

Modified neutrosophic fuzzy optimization model for optimal closed-loop supply chain management under uncertainty

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15.1 Introduction

Needs for commodities or products for human life explored and enhanced a revolution in industrial sectors. An initially integrated mechanism for production to consumption of commodities was only the goals of any firm. Decision policies concerning the production and use of products were the prime concern. Therefore, systematic and business-oriented managerial practices were designed for the flow of products, termed as supply chain management (SCM). SCM is the procedure of procurement, processing, distribution, and consumption of finished products in a clear planning timescale. The general structural domain of SCM includes a raw material supplier point, a manufacturing plant, a distribution center, and the end-users or customers. These echelons are interconnected or interlocked to each other for the movement of different materials and products. The organizational and managerial perspective of SCM terminates at the end-users of finished products and terminates from ultimately the next stages related to the three R's (reduce, reuse, and recycle). End-of-use and end-of-life products create various environmental issues due to improper management of used products. Consequently, harmful impacts due to landfills, contamination of freshwater resources, and toxic air pollution generated on a large scale influenced human life drastically. These issues could not be compensated at any cost. To ensure that environmental questions and social concerns arise during supply chain design, a government has taken the initiative and established laws that include wholesome supply chain practices, termed as the closed loop supply chain (CLSC) network. The CLSC design helps in strengthening the ecofriendly practices with end-of-use products and reduces environmental impacts. Therefore, to reveal pervasiveness in SCM, extension of echelons has been located. Hence the concept of the reverse chain has been identified to execute backward processes for used products. Generally, the reverse chain consists of different echelons, such as the collection center, recycling

point, and disposal sites. The CLSC design contemplates the flow of different materials, products, and parts of the used commodity in a well-defined interconnected path. Various facility centers in the reverse chain reduce the environmental impacts and substantiates ecofriendly production-consumption planning scenarios. For successful completion of sustainable trade practices, the significant role of the CLSC design network would be crucial or at least prominent. Ultimate destinations for end-of-use and end-of-life products would be more rigorously depicted in CLSC design. Refurbishing and recycling centers inevitably provide services to the used products and parts to transform into their useful life. The marginal reduction in different kinds of costs and a significant increase in revenues are the counterpart for enhancement in net profit throughout the CLSC planning network.

Consumerism has been a considerable part of the sustainability problem for years by imposing a burden with harmful waste through flooding and landfill issues. The CLSC business model implements highly efficient management of materials and waste minimization strategies that lead to zero-waste generation. The CLSC management network includes either putting all outputs back into the system or incineration. A combination of forwarding and reverse material flows to reuse and recycle all metals and transform waste into energy. The CLSC can enable manufacturers to take a proactive stance toward and ensure easy compliance with electronic waste regulations. Environmental value is the ease of agreement to be more conscious about the environment. A CLSC can allow the business to respond to ecological concerns by saving energy and decreasing the input of new materials. Consumer value can be achieved by a well-organized customer product returns system that can help ensure hassle-free warranties and improve customer loyalty. Improved parts management helps the business deliver extended warranties and service agreements that can boost customer satisfaction. The acquisition process in CLSC management provides valuable data on common production issues, supply defects, failure rates, product life-cycle, consumer complaints, and consumer usage patterns. This information can be used to improve product design and development. Minimize wastewater and industrial sludge production by reducing the amount of water needed for the manufacturing process. Procure raw material in bulk (where possible) to reduce the amount of packaging material that enters the waste stream. Assure precautions to avoid the process that causes hazardous waste to be mixed with nonhazardous waste, minimizing the amount of dangerous waste that must be stored, treated, and disposed of. Practice quality control strategies like ISI 14001 and Six Sigma to help minimize product defects.

The implicitness of uncertainty is trivial in real-life scenarios. Inconsistent, incomplete, inappropriate, inexact, and improper information about various input parameters such as costs, capacity, and demand in the CLSC design network lead to the existence of uncertainty theory. Several aspects inherently affect the modeling and optimizing procedure of real-life optimization problem. Abrupt changes in the prices of raw materials, hike in fuel rates, increases in required facility locations, behavior of fluctuating markets, competition among different companies' policies for customer satisfaction, environmental conditions, failing in timely shipment of ordered products,

political and governmental policies regarding various taxes over procurement, production, distribution, and management of end-of-use products are the most dominating factors for causing uncertainty in modeling approaches. Impreciseness may be represented in different forms. The difficulty involved in parameters due to vague information can be dealt with by different fuzzy techniques. Fuzziness among parameters most frequently encounters and results in uncertainty modeling. To reflect the most common aspect of uncertainty, we have assumed that all the input parameters are a triangular and trapezoidal fuzzy number rather than stochastic random variables. Defuzzification or the ranking function executes the process of obtaining the crisp or deterministic version of a fuzzy number. A robust technique has been used, which covers an extensive range of feasibility degrees. Most of the conventional methods are limited to fuzzy-based solution schemes by defining the marginal evaluation of each objective using the membership function. Apart from metaheuristic techniques, a tremendous number of research papers have investigated and implemented the different fuzzy optimization techniques to obtain the global compromise solution of the CLSC planning problems. A detailed list of such fuzzy approaches can be found in Govindan et al. [1] and Govindan and Bouzon [2]. Here in this study, a neutrosophic fuzzy programming approach (NFPA) based on the neutrosophic decision has been suggested to solve the proposed CLSC design problem. Intuitionistic fuzzy imprecise preference relations among different objectives have also been investigated and successfully incorporated with an NFPA which is together termed as modified NFPA with intuitionistic fuzzy importance relations.

The rest part of this chapter is as follows: In [Section 15.2](#), a literature review related to the CLSC network is presented whereas [Section 15.3](#) highlights the significant research contribution. [Section 15.4](#) discusses the modeling CLSC design network under uncertainty while [Section 15.5](#) represents the solution methodology to solve the final model. A real-life case study based on a laptop manufacturing firm is examined in [Section 15.6](#), which shows the applicability and validity of the proposed approach efficiently. Finally, conclusions are highlighted based on the present work in [Section 15.7](#).

15.2 Literature review

The CLSC planning problem has rapidly gained popularity among many researchers. The complex and challenging situation during the flow of goods and products from different sources to destination points has immensely attracted attention toward emerging research scope for the optimal policy implementation or decision-making processes to CLSC planning problems. Consequently, different approaches to solve the CLSC planning model have been introduced, along with their promising features in the context of optimality and applicability under different environments. Thus, here we review some existing CLSC models under different uncertainty and discuss the approaches adopted to solve them.

A well-defined set of the interconnected network for the flow of multiple products has also created a very complex configuration of multiechelon CLSC design. Most of the existing studies have been presented on multiproduct and multiechelon CLSC planning problems. Gupta and Evans [3] have addressed multiple-echelon CLSC frameworks for electrical and electronic gadget scrap products. They designed a weighted nonprimitive goal programming model for the CLSC model and solved the proposed model with the aid of a discrete weighting scheme to the corresponding goal preference. Pishvaei et al. [4] designed a robust optimization model for CLSC configuration under randomly distributed parameters. The developed modeling approach then turned into the deterministic mixed-integer linear programming model and they solved this using a robust optimization technique. Özceylan and Paksoy [5] also presented a mixed-integer fuzzy mathematical model for CLSC under uncertainty with multiparts and multiperiods. The fuzzy solution approach has been applied for both fuzzy objectives and parameters with the help of a linear membership function. Özkır and Başlıgil [6] developed a multiobjective CLSC model with particular emphasis on the satisfaction level of trade, customer, and net profit incurred over the current product's lifetime in the supply chain network. They adopted a fuzzy set (FS) theory-based solution method to deal with the proposed CLSC model. Yin and Nishi [7] also discussed an SCM problem with a quantity discount and uncertain demand at each echelon. The constructed SCM model resulted in the form of a mixed-integer nonlinear programming problem (MINLPP) with integral functions. An outer-approximation method has been suggested to solve the MINLPP. An improvement in efficiency performance has been achieved by reconstructing the MINLPP model into a stochastic programming model with the replacement of integral functions by incorporating the normalization method. Özceylan and Paksoy [8] addressed the CLSC planning model under tactical and strategic decision scenarios. The developed CLSC planning model has emerged as an MINLPP. They applied a fuzzy interactive solution approach to solve the propounded CLSC network design. Garg et al. [9] also designed a sustainable CLSC network with the core emphasis on environmental issues raised after the end of use and end of life of the used products. They formulated a bi-objective integer nonlinear programming problem for the proposed CLSC network. The solution scheme has been adopted and applied by balancing the trade-off between socioeconomic and environmental aspects. The interactive multiobjective programming approach has been used to obtain the optimal allocation of different products. Alshamsi and Diabat [10] presented the reverse logistic (RL) system in the CLSC design network. The proposed RL texture initiates at the customer level and terminates at remanufacturing facilities in the reverse supply chain. The presented study was found to be limited to the RLs system. They modeled the deterministic mixed-integer linear programming problem with a single objective. A sustainable supply chain network has been designed by Arampantzi and Minis [11] and incorporates various factors, such as social, capital investment, environmental, political, etc., that affect the supply chain network directly and indirectly. They formulated a multiobjective mixed-integer linear programming problem and solved it by using two different conventional techniques: the goal programming method and the ϵ -constrained method.

