\ \left. \frac{1}{1+\{ \lambda(\frac{1-I_1^+}{I_1^+})^\rho \}^{1/\rho}} \right], \left[\frac{1}{1+\{ \lambda(\frac{1-F_1^-}{F_1^-})^\rho \}^{1/\rho}}, \frac{1}{1+\{ \lambda(\frac{1-F_1^+}{F_1^+})^\rho \}^{1/\rho}} \right] \left. \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{ \lambda(\frac{t_1}{1-t_1})^\rho \}^{1/\rho}}, \frac{1}{1+\{ \lambda(\frac{1-i_1}{i_1})^\rho \}^{1/\rho}}, \frac{1}{1+\{ \lambda(\frac{1-f_1}{f_1})^\rho \}^{1/\rho}} \right\rangle \end{array} \right); \quad (12)$$

$$(iv) \quad x_1^\lambda = \left(\begin{array}{c} \left\langle \left[\frac{1}{1+\{ \lambda(\frac{1-T_1^-}{T_1^-})^\rho \}^{1/\rho}}, \frac{1}{1+\{ \lambda(\frac{1-T_1^+}{T_1^+})^\rho \}^{1/\rho}} \right], \left[1 - \frac{1}{1+\{ \lambda(\frac{I_1^-}{1-I_1^-})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ \lambda(\frac{I_1^+}{1-I_1^+})^\rho \}^{1/\rho}} \right], \right. \\ \left[1 - \frac{1}{1+\{ \lambda(\frac{F_1^-}{1-F_1^-})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ \lambda(\frac{F_1^+}{1-F_1^+})^\rho \}^{1/\rho}} \right] \left. \right\rangle, \\ \left\langle \frac{1}{1+\{ \lambda(\frac{1-t_1}{t_1})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ \lambda(\frac{i_1}{1-i_1})^\rho \}^{1/\rho}}, 1 - \frac{1}{1+\{ \lambda(\frac{f_1}{1-f_1})^\rho \}^{1/\rho}} \right\rangle \end{array} \right). \quad (13)$$

If some of the memberships $(T_j^-, T_j^+, I_j^-, I_j^+, F_j^-, F_j^+, t_j, i_j, f_j)$ are 0 or 1, then the above Dombi operations of NCNs are calculated by conjunction and disjunction according to Definition 7.

Example 2. *Let Ψ_3 and Ψ_4 be two NCNs. Assume that $\Psi_3 = (<[0, 0.5], [0, 0.2], [0, 0.4]>, <0.5, 0, 0>)$, $\Psi_4 = (<[0.5, 1], [0.7, 1], [0, 0]>, <1, 1, 0>)$ and $\rho = 1$. According to Definitions 7 and 8, $\Psi_3 \oplus \Psi_4$ is calculated as follows:*

$$\Psi_3 \oplus \Psi_4 = \left(\left\langle \left[1 - \frac{1}{1+\{0 + (\frac{0.5}{1-0.5})\}}, 1 \right], \left[0, \frac{1}{1+\{ (\frac{1-0.2}{0.2}) + 0\}} \right], [0, 0] \right\rangle, <1, 0, 0> \right)$$

$$= (<[0.5, 1], [0, 0.2], [0, 0]>, <1, 0, 0>).$$

4. Dombi Weighted Aggregation Operators of NCSs

In this section, two Dombi weighted aggregation operators of NCNs are proposed based on the Dombi operators of NCNs in Definition 8 and then their properties are investigated.

Definition 9. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a neutrosophic cubic set, where $x_j = (< [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] >, < t_j, i_j, f_j >)$ for $j = 1, 2, \ldots, n$ is a collection of neutrosophic cubic numbers and their corresponding weight vector is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, satisfying $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$. Then, the neutrosophic cubic Dombi weighted arithmetic average and neutrosophic cubic Dombi weighted geometric average operators are defined, respectively, as follows:*

$$NCDWAA(x_1, x_2, \ldots, x_n) = \overset{n}{\underset{j=1}{\oplus}} \omega_j x_j, \quad (14)$$

$$NCDWGA(x_1, x_2, \ldots, x_n) = \overset{n}{\underset{j=1}{\otimes}} x_j^{\omega_j}. \tag{15}$$

Theorem 1. *Let* $x_j = (< [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] >, < t_j, i_j, f_j >)$ *(j = 1, 2, . . . , n) is a collection of NCNs and their corresponding weight vector is* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, *satisfying* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \omega_j = 1$. *Then, the aggregated value of the NCDWAA operator is still a NCN, which can be calculated as follows:*

$$NCDWAA(x_1, x_2, \ldots, x_n) = \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \\ \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{n} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right). \tag{16}$$

We can prove Theorem 1 by the mathematical induction.

Proof. If *n* = 2, according to the Dombi operations of NCNs in Definition 8, we can get the following result:

$$NCDWAA(x_1, x_2) = \omega_1 x_1 \oplus \omega_2 x_2$$

$$= \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \dfrac{1}{1+\{\omega_1(\frac{T_1^-}{1-T_1^-})^\rho + \omega_2(\frac{T_2^-}{1-T_2^-})^\rho\}^{1/\rho}}, 1 - \dfrac{1}{1+\{\omega_1(\frac{T_1^+}{1-T_1^+})^\rho + \omega_2(\frac{T_2^+}{1-T_2^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\omega_1(\frac{1-I_1^-}{I_1^-})^\rho + \omega_2(\frac{1-I_2^-}{I_2^-})^\rho\}^{1/\rho}}, \\ \dfrac{1}{1+\{\omega_1(\frac{1-I_1^+}{I_1^+})^\rho + \omega_2(\frac{1-I_2^+}{I_2^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\omega_1(\frac{1-F_1^-}{F_1^-})^\rho + \omega_2(\frac{1-F_2^-}{F_2^-})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\omega_1(\frac{1-F_1^+}{F_1^+})^\rho + \omega_2(\frac{1-F_2^+}{F_2^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \dfrac{1}{1+\{\omega_1(\frac{t_1}{1-t_1})^\rho + \omega_2(\frac{t_2}{1-t_2})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\omega_1(\frac{1-i_1}{i_1})^\rho + \omega_2(\frac{1-i_2}{i_2})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\omega_1(\frac{1-f_1}{f_1})^\rho + \omega_2(\frac{1-f_2}{f_2})^\rho\}^{1/\rho}} \right\rangle \end{array} \right)$$

$$= \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \\ \begin{bmatrix} \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{2} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right). \tag{17}$$

