International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

Fuzzy Neutrosophic Quadratic Programming Problem As A Linear Complementarity Problem

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Abstract— In this paper, a new approach for solving fuzzy neutrosophic quadratic programming problem (FNQPP) is suggested. Here, all the fuzzy coefficients in the objective functions and constraints are replaced by single valued triangular neutrosphic numbers. The problem of quadratic programming with triangular neutrosphic number coefficients is reduced to a Fuzzy Neutrosophic Linear Complementarity problem (FNLCP) and an algorithm is proposed to solve the formulated model. The effectiveness of the proposed method is illustrated by means of an numerical example.

Keywords— Fuzzy Quadratic programming problem, Fuzzy Linear Complementarity problem, Single valued Triangular Neutrosophic Number, Lemke's method.

1. INTRODUCTION

The concept of Neutrosophic Set (NS) was first introduced by Smarandache [4] which is a generalisation of classical sets, fuzzy set, intuitionistic fuzzy set etc. Zadeh's [1] classical concept of fuzzy set is a strong mathematical tool to deal with the complexity generally arising from uncertainty in the form of ambiguity in real life scenario. Many Linear Programming techniques are in fact more complicated. Moreover, many practical problems cannot be represented by non-linear programming model. Therefore, attempts were made to develop more general mathematical programming methods and many significant advances have been made in the area of fuzzy quadratic programming problems.

The linear complementarity problem (LCP) is a well-known problem in mathematical programming and it has been studied by many researchers. LCP is a general problem that unifies linear, quadratic programs and bimatrix games. In 1968, Lemke[7] proposed a complementary pivoting algorithm for solving linear complementarity problem and Katta.G.Murthy [6]. Since, the KKT conditions for quadratic programming problems can be written as a LCP, Lemke's algorithm can be used to solve quadratic programs. This paper provides a new technique for solving fuzzy neutrosophic quadratic programming problem by converting it into a fuzzy neutrosophic linear complementarity problem.

The neutrosophic sets reflect on the truth membership, indeterminacy membership and falsity membership concurrently, which is more practical and adequate than Fuzzy sets and Intuitionistic fuzzy sets. Single valued neutrosophic sets are an extension of neutrosophic sets which were introduced by Wang [5]. The paper is organized as follows. In Section 2, Single Valued Triangular Neutrosophic numbers (SVTN) and the fuzzy arithmetical operators are provided. Section 3 deals with an algorithm for solving a fuzzy neutrosophic linear complementarity problem. In section 4, the conversion of FQPP to FNLCP is discussed.

2. PRELIMINARIES

Definition 2.1

Let E be a universe. A Neutrosophic set A over E is defined by $A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in E\}$,Where $T_A(x), I_A(x), F_A(x)$ are called the truth – membership function, indeterminacy membership function and falsity membership function respectively. They are respectively defined by $T_A : E \to]\,^-0, 1^+[$, $I_A : E \to]\,^-0, 1^+[$, $F_A : E \to]\,^-0, 1^+[$,Such that $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$

Definition 2.2

A Single Valued Triangular Neutrosophic number (SVTN) $\tilde{A} = \langle (a, b, c);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is a special neutrosophic set on the real number set R , whose truth – membership, indeterminacy – membership and a falsity membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-a)w_{\tilde{a}}}{(b-a)} & , a \leq x \leq b \\ w_{\tilde{a}} & , x = b \\ \frac{(c-x)w_{\tilde{a}}}{(c-b)} & , b \leq x \leq c \\ 0 & , otherwise \end{cases},$$

$$v_{\tilde{a}}(x) = \begin{cases} \frac{(b-x+u_{\tilde{a}}(x-a))}{(b-a)} & , a \leq x \leq b \\ \mu_{\tilde{a}} & , x = b \\ \frac{(x-b+u_{\tilde{a}}(c-x))}{(c-b)} & , b \leq x \leq c \\ 0 & , otherwise \end{cases},$$

$$\lambda_a(x) = \begin{cases} \frac{(b-x+y_{\tilde{a}}(x-a))}{(b-a)} & , a \leq x \leq b \\ y_{\tilde{a}} & , x = b \\ \frac{(x-b+y_{\tilde{a}}(c-x))}{(c-b)} & , b \leq x \leq c \\ 0 & , otherwise \end{cases}$$

2.3 Arithmetic Operations on Triangular Neutrosophic Numbers:

Let $\tilde{a} = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (a_2, b_2, c_2);\ w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$.

Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

Multiplication: $\tilde{a}\tilde{b} =$

$$\begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle\ , (c_1 > 0, c_2 > 0) \\ \langle (a_1c_2, b_1b_2, c_1a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle\ , (c_1 < 0, c_2 > 0) \\ \langle (c_1c_2, b_1b_2, a_1a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle\ , (c_1 < 0, c_2 < 0) \end{cases}$$

Division: $\tilde{a} \ / \ \tilde{b} =$

$$\begin{cases} \langle (a_1/c_2, b_1/b_2, c_1/a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle\ , (d_1 > 0, d_2 > 0) \\ \langle (c_1/c_2, b_1/b_2, a_1/a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle\ , (d_1 < 0, d_2 > 0) \\ \langle (c_1/a_2, b_1/b_2, a_1/c_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle\ , (d_1 < 0, d_2 < 0) \end{cases}$$

Scalar Multiplication:

$$\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle\ , (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle\ , (\gamma < 0) \end{cases}$$

Inverse: $\tilde{a}^{-1} = \langle \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle\ , (\tilde{a} \neq 0)$

3. FUZZY LINEAR COMPLEMENTARITY PROBLEM

The following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with Single Valued Triangular Neutrosophic numbers.

$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q}$ (1)

$\tilde{W}_j \geq 0, Z_j \geq 0,\ j = 1, 2, 3, \ldots\ldots\ldots.n$ (2)

$\tilde{W}_j\tilde{Z}_j = 0,\ j = 1, 2, 3, \ldots\ldots\ldots\ldots n$ (3)

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of fuzzy Linear complementary variables.

Definition 3.2

A solution $(\tilde{W}, \tilde{Z})$ to the above system (1) - (3) is called a Neutrosophic complementary feasible solution, if $(\tilde{W}, \tilde{Z})$ is a Neutrosophic basic feasible solution to (1) and (2) with one of the pair $(\tilde{W}_j, \tilde{Z}_j)$ basic for each j =1, 2, 3…n.

3.3 Algorithm for Neutrosophic Linear Complementarity Problem

Consider the Neutrosophic Linear complementarity problem $(\tilde{q}, \tilde{M})$, where the Neutrosophic fuzzy matrix $\tilde{M}_j$ is a positive semi definite matrix of order *n*. The original table for this version of the algorithm is:

$\tilde{w}$	$\tilde{Z}$	$\tilde{Z}_0$	
$\tilde{I}$	$-\tilde{M}$	$-\tilde{d}$	$\tilde{q}$

This method deals only with Neutrosophic complementary basic vectors, beginning with $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_n)$ as the initial Neutrosophic complementary basic vector. All the Neutrosophic complementary basic vectors obtained in the method, except the terminal one, will be infeasible. When a Neutrosophic complementary feasible basic vector for (4) is obtained, the method terminates.

If $\tilde{q} \geq 0$, then we have the solution satisfying (1)-(3), by letting $\tilde{w} = \tilde{q}$ and $\tilde{Z} = 0$

If $\tilde{q} < 0$, we will consider the following system

$$\widetilde{W} - \widetilde{M}\tilde{Z} - \tilde{e}\tilde{Z}_0 = \tilde{q} \qquad (4)$$

$$\widetilde{W}_j \geq 0, Z_j \geq 0, j = 1,2,3, \dots \dots, n \qquad (5)$$

$$\widetilde{W}_j\tilde{Z}_j = 0 \,, j = 1,2,3, \dots \dots, n \qquad (6)$$

Where Z_0 is an artificial Neutrosophic fuzzy variable and $\tilde{e}$ is an n-vector with all components equal to any constant. Letting $\tilde{Z}_0$ = maximum $\{\tilde{q}_i/1 \leq i \leq n\}, \tilde{Z} = 0$ and $\widetilde{W} = \tilde{q} + \tilde{e}\tilde{Z}_0$. We obtain a starting solution to the system (4)-(6). Through a sequence of pivots, we attempt to drive the Neutrosophic fuzzy artificial variable $\tilde{Z}_0$ to level zero, thus obtaining a solution to the Neutrosophic linear complementarity problem (NLCP).