Ma and Li [12] discussed a CLSC model for hazardous products under different uncertain parameters. The motive was to determine the optimal quantity of a shipment under a probabilistic environment. To address the scenario efficiently, the proposed model has been reformulated as a two-stage stochastic programming model along with risk and reward constraints. The two solution approaches, the Parallel Enumeration Method and the Genetic Algorithm (GA), have been applied to solve the designed CLSC model.

Fard and Hajaghaei-Keshteli [13] also addressed a tri-level location-allocation planning problem for a CLSC network. The modeling study undertaken comprises three echelons: distribution center, customer zone, and recovery facility. The propounded tri-level CLSC planning model has been solved by using a Variable Neighborhood Search, Tabu Search (TS), and Particle Swarm Optimization in addition to these approaches; Fard and Hajaghaei-Keshteli further applied two recent meta-heuristic algorithms, the Keshtel Algorithm and Water Wave Optimization, to obtain a feasible solution to the location-allocation problem. Zhen et al. [14] also designed a CLSC model with the capacitated allocation of products under uncertain demand for new and returned merchandise. The proposed decision-making model turned into a two-stage stochastic mixed-integer nonlinear programming problem (SMINLPP). Thus, the transformed model resulted in the deterministic demand and capacities parameters involved in the designed CLSC model. They also implemented the TS algorithm to solve the SMINLPP. Tsao et al. [15] formulated a sustainable supply chain design under economic and environmental objectives. The proposed supply chain model has taken the form of a multiobjective mathematical programming problem under stochastic demand and fuzzy costs. An interactive two-phase fuzzy probabilistic multiobjective programming problem has been introduced to deal with both sorts of uncertainty. Hasanov et al. [16] addressed the optimal quantity of products under four-level CLSC with a hybrid remanufacturing facility. The reverse chain includes the recovered process, which ensures the reuse of used products at a different level. The mathematical modeling framework has been carried out with a particular emphasis on remanufactured or returned products, or both. The developed modeling approach is aiming to minimize the overall cost incurred over the policies implemented during a single time horizon. Fakhrazad et al. [17] presented multiple products, periods, levels, and indices in the green CLSC planning model under uncertainty. The propounded model was then transformed into the multiobjective mixed-integer linear programming problem. Since the proposed model was NP-hard, to deal with it Nondominated Sorting Genetic Algorithm-II (NSGA-II) has been adopted to solve the proposed green CLSC network. Singh and Goh [18] also discussed the multiobjective mixed-integer linear programming problem under intuitionistic fuzzy parameters. Further, they transformed the multiobjective optimization problem into a single objective to solve the model. To achieve an acceptable satisfaction degree, different scalarization techniques such as the γ -connective approach and minimum sum bounded operator have been used. The proposed solution scheme has also been implemented to solve the pharmaceutical SCM model. Fathollahi-Fard et al. [19] designed a multiobjective stochastic CLSC model with an exclusive focus on the

social issues associated with individual requirement and responsibility (such as job opportunity). The addressed stochastic CLSC model has been solved by using a couple of different nature-inspired algorithms and hybridized into the benefits of both, that is, social and environmental domains. Liao [20] presented a reverse logistics network design (RLND) for product recovery and remanufacturing processes. The proposed model emerged into a conventional mixed-integer nonlinear programming model for RLND under multiple echelons. The GA has been adopted as the solution method of the proposed RLND model. The formulated modeling structure has been validated and implemented with the help of the recycling bulk waste example in Taiwan. Zorbakhshnia et al. [21] have also discussed the green closed loop logistics network model as the mixed-integer linear programming problem. The undertaken study has mainly been concerned with the multiple stages, products, and objectives in the proposed model. A solution scheme, the ϵ -constraint method, has been chosen to solve numerous targets. Dominguez et al. [22] also investigated the role of manufactured and remanufactured products in the CLSC with capacitated constraints. The research background explicitly reveals the four relevant uncertain factors to determine the efficiency of executed policies in the system. A managerial insight has been propounded that could contribute to understanding decision-making processes. Eskandarpour et al. [23] presented a study on the literature review of approximately 80 research papers in the field of CLSC planning problems. The chosen study area has been classified based on four questions: (i) What kind of socioeconomic and environmental issues have been included? (ii) How the problems related to the matters discussed have been unified or integrated in the supply chain model? (iii) What sort of solution schemes have been applied to solve the modeling problem? and (iv) Which numerical illustrations or computational studies have been taken from real-life applications? Furthermore, the shortcomings and drawbacks of different models have been pointed out, and consequently, the scope for future research has also been intimated. The interested reader may refer to the recent publications by Govindan et al. [1] and Govindan and Bouzon [2], based on reviewed work in the reverse logistic barriers and drivers.

15.3 Research contribution

A tremendous amount of work has been developed and applied successfully on the CLSC network in the last few decades. Only a few research works are available that have included the testing center as a facility location for the disassembled parts/components [3, 9]. Therefore, this chapter has put more emphasis on the reverse chain and is mainly concerned with end-of-use products and end-of-life. The modified neutrosophic fuzzy optimization techniques have been used for the first time in the field of SCM.

The following are the significant and remarkable contributions to this presented research work.

- The proposed CLSC planning model has been designed for multidimensional echelons, in which five multiple echelons have been included in the forwarding chain, whereas six

various echelons have been integrated into the reverse chain which shows the great concern or influence regarding the end-of-use and end-of-life products. The different facility centers in the reverse chain ensure that the CLSC planning model is socioeconomic and environmentally friendly.

- The different objective functions have been presented to analyze the shares in total capital investment over the raw materials and products in the forward and reverse chain individually. A new preference scheme has been investigated to achieve better outcomes for the preferred objective functions.
- The uncertainty among parameters has been represented with fuzzy numbers and dealt with the expected interval and expected values of the involved parameters. Three constraints have been depicted with fuzzy equality in restrictions, which reveals the reality more closely. The fuzzy equality constraints are then efficiently transformed into two subconstraints.
- The NFPA has been developed to solve the proposed CLSC designed model. The proposed solution approach has been inspired by the indeterminacy degree that emerged in decision-making processes. Indeterminacy/neutral thoughts are the region of negligence for propositions' values, between the degree of acceptance and rejection. It is the first time that the NFPA has been applied to solve the CLSC planning model.
- A novel intuitionistic fuzzy linguistic preference scheme has been investigated to assign weight/preference to the most preferred objective functions. The intuitionistic fuzzy linguistic preference relations have been efficiently integrated with an NFPA and termed as a modified NFPA.
- The proposed CLSC designed model has been implemented on real case study data to show the validity and applicability of the proposed solution methods. A variety of different solutions sets has been generated and summarized under the optimal choices of quantity allocation.
- The sensitivity analysis has also been performed on the obtained solution results based on the feasibility degree β and crisp weight parameter α by tuning them at different values between 0 and 1.
- The significance of the obtained results has been analyzed along with the remarkable findings. Conclusions and future research scope have been set out based on the present study.