If *n* = *k*, by Equation (16), we obtain the following formula:

$$NCDWAA(x_1, x_2, \ldots, x_k) = \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \\ \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum_{j=1}^{k} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right). \tag{18}$$

If $n = k + 1$, based on Equations (17) and (18), we have the following result:

$$NCDWAA(x_1, x_2, \ldots, x_k, x_{k+1}) = \left(\begin{array}{c} \left\langle \left[1 - \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \right. \right. \\ \left. \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \right] \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{k} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right) \oplus \omega_{k+1} x_{k+1}$$

$$ = \left(\begin{array}{c} \left\langle \left[1 - \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \right. \right. \\ \left. \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \right] \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{k+1} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right)$$

Thus, Equation (16) holds for all n. Hence, Theorem 1 is true. The proof is finished. $\square$

Then, the NCDWAA operator contains the following properties:

(i) Reducibility: If $\omega = (1/n, 1/n, \ldots, 1/n)$, then there exists

$$NCDWAA(x_1, x_2, \ldots, x_n) = \left(\begin{array}{c} \left\langle \left[1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \right. \right. \\ \left. \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \right] \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n} (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right). \quad (19)$$

(ii) Idempotency: Let $x_j = (< [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] >, < t_j, i_j, f_j >)$ $(j = 1, 2, ..., n)$ be a group of NCNs. When $x_j = x$ for $j = 1, 2, \ldots, n$, there is NCDWAA $(x_1, x_2, \ldots, x_n) = x$.

(iii) Commutativity: Suppose the NCSs $(x_1', x_2', \ldots, x_n')$ be any permutation of $(x_1, x_2, \ldots, x_n)$. Then, $NCDWAA (x_1', x_2', \ldots, x_n') = NCDWAA (x_1, x_2, \ldots, x_n)$.

(iv) Boundedness: Let $x_{\min} = (< [\min\limits_{j}(T_j^-), \min\limits_{j}(T_j^+)], [\max\limits_{j}(I_j^-), \max\limits_{j}(I_j^+)], [\max\limits_{j}(F_j^-), \max\limits_{j}(F_j^+)] >, < \min\limits_{j}(t_j), \max\limits_{j}(i_j), \max\limits_{j}(f_j) >)$ and $x_{\max} = (< [\max\limits_{j}(T_j^-), \max\limits_{j}(T_j^+)], [\min\limits_{j}(I_j^-), \min\limits_{j}(I_j^+)], [\min\limits_{j}(F_j^-), \min\limits_{j}(F_j^+)] >, < \max\limits_{j}(t_j), \min\limits_{j}(i_j), \min\limits_{j}(f_j) >)$. Then, $x_{\min} \leq$ NCDWAA $(x_1, x_2, \ldots, x_n) \leq x_{\max}$.

Proof. (i) The property is obvious by Equation (16).

(ii) Let $x_j = (< [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] >, < t_j, i_j, f_j >) = x$ $(j = 1, 2, ..., n)$. Then, based on Equation (16), we can get the result as follows:

$$
NCDWAA(x_1, x_2, \ldots, x_n) = \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{T_j^-}{1-T_j^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{T_j^+}{1-T_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-I_j^-}{I_j^-})^\rho\}^{1/\rho}}, \\ \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-I_j^+}{I_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-F_j^-}{F_j^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-F_j^+}{F_j^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right)
$$

$$
= \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \frac{1}{1+\{(\frac{T^-}{1-T^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{(\frac{T^+}{1-T^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\{(\frac{1-I^-}{I^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-I^+}{I^+})^\rho\}^{1/\rho}} \end{bmatrix}, \\ \begin{bmatrix} \frac{1}{1+\{(\frac{1-F^-}{F^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-F^+}{F_j^+})^\rho\}^{1/\rho}} \end{bmatrix} \end{array} \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{(\frac{t}{1-t})^\rho\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-i}{i})^\rho\}^{1/\rho}}, \frac{1}{1+\{(\frac{1-f}{f})^\rho\}^{1/\rho}} \right\rangle \end{array} \right)
$$

$$
= \left(\begin{array}{c} \left\langle \begin{bmatrix} 1 - \frac{1}{1+\frac{T^-}{1-T^-}}, 1 - \frac{1}{1+\frac{T^+}{1-T^+}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\frac{1-I^-}{I^-}}, \frac{1}{1+\frac{1-I^+}{I^+}} \end{bmatrix}, \begin{bmatrix} \frac{1}{1+\frac{1-F^-}{F^-}}, \frac{1}{1+\frac{1-F^+}{F^+}} \end{bmatrix} \right\rangle, \\ \left\langle 1 - \frac{1}{1+\frac{t}{1-t}}, \frac{1}{1+\frac{1-i}{i}}, \frac{1}{1+\frac{1-f}{f}} \right\rangle \end{array} \right)
$$

$$
= (< [T^-, T^+], [I^-, I^+], [F^-, F^+] >, < t, i, f >) = x
$$

Hence, NCDWAA $(x_1, x_2, \ldots, x_n) = x$ holds.

(iii) The property is obvious.

