Int. J. Math. And Appl., 7(4)(2019), 37–46
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Homomorphism of Neutrosophic Fuzzy L-ideals

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Abstract: In this paper, we define the concept of homomorphism of Neutrosophic fuzzy L-Ideals and some related properties are discussed. Finally, some results on Neutrosophic fuzzy L-ideals are investigated.

Keywords: Neutrosophic set, Neutrosophic Lattices, Neutrosophic fuzzy L-Ideals, Homomorphism of Neutrosophic fuzzy L-ideals.

1. Introduction

Zadeh [9], first of all introduced the notion of Fuzzy set in 1965. Rosenfield established the fuzzy group. K.T.Atanassov introduced concept of Intuitionistic fuzzy set as a generalization of the notion of fuzzy set. M.Mullai [2] defined the notion of fuzzy L-ideals. N.Palaniappan established Intuitionistic L-fuzzy ideal and homomorphism of Intuitionistic L-fuzzy ideal. K.V.Thomas [5] developed the hypothesis of Intuitionistic fuzzy sublattice. Florentin Smarandache [7] introduced the notion of Neutrosophy as a new branch of philosophy. Neutrosophy is a base of Neutrosophic logic which is an extension of fuzzy logic in which Indeterminacy is included. In Neutrosophic logic, each intention is likely to have the proportion of truth in a subset T, proportion of Indeterminacy in a subset I, and the proportion of falsity in a subset F. The hypothesis of Neutrosophic set have achieved great success in a variety of fields like Medical analysis, Topology, Image dispensation, Decision making problem, Robotics and etc. The Neutrosophic set is a great instrument to contract with undetermined and incompatible data. The main principle of this work is to study the generalization of the concept of Intuitionistic fuzzy L-ideals. The main inspiration of this work is to introduce the concept of Neutrosophic fuzzy L-ideals and recognized some results on it.

2. Preliminaries

Definition 2.1. *A Poset in which every pair of elements has both a least upper bound and greatest lower bound is called Lattice* $(L, \vee, \wedge)$.

(1). "$\vee$"-The Join of two elements is their least upper bound.

(2). "$\wedge$"-The Meet of two elements is their greatest lower bound.

Definition 2.2. *Let L be a Lattice and μ is a fuzzy set. Then, μ be a fuzzy Lattice. if, $\forall\ x, y \in L$.*

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(1). $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$.

(2). $\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2.3. *Let* $(L, \vee, \wedge)$ *and* $(L', \vee, \wedge)$ *be two lattices. A function* $f : L \to L'$ *is a lattice homomorphism. If,* $\forall\, x, y \in L$*. Then following conditions are satisfied,*

(1). $f(x \vee y) = f(x) \vee f(y)$.

(2). $f(x \wedge y) = f(x) \wedge f(y)$.

Definition 2.4. *A Neutrosophic fuzzy set A on Universe set X describe by a Truth characteristic function* $T_A(x)$*, an Indeterminacy characteristic function* $I_A(x)$*, and a Falsity characteristic function* $F_A(x)$ *is defined as,* $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in X\}$*. Where* $T_A,\ I_A,\ F_A : X \to [0,1]$ *and* $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.5. *Let A and B be Neutrosophic fuzzy Lattice sets. Then,*

(1). $A \cup B = \{\langle x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)\rangle\}$, *where* $T_{A\cup B}(x) = \max\{T_A(x), T_B(x)\}$, $I_{A\cup B}(x) = \max\{I_A(x), I_B(x)\}$, $F_{A\cup B}(x) = \min\{F_A(x), F_B(x)\}$, $\forall\ x \in L$.

(2). $A \cap B = \{\langle x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)\rangle\}$, *where* $T_{A\cap B}(x) = \min\{T_A(x), T_B(x)\}$, $I_{A\cap B}(x) = \min\{I_A(x), I_B(x)\}$, $F_{A\cap B}(x) = \max\{F_A(x), F_B(x)\}$, $\forall\ x \in L$.

(3). Complement:

(a). $T_{A^c}(x) = F_A(x)$

(b). $I_{A^c}(x) = 1 - I_A(x)$

(c). $F_{A^c}(x) = T_A(x)$.