15.4 Description of CLSC network

A well-organized systematic and interconnected network for the flow of materials, products, and parts is much needed to survive in the competitive market. Production processes explicitly adhere to the different perspectives of the finished products. The conventional supply chain design initiates with the availability of raw resources to finished goods and terminates at the consumption points. The globalization of markets, governmental legislation, and environmental practices creates many concerns for the used products and leads to the existence of a CLSC that inherently ensures the best management of end-of-use and end-of-life products. The efficiently expanded texture of the supply chain network designed has been widely adopted by the decision maker(s) with the inclusion of the reverse chain. Therefore, the CLSC network consists of two phases: forward chain and reverse chain for the flow of material, products, and used parts. In this study, a CLSC design is presented, which consists of five echelons in the forward chain and six echelons in the reverse chain, which is shown in [Fig. 15.1](#).

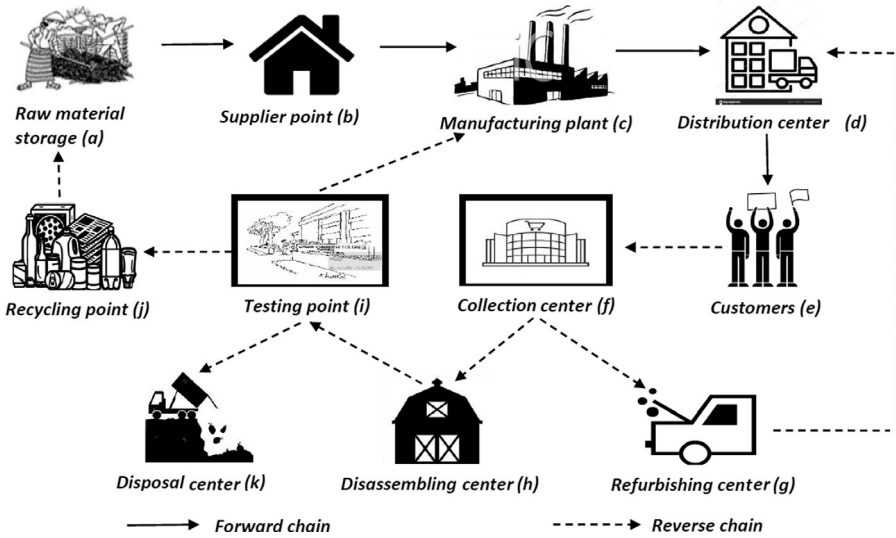


Fig. 15.1 Proposed closed loop supply chain network design.

The initialization of production processes starts with the procurement of raw materials from the storage center to the supplier point, which in turn supplies the relevant raw materials to the manufacturing plant for the production of new products. Afterward, the finished products are delivered to the distribution center to fulfill the demand of customers or markets. Unlike the forward logistics flow, the reverse logistics flow consists of a few more steps. The first step involves collecting defective products from customers at the collection center. The end-of-use products are outsourced from different customers, either directly or via markets. Collection of used products initiates the sustainable reverse chain, and collecting the used products maintains the flow cycle of products into a different facility phase. The collection center is responsible for the optimal distribution of used products for further required services. In most cases, returns processors collect fewer defective products, undertake repairs at refurbishing centers, and return them to the buyers. At this point, it is worth noting that returns processors may remanufacture defective products and ship them back to retailers and distributors, who in turn sell them to end users. Alternatively, returns processors may recycle defective products to extract materials and parts that can be reused in the production process by sending them to a disassembling center. Further, the parts and materials are taken to a testing point for the inspection of their further utility, and from there the elements that can be used to make new goods are sent back to the hybrid manufacturing plant. On the other hand, only the parts that are recyclable are shipped to the recycling point, and move forward through the supply chain until they reach the end users. The final step involves any materials or parts that are not utilized throughout the steps discussed earlier, which reach the disposal center for incineration or dumping purposes.

To and fro movement of materials, products, and parts throughout the CLSC network contemplates over multifarious objectives associated with the entire phenomenon. Procurement, processing, distribution, and transportation processes turn into a significant investment in costs which should be optimized under optimal allocation of the commodity. The cost of purchasing raw materials and used products is also a measure of great concern. Delivery time of the finished products to the customers must be reduced to overcome cancellations of ordered products. Revenues from sales of new products and recyclable parts encourage the enhancement of shares in the net profit. Hence the proposed CLSC model comprises multiple conflicting objectives such as minimization of processing, purchasing, and transportation costs, minimization of expected product delivery time, and maximization revenue from the selling of the products.

The propounded CLSC planning network configuration is based on the following postulated assumptions:

- The propounded CLSC network has been designed for multiple raw materials/parts multi-products, and multiechelons along with single time horizons. Each facility location is well established and functional for the associated services over the stipulated period.
- Movement of new products initiates from manufacturing plants to customers, and the flow of used products starts from customers to the disassembling center. Meanwhile, the recovered products are also shipped after renovation to the distribution center. Therefore, the demand for new and refurbished products is met through the distribution center only.
- Set-up costs associated with different echelons are assumed to be included in the processing costs. Revenues are only derived from the selling prices of new products and recyclable products, which turn into a contribution to the net profit.
- The disposal facility is the only route to remove the scrap parts/components from the proposed CLSC planning model. The rest of the quantity is assumed to remain in its useful life.
- Uncertainty among different parameters has been considered as fuzzy numbers. The fuzzy linguistic term has assigned the preference among different objective functions.

Indices	Descriptions
<i>a</i>	The number of raw materials/parts/components storage points ($a = 1, 2, \dots, A$)
<i>b</i>	The number of supplier points ($b = 1, 2, \dots, B$)
<i>c</i>	The number of manufacturing/remanufacturing plants ($c = 1, 2, \dots, C$)
<i>d</i>	The number of distribution center ($d = 1, 2, \dots, D$)
<i>e</i>	The number of customers/markets ($e = 1, 2, \dots, E$)
<i>f</i>	The number of collection center ($f = 1, 2, \dots, F$)
<i>g</i>	The number of refurbishing/repairing center ($g = 1, 2, \dots, G$)
<i>h</i>	The number of disassembling center ($h = 1, 2, \dots, H$)
<i>i</i>	The number of raw materials/parts/components testing points ($i = 1, 2, \dots, I$)
<i>j</i>	The number of recycling points ($j = 1, 2, \dots, J$)
<i>k</i>	The number of disposal centers ($k = 1, 2, \dots, K$)
<i>l</i>	The number of different products ($l = 1, 2, \dots, L$)

m	The number of raw materials/parts/components storage points ($m = 1, 2, \dots, M$)
Decision variables	Descriptions
$X1_{m,a,b}$	The quantity of raw material m shipped from raw material storage point a to supplier point b
$X2_{m,b,c}$	The quantity of raw material m shipped from supplier point b to manufacturing plant c
$X3_{l,c,d}$	The quantity of different products l shipped from manufacturing plant c to different distribution center d
$X4_{l,d,e}$	The quantity of different products l shipped from different distribution center d to different customers/markets e
$X5_{l,e,f}$	The quantity of different used products l shipped from different customers/markets e to collection center f
$X6_{l,f,g}$	The quantity of different repairable products l shipped from collection center f to refurbishing center g
$X7_{l,g,d}$	The quantity of different recovered products l shipped from refurbishing center g to different distribution center d
$X8_{l,f,h}$	The quantity of different unrepairable products l shipped from collection center f to disassembling center h
$X9_{m,h,i}$	The quantity of parts/components m shipped from disassembling center h to testing point i
$X10_{m,i,c}$	The quantity of raw material m shipped from testing point i to manufacturing plant c
$X11_{m,i,j}$	The quantity of recyclable parts/components m shipped from testing point i to recycling point j
$X12_{m,i,k}$	The quantity of scrap parts/components m shipped from testing point i to disposal center k
$X13_{m,j,a}$	The quantity of recovered parts/components m shipped from recycling point j to raw materials storage point a
Parameters	Descriptions
rf_l	Recovery rate of used products l at refurbishing center
$rc_{l,e}$	Collection rate of used products l from customer or market e
rt_m	Testing rate of different parts/components m at testing center
rm_m	Reuse rate of different tested parts/components m at manufacturing plant
rr_m	Recycling rate of different recyclable parts/components m at recycling center
rd_m	Disposal rate of raw materials/parts/components m at disposal center