(iv) Since $\min\limits_{j}(T_j^-) \leq T_j^- \leq \max\limits_{j}(T_j^-), \min\limits_{j}(T_j^+) \leq T_j^+ \leq \max\limits_{j}(T_j^+), \min\limits_{j}(I_j^-) \leq I_j^- \leq \max\limits_{j}(I_j^-), \min\limits_{j}(F_j^-) \leq F_j^- \leq \max\limits_{j}(F_j^-), \min\limits_{j}(t_j) \leq t_j \leq \max\limits_{j}(f_j), \min\limits_{j}(i_j) \leq i_j \leq \max\limits_{j}(f_j),$ and $\min\limits_{j}(f_j) \leq f_j \leq \max\limits_{j}(f_j).$ Then we have the following inequalities:

$$
1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{\min\limits_{j}(t_j)}{1-\min\limits_{j}(t_j)})^\rho\}^{1/\rho}} \leq 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{t_j}{1-t_j})^\rho\}^{1/\rho}} \leq 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{\max\limits_{j}(t_j)}{1-\max\limits_{j}(t_j)})^\rho\}^{1/\rho}}
$$

$$
\frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-\min\limits_{j}(i_j)}{\min\limits_{j}(i_j)})^\rho\}^{1/\rho}} \leq \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-i_j}{i_j})^\rho\}^{1/\rho}} \leq \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-\max\limits_{j}(i_j)}{\max\limits_{j}(i_j)})^\rho\}^{1/\rho}}
$$

$$
\frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-\min\limits_{j}(f_j)}{\min\limits_{j}(f_j)})^\rho\}^{1/\rho}} \leq \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-f_j}{f_j})^\rho\}^{1/\rho}} \leq \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-\max\limits_{j}(f_j)}{\max\limits_{j}(f_j)})^\rho\}^{1/\rho}}
$$

We can obtain the similar inequalities for $T_j^-, T_j^+, I_j^-, I_j^+, F_j^-,$ and F_j^+. Hence, $x_{\min} \leq$ NCDWAA $(x_1, x_2, \ldots, x_n) \leq x_{\max}$ holds. □

Theorem 2. *Let* $x_j = (< [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] >, < t_j, i_j, f_j >)$ *(j = 1, 2, ..., n) be a group of NCNs. The weight vector of NCN* x_j *is* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$, *satisfying* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \omega_j = 1$. *Then, the aggregated value of the NCDWGA operator is still a NCN, which can be calculated as follows:*

$$
NCDWGA(x_1, x_2, \ldots, x_n) = \left(\begin{array}{c} \left\langle \begin{bmatrix} \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-T_j^-}{T_j^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-T_j^+}{T_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{I_j^-}{1-I_j^-})^\rho\}^{1/\rho}}, \\ 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{I_j^+}{1-I_j^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{F_j^-}{1-F_j^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{F_j^+}{1-F_j^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \\ \left\langle \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{1-t_j}{t_j})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{i_j}{1-i_j})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_j (\frac{f_j}{1-f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right). \quad (20)
$$

Theorem 2 can be proved by a similar proof process as Theorem 1. Hence, it is not repeated here. Obviously, the NCDWGA operator also satisfies the following properties:

(i) Reducibility: If $\omega = (1/n, 1/n, \dots, 1/n)$, then there exists

$$NCDWGA(x_1, x_2, \dots, x_n) = \left(\begin{array}{c} \left\langle \begin{array}{c} \left[\frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{1-T_j^-}{T_j^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{1-T_j^+}{T_j^+})^\rho\}^{1/\rho}} \right], \left[1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{I_j^-}{1-I_j^-})^\rho\}^{1/\rho}}, \right. \\ \left. 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{I_j^+}{1-I_j^+})^\rho\}^{1/\rho}} \right], \left[1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{F_j^-}{1-F_j^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{F_j^+}{1-F_j^+})^\rho\}^{1/\rho}} \right] \end{array} \right\rangle, \\ \left\langle \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{1-t_j}{t_j})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{i_j}{1-i_j})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \frac{1}{n}(\frac{f_j}{1-f_j})^\rho\}^{1/\rho}} \right\rangle \end{array} \right). \quad (21)$$

(ii) Idempotency: Let $x_j = x$ for $j = 1, 2, \dots, n$, there is NCDWGA $(x_1, x_2, \dots, x_n) = x$.

(iii) Commutativity: Suppose the NCSs $(x_1', x_2', \dots, x_n')$ be any permutation of $(x_1, x_2, \dots, x_n)$. Then, $NCDWGA(x_1', x_2', \dots, x_n') = NCDWGA(x_1, x_2, \dots, x_n)$.

(iv) Boundedness: Let $x_{\min} = (< [\min\limits_j(T_j^-), \min\limits_j(T_j^+)], [\max\limits_j(I_j^-), \max\limits_j(I_j^+)], [\max\limits_j(F_j^-), \max\limits_j(F_j^+)] >,$ $< \min\limits_j(t_j), \max\limits_j(i_j), \max\limits_j(f_j) >)$ and $x_{\max} = (< [\max\limits_j(T_j^-), \max\limits_j(T_j^+)], [\min\limits_j(I_j^-), \min\limits_j(I_j^+)],$ $[\min\limits_j(F_j^-), \min\limits_j(F_j^+)] >, < \max\limits_j(t_j), \min\limits_j(i_j), \min\limits_j(f_j) >)$. Then, $x_{\min} \leq$ NCDWGA $(x_1, x_2, \dots, x_n) \leq x_{\max}$.

We can prove these properties by the same way as that of Theorem 1. Thus, they are omitted here.

5. MADM Method Using the NCDWAA or NCDWGA Operators

In this section, a MADM method based on the NCDWAA operator or the NCDWGA operator is proposed to handle MADM problems with neutrosophic cubic information.