3. Neutrosophic Fuzzy L-Ideals

Definition 3.1. *Let L be a lattice and* $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in L\}$*. Where,* $T_A(x) \in [0,1]$, $I_A(x) \in [0,1]$, $F_A(x) \in [0,1]$*. Then A Neutrosophic set A of L is called Neutrosophic fuzzy L-ideal. if,*

(1). $T_A(x \vee y) \geq \min\{T_A(x), T_A(y)\}$.

(2). $T_A(x \wedge y) \geq \max\{T_A(x), T_A(y)\}$.

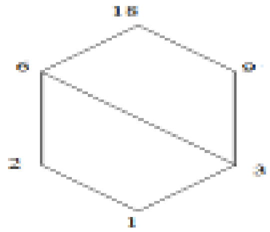
(3). $I_A(x \vee y) \geq \min\{I_A(x), I_A(y)\}$.

(4). $I_A(x \wedge y) \geq \max\{I_A(x),\ I_A(y)\}$.

(5). $F_A(x \vee y) \leq \max\{F_A(x), F_A(y)\}$.

(6). $F_A(x \wedge y) \leq \min\{F_A(x), F_A(y)\}$, $\forall\ \ x, y \in L$.

Example 3.2. *Consider the Lattice* $L = \{1, 2, 3, 6, 9, 18\}$ *is divisors of 18. A is a Neutrosophic Fuzzy L-ideal.*



We define $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle / x \in L\}$ *by* $\{\langle 1, 0.4, 0.6, 0.9\rangle, \langle 2, 0.2, 0.5, 0.4\rangle, \langle 3, 0.3, 0.7, 0.6\rangle, \langle 6, 0.1, 0.7, 0.8\rangle, \langle 9, 0.9, 0.6, 0.4\rangle, \langle 18, 0.8, 0.5, 0.3\rangle\}$.

Theorem 3.3. *Let A and B be Neutrosophic fuzzy L-ideals and if, one is contained another. Then* $A \cup B$ *is a Neutrosophic fuzzy L-ideal.*

Proof. If, $\forall\ x, y \in L$. Then,

(1). $T_{A\cup B}(x \vee y) = \max\{T_A(x \vee y), T_B(x \vee y)\},$

$$= \max\{\min(T_A(x), T_A(y)), \min(T_B(x), T_B(y))\}$$

$$\geq \min\{\max(T_A(x), T_B(x)), \max(T_A(y), T_B(y))\}$$

$$\geq \min\{T_{A\cup B}(x), T_{A\cup B}(y)\}.$$

(2). $T_{A\cup B}(x \wedge y) = \max\{T_A(x \wedge y), T_B(x \wedge y)\},$

$$= \max\{\max(T_A(x), T_A(y)), \max(T_B(x), T_B(y))\}$$

$$\geq \max\{\max(T_A(x), T_B(x)), \max(T_A(y), T_B(y))\}$$

$$\geq \max\{T_{A\cup B}(x), T_{A\cup B}(y)\}.$$

(3). $I_{A\cup B}(x \vee y) = \max\{I_A(x \vee y), I_B(x \vee y)\},$

$$= \max\{\min(I_A(x), I_A(y)), \min(I_B(x), I_B(y))\}$$

$$\geq \min\{\max(I_A(x), I_B(x)), \max(I_A(y), I_B(y))\}$$

$$\geq \min\{I_{A\cup B}(x), I_{A\cup B}(y)\}.$$

(4). $I_{A\cup B}(x \wedge y) = \max\{I_A(x \vee y), I_B(x \vee y)\},$

$$= \max\{\max(I_A(x), I_A(y)), \max(I_B(x), I_B(y))\}$$

$$\geq \max\{\max(I_A(x), I_B(x)), \max(I_A(y), I_B(y))\}$$

$$\geq \max\{I_{A\cup B}(x), I_{A\cup B}(y)\}.$$

(5). $F_{A\cup B}(x \vee y) = \min\{F_A(x \vee y), F_B(x \vee y)\},$

$$= \min\{\max(F_A(x), F_A(y)), \max(F_B(x), F_B(y))\}$$

$$\leq \max\{\min(F_A(x), F_B(x)), \min(F_A(y), F_B(y))\}$$

$$\leq \max\{F_{A\cup B}(x), F_{A\cup B}(y)\}.$$

(6). $F_{A\cup B}(x\wedge y) = \min\{F_A(x\vee y), F_B(x\vee y)\},$

$$= \min\{\min(F_A(x), F_A(y)), \min(F_B(x), F_B(y))\}$$

$$\leq \min\{\min(F_A(x), F_B(x)), \min(F_A(y), F_B(y))\}$$

$$\leq \min\{F_{A\cup B}(x), F_{A\cup B}(y)\}.$$

Hence the theorem proved. □

Remark 3.4. *The Union of any family of Neutrosophic fuzzy L-ideal is also a Neutrosophic fuzzy L-ideal.*