Parameters	Descriptions
$PC1_{m,a}$	Unit storage cost incurred over raw material m at raw material storage center a
$PC2_{m,b}$	Unit safety cost incurred over raw material m at supplier point b
$PC3_{l,c}$	Unit production cost levied over product l at manufacturing plant c
$PC4_{l,d}$	Unit inventory holding cost levied over product l at distribution center d
$PC5_{l,f}$	Unit collection facility cost levied over product l at collection center f
$PC6_{l,g}$	Unit refurbishing cost levied over product l at refurbishing center g
$PC7_{l,h}$	Unit disassembling cost levied over product l at disassembling center h
$PC8_{m,i}$	Unit testing cost levied over each component m at testing center i

$PC9_{m,j}$	Unit recycling cost levied over raw material m at recycling point j
$PC10_{m,k}$	Unit disposal cost levied over each component m at disposal center k
$TC1_{m,a,b}$	Unit transportation cost of raw material m shipped from raw material storage point a to supplier point b
$TC2_{m,b,c}$	Unit transportation cost of raw material m shipped from supplier point b to manufacturing plant c
$TC3_{l,c,d}$	Unit transportation cost of different products l shipped from manufacturing plant c to different distribution center d
$TC4_{l,d,e}$	Unit transportation cost of different products l shipped from different distribution center d to different customer/market e
$TC5_{l,e,f}$	Unit transportation cost of different used products l shipped from different customers/markets e to collection center f
$TC6_{l,f,g}$	Unit transportation cost of different repairable products l shipped from collection center f to refurbishing center g
$TC7_{l,g,d}$	Unit transportation cost of different recovered products l shipped from refurbishing center g to different distribution center d
$TC8_{l,f,h}$	Unit transportation cost of different unrepairable products l shipped from collection center f to disassembling center h
$TC9_{m,h,i}$	Unit transportation cost of parts/components m shipped from disassembling center h to testing point i
$TC10_{m,i,c}$	Unit transportation cost of parts/components m shipped from testing point i to manufacturing plant c
$TC11_{m,i,j}$	Unit transportation cost of different recyclable parts/components m shipped from testing point i to recycling point j
$TC12_{m,i,k}$	Unit transportation cost of disposable parts/components m shipped from testing point i to disposal center k
$TC13_{m,j,a}$	Unit transportation cost of recovered parts/components m shipped from recycling point j to raw materials storage point a
$T_{l,d,e}$	Unit transportation time required to ship different products l from distribution center d to different customers/markets e
$PU1_m$	Unit purchasing cost of raw materials/parts/components m
$PU2_l$	Unit purchasing cost of different used products l
$SP1_m$	Unit selling price of different recyclable parts/components m
$SP2_l$	Unit selling price of different new products l
$MC1_{m,a}$	Maximum available quantity of raw material m at raw material storage center a
$MC2_{m,b}$	Maximum available quantity of raw material m at supplier b
$MC3_{m,c}$	Minimum required quantity of raw material m at manufacturing plant c
$MC4_{l,d}$	Maximum available quantity of new products l at distribution center d
$MC5_{l,e}$	Minimum demand quantity of different new products l by customers or at markets e
$MC6_{l,f}$	Maximum collection capacity of different used products l at collection center f
$MC7_{l,g}$	Maximum refurbishing capacity of different repairable products l at refurbishing center g
$MC8_{m,h}$	Maximum disassembling capacity of different parts/components m at disassembling center h

$MC9_{m,i}$	Maximum testing capacity different scrap parts/components m at testing point i
$MC10_{m,j}$	Maximum capacity of recyclable parts/components m at recycling point j
$MC11_{m,k}$	Maximum disposal capacity of disposable parts/components m at disposal center k

15.4.1 Multiple objective function

The typical and efficient CLSC model always comprises multiple conflicting objectives for both forward and reverse chains, which are to be attained simultaneously. Here, we highlight the different costs associated with ahead and change strings separately to analyze the echelon-wise effects in terms of expenditure on the overall CLSC planning problem.

Objective 1: Total processing costs. Initially, the raw materials have been stored at a raw material storage center to ensure the smooth running of the CLSC design. The processing cost indicates the different sort of value at each echelon such as storage cost at the raw material storage center, safety cost at the supplier point, production cost at the manufacturing center and inventory or distribution cost at the distribution center, levied on the unit raw material or new products. The significant reduction in these processing costs automatically results in the maximum margin of profit. The reverse chain also contains multiple echelons with different processing costs associated with them. Here, the processing cost refers to the value of the collection at the collection center, the cost of disassembly at the disassembling center, the refurbishing cost at the refurbishing center, the cost of testing at the testing center, the cost of recycling at the recycling center, and the disposal cost at the disposal point, respectively. The designed network facility executed by each echelon ensures that the commonly used products in the reverse supply chain survive at their end-of-life use or disposable condition. Thus the first objective function ensures the minimization of the processing costs at different echelon in the forward chain under the optimal quantity allocation.

$$\begin{aligned}
 \text{Minimize } Z_1 = & \sum_{m=1}^M \sum_{a=1}^A PC1_{m,a} X1_{m,a,b} + \sum_{m=1}^M \sum_{b=1}^B PC2_{m,b} X2_{m,b,c} \\
 & + \sum_{l=1}^L \sum_{c=1}^C PC3_{l,c} X3_{l,c,d} + \sum_{l=1}^L \sum_{d=1}^D PC4_{l,d} X4_{l,d,e} \\
 & + \sum_{l=1}^L \sum_{f=1}^F PC5_{l,f} X5_{l,e,f} + \sum_{l=1}^L \sum_{g=1}^G PC6_{l,g} X6_{l,f,g} \\
 & + \sum_{l=1}^L \sum_{h=1}^H PC7_{l,h} X8_{l,f,h} + \sum_{i=1}^M \sum_{i=1}^I PC8_{m,i} X10_{m,i,c} \\
 & + \sum_{m=1}^M \sum_{j=1}^J PC9_{m,j} X11_{m,i,j} + \sum_{m=1}^M \sum_{k=1}^K PC10_{m,k} X12_{m,i,k} \quad \forall c, f, g, h, a, b, c, d, e, i.
 \end{aligned}$$

Objective 2: Total transportation costs.

The transportation cost is one of the well-known objective functions under CLSC design. Typical and interconnected transportation networks within each echelon in CLSC design yield high transportation costs. In the forward chain, the shipment of raw material from the raw material storage point to the supplier point and from the supplier point to the manufacturing plant integrates the marginal shares in the total transportation cost. The delivery of new products from the manufacturing plant to the distribution center and from the distribution center to customers also has a significant role in attaining the gross profit in the proposed CLSC network. The propounded CLSC network has put more emphasis on the reverse chain by including more facility locations compared to the forward chain. The to and fro shipment of used products and raw parts/components results in high transportation costs. The reverse chain network allows the recovered products and tested parts/components to enter into the forward chain directly from the refurbishing center to the distribution center and from the testing point to the manufacturing plant without touching the recycling facility. Hence to and fro shipment of products and parts/components from multiple different echelons is turned into high transportation costs. Therefore, the second objective function results in the minimization of to and fro transportation costs to varying echelons in the forward chain for the maximum shipment quantity of products under the optimal allocation policy.