In a MADM problem with NCN information, let $X = \{X_1, X_2, \dots, X_m\}$ be a set of m alternatives and $Y = \{Y_1, Y_2, \dots, Y_n\}$ be a set of attributes. Suppose that $\omega_Y = (\omega_{Y1}, \omega_{Y2}, \dots, \omega_{Yn})$ is the weight vector of the attributes Y_j ($j = 1, 2, \dots, n$) with $\omega_{Yj} \in [0, 1]$ and $\sum_{j=1}^n \omega_{Yj} = 1$. The evaluation value of an alternative X_k ($k = 1, 2, \dots, m$) under an attribute Y_j ($j = 1, 2, \dots, n$) is expressed by a NCN $x_{kj} = (< [T_{kj}^-, T_{kj}^+], [I_{kj}^-, I_{kj}^+], [F_{kj}^-, F_{kj}^+] >, < t_{kj}, i_{kj}, f_{kj} >)$ ($k = 1, 2, \dots, m; j = 1, 2, \dots, n$), where $[T_{kj}^-, T_{kj}^+], [I_{kj}^-, I_{kj}^+], [F_{kj}^-, F_{kj}^+] \subseteq$, and $t_{kj}, i_{kj}, f_{kj} \in [0, 1].[0, 1]$ Then, we can construct a NCN decision matrix $M = (x_{kj})_{m \times n}$.

In this case, we present a MADM method based on the NCDWAA operator or the NCDWGA operator to handle MADM problems with NCN information and the decision steps can be described as following:

Step 1. Derive the collective NCN x_k ($k = 1, 2, \dots, m$) for the alternative X_k ($k = 1, 2, \dots, m$) by using the NCDWAA operator:

$$x_k = NCDWAA(x_{k1}, x_{k2}, \dots, x_{kn})$$
$$= \left(\begin{array}{c} \left\langle \begin{array}{c} \left[1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{T_{kj}^-}{1-T_{kj}^-})^\rho\}^{1/\rho}}, 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{T_{kj}^+}{1-T_{kj}^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-I_{kj}^-}{I_{kj}^-})^\rho\}^{1/\rho}}, \right. \\ \left. \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-I_{kj}^+}{I_{kj}^+})^\rho\}^{1/\rho}} \right], \left[\frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-F_{kj}^-}{F_{kj}^-})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-F_{kj}^+}{F_{kj}^+})^\rho\}^{1/\rho}} \right] \end{array} \right\rangle, \\ \left\langle 1 - \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{t_{kj}}{1-t_{kj}})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-i_{kj}}{i_{kj}})^\rho\}^{1/\rho}}, \frac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-f_{kj}}{f_{kj}})^\rho\}^{1/\rho}} \right\rangle \end{array} \right) \quad (22)$$

or by using the NCDWGA operator:

$$
\begin{aligned}
&x_k = NCDWGA(x_{k1}, x_{k2}, \ldots, x_{kn}) \\
&= \left(\left\langle \begin{bmatrix} \dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-T_{kj}^-}{T_{kj}^-})^\rho\}^{1/\rho}}, \dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-T_{kj}^+}{T_{kj}^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} 1-\dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{I_{kj}^-}{1-I_{kj}^-})^\rho\}^{1/\rho}}, \\ 1-\dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{I_{kj}^+}{1-I_{kj}^+})^\rho\}^{1/\rho}} \end{bmatrix}, \begin{bmatrix} 1-\dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{F_{kj}^-}{1-F_{kj}^-})^\rho\}^{1/\rho}}, 1-\dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{F_{kj}^+}{1-F_{kj}^+})^\rho\}^{1/\rho}} \end{bmatrix} \right\rangle, \right. \\
&\left. \left\langle \dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{1-t_{kj}}{t_{kj}})^\rho\}^{1/\rho}}, 1-\dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{i_{kj}}{1-i_{kj}})^\rho\}^{1/\rho}}, 1-\dfrac{1}{1+\{\sum\limits_{j=1}^{n} \omega_{Yj}(\frac{f_{kj}}{1-f_{kj}})^\rho\}^{1/\rho}} \right\rangle \right),
\end{aligned}
\tag{23}
$$

where $\omega_{Yj} \in [0, 1]$ and $\sum_{j=1}^{n} \omega_{Yj} = 1$ for $j = 1, 2, \ldots, n$.

Step 2. Calculate the score values of $S(x_k)$ (the accuracy values of $A(x_k)$ or certainty values $C(x_k)$ if necessary) of the collective NCN x_k ($k = 1, 2, \ldots, m$) by using Equations (5)–(7).

Step 3. Rank all the alternatives and select the best one(s) according to the values of $S(x_k)$, $A(x_k)$ and $C(x_k)$.

Step 4. End.

6. Illustrative Examples and Comparison Analysis

6.1. Illustrative Examples

In order to demonstrate the application of the proposed MADM method, in this section, we provide two illustrative examples with neutrosophic cubic information adapted from [29].

Example 3 ([29]). *A passenger needs to make a travel decision from four possible vans (alternatives) X_j ($j = 1, 2, 3, 4$). The customer needs to evaluate the four alternatives according to the following four attributes: (1) Y_1 is the facility; (2) Y_2 is the rent saving; (3) Y_3 is the comfort; (4) Y_4 is the safety. The weight vector of the attributes is given by ω_Y = (0.5, 0.0.25, 0.125, 0.125). Thus, the decision matrix can be constructed using the form of NCNs as follows:*