Theorem 3.5. *Let A and B be Neutrosophic fuzzy L-ideals. Then $A\cap B$ is a Neutrosophic fuzzy L-ideal.*

Proof. If, $\forall\ x, y \in L$, we have,

(1). $T_{A\cap B}(x\vee y) = \min\{T_A(x\vee y), T_B(x\vee y)\},$

$$= \min\{\min(T_A(x), T_A(y)), \min(T_B(x), T_B(y))\}$$

$$\geq \min\{\min(T_A(x), T_B(x)), \min(T_A(y), T_B(y))\}$$

$$\geq \min\{T_{A\cap B}(x), T_{A\cap B}(y)\}.$$

(2). $T_{A\cap B}(x\wedge y) = \min\{T_A(x\wedge y), T_B(x\wedge y)\},$

$$= \min\{\max(T_A(x), T_A(y)), \max(T_B(x), T_B(y))\}$$

$$\geq \max\{\min(T_A(x), T_B(x)), \min(T_A(y), T_B(y))\}$$

$$\geq \max\{T_{A\cap B}(x), T_{A\cap B}(y)\}.$$

(3). $I_{A\cap B}(x\vee y) = \min\{I_A(x\vee y), I_B(x\vee y)\},$

$$= \min\{\min(I_A(x), I_A(y)), \min(I_B(x), I_B(y))\}$$

$$\geq \min\{\min(I_A(x), I_B(x)), \min(I_A(y), I_B(y))\}$$

$$\geq \min\{I_{A\cap B}(x), I_{A\cap B}(y)\}.$$

(4). $I_{A\cap B}(x\wedge y) = \min\{I_A(x\vee y), I_B(x\vee y)\},$

$$= \min\{\max(I_A(x), I_A(y)), \max(I_B(x), I_B(y))\}$$

$$\geq \max\{\min(I_A(x), I_B(x)), \min(I_A(y), I_B(y))\}$$

$$\geq \max\{I_{A\cap B}(x), I_{A\cap B}(y)\}.$$

(5). $F_{A\cap B}(x\vee y) = \max\{F_A(x\vee y), F_B(x\vee y)\},$

$$= \max\{\max(F_A(x), F_A(y)), \max(F_B(x), F_B(y))\}$$

$$\leq \max\{\max(F_A(x), F_B(x)), \max(F_A(y), F_B(y))\}$$

$$\leq \max\{F_{A\cap B}(x), F_{A\cap B}(y)\}.$$

(6). $F_{A\cap B}(x\wedge y) = \max\{F_A(x\vee y), F_B(x\vee y)\},$

$$= \max\{\min(F_A(x), F_A(y)), \min(F_B(x), F_B(y))\}$$

$$\leq \min\{\max(F_A(x), F_B(x)), \max(F_A(y), F_B(y))\}$$

$$\leq \min\{F_{A\cap B}(x), F_{A\cap B}(y)\}.$$

Hence the theorem proved. □

Remark 3.6. *The intersection of any family of Neutrosophic fuzzy L-ideal is also a Neutrosophic fuzzy L-ideal.*

Proposition 3.7. *A be a Neutrosophic fuzzy L-ideal if and only if* $[A]$, $\langle A\rangle$ *and* (A) *are Neutrosophic fuzzy L-ideals.*

Proof. Firstly assume that, A be a Neutrosophic fuzzy L-ideal. We have $[A] = \{\langle x, T_A(x), I_A(x), T_{A^c}(x)\rangle; x \in L\}$, where $T_{A^c}(x) = F_A(x)$, $\forall\ x, y \in L$.