$$\begin{aligned}
 \text{Minimize } Z_2 = & \sum_{m=1}^M \sum_{a=1}^A \sum_{b=1}^B TC1_{m,a,b} X1_{m,a,b} + \sum_{m=1}^M \sum_{b=1}^B \sum_{c=1}^C TC2_{m,b,c} X2_{m,b,c} \\
 & + \sum_{l=1}^L \sum_{c=1}^C \sum_{d=1}^D TC3_{l,c,d} X3_{l,c,d} + \sum_{l=1}^L \sum_{d=1}^D \sum_{e=1}^E TC4_{l,d,e} X4_{l,d,e} \\
 & + \sum_{l=1}^L \sum_{e=1}^E \sum_{f=1}^F TC5_{l,e,f} X5_{l,e,f} + \sum_{l=1}^L \sum_{f=1}^F \sum_{g=1}^G TC6_{l,f,g} X6_{l,f,g} \\
 & + \sum_{l=1}^L \sum_{g=1}^G \sum_{d=1}^D TC7_{l,g,d} X7_{l,g,d} \\
 & + \sum_{l=1}^L \sum_{f=1}^F \sum_{h=1}^H TC8_{l,f,h} X8_{l,f,h} + \sum_{m=1}^M \sum_{i=1}^I \sum_{h=1}^H TC9_{m,h,i} X9_{m,h,i} \\
 & + \sum_{m=1}^M \sum_{i=1}^I \sum_{c=1}^C TC10_{m,i,c} X10_{m,i,c} \\
 & + \sum_{m=1}^M \sum_{i=1}^I \sum_{j=1}^J TC11_{m,i,j} X11_{m,i,j} + \sum_{m=1}^M \sum_{i=1}^I \sum_{k=1}^K TC12_{m,i,k} X12_{m,i,k} \\
 & + \sum_{m=1}^M \sum_{j=1}^J \sum_{a=1}^A TC13_{m,j,a} X13_{m,j,a}
 \end{aligned}$$

Objective 3: Total purchasing cost of used products and raw materials.

In this proposed CLSC design, the purchasing of raw material and used products at two echelons has been allowed. The purchasing cost of raw materials from the supplier point and the purchasing cost of used products from customers yields the total purchasing cost. However, these costs leave a significant margin among the new out-sourced products by contributing less operational costs on the recovered products. Therefore, the third objective function ensures the minimization of the total purchasing cost of raw materials and different used products from suppliers and customers to maintain the efficiency of the manufacturing plant.

$$\text{Minimize } Z_3 = \sum_{m=1}^M PU1_m X2_{m,b,c} + \sum_{l=1}^L PU2_l X5_{l,e,f} \quad \forall b, c, e, f.$$

Objective 4: Products delivery time.

The most critical issue in CLSC design is to determine the optimal time policy during the whole process. Notably, the shipment time of new products from the distribution center to different customers must be attained under the stipulated delivery period at the time of the ordered quantity. The goodwill and reputation of the company are strongly connected with delivery time. The latter also reduces the loss of any perishable products that happens due to delay. Moreover, cancelation from the customers' side would be almost negligible with the timely transshipment of the products. Henceforth, the fourth objective dynamically ensures the minimization of the total shipment time of different new products from the distributor to customers to maintain the reputation and reliability of the company.

$$\text{Minimize } Z_4 = \sum_{l=1}^L \sum_{d=1}^D \sum_{e=1}^E T_{l,d,e} X4_{l,d,e} \quad \forall d, e.$$

Objective 5: Revenues from the sale of new products and recyclable parts/components.

By the significant increase in the sales ratio of new products and recyclable parts/components, a marginal profit could be extracted. Selling of new products at higher quantities covers the maximum part of the capital investment during the production and distribution processes—recyclable parts/components are also a reliable source of profit from its sales. The selling price of the new products has a significant contribution toward the net profit and simultaneously yields in the contribution to gross profit. Thus the fifth or last objective function ensures the maximization of new products selling to survive in the competitive market with the maximum turnover of the new products under the optimal production policy.

$$\text{Maximize } Z_5 = \sum_{m=1}^M SP1_m X1_{m,i,j} + \sum_{l=1}^L SP2_l X4_{l,d,e} \quad \forall i, j, d, e.$$

15.4.2 Constraints

The following are the relevant constraints or restrictions under which the objective functions are to be optimized by yielding the most promising and systematic strategies for allocating different raw materials or parts/components and various products among multiple echelons in the proposed CLSC designed model. For the sake of convenience, we have categorized all the constraints under six different groups, and these can be summarized as follows.

15.4.2.1 Constraints related to the capacity of different echelons in the CLSC network

The procurement of raw materials initiates from the raw material storage center where the abundance or stock of raw materials has been kept to fulfill the demand from suppliers. Therefore, the total shipment quantity of different raw materials from the raw materials storage center to the supplier must not exceed its capacities and can be represented by Eq. (15.1). Supplier points also have a limited ability for the flow of different raw materials to maintain the intake and outsourced ratio. It is essential for the supplier to hold some raw materials for distribution at times of scarcity, when raw material storage functioning is interrupted over a stipulated time. Hence the constraints imposed over the number of raw materials shipped from the supplier point to a different manufacturing plant must less than or equal to the capacity of suppliers and can be presented by Eq. (15.2). The collection of used products from different customers starts the key functioning role of the reverse chain. It is the very first stage at which the end-of-use products are collected by the collection center. It must be assured that the accumulation quantity of used products from different customers must be less than or equal to the capacity of various collection centers and can be represented by Eq. (15.3). A well-organized system of collection centers provides frequent services to the used products so that all the end-of-use products are refurbished and can be used further without significantly affecting the demand. After ensuring the required services for used products, it has been allowed to ship the used merchandise from the collection center to the refurbishing center for renovating processes. Hence the total quantity of used products must not exceed the capacity of the refurbishing center and can be given in Eq. (15.4). The number of used products that need testing services for their further utilization has been shipped to the disassembling facility to disassemble the used products into different components or parts. To ensure that the number of used products which have been sent for dismantling purpose must be less than or equal to its capacity and can be represented by Eq. (15.5). After completing the required test for the parts/components, the recyclable quantity of parts/components has been sent to the recycling facility, which denotes the last echelon of the CLSC network. To ensure the number of recyclable parts/components received from different testing points must not exceed the maximum capacity of the recycling center and can be stated in Eq. (15.6). After testing procedures, end-of-life parts or components are declared as disposable parts/components and shipped to the disposal center in good time to reduce environmental issues. Hence to avoid the burden on landfills and underground

disposal, the number of disposable parts/components must not exceed the maximum disposal capacity at the disposal center and this can be represented by Eq. (15.7). After recycling processes, the parts/components are transformed into new raw materials and ready the shipment to the raw material storage center. To fulfill the stock capacity of a natural material storage center, the number of raw materials must be greater than or equal to its minimum storage capacity for the smooth running of the production system, and this can be represented by Eq. (15.8).

$$\sum_{b=1}^B X1_{m,a,b} \leq MC1_{m,a} \quad \forall m,a, \quad (15.1)$$

$$\sum_{c=1}^C X2_{m,b,c} \leq MC2_{m,b} \quad \forall m,b, \quad (15.2)$$

$$\sum_{e=1}^E rc_{l,e} X5_{l,e,f} \leq MC6_{l,f} \quad \forall l,f, \quad (15.3)$$

$$\sum_{g=1}^G X6_{l,f,g} \leq MC7_{l,g} \quad \forall l,g, \quad (15.4)$$

$$\sum_{h=1}^H X8_{l,f,h} \leq MC8_{l,h} \quad \forall l,h, \quad (15.5)$$

$$\sum_{i=1}^I rr_m X11_{m,i,j} \leq MC10_{m,j} \quad \forall m,j, \quad (15.6)$$

$$\sum_{i=1}^I rd_m X12_{m,i,k} \leq MC11_{m,k} \quad \forall m,k, \quad (15.7)$$

$$\sum_{j=1}^J X13_{m,j,a} \geq MC1_{m,a} \quad \forall m,a. \quad (15.8)$$

15.4.2.2 Constraints related to production requirement

An efficient production system is an integral part of the CLSC network. Hybrid manufacturing/remanufacturing plants play a vital role in the optimal production of new products. Therefore, particular minimum requirements must be met to start the production processes. To ascertain the minimum condition of raw materials from two sources, the supplier point and testing center, the number of raw materials from two references must be greater than or equal to their production capacity at different manufacturing plants, and this can be represented by Eq. (15.9).

$$\sum_{b=1}^B X2_{m,b,c} + \sum_{i=1}^I rm_m X10_{m,i,c} \geq MC3_{m,c} \quad \forall m,c. \quad (15.9)$$