$$
M_1 = \begin{bmatrix}
(<[0.2,0.5],[0.3,0.7],[0.1,0.2]>,<0.9,0.7,0.2>) & (<[0.2,0.4],[0.4,0.5],[0.2,0.5]>,<0.7,0.4,0.5>) \\
(<[0.3,0.9],[0.2,0.7],[0.3,0.5]>,<0.5,0.7,0.5>) & (<[0.3,0.7],[0.6,0.8],[0.2,0.4]>,<0.7,0.6,0.8>) \\
(<[0.3,0.4],[0.4,0.8],[0.2,0.6]>,<0.1,0.4,0.2>) & (<[0.2,0.4],[0.2,0.3],[0.2,0.5]>,<0.2,0.2,0.2>) \\
(<[0.5,0.9],[0.1,0.8],[0.2,0.6]>,<0.1,0.7,0.2>) & (<[0.3,0.5],[0.5,0.7],[0.1,0.2]>,<0.3,0.5,0.2>) \\
(<[0.2,0.7],[0.4,0.9],[0.5,0.7]>,<0.7,0.7,0.5>) & (<[0.1,0.6],[0.3,0.4],[0.5,0.8]>,<0.5,0.5,0.7>) \\
(<[0.3,0.9],[0.4,0.6],[0.6,0.8]>,<0.9,0.4,0.6>) & (<[0.2,0.5],[0.4,0.9],[0.5,0.8]>,<0.5,0.2,0.7>) \\
(<[0.4,0.9],[0.1,0.2],[0.4,0.5]>,<0.9,0.5,0.5>) & (<[0.6,0.7],[0.3,0.6],[0.3,0.7]>,<0.7,0.5,0.3>) \\
(<[0.5,0.6],[0.2,0.4],[0.3,0.5]>,<0.5,0.4,0.5>) & (<[0.3,0.7],[0.7,0.8],[0.6,0.7]>,<0.4,0.2,0.8>)
\end{bmatrix}
$$

Then, we apply the NCDWAA operator or the NCDWGA operator to solve the MADM problem with NCN information.

Now, we use the NCDWAA operator to handle this decision-making problem as follows:

Step 1. By using Equation (22) for $\rho = 1$, the collective NCNs for the alternatives X_j ($j = 1, 2, 3, 4$) can be obtained based on the NCDWAA as follows:

X_1 = (<[0.1887, 0.5340], [0.3310, 0.6004], [0.1481, 0.2999]>, <0.8462, 0.5657, 0.2887>)

X_2 = (<[0.2889, 0.8636], [0.2824, 0.7278], [0.2963, 0.5161]>, <0.7000, 0.4835, 0.5283>)

X_3 = (<[0.3538, 0.6571], [0.2400, 0.4364], [0.2233, 0.5676]>, <0.6055, 0.3333, 0.2308>)

X_4 = (<[0.3824, 0.8395], [0.1586, 0.6892], [0.1778, 0.3981]>, <0.2706, 0.4647, 0.2564>)

Step 2. By using Equation (5), the score values of $S(X_j)$ of the collective NCN for the alternatives X_j (j = 1, 2, 3, 4) can be calculated as the following results:

$$S(X_1) = 0.5928, S(X_2) = 0.5576, S(X_3) = 0.6206, S(X_4) = 0.5942.$$

Step 3. According to the above score values, the ranking order of the alternatives is $X_3 \succ X_4 \succ X_1 \succ X_2$ and thus X_3 is the best alternative.

Or we can use the NCDWGA operator for the MADM problem as follows:

Step 1′. By using Equation (23) for ρ = 1, the collective NCNs for the alternatives X_j (j = 1, 2, 3, 4) can be obtained based on the NCDWGA as follows:

$$X_1 = (<[0.1778, 0.4970], [0.3412, 0.7241], [0.2690, 0.5385]>, <0.7456, 0.6364, 0.4419>)$$

$$X_2 = (<[0.2824, 0.7683], [0.4000, 0.7767], [0.3708, 0.6250]>, <0.5727, 0.6235, 0.6643>)$$

$$X_3 = (<[0.2909, 0.4561], [0.3166, 0.6993], [0.2449, 0.5862]>, <0.1523, 0.3924, 0.2680>)$$

$$X_4 = (<[0.4000, 0.6933], [0.3859, 0.7600], [0.2826, 0.5514]>, <0.1564, 0.6049, 0.4483>)$$

Step 2′. By using Equation (5), the score values of $S(X_j)$ of the collective NCN for the alternatives X_j (j = 1, 2, 3, 4) can be calculated as the following results:

$$S(X_1) = 0.4966, S(X_2) = 0.4626, S(X_3) = 0.4880, S(X_4) = 0.4685.$$

Step 3′. According to the above score values, the ranking order of the alternatives is $X_1 \succ X_3 \succ X_4 \succ X_2$ and thus X_1 is the best alternative.

Further, all the ranking results of alternatives are listed in Tables 1 and 2 when the parameter ρ is changed from 1 to 5 in the NCDWAA and NCWGA operators.

Table 1. Ranking results of the NCDWAA operator for different operational parameters.

ρ	$S(x_1)$ $S(x_2)$ $S(x_3)$ $S(x_4)$	Ranking Order	The Best Alternative
1	0.5928, 0.5576, 0.6206, 0.5942	$X_3 \succ X_4 \succ X_1 \succ X_2$	X_3
2	0.6176, 0.5896, 0.6763, 0.6360	$X_3 \succ X_4 \succ X_1 \succ X_2$	X_3
3	0.6334, 0.6091, 0.7047, 0.6631	$X_3 \succ X_4 \succ X_1 \succ X_2$	X_3
4	0.6441, 0.6210, 0.7215, 0.6802	$X_3 \succ X_4 \succ X_1 \succ X_2$	X_3
5	0.6516, 0.6289, 0.7323, 0.6916	$X_3 \succ X_4 \succ X_1 \succ X_2$	X_3

Table 2. Ranking results of the NCDWGA operator for different operational parameters.