Step: 1

Introduce the Neutrosophic artificial variable $\tilde{Z}_0$ and consider the system (4)-(6).

(i) If $\tilde{q} \geq 0$ stop; then $(\widetilde{W}, \tilde{Z}) = (\tilde{q}, \tilde{0})$ is a Neutrosophic complementary basic feasible solution.

(ii) If $\tilde{q} < 0$ display the system (4),(5) as given in the simplex method.
Let $-q_s$= maximum $\{-q_i / 1 \leq i \leq n\}$, and update the table by pivoting at row s and the $\tilde{Z}_0$ column. Thus the Neutrosophic basic variables $\tilde{Z}_0$ and $\widetilde{W}_s$ for j= 1, 2, 3…n and $j \neq s$ are positive.

Let $\tilde{y}_s = \tilde{Z}_0$ and go to step: 2.

Step: 2

Let $\tilde{d}_s$ be the updated column in the current table under the variable $\tilde{y}_s$.

If $\tilde{d}_s \leq 0$, go to step:5, otherwise determine the index r by the following minimum ratio test, $\frac{\tilde{\bar{q}}}{\bar{d}_{rs}} = \min\left\{\frac{\tilde{\bar{q}}_i}{\tilde{d}_{is}}, \tilde{d}_{is} > 0\right\}$, where $\tilde{\bar{q}}$ is the updated right-hand side column denoting the values of the Neutrosophic basic variables.

If the Neutrosophic basic variable at row r is $\tilde{Z}_0$, go to step: 4, otherwise, go to step: 3.
Step: 3
The Neutrosophic basic variable at row r is either $\widetilde{W}_l or\ \tilde{Z}_l$ for some $l \neq s$. The variable $\tilde{y}_s$ enters the basis and the table is updated by pivoting at row r and $\tilde{y}_s$ the column. If $\widetilde{W}_l$ leaves the basis, then let $\tilde{y}_s = \tilde{Z}_l$; and if $\tilde{Z}$ leaves the basis, then let $\tilde{y}_s = \widetilde{W}_l$; Return to step: 2.
Step: 4
Here $\tilde{y}_s$ enters the basis, and $\tilde{Z}_0$ leaves the basis. Pivot at the $\tilde{y}_s$ column and the $\tilde{Z}_0$ row, producing a Neutrosophic complementary basic feasible solution. Stop.
Step: 5
Stop with ray termination.
A ray $R = \{(\widetilde{W}, \tilde{Z}, \tilde{Z}_0) + \lambda\tilde{d}/\lambda \geq 0\}$ is found such that every point in R satisfying (4), (5) and (6), where $(\widetilde{W}, \tilde{Z}, \tilde{Z}_0)$ is the almost Neutrosophic complementary basic feasible solution, and $\tilde{d}$ is an extreme direction of the set defined by (4) and (5) having a $\tilde{1}$ in the row corresponding to $\tilde{y}_s$, $-\tilde{d}_s$ in the rows of the current basic variables and zeros everywhere else.

Fuzzy Quadratic Programming Problem (FQPP) as a Fuzzy Linear Complementarity Problem (FLCP)
Consider the following Quadratic Programming Problem

Minimize $\tilde{f}(\tilde{x}) = \tilde{c}^t\tilde{x} + \frac{1}{2}\tilde{x}^t\widetilde{H}\tilde{x}$

Subject to $\tilde{A}\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$

Where $\tilde{c}$ an n- vector of fuzzy numbers is, $\tilde{b}$ is an m – vector. $\tilde{A}$ is an $m \times n$ fuzzy matrix and $\widetilde{H}$ is an $n \times n$ fuzzy symmetric matrix. Let $\tilde{y}$ denotes the vector of slack variables and $\tilde{u}, \tilde{v}$ be the Lagrangian multiplier vectors of the constraints $\tilde{A}\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$ respectively, then the Kuhn-Tucker conditions can be written as

$$\tilde{A}\tilde{x} + \tilde{y} = \tilde{b}$$

$$-\widetilde{H}\tilde{x} - \tilde{A}^t\tilde{u} + \tilde{v} = \tilde{c}$$

$$\tilde{x}^t\tilde{v} = 0, \tilde{u}^t\tilde{y} = 0 \text{ And } \tilde{x}, \tilde{y}, \tilde{u}, \tilde{v} \geq 0$$