15.4.2.3 Constraints related to maximum inventory at the distribution center

The distribution center is responsible for the shipment of products to different customers/markets. The demand for a new product is uncertain and only can be predicted based on previous information. Thus, to avoid the inventory cost and ascertain the maximum capacity restriction at the distribution center, the incoming products from manufacturing plants as well as refurbishing centers must be less than or equal to the maximum capacity of inventory at the distribution center, and this can be achieved by Eq. (15.10).

$$\sum_{c=1}^C X3_{l,c,d} + \sum_{g=1}^G r_{fg} X7_{l,g,d} \leq MC4_{l,d} \quad \forall l, d. \quad (15.10)$$

15.4.2.4 Constraints related to demand of new and refurbished products

The most important and critical aspect of integrated CLSC is to fulfill the demand of customers or markets. The need for products is seldom stable. However, it can be predicted through prior information from the demand pattern. The only distribution center is responsible for the delivery of new products to the customers in this proposed CLSC network. To ensure this, the number of shipped products from the distribution center to different markets must be higher than its tentative demand over the stipulated ordered period, and this can be represented by Eq. (15.11).

$$\sum_{e=1}^E X4_{l,d,e} \geq MC5_{l,e} \quad \forall l, e. \quad (15.11)$$

15.4.2.5 Constraints related to the testing capacity at testing facility centers

The testing facility has been designed for taking the final decision over the parts or components regarding at which echelon they are to be transported. From a testing point, there are three facility options for the processing of tested parts/components. The manufacturing plant, recycling center, and disposal center have been structured for the final termination of the reverse supply chain. Hence the total sum of the number of parts/components that are transported from the testing plant to different facility locations must be less than or equal to the maximum capacity of the testing point, and this can be represented by Eq. (15.12).

$$\sum_{c=1}^C r_{tm} X10_{m,i,c} + \sum_{j=1}^J r_{tm} X11_{m,i,j} + \sum_{k=1}^K r_{tm} X12_{m,i,k} \leq MC9_{m,i} \quad \forall m, i. \quad (15.12)$$

15.4.3 Proposed CLSC model formulation under uncertainty

The formulation of different conflicting objective functions and with some dynamic constraints under the proposed CLSC network has been presented in previous sections. Usually, the modeling texture of the CLSC network has been regarded as deterministic, which means that all the introduced parameters and constraints are known and predetermined well in advance. However, it is often observed that a deterministic modeling approach under CLSC design may not be an appropriate framework in decision-making processes. The typical multiechelon interconnected CLSC design model inherently yields some uncertainty. Impreciseness, vagueness, ambiguousness, randomness, incompleteness, etc., are the most common and frequent issues in the CLSC model. Different factors are responsible for the creation of uncertainty in the modeling of the CLSC network. Random fluctuation in the demand quantity, competitive market scenario, natural tragedy, variation in different kinds of costs, etc., laid down the base of uncertainty. In various adverse circumstances, the complete information about different parameters is not predetermined, but some inconsistent, improper, and incomplete information may be available to determine the deterministic value of the parameters. Uncertainty may exist in different forms, such as fuzzy, stochastic, and other types of risk. Vagueness or ambiguousness is responsible for fuzzy parameters which can be dealt with using the fuzzy techniques, whereas randomness gives birth to the stochastic parameters and can be quickly sorted out by using stochastic programming techniques with known means and variances of the parameters. Therefore, to highlight the most critical insight of the uncertainty, we have incorporated fuzzy parameters and few fuzzy equality constraints in the proposed CLSC designed network. Various cost parameters, such as processing costs, transportation costs, purchasing cost, selling prices, and time, have been taken as fuzzy parameters. The capacities or volumes of different echelons are also considered as fuzzy numbers. Inequality restrictions imposed over different constraints may avoid some aspects of getting better results from the CLSC planning model. Flexibility, among some preferred limitations, has been postulated to reveal reality more clearly. Hence we have developed a couple of fuzzy equality constraints (\cong) which means “essentially equal to” which signifies that the restrictions should more or less be satisfied and are more flexible than inequality constraints (Eqs. 15.22–15.24). The customer demand constraint has been assured with fuzzy equality constraints due to the change in utility or satisfaction behavior of the customers. The disposal facility is a single way for the removal of scrap parts/components out of the CLSC network. The testing facility plays a vital role in inspecting different parts/components. The optimum allocation of used parts/products has been decided at the testing facility point. Three various service destinations have been designed for the parts/components according to their potential utility after inspection. Therefore, more or less shipment quantity of parts/components is justifiable to ensure the optimum allocation to different facility centers. Hence, the proposed model with multiple objective functions and various constraints under uncertainty has been presented in model M_1 .

$$\begin{aligned}
M_1 : \text{Minimize } Z_1 = & \sum_{m=1}^M \sum_{a=1}^A \widetilde{PC1}_{m,a} X_{1_{m,a,b}} + \sum_{m=1}^M \sum_{b=1}^B \widetilde{PC2}_{m,b} X_{2_{m,b,c}} \\
& + \sum_{l=1}^L \sum_{c=1}^C \widetilde{PC3}_{l,c} X_{3_{l,c,d}} + \sum_{l=1}^L \sum_{d=1}^D \widetilde{PC4}_{l,d} X_{4_{l,d,e}} \\
& + \sum_{l=1}^L \sum_{f=1}^F \widetilde{PC5}_{l,f} X_{5_{l,e,f}} + \sum_{l=1}^L \sum_{g=1}^G \widetilde{PC6}_{l,g} X_{6_{l,f,g}} \\
& + \sum_{l=1}^L \sum_{h=1}^H \widetilde{PC7}_{l,h} X_{8_{l,f,h}} + \sum_{i=1}^M \sum_{i=1}^I \widetilde{PC8}_{m,i} X_{10_{m,i,c}} \\
& + \sum_{m=1}^M \sum_{j=1}^J \widetilde{PC9}_{m,j} X_{11_{m,i,j}} + \sum_{m=1}^M \sum_{k=1}^K \widetilde{PC10}_{m,k} X_{12_{m,i,k}}
\end{aligned}$$

$$\begin{aligned}
\text{Minimize } Z_2 = & \sum_{m=1}^M \sum_{a=1}^A \sum_{b=1}^B \widetilde{TC1}_{m,a,b} X_{1_{m,a,b}} + \sum_{m=1}^M \sum_{b=1}^B \sum_{c=1}^C \widetilde{TC2}_{m,b,c} X_{2_{m,b,c}} \\
& + \sum_{l=1}^L \sum_{c=1}^C \sum_{d=1}^D \widetilde{TC3}_{l,c,d} X_{3_{l,c,d}} + \sum_{l=1}^L \sum_{d=1}^D \sum_{e=1}^E \widetilde{TC4}_{l,d,e} X_{4_{l,d,e}} \\
& + \sum_{l=1}^L \sum_{e=1}^E \sum_{f=1}^F \widetilde{TC5}_{l,e,f} X_{5_{l,e,f}} + \sum_{l=1}^L \sum_{f=1}^F \sum_{g=1}^G \widetilde{TC6}_{l,f,g} X_{6_{l,f,g}} \\
& + \sum_{l=1}^L \sum_{g=1}^G \sum_{d=1}^D \widetilde{TC7}_{l,g,d} X_{7_{l,g,d}} \\
& + \sum_{l=1}^L \sum_{f=1}^F \sum_{h=1}^H \widetilde{TC8}_{l,f,h} X_{8_{l,f,h}} + \sum_{m=1}^M \sum_{i=1}^I \sum_{h=1}^H \widetilde{TC9}_{m,h,i} X_{9_{m,h,i}} \\
& + \sum_{m=1}^M \sum_{i=1}^I \sum_{c=1}^C \widetilde{TC10}_{m,i,c} X_{10_{m,i,c}} + \sum_{m=1}^M \sum_{i=1}^I \sum_{j=1}^J \widetilde{TC11}_{m,i,j} X_{11_{m,i,j}} \\
& + \sum_{m=1}^M \sum_{i=1}^I \sum_{k=1}^K \widetilde{TC12}_{m,i,k} X_{12_{m,i,k}} + \sum_{m=1}^M \sum_{j=1}^J \sum_{a=1}^A \widetilde{TC13}_{m,j,a} X_{13_{m,j,a}}
\end{aligned}$$