ρ	$S(x_1)$ $S(x_2)$ $S(x_3)$ $S(x_4)$	Ranking Order	The Best Alternative
1	0.4966, 0.4626, 0.4880, 0.4685	$X_1 \succ X_3 \succ X_4 \succ X_2$	X_1
2	0.4524, 0.4246, 0.4645, 0.4112	$X_3 \succ X_1 \succ X_2 \succ X_4$	X_3
3	0.4238, 0.3980, 0.4483, 0.3781	$X_3 \succ X_1 \succ X_2 \succ X_4$	X_3
4	0.4053, 0.3803, 0.4364, 0.3584	$X_3 \succ X_1 \succ X_2 \succ X_4$	X_3
5	0.3925, 0.3680, 0.4274, 0.3456	$X_3 \succ X_1 \succ X_2 \succ X_4$	X_3

Example 4 ([29]). *A customer wishes to buy a mobile phone and needs to evaluate three models (alternatives) Q_k (k = 1, 2, 3) according to the following three attributes (specifications): H_1 = Processor; H_2 = Camera; (3)*

$H_3 = Battery.$ *The weight vector of the attributes is given by* $\omega_H = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6} \right)$. *The decision matrix can be constructed under the NCS environment as follows:*

$$
M_2 = \begin{bmatrix}
(< [0.2, 0.7], [0.3, 0.7], [0.3, 0.8] >, < 0.3, 0.4, 0.1 >) & (< [0.4, 0.7], [0.3, 0.7], [0.5, 0.8] >, < 0.2, 0.4, 0.5 >) \\
(< [0.2, 0.7], [0.3, 0.7], [0.4, 0.6] >, < 0.9, 0.6, 0.2 >) & (< [0.2, 0.3], [0.3, 0.6], [0.1, 0.4] >, < 0.6, 0.7, 0.6 >) \\
(< [0.2, 0.5], [0.2, 0.7], [0.1, 0.2] >, < 0.5, 0.7, 0.2 >) & (< [0.1, 0.6], [0.2, 0.6], [0.3, 0.4] >, < 0.4, 0.5, 0.6 >) \\
(< [0.2, 0.8], [0.2, 0.7], [0.1, 0.6] >, < 0.1, 0.3, 0.5 >) & \\
(< [0.2, 0.7], [0.4, 0.7], [0.1, 0.3] >, < 0.3, 0.5, 0.7 >) & \\
(< [0.2, 0.5], [0.3, 0.4], [0.3, 0.4] >, < 0.2, 0.4, 0.6 >) &
\end{bmatrix}
$$

Then, we use the NCDWAA operator or the NCDWGA operator to solve the MADM problem with NCN information. By the same steps as that of Example 2, we obtain the ranking results of the alternatives. Tables 3 and 4 list the ranking results of the NCDWAA operator and NCWGA operator, respectively, when the parameter ρ is changed from 1 to 5.

Table 3. Ranking results of the NCDWAA operator for different operational parameters.

ρ	$S(Q_1)\ S(Q_2)\ S(Q_3)$	Ranking Order	The Best Alternative
1	0.5241, 0.5739, 0.5437	$Q_2 \succ Q_3 \succ Q_1$	Q_2
2	0.5410, 0.5934, 0.5474	$Q_2 \succ Q_3 \succ Q_1$	Q_2
3	0.5534, 0.6041, 0.5513	$Q_2 \succ Q_1 \succ Q_3$	Q_2
4	0.5626, 0.6109, 0.5547	$Q_2 \succ Q_1 \succ Q_3$	Q_2
5	0.5697, 0.6158, 0.5574	$Q_2 \succ Q_1 \succ Q_3$	Q_2

Table 4. Ranking results of the NCDWGA operator for different operational parameters.

ρ	$S(Q_1)\ S(Q_2)\ S(Q_3)$	Ranking Order	The Best Alternative
1	0.4760, 0.4856, 0.5095	$Q_3 \succ Q_2 \succ Q_1$	Q_3
2	0.4604, 0.4502, 0.4883	$Q_3 \succ Q_1 \succ Q_2$	Q_3
3	0.4509, 0.4300, 0.4737	$Q_3 \succ Q_1 \succ Q_2$	Q_3
4	0.4448, 0.4176, 0.4637	$Q_3 \succ Q_1 \succ Q_2$	Q_3
5	0.4406, 0.4093, 0.4567	$Q_3 \succ Q_1 \succ Q_2$	Q_3

6.2. Comparison Analysis

From Tables 1–4, we see that the ranking orders corresponding to the NCDWAA and NCDWGA operators show obvious difference in the MADM problem. In Example 3, Table 1 indicates that the different parameters of ρ may not influence the ranking orders corresponding to the NCDWAA operator; while Table 2 shows the different parameters of ρ can change the ranking orders based on the NCDWGA operator. In Table 2, when $\rho = 1$, the best alternative is X_1, while the worst alternative is X_2; when $\rho = 2$, $\rho = 3$, $\rho = 4$ and $\rho = 5$, the ranking order is changed and the best alternative is X_3 and the worst alternative is X_4. In Example 4, Tables 3 and 4 indicate that the different values of ρ can change the ranking orders based on the NCDWGA and NCDWGA operators. In Table 3, when $\rho = 1$ and $\rho = 2$, Q_1 is the worst alternative; when $\rho = 3$, $\rho = 4$ and $\rho = 5$, the ranking order is changed and Q_3 is the worst alternative. In Table 4, when $\rho = 1$, Q_1 is the worst alternative; when $\rho = 2$, $\rho = 3$, $\rho = 4$ and $\rho = 5$, the ranking order is changed and Q_2 is the worst alternative.

From the results of Tables 1–4, we can say that the NCDWAA and NCDWGA operators are sensitive to ρ. Hence, decision makers can specify some parameter ρ according to actual requirements and/or their preference.

Compared with the existing MADM method for NCSs introduced in [29], Table 5 lists the MADM results using NCDWAA and NCDWGA operators proposed in this paper and the weighted average operator ($\mathcal{A}_W$) of NCSs in the relevant paper [29], respectively. From Table 5, we see that the ranking orders based on the Dombi operators proposed in this paper and the weighted average operator ($\mathcal{A}_W$) of

NCSs have obvious difference since different aggregation operators may be result in different ranking orders. Due to no parameter selected in [29], the proposed MADM based on Dombi aggregation operators is more flexible than the approach provided in [29].

Table 5. Decision results of MADM problem with neutrosophic cubic information.