$$\begin{aligned}
T_A(x \vee y) &\geq \min\{T_A(x), T_A(y)\}\\
T_A(x \wedge y) &\geq \max\{T_A(x), T_A(y)\}\\
T_{A^c}(x \vee y) &= F_A(x \vee y)\\
&\leq \max\{F_A(x), F_A(y)\}\\
&\leq \max\{T_{A^c}(x), T_{A^c}(y)\}.
\end{aligned}$$

Similarly, $T_{A^c}(x \wedge y) \leq \min\{T_{A^c}(x), T_{A^c}(y)\}$. Hence, $[A]$ be a Neutrosophic fuzzy L-ideal. We have $\langle A\rangle = \{\langle x, T_A(x), 1 - I_A(x), F_A(x)\rangle; x \in L\}$, where $I_{A^c}(x) = 1 - I_A(x)$, $\forall\ x, y \in L$.

$$\begin{aligned}
I_A(x \vee y) &\geq \min\{I_A(x), I_A(y)\}\\
I_A(x \wedge y) &\geq \max\{I_A(x), I_A(y)\}\\
I_{A^c}(x \vee y) &= 1 - I_A(x \vee y)\\
&\geq 1 - \min\{I_A(x), I_A(y)\}\\
&\geq \max\{1 - I_A(x), 1 - I_A(y)\}\\
&\geq \max\{I_{A^c}(x), I_{A^c}(y)\}.
\end{aligned}$$

Similarly, $I_{A^c}(x \wedge y) \geq \min\{I_{A^c}(x), I_{A^c}(y)\}$. Hence, $\langle A\rangle$ be a Neutrosophic fuzzy L-ideal. We have $(A) = \{\langle x, F_{A^c}(x), I_A(x), F_A(x)\rangle; x \in L\}$, where $F_{A^c}(x) = T_A(x)$, $\forall\ x, y \in L$.

$$\begin{aligned}
F_A(x \vee y) &\leq \max\{F_A(x), F_A(y)\}\\
F_A(x \wedge y) &\leq \min\{F_A(x), F_A(y)\}\\
F_{A^c}(x \vee y) &= T_A(x \vee y)\\
&\geq \min\{T_A(x), T_A(y)\}\\
&\geq \min\{F_{A^c}(x), F_{A^c}(y)\}.
\end{aligned}$$

Similarly, $F_{A^c}(x \wedge y) \geq \max\{F_{A^c}(x), F_{A^c}(y)\}$. Hence, (A) be a Neutrosophic fuzzy L-ideal.

Conversely, assume that $[A]$, $< A >$ and (A) are Neutrosophic fuzzy L-ideals.

To prove that, A is a Neutrosophic fuzzy L-ideal.

$$\begin{aligned}
T_{A^c}(x \vee y) &\leq \max\{T_{A^c}(x), T_{A^c}(y)\}\\
&\leq \max\{F_A(x), F_A(y)\}\\
&= F_A(x \vee y), \quad \forall\ \ x, y \in L.
\end{aligned}$$

Similarly, $T_{A^c}(x \wedge y) = F_A(x \wedge y)$, $\forall\ \ x, y \in L$.

$$I_{A^c}(x \vee y) \geq \max\{I_{A^c}(x), I_{A^c}(y)\}$$

$$\geq \max\{1 - I_A(x), 1 - I_A(y)\}$$
$$\geq 1 - \min\{I_A(x), I_A(y)\}$$
$$= 1 - I_A(x \vee y), \quad \forall \;\; x, y \in L.$$

Similarly, $I_{A^c}(x \wedge y) = 1 - I_A(x \wedge y)$, $\forall \;\; x, y \in L$.

$$\begin{aligned} F_{A^c}(x \vee y) &\geq \min\{F_{A^c}(x), F_{A^c}(y)\} \\ &\geq \min\{T_A(x), T_A(y)\} \\ &= T_A(x \vee y), \quad \forall \;\; x, y \in L. \end{aligned}$$

Similarly, $F_{A^c}(x \wedge y) = T_A(x \wedge y)$, $\forall \;\; x, y \in L$. Hence, A be a Neutrosophic fuzzy L-ideal. □

4. Homomorphism of Neutrosophic Fuzzy L-Ideals

Definition 4.1. *Let L and L' be two non-empty sets and $f : L \to L'$ be a function.*