$$\text{Minimize } Z_3 = \sum_{m=1}^M \widetilde{PU1}_m X_{2_{m,b,c}} + \sum_{l=1}^L \widetilde{PU2}_l X_{5_{l,e,f}}$$

$$\text{Minimize } Z_4 = \sum_{l=1}^L \sum_{d=1}^D \sum_{e=1}^E \widetilde{T}_{l,d,e} X_{4_{l,d,e}}$$

$$\text{Maximize } Z_5 = \sum_{m=1}^M \widetilde{SP1}_m X_{2_{m,b,c}} + \sum_{l=1}^L \widetilde{SP2}_l X_{5_{l,e,f}}$$

subject to

$$\sum_{b=1}^B X1_{m,a,b} \leq \widetilde{MC}1_{m,a}, \quad (15.13)$$

$$\sum_{c=1}^C X2_{m,b,c} \leq \widetilde{MC}2_{m,b}, \quad (15.14)$$

$$\sum_{b=1}^B X2_{m,b,c} + \sum_{i=1}^I r m_m X10_{m,i,c} \geq \widetilde{MC}3_{m,c}, \quad (15.15)$$

$$\sum_{c=1}^C X3_{l,c,d} + \sum_{g=1}^G r f_l X7_{l,g,d} \leq \widetilde{MC}4_{l,d}, \quad (15.16)$$

$$\sum_{e=1}^E r c_{l,e} X5_{l,e,f} \leq \widetilde{MC}6_{l,f}, \quad (15.17)$$

$$\sum_{g=1}^G X6_{l,f,g} \leq \widetilde{MC}7_{l,g}, \quad (15.18)$$

$$\sum_{h=1}^H X8_{l,f,h} \leq \widetilde{MC}8_{l,h}, \quad (15.19)$$

$$\sum_{i=1}^I r r_m X11_{m,i,j} \leq \widetilde{MC}10_{m,j}, \quad (15.20)$$

$$\sum_{j=1}^J X13_{m,j,a} \geq \widetilde{MC}1_{m,a}, \quad (15.21)$$

$$\sum_{e=1}^E X4_{l,d,e} \cong \widetilde{MC}5_{l,e}, \quad (15.22)$$

$$\sum_{i=1}^I r d_m X12_{m,i,k} \cong \widetilde{MC}11_{m,k}, \quad (15.23)$$

$$\sum_{c=1}^C r t_m X10_{m,i,c} + \sum_{j=1}^J r t_m X11_{m,i,j} + \sum_{k=1}^K r t_m X12_{m,i,k} \cong \widetilde{MC}9_{m,i}. \quad (15.24)$$

Where notations (\cdot) over different parameters represent the triangular/trapezoidal fuzzy number for all indices' sets, the fuzzy crisp inequality constraint has been described by (\leq, \geq) . The fuzzy equality constraints indicate that more or less attainment has been represented by (\cong) for the given indices' sets.

15.5 Solution methodology

15.5.1 Treating fuzzy parameters and constraints

The addressed CLSC mathematical model inherently involves some vagueness and ambiguousness in the value of different parameters such as costs, capacity, revenues, etc. Defuzzification and the ranking function are the processes to obtain crisp versions of the fuzzified parameters based on the upper and lower magnitude of the vague parameters. On the other hand, the vagueness or uncertainty present in the equality or inequality constraints also needs to be defuzzified, and then converted into the strict crisp equality or inequality form of the constraints. To deal with vague or fuzzy parameters and constraints, different defuzzification techniques have been used in the literature. Among all the defuzzification approaches for uncertain parameters and constraints, Jiménez [24] and Jiménez et al. [25] discussed the combo defuzzification or ranking approach, which deals efficiently with the vague parameters as well as vague constraints. They also elaborately discussed the strong justification for ranking approaches with the help of different properties such as robustness, distinguishability, fuzzy or linguistic notations, and rationality. Later on, it has been extensively used by many researchers (see [25–27]). Without more justification on the ranking function, this chapter has adopted the defuzzification or ranking function for both vague parameters and constraints based on the Jiménez [24] approaches.

Definition 15.1. Jiménez et al. [25]

An FS defined over any universe of discourse is said to be a fuzzy number if the membership function is increasing semicontinuously in the upper interval and decreasing semicontinuously in the lower range, respectively. Therefore, the membership function of a fuzzy number along with $f_\phi(x)$ and $g_\phi(x)$, which are the left- and right-hand sides of the membership function, can be given as follows:

$$\mu_\phi(x) = \begin{cases} 0 & \text{if } x \leq \phi_1 \text{ or } x \geq \phi_4 \\ f_\phi(x) & \text{if } \phi_1 \leq x \leq \phi_2 \\ g_\phi(x) & \text{if } \phi_3 \leq x \leq \phi_4, \\ 1 & \text{if } \phi_2 \leq x \leq \phi_3 \end{cases} \quad (15.25)$$

where $\tilde{\phi} = (\phi_1, \phi_2, \phi_3, \phi_4; 1)$ represents a fuzzy number. A fuzzy number $\tilde{\phi} = (\phi_1, \phi_2, \phi_3, \phi_4)$ is said to be trapezoidal if $f_\phi(x)$ and $g_\phi(x)$ exist. Also, if $\phi_2 = \phi_3$, then one can obtain a triangular fuzzy number.

Definition 15.2. Jiménez et al. [25]

The representation of an expected interval for the fuzzy number $\tilde{\phi}$ can be provided as follows:

$$EI(\tilde{\phi}) = [E_1^\phi, E_2^\phi] = \left[\int_0^1 f_\phi^{-1}(x) dx, \int_0^1 g_\phi^{-1}(x) dx \right]. \quad (15.26)$$

The half point of the expected interval of the fuzzy number $\tilde{\phi}$ is termed as its expected value and can be shown as follows:

$$EV(\tilde{\phi}) = \left[\frac{E_1^\phi + E_2^\phi}{2} \right]. \tag{15.27}$$

Hence the expected interval and expected value for a trapezoidal fuzzy number $\tilde{\phi} = (\phi_1, \phi_2, \phi_3, \phi_4)$ can be obtained as follows:

$$EI(\phi) = \left[\frac{\phi_1 + \phi_2}{2}, \frac{\phi_3 + \phi_4}{2} \right], \tag{15.28}$$

$$EV(\phi) = \left[\frac{\phi_1 + \phi_2 + \phi_3 + \phi_4}{4} \right]. \tag{15.29}$$

For any trapezoidal fuzzy number $\tilde{\phi} = (\phi_1, \phi_2, \phi_3, \phi_4)$, if $\phi_2 = \phi_3$ (say ϕ) then it reduces into a triangular fuzzy number $\tilde{\phi} = (\phi_1, \phi, \phi_4)$ and; its expected interval and expected value can be derived as follows:

$$EI(\phi) = \left[\frac{\phi_1 + \phi}{2}, \frac{\phi + \phi_4}{2} \right], \tag{15.30}$$

$$EV(\phi) = \left[\frac{\phi_1 + 2\phi + \phi_4}{4} \right]. \tag{15.31}$$

Definition 15.3. Jiménez et al. [25]

Suppose that there are two fuzzy $\tilde{\phi}$ and $\tilde{\psi}$ such that both have semicontinuous increasing and decreasing membership functions for upper and lower intervals, then the degree in which $\tilde{\phi}$ is greater than $\tilde{\psi}$ can be easily pointed out by constructing the following membership function:

$$\delta_V(\tilde{\phi}, \tilde{\psi}) = \begin{cases} 0 & \text{if } E_2^\phi - E_1^\psi < 0 \\ \frac{E_2^\phi - E_1^\psi}{E_2^\phi - E_1^\psi - (E_1^\phi - E_2^\psi)} & \text{if } 0 \in [E_1^\phi - E_2^\psi, E_2^\phi - E_1^\psi], \\ 1 & \text{if } E_2^\phi - E_1^\psi > 0 \end{cases}, \tag{15.32}$$

where $[E_1^\phi, E_2^\phi]$ and $[E_1^\psi, E_2^\psi]$ represent the expected intervals of $\tilde{\phi}$ and $\tilde{\psi}$. If $\delta_V(\tilde{\phi}, \tilde{\psi}) = 0.5$, then one can say that both $\tilde{\phi}$ and $\tilde{\psi}$ are indifferent.

