GENERALIZED β – CONTINUOUS FUNCTION IN NEUTROSOPHIC BITOPOLOGICAL SPACES

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ABSTRACT

In this paper we address the continuous function in neutrosophic bitopological spaces. The focus of this paper is to introduce gβ- continuous in neutrosophic bitopological spaces. Some of the fundamental properties and relationship between these continuous functions are investigated in this paper.

Key words: Ngβ – continuous maps in neutrosophic bitopological spaces.

INTRODUCTION:

In the neutrosophic set, all the elements have the degree of membership, indeterminacy and degree of non-membership. The neutrosophic closed sets and neutrosophic continuous functions were introduced by salama,smarandache and valeri [16] in 2014. D.Andrijevic,[2] introduced " semi preopen sets" in 1986. In 1983, Abd EI – Monsef [1] introduced the classes of beta open sets and beta continuous mappings. In 2014 Jayanthi [7] introduced the generalized β– closed set in intuitionistic fuzzy topological spaces.F.H.Khedr,S.M.AI-Areefi, and T.Noiri,[8] introduced generalized the notions of β – open sets and investigated β - continuous functions in bitopological spaces.

2. Preliminaries:

Definition 2.1 [10] Neutrosophic topological spaces

Let τ be a collection of all neutrosophic subsets on X. Then τ is called a neutrosophic topology in X if the following conditions hold

i. 0_X and 1_X belong to τ.

ii. Union of any number of neutrosophic sets in τ again belong to .

iii. Intersection of any two neutrosophic set in τ belong to .

Then the pair (X, τ) is called neutrosophic topology on X.

Definition 2.2 [17]

A neutrosophic set A on the universe of discourse X is defined as $A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \epsilon X\}$

Where μ_A, σ_A,:$X \longrightarrow]0-, 1+[$ and $0- \leq \mu_A + \sigma_A + \gamma_A \leq 3+$, μ_A represents degrees of membership function, σ_A is the degree of indeterminacy and γ_A is the degree of non-membership function.

Definition 2.2 [3]

A map f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) is (i,j) neutrosophic generalized β σ_k - continuous [(i,j) NGβ- σ_k-continuous] if f^{-1}(U) is (i, j) NGβ closed in (X, τ_1,τ_2) for each σ_k- closed set U in (Y,σ_1, σ_2) where i, j, k =1,2 and i ≠ j

Definition 2.3

Let (X,τ_1,τ_2) and (Y,σ_1, σ_2) be any two bitopological spaces. A map f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) is said to be

i) Semi-continuous [9] if f^{-1} (U) is semi-closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

ii) pre-continuous [11] if f^{-1} (U) is pre-closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

iii) semi pre-continuous [14] if f^{-1} (U) is semi pre-closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

iv) α - continuous [12] if f^{-1} (U) is α-closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

Definition 2.4

Let (X,τ_1,τ_2) and (Y,σ_1, σ_2) be any two bitopological spaces. A map f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) is said to be

i) generalized continuous [5] if f^{-1} (U) is generalized closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

ii) generalized β- continuous [13] if f^{-1} (U) is generalized β- closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

iii) semi generalized-continuous [4] if f^{-1} (U) is semi generalized closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

iv) generalized semi-continuous [6] if f^{-1} (U) is generalized semi closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

v) regular generalized-continuous [15] if f^{-1} (U) is regular generalized closed set in (X,τ_1,τ_2) for every closed set U of (Y,σ_1, σ_2).

3. Generalized β - continuous maps in neutrosophic bitopological spaces

Definition 3.1:

A map f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) is (i,j) neutrosophic generalized β σ_k - continuous [(i,j) NGβ- σ_k-continuous] if f^{-1}(U) is (i, j) NGβ closed in (X, τ_1,τ_2) for each σ_k- closed set U in (Y,σ_1, σ_2) where i, j, k =1,2 and i ≠ j

Theorem 3.2:

Every σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a σ_k-neutrosophic continuous map .

Let us prove that f is (i,j) NG β σ_k- continuous.Let U be a σ_k- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is σ_k -Nclosed in (X, τ_1,τ_2).Since every σ_k -NC set is (i,j) NGβ closed.

f^{-1}(U) is NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.3:

Every (i,j) Ng-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof :

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Ng- σ_k-continuous map .

Let us prove that f is (i,j) NG β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is (i,j) Ng closed in (X, τ_1,τ_2).Since every (i,j)Ng closed set is (i,j) NGβ closed.

f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.4:

Every (i,j) Nsg-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Nsg- σ_k- continuous map .

Let us prove that f is (i,j) Ng β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2).

Then f^{-1}(U) is (i,j) Nsg closed in (X, τ_1,τ_2).Since every (i,j)Nsg closed set is (i,j) NGβ closed.f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.5:

Every (i,j) Ngs-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Ngs- σ_k- continuous map .

Let us prove that f is (i,j) Ng β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is (i,j) Ngs -closed in (X, τ_1,τ_2).Since every (i,j)Ngs-closed set is (i,j) NGβ closed.

f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.6:

Every (i,j) Nβ-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Nβ- σ_k- continuous map .

Let us prove that f is (i,j) Ng β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is (i,j) Nβ -closed in (X, τ_1,τ_2).Since every (i,j)Nβ-closed set is (i,j) NGβ closed.

f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.7:

Every (i,j) Nαg-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Nαg - σ_k- continuous map .

Let us prove that f is (i,j) Ng β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is (i,j) Nαg -closed in (X, τ_1,τ_2).Since every (i,j) Nαg -closed set is (i,j) NGβ closed.

f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.8:

Every (i,j) Nα-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof :

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Nα - σ_k- continuous map .

Let us prove that f is (i,j) Ng β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is (i,j) Nα -closed in (X, τ_1,τ_2).Since every (i,j) Nα -closed set is (i,j) NGβ closed.

f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

Theorem 3.9:

Every (i,j) Nrg-σ_k -neutrosophic continuous is (i,j) NG β σ_k- continuous.

Proof :

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a (i,j) Nrg - σ_k- continuous map .

Let us prove that f is (i,j) Ng β σ_k- continuous.Let U be a $N\sigma_k$- closed set in (Y,σ_1, σ_2)

Then f^{-1}(U) is (i,j) Nrg -closed in (X, τ_1,τ_2).Since every (i,j) Nrg -closed set is (i,j) NGβ closed. f^{-1}(U) is (i,j) NGβ closed and hence f is (i,j) NG β σ_k- continuous.

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