(1). If A be a Neutrosophic fuzzy Lattice set in L, the Homomorphic image of A under f is denoted by $f(A)$ is the Neutrosophic fuzzy lattice set in L' defined by,

$$f(A) = \left\{ \langle y, f(T_A)(y), f(I_A)(y), f(F_A)(y) \rangle / y \in L' \right\}.$$

Where,

$$f(T_A)(y) = \begin{cases} \sup\left\{T_A(x) / x \in f^{-1}(y)\right\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$f(I_A)(y) = \begin{cases} \sup\left\{I_A(x) / x \in f^{-1}(y)\right\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$f(F_A)(y) = \begin{cases} \inf\left\{F_A(x) / x \in f^{-1}(y)\right\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

where, $(f(F_A))(y) = (1 - f(1 - F_A(y)))$.

(2). If B be a Neutrosophic fuzzy Lattice set in L', the Homomorphic pre-image of B under f, denote by $f^{-1}(B)$ is the Neutrosophic fuzzy Lattice set in L, define by

$$f^{-1}(B) = \left\{ < x, f^{-1}(T_B)(x), f^{-1}(I_B)(x), f^{-1}(F_B)(x) > / x \in L \right\}.$$

Where, $(f^{-1}(T_B))(x) = T_B(f(x))$ and so on.

Theorem 4.2. *Let L and L' is two lattices and f is homomorphism of L to L'. If A' is a Neutrosophic fuzzy L-ideal of L', then Pre-image of A' is Neutrosophic fuzzy L-ideal of L.*

Proof. Let, $\forall \;\; x, y \in L$, we have

$$
\begin{aligned}
(1).\ f^{-1}\left(T_{A'}\left(x\vee y\right)\right) &= T_{A'}(f\left(x\vee y\right)) \\
&= T_{A'}(f\left(x\right)\vee f\left(y\right)) \\
&\geq \min\{T_{A'}\left(f\left(x\right)\right), T_{A'}\left(f\left(y\right)\right)\} \\
&\geq \min\{f^{-1}\left(T_{A'}\left(x\right)\right), f^{-1}\left(T_{A'}\left(y\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(2).\ f^{-1}\left(T_{A'}\left(x\wedge y\right)\right) &= T_{A'}(f\left(x\wedge y\right)) \\
&= T_{A'}(f\left(x\right)\wedge f\left(y\right)) \\
&\geq \max\{T_{A'}\left(f\left(x\right)\right), T_{A'}\left(f\left(y\right)\right)\} \\
&\geq \max\{f^{-1}\left(T_{A'}\left(x\right)\right), f^{-1}\left(T_{A'}\left(y\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(3).\ f^{-1}\left(I_{A'}\left(x\vee y\right)\right) &= I_{A'}(f\left(x\vee y\right)) \\
&= I_{A'}(f\left(x\right)\vee f\left(y\right)) \\
&\geq \min\{I_{A'}\left(f\left(x\right)\right), I_{A'}\left(f\left(y\right)\right)\} \\
&\geq \min\{f^{-1}\left(I_{A'}\left(x\right)\right), f^{-1}\left(I_{A'}\left(y\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(4).\ f^{-1}\left(I_{A'}\left(x\wedge y\right)\right) &= I_{A'}(f\left(x\wedge y\right)) \\
&= I_{A'}(f\left(x\right)\wedge f\left(y\right)) \\
&\geq \max\{I_{A'}\left(f\left(x\right)\right), I_{A'}\left(f\left(y\right)\right)\} \\
&\geq \max\{f^{-1}\left(I_{A'}\left(x\right)\right), f^{-1}\left(I_{A'}\left(y\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(5).\ f^{-1}\left(F_{A'}\left(x\vee y\right)\right) &= F_{A'}(f\left(x\vee y\right)) \\
&= F_{A'}(f\left(x\right)\vee f\left(y\right)) \\
&\leq \max\{F_{A'}\left(f\left(x\right)\right), F_{A'}\left(f\left(y\right)\right)\} \\
&\leq \max\{f^{-1}\left(F_{A'}\left(x\right)\right), f^{-1}\left(F_{A'}\left(y\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(6).\ f^{-1}\left(F_{A'}\left(x\wedge y\right)\right) &= F_{A'}(f\left(x\wedge y\right)) \\
&= F_{A'}(f\left(x\right)\wedge f\left(y\right)) \\
&\leq \min\{F_{A'}\left(f\left(x\right)\right), F_{A'}\left(f\left(y\right)\right)\} \\
&\leq \min\{f^{-1}\left(F_{A'}\left(x\right)\right), f^{-1}\left(F_{A'}\left(y\right)\right)\}.
\end{aligned}
$$