Example	MADM Method	Ranking Order	The Best Alternative
Example 3	NCDWAA ($\rho = 1$)	$X_3 \succ X_4 \succ X_1 \succ X_2$	X_3
	NCDWGA ($\rho = 1$)	$X_1 \succ X_3 \succ X_4 \succ X_2$	X_1
	Weighted average operator ($\mathcal{A}_W$) [29]	$X_4 \succ X_2 \succ X_3 \succ X_1$	X_4
Example 4	NCDWAA ($\rho = 1$)	$Q_2 \succ Q_3 \succ Q_1$	Q_2
	NCDWGA ($\rho = 1$)	$Q_3 \succ Q_2 \succ Q_1$	Q_3
	Weighted average operator ($\mathcal{A}_W$) [29]	$Q_3 \succ Q_1 \succ Q_2$	Q_3

For further comparison, the existing related decision-making approaches [51–53] based on some Dombi operations cannot deal with the decision-making problem with NCSs. However, the decision-making method presented in this paper can describe attributes with interval neutrosophic sets and single valued neutrosophic sets information simultaneously. Therefore, the paper provides a new effective way for decision makers to deal with MADM problems under neutrosophic cubic environment.

7. Conclusions

This paper proposed the NCDWAA and NCDWGA operators and discussed their properties. Then, we presented a MADM method based on the NCDWAA and NCDWGA operators to handle MADM problems under a NCN environment, in which attribute values of the alternatives were ranked and the best one(s) was determined according to their score (accuracy) function values. Finally, two illustrative examples were provided to illustrate the application and effectiveness of the established MADM method. The developed MADM method can effectively solve decision-making problems with flexible operational parameter under neutrosophic cubic environments. In future work, we will further develop more aggregation operators for hesitant neutrosophic cubic sets and apply them in these areas, such as decision-making problems and fault diagnosis.

Author Contributions: Lilian Shi proposed the NCDWAA and NCDWGA operators and their decision-making method; Jun Ye provided the decision-making example and comparative analysis; we wrote this paper together.

Conflicts of Interest: The author declares no conflict of interests.

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Turksen, I.B. Interval-valued fuzzy sets based on normal forms. *Fuzzy Sets Syst.* **1986**, *20*, 191–210. [CrossRef]
3. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
4. Atanassov, K.; Gargov, G. Interval valued intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1989**, *31*, 343–349. [CrossRef]
5. Smarandache, F. *Neutrosophy: Neutrosophic Probability, Set, and Logic*; American Research Press: Rehoboth, DE, USA, 1998.
6. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*; Hexis: Phoenix, AZ, USA, 2005.
7. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets. *Multisp. Multistruct.* **2010**, *4*, 410–413.

8.   Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst.* **2014**, *26*, 2459–2466. [CrossRef]

9.   Ye, J. Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. *Neural Comput. Appl.* **2017**, *28*, 1021–1029. [CrossRef]

10. Ye, J. Multiple-attribute group decision-making method under a neutrosophic number environment. *J. Intell. Fuzzy Syst.* **2016**, *25*, 377–386. [CrossRef]

11. Tian, Z.P.; Wang, J.Q.; Zhang, H.Y. Hybrid single-valued neutrosophic MCGDM with QFD for market segment evaluation and selection. *J. Intell. Fuzzy Syst.* **2018**, *34*, 177–187. [CrossRef]

12. Wang, J.Q.; Zhang, X.; Zhang, H.Y. Hotel recommendation approach based on the online consumer reviews using interval neutrosophic linguistic numbers. *J. Intell. Fuzzy Syst.* **2018**, *34*, 381–394. [CrossRef]

13. Ye, J. Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making. *J. Intell. Fuzzy Syst.* **2014**, *27*, 2453–2462. [CrossRef]

14. Shi, L. Correlation coefficient of simplified neutrosophic sets for bearing fault diagnosis. *Shock Vib.* **2016**, *2016*, 5414361. [CrossRef]

15. Peng, J.J.; Wang, J.Q. Multi-valued Neutrosophic Sets and Its Application in Multi-criteria Decision-Making Problems. *Neutrosophic Sets Syst.* **2015**, *10*, 3–17. [CrossRef]

16. Smarandache, F. n-Valued refined neutrosophic logic and its applications in physics. *Prog. Phys.* **2013**, *4*, 143–146.

17. Broumi, S.; Smarandache, F. Intuitionistic neutrosophic soft set. *J. Inf. Comput. Sci.* **2013**, *8*, 130–140.

18. Broum, S.; Smarandache, F.; Dhar, M. Rough neutrosophic sets. *Neutrosophic Sets Syst.* **2014**, *3*, 62–67.

19. Ye, J. Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment. *J. Intell. Fuzzy Syst.* **2015**, *24*, 23–36. [CrossRef]

20. Ye, J.; Smarandache, F. Similarity measure of refined single-valued neutrosophic sets and its multicriteria decision making method. *Neutrosophic Sets Syst.* **2016**, *12*, 41–44.

21. Karaaslan, F. Correlation coefficient between possibility neutrosophic soft sets. *Math. Sci. Lett.* **2016**, *5*, 71–74. [CrossRef]

22. Fang, Z.B.; Ye, J. Multiple attribute group decision-making method based on linguistic neutrosophic number. *Symmetry* **2017**, *9*, 111. [CrossRef]

23. Shi, L.; Ye, J. Cosine measures of linguistic neutrosophic numbers and their application in multiple attribute group decision-making. *Information* **2017**, *8*. [CrossRef]

24. Ye, J. Multiple attribute decision-making method using correlation coefficients of normal neutrosophic sets. *Symmetry* **2017**, *9*, 80. [CrossRef]

25. Şahin, R.; Liu, P. Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. *Neural Comput. Appl.* **2017**, *28*, 1387–1395. [CrossRef]

26. Jun, Y.B.; Kim, C.S.; Yang, K.O. Cubic sets. *Ann. Fuzzy Math. Inf.* **2012**, *4*, 83–98.

27. Ali, M.; Deli, I.; Smarandache, F. The theory of neutrosophic cubic sets and their applications in pattern recognition. *J. Intell. Fuzzy Syst.* **2016**, *30*, 1957–1963. [CrossRef]

28. Jun, Y.B.; Smarandache, F.; Kim, C.S. P-union and P-intersection of neutrosophic cubic sets. *An. St. Univ. Ovidius Constanta* **2017**, *25*, 99–115. [CrossRef]

29. Zhan, J.; Khan, M.; Gulistan, M.; Ali, A. Applications of neutrosophic cubic sets in multi-criteria decision making. *Int. J. Uncertain. Quantif.* **2017**, *7*, 377–394. [CrossRef]

30. Banerjee, D.; Giri, B.C.; Pramanik, S.; Smarandache, F. GRA for multi attribute decision making in neutrosophic cubic set environment. *Neutrosophic Sets Syst.* **2017**, *15*, 64–73.