Now letting $\widetilde{M} = \begin{bmatrix} \tilde{0} & -\tilde{A} \\ \tilde{A}^t & \widetilde{H} \end{bmatrix}, \tilde{q} = \begin{bmatrix} \tilde{b} \\ \tilde{c} \end{bmatrix}, \widetilde{w} = \begin{bmatrix} \tilde{y} \\ \tilde{v} \end{bmatrix} and\ \tilde{z} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}$ the Kuhn-Tucker conditions can be expressed as the LCP.

$$\widetilde{W} - \widetilde{M}\tilde{Z} = \tilde{q}$$

$$\widetilde{W}^t\tilde{Z} = 0$$

$(\widetilde{W}, \tilde{Z}) \geq 0$

Numerical Example

Consider the following fuzzy quadratic programming problem

$$Minimize\ \tilde{z} = -\tilde{4}\tilde{x}_1 + \tilde{x}_1{}^2 - \tilde{2}\tilde{x}_1\tilde{x}_2 + \tilde{2}\tilde{x}_2{}^2$$

Subject to constraint $\tilde{2}\tilde{x}_1 + \tilde{x}_2 \leq \tilde{6}$, $\tilde{x}_1 - \tilde{4}\tilde{x}_2 \leq \tilde{0}$ and $\tilde{x}_1, \tilde{x}_2 \geq \tilde{0}$

Where $\tilde{A} = \begin{bmatrix} \tilde{2} & \tilde{1} \\ \tilde{1} & -\tilde{4} \end{bmatrix}, \widetilde{H} = \begin{bmatrix} \tilde{2} & -\tilde{2} \\ -\tilde{2} & \tilde{4} \end{bmatrix}, \tilde{b} = \begin{bmatrix} \tilde{6} \\ \tilde{0} \end{bmatrix}, \tilde{c} = \begin{bmatrix} -\tilde{4} \\ \tilde{0} \end{bmatrix}$ in which the single valued Neutrosophic coefficients are assumed to be

$$\tilde{1} = \langle (0,1,2); 0.2,0.3,0.5 \rangle\ , \tilde{2} = \langle (0,2,4); 0.6,0.4,0.1 \rangle\ ,$$

$-\tilde{1} = \langle (-3, -1, -0.5); 0.6,0.4,0.1 \rangle$

$\tilde{2} = \langle (1,2,3); 0.2,0.3,0.5 \rangle$, $\tilde{4} = \langle (0,4,8); 0.4,0.3,0.2 \rangle$ and $\tilde{6} = \langle (4,6,8); 0.6,0.4,0.1 \rangle$

Neutrosophic linear complementary problem is solved by the proposed algorithm and the results are tabulated.

Hence the solution of this Neutrosophic Linear complementary Problem (q, M) is

$z_1 = \langle (2,0.6,0); 0.2,0.4,0.5 \rangle, z_2 = \langle (0,0,0); 1,0,0 \rangle, z_3 = \langle (0,2.47,0); 0.2,0.4,0.5 \rangle,$

$z_4 = \langle 0,1.08,4; 0.2,0.4,0.5 \rangle$

4. CONCLUSION

In this paper, a new approach for solving a fuzzy quadratic programming problem as a fuzzy linear complementarity with neutrosophic fuzzy parameters is suggested. Here Linear Complementarity Problem (q, M) is solved by Principal Pivoting method with Single Valued Triangular Neutrosophic numbers (SVTN). The proposed method can be extended to non-linear and multi objective programming with neutrosophic fuzzy coefficients in our upcoming study.

ACKNOWLEDGMENT

We authors are very much grateful to the unknown referees for their valuable suggestions and comments to improve the quality of this research work.

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