Hence the theorem proved. □

Theorem 4.3. *Let L and L' is two lattices and f be a homomorphism of L to L'. If A be a Neutrosophic fuzzy L-ideal of L, then image of A be a Neutrosophic fuzzy L-ideal of L'.*

Proof. Let $A = \{\langle x, T_A\left(x\right), I_A\left(x\right), F_A(x)\rangle / x \in L\}$ be a Neutrosophic fuzzy L-ideal of L, then $f\left(A\right) = \{\langle y, f\left(T_A\right)\left(y\right), f\left(I_A\right)\left(y\right), f(F_A(y)\rangle / y \in L'\}$. If, $\forall\ x_1, x_2 \in L$ and $\forall\ y_1, y_2 \in L'$, we have,

$$
\begin{aligned}
(1).\ f\left(T_A\left(y_1\vee y_2\right)\right) &= \sup\{T_A\left(x_1\vee x_2\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \sup\left\{\min\left(T_A\left(x_1\right), T_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\right\} \\
&\geq \min\{\sup(T_A\left(x_1\right)), \sup\left(T_A\left(x_2\right)\right)\ /x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \min\{f(T_A\left(y_1\right)), f\left(T_A\left(y_2\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(2).\ \ f\left(T_A\left(y_1 \wedge y_2\right)\right) &= \sup\{T_A\left(x_1 \wedge x_2\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \sup\left\{\max(T_A\left(x_1\right), T_A\left(x_2\right))/x_1, x_2 \in f^{-1}\left(L'\right)\right\} \\
&\geq \max\{\sup(T_A\left(x_1\right)), \sup\left(T_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \max\{f(T_A\left(y_1\right)), f\left(T_A\left(y_2\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(3).\ \ f\left(I_A\left(y_1 \vee y_2\right)\right) &= \sup\{I_A\left(x_1 \vee x_2\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \sup\left\{\min\left(I_A\left(x_1\right), I_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\right\} \\
&\geq \min\{\sup(I_A\left(x_1\right)), \sup\left(I_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \min\{f(I_A\left(y_1\right)), f\left(I_A\left(y_2\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(4).\ \ f\left(I_A\left(y_1 \wedge y_2\right)\right) &= \sup\{I_A\left(x_1 \wedge x_2\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \sup\left\{\max(I_A\left(x_1\right), I_A\left(x_2\right))/x_1, x_2 \in f^{-1}\left(L'\right)\right\} \\
&\geq \max\{\sup(I_A\left(x_1\right)), \sup\left(I_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\geq \max\{f(I_A\left(y_1\right)), f\left(I_A\left(y_2\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(5).\ \ f\left(F_A\left(y_1 \vee y_2\right)\right) &= \inf\{F_A\left(x_1 \vee x_2\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\leq \inf\left\{\max(F_A\left(x_1\right), F_A\left(x_2\right))/x_1, x_2 \in f^{-1}\left(L'\right)\right\} \\
&\leq \max\{\inf(F_A\left(x_1\right)), \inf\left(F_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\leq \max\{f(F_A\left(y_1\right)), f\left(F_A\left(y_2\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
(6).\ \ f\left(F_A\left(y_1 \wedge y_2\right)\right) &= \inf\{F_A\left(x_1 \wedge x_2\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\leq \inf\left\{\min\left(F_A\left(x_1\right), F_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\right\} \\
&\leq \min\{\inf(F_A\left(x_1\right)), \sup\left(F_A\left(x_2\right)\right)/x_1, x_2 \in f^{-1}\left(L'\right)\} \\
&\leq \min\left\{f\left(F_A\left(y_1\right)\right), f\left(F_A\left(y_2\right)\right)\right\}
\end{aligned}
$$

Hence the theorem proved. □

Theorem 4.4. *If $f : L \to L'$ be homomorphic map and A and A' be Neutrosophic fuzzy sets of the Lattices L and L' respectively. Then,*

(1). $f\left(f^{-1}\left(A'\right)\right) = A'$,

(2). $A = f^{-1}(f\left(A\right))$.