31. Lu, Z.; Ye, J. Cosine measures of neutrosophic cubic sets for multiple attribute decision-making. *Symmetry* **2017**, *9*. [CrossRef]

32. Pramanik, S.; Dalapati, S.; Alam, S.; Roy, T.K.; Smarandache, F. Neutrosophic cubic MCGDM method based on similarity measure. *Neutrosophic Sets Syst.* **2017**, *16*, 44–56.

33. Pramanik, S.; Dalapati, S.; Alam, S.; Roy, T.K. NC-TODIM based MAGDM under neutrosophic cubic set environment. *Information* **2017**, *8*. [CrossRef]

34. Wei, G.W. Some harmonic aggregation operators with 2-tuple linguistic assessment information and their application to multiple attribute group decision making. *Int. J. Uncertain. Quantif. Fuzz. Knowl. Based Syst.* **2011**, *19*, 977–998. [CrossRef]

35. Das, S.; Guha, D. Power harmonic aggregation operator with trapezoidal intuitionistic fuzzy numbers for solving MAGDM problems. *Iran. J. Fuzzy Syst.* **2015**, *12*, 41–74.

36. Liu, P.D.; Wang, Y.M. Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Comput. Appl.* **2014**, *25*, 2001–2010. [CrossRef]

37. Verma, R. Generalized Bonferroni mean operator for fuzzy number intuitionistic fuzzy sets and their application to multi attribute decision making. *Int. J. Intell. Syst.* **2015**, *30*, 499–519. [CrossRef]

38. Wang, L.; Zhang, H.Y.; Wang, J.Q. Frank Choquet Bonferroni mean operators of bipolar neutrosophic sets and their application to multi-criteria decision-making problems. *Int. J. Fuzzy Syst.* **2017**. [CrossRef]

39. Liu, P.D.; Li, H.G. Multiple attribute decision-making method based on some normal neutrosophic Bonferroni mean operators. *Neural Comput. Appl.* **2017**, *28*, 179–194. [CrossRef]

40. Verma, R.; Sharma, B.D. Intuitionistic fuzzy Einstein prioritized weighted operators and their application to multiple attribute group decision making. *Appl. Math. Inf. Sci.* **2015**, *9*, 3095–3107. [CrossRef]

41. Zhao, A.W.; Du, J.G.; Guan, H.J. Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator. *J. Intell. Fuzzy Syst.* **2015**, *29*, 2697–2706. [CrossRef]

42. Sun, H.X.; Yang, H.X.; Wu, J.Z.; Yao, O.Y. Interval neutrosophic numbers Choquet integral operator for multi-criteria decision making. *J. Intell. Fuzzy Syst.* **2015**, *28*, 2443–2455. [CrossRef]

43. Zhou, H.; Xu, Z.; Cui, F. Generalized hesitant fuzzy harmonic mean operators and their applications in group decision making. *Int. J. Fuzzy Syst.* **2016**, *18*, 685–696. [CrossRef]

44. Liao, H.; Xu, Z. Extended hesitant fuzzy hybrid weighted aggregation operators and their application in decision making. *Soft Comput.* **2015**, *19*, 2551–2564. [CrossRef]

45. Wang, X.F.; Wang, J.Q.; Deng, S.Y. Some geometric operators for aggregating intuitionistic linguistic information. *Int. J. Fuzzy Syst.* **2015**, *17*, 268–278. [CrossRef]

46. Verma, R. Multiple attribute group decision making based on generalized trapezoid fuzzy linguistic prioritized weighted average operator. *Int. J. Mach. Learn. Cybern.* **2016**, *8*, 1993–2007. [CrossRef]

47. Liu, P.D.; Wang, Y.M. Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making. *J. Syst. Sci. Complex* **2016**, *29*, 681–697. [CrossRef]

48. Ye, J. Exponential operations and aggregation operators of interval neutrosophic sets and their decision making methods. *Springerplus* **2016**, *5*, 1488. [CrossRef] [PubMed]

49. Wang, L.; Shen, Q.; Zhu, L. Dual hesitant fuzzy power aggregation operators based on archimedean T-conorm and T-norm and their application to multiple attribute group decision making. *Appl. Soft Comput.* **2016**, *38*, 23–50. [CrossRef]

50. Wu, X.H.; Wang, J.Q.; Peng, J.J.; Chen, X.H. Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. *J. Intell. Fuzzy Syst.* **2016**, *18*, 1104–1116. [CrossRef]

51. Dombi, J. A general class of fuzzy operators, the demorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. *Fuzzy Sets Syst.* **1982**, *8*, 149–163. [CrossRef]

52. Liu, P.D.; Liu, J.L.; Chen, S.M. Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making. *J. Oper. Res. Soc.* **2017**. [CrossRef]

53. Chen, J.Q.; Ye, J. Some single-valued neutrosophic Dombi weighted aggregation operators for multiple attribute decision-making. *Symmetry* **2017**, *9*, 82. [CrossRef]

54. Ye, J. Linguistic neutrosophic cubic numbers and their multiple attribute decision-making method. *Information* **2017**, *8*, 110. [CrossRef]