Proof.

(1). If $\forall\ \ y \in L'$ followed by,

$$
\begin{aligned}
f\left(f^{-1}\left(T_{A'}\right)\left(y\right)\right) &= \sup\left\{f^{-1}\left(T_{A'}\right)\left(x\right)/x \in f^{-1}\left(y\right)\right\} \\
&= \sup\{T_{A'} f\left(x\right)/x \in L, f\left(x\right) = y\} \\
&= T_{A'}(y). \\
f\left(f^{-1}\left(I_{A'}\right)\left(y\right)\right) &= \sup\{f^{-1}\left(I_{A'}\right)\left(x\right)/x \in f^{-1}\left(y\right)\} \\
&= \sup\{I_{A'} f\left(x\right)/x \in L,\ f\left(x\right) = y\}
\end{aligned}
$$

$$
\begin{aligned}
&= I_{A'}(y) . \\
f\left(f^{-1}(F_{A'})(y)\right) &= \inf\{f^{-1}(F_{A'})(x) / x \in f^{-1}(y)\} \\
&= \inf\{F_{A'} f(x) / x \in L,\ f(x) = y\} \\
&= F_{A'}(y) .
\end{aligned}
$$

Hence the part is proved.

(2). If $\forall\ x \in L$, we have,

$$
\begin{aligned}
f^{-1}(f(T_A))(x) &= f(T_A)(f(x)) \\
&= \sup\{T_A(x) / x \in f^{-1}(x)\} \\
&= T_A(x) . \\
f^{-1}(f(I_A))(x) &= f(I_A)(f(x)) \\
&= \sup\{I_A(x) / x \in f^{-1}(x)\} \\
&= I_A(x) . \\
f^{-1}(f(F_A))(x) &= f(F_A)(f(x)) \\
&= \inf\{F_A(x) / x \in f^{-1}(x)\} \\
&= F_A(x) .
\end{aligned}
$$

Hence the theorem proved. □

5. Conclusion

In this paper, we studied the concept of Neutrosophic fuzzy L-ideals and discussed some algebraic properties. We have proved that intersection of two Neutrosophic fuzzy L-ideal is a Neutrosophic fuzzy L-ideal. Then we have studied the homomorphism of Neutrosophic fuzzy L-ideals and discussed some basic algebraic properties of lattices. Also, we can extend the result for Neutrosophic fuzzy L-filters.

Acknowledgements

The author would like to thank the referee for gives helpful suggestion for raising the work.

References

[1] I. Arockiarani and C. Antony Crispin Sweety, *Rough Neutrosophic set in a lattice*, International Journal of Applied Research, 2(5)(2016), 143-150.

[2] M. Mullai and B. Chellappa, *Fuzzy L-ideal*, ActaCiencia, XXXVM(2)(2009).

[3] K. R. Sasireka, K. E. Sathappan and B. Chellappa, *Intuitionistic Fuzzy L-Filters*, International Journal of Mathematics and its Applications, 4(2-C)(2016), 171-178.

[4] A. Solairaju and S. Thiruveni, *Neutrosophic Fuzzy Ideal of Near rings*, International Journal of Pure and Applied Mathematics, 118(6)(2018), 527-539.

[5] K. V. Thomas and S. Latha Nair, *Intuitionistic fuzzy sublattice and ideals*, Fuzzy information and Engineering, 3(2011), 321-331.

[6] Vakkas Ulucay, Mehmet Sahin, Necati Olgun and Adem killicman, *On Neutrosophic soft Lattices*, African Mathematical Union, (2016).

[7] Vasantha Kandasamy and Florentin Smarandache, *Neutrosophic Lattices*, Neutrosophic sets and systems, 2(2013).

[8] Vildan Cetkin and Halis Aygun, *An approach to Neutrosophic Ideals*, Universal Journal of Mathematics and Applications, 1(2)(2018), 132-136.

[9] L. A. Zadeh, *Fuzzy sets*, Inform. Control, 8(1965), 338-353.