Consequently, if $\delta_V(\tilde{\phi}, \tilde{\psi}) \geq \beta$, then one can say that $\tilde{\phi}$ is greater than or equal to $\tilde{\psi}$, at least in a degree β , and can be mathematically represented as $\tilde{\phi}_i \geq_{\beta} \tilde{\psi}_i$.

Definition 15.4. Jiménez et al. [25]

Introducing a decision vector X such that $x \in R^n$, then we can assign a feasibility degree β if for at least

$$\min_{i \in V} [\delta_V(\tilde{\phi}_i X, \tilde{\psi}_i)] = \beta, \tag{15.33}$$

where $\tilde{\phi}_i = (\tilde{\phi}_{i1}, \tilde{\phi}_{i2}, \dots, \tilde{\phi}_{iv})$.

Intuitively, in another sense, it can be written as

$$\tilde{\phi}_i X \geq_{\beta} \tilde{\psi}_i \quad \forall i = 1, 2, \dots, v. \tag{15.34}$$

Incorporating the concept of (Jiménez et al. [25]) in the above inequality, equivalently we have

$$\frac{E_2^{\phi_i X} - E_1^{\psi_i}}{E_2^{\phi_i X} - E_1^{\psi_i} - (E_1^{\phi_i X} - E_2^{\psi_i})} \geq \beta \quad \forall i = 1, 2, \dots, v. \tag{15.35}$$

On simplifying the above inequality equation, the equivalent inequality relations with feasibility degree β have been derived as follows:

$$((1 - \beta)E_2^{\phi_i} + \beta E_1^{\phi_i}) X \geq (\beta E_2^{\psi_i} + (1 - \beta)E_1^{\psi_i}). \tag{15.36}$$

Furthermore, it can be concluded that the β -feasible fuzzy equalities, such as

$$\tilde{\phi}_i X \cong_{\beta} \tilde{\psi}_i \quad \forall i = v + 1, v + 2, \dots, V, \tag{15.37}$$

can also be defuzzified in a similar fashion to the ranking function approach for fuzzy inequalities and can be given as follows:

$$\left(\left(1 - \frac{\beta}{2} \right) E_2^{\phi_i} + \frac{\beta}{2} E_1^{\phi_i} \right) X \geq \left(\frac{\beta}{2} E_2^{\psi_i} + \left(1 - \frac{\beta}{2} \right) E_1^{\psi_i} \right), \tag{15.38}$$

$$\left(\frac{\beta}{2} E_2^{\phi_i} + \left(1 - \frac{\beta}{2} \right) E_1^{\phi_i} \right) X \leq \left(\left(1 - \frac{\beta}{2} \right) E_2^{\psi_i} + \frac{\beta}{2} E_1^{\psi_i} \right). \tag{15.39}$$

Therefore, the fuzzy equality constraints result in the doubly crisp auxiliary inequality constraints for representing the restrictions with half of the β -feasibility degree by balancing an equilibrium state for the fuzzy equality constraints.

In order to obtain the crisp version of the proposed CLSC model, we have used the expected values [25] of the triangular fuzzy parameters present in the objective functions such as transportation cost, processing cost, purchasing cost, time, and revenues, whereas the trapezoidal fuzzy parameters such as different capacities involved in the constraints have been defuzzified by using the concept of the expected interval [25] of the parameters. Based on the above-discussed defuzzification approaches, the fuzzy

parameters and constraints have been converted into their crisp versions, which has been also shown in [Table 15.1](#).

$$\begin{aligned}
 M_2 : \text{Minimize } Z_1 = & \sum_{m=1}^M \sum_{a=1}^A EV(\widetilde{PC1})_{m,a} X_{1m,a,b} + \sum_{m=1}^M \sum_{b=1}^B EV(\widetilde{PC2})_{m,b} X_{2m,b,c} \\
 & + \sum_{l=1}^L \sum_{c=1}^C EV(\widetilde{PC3})_{l,c} X_{3l,c,d} + \sum_{l=1}^L \sum_{d=1}^D EV(\widetilde{PC4})_{l,d} X_{4l,d,e} \\
 & + \sum_{l=1}^L \sum_{f=1}^F EV(\widetilde{PC5})_{l,f} X_{5l,e,f} + \sum_{l=1}^L \sum_{g=1}^G EV(\widetilde{PC6})_{l,g} X_{6l,f,g} \\
 & + \sum_{l=1}^L \sum_{h=1}^H EV(\widetilde{PC7})_{l,h} X_{8l,f,h} + \sum_{i=1}^M \sum_{i=1}^I EV(\widetilde{PC8})_{m,i} X_{10m,i,c} \\
 & + \sum_{m=1}^M \sum_{j=1}^J EV(\widetilde{PC9})_{m,j} X_{11m,i,j} + \sum_{m=1}^M \sum_{k=1}^K EV(\widetilde{PC10})_{m,k} X_{12m,i,k}
 \end{aligned}$$

$$\begin{aligned}
 \text{Minimize } Z_2 = & \sum_{m=1}^M \sum_{a=1}^A \sum_{b=1}^B EV(\widetilde{TC1})_{m,a,b} X_{1m,a,b} + \sum_{m=1}^M \sum_{b=1}^B \sum_{c=1}^C EV(\widetilde{TC2})_{m,b,c} X_{2m,b,c} \\
 & + \sum_{l=1}^L \sum_{c=1}^C \sum_{d=1}^D EV(\widetilde{TC3})_{l,c,d} X_{3l,c,d} + \sum_{l=1}^L \sum_{d=1}^D \sum_{e=1}^E EV(\widetilde{TC4})_{l,d,e} X_{4l,d,e} \\
 & + \sum_{l=1}^L \sum_{e=1}^E \sum_{f=1}^F EV(\widetilde{TC5})_{l,e,f} X_{5l,e,f} + \sum_{l=1}^L \sum_{f=1}^F \sum_{g=1}^G EV(\widetilde{TC6})_{l,f,g} X_{6l,f,g} \\
 & + \sum_{l=1}^L \sum_{g=1}^G \sum_{d=1}^D EV(\widetilde{TC7})_{l,g,d} X_{7l,g,d} + \sum_{l=1}^L \sum_{f=1}^F \sum_{h=1}^H EV(\widetilde{TC8})_{l,f,h} X_{8l,f,h} \\
 & + \sum_{m=1}^M \sum_{i=1}^I \sum_{h=1}^H EV(\widetilde{TC9})_{m,h,i} X_{9m,h,i} + \sum_{m=1}^M \sum_{i=1}^I \sum_{c=1}^C EV(\widetilde{TC10})_{m,i,c} X_{10m,i,c} \\
 & + \sum_{m=1}^M \sum_{i=1}^I \sum_{j=1}^J EV(\widetilde{TC11})_{m,i,j} X_{11m,i,j} + \sum_{m=1}^M \sum_{i=1}^I \sum_{k=1}^K EV(\widetilde{TC12})_{m,i,k} X_{12m,i,k} \\
 & + \sum_{m=1}^M \sum_{j=1}^J \sum_{a=1}^A EV(\widetilde{TC13})_{m,j,a} X_{13m,j,a}
 \end{aligned}$$

$$\text{Minimize } Z_3 = \sum_{m=1}^M EV(\widetilde{PU1})_m X_{2m,b,c} + \sum_{l=1}^L EV(\widetilde{PU2})_l X_{5l,e,f}$$

$$\text{Minimize } Z_4 = \sum_{l=1}^L \sum_{d=1}^D \sum_{e=1}^E EV(\widetilde{T})_{l,d,e} X_{4l,d,e}$$

$$\text{Maximize } Z_5 = \sum_{m=1}^M EV(\widetilde{SP1})_m X_{2m,b,c} + \sum_{l=1}^L EV(\widetilde{SP2})_l X_{5l,e,f}$$

Table 15.1 Information regarding triangular/trapezoidal fuzzy parameters.

Fuzzy parameter	Triangular/trapezoidal fuzzy number	$EI(.) = [E_1^{(.)}, E_2^{(.)}]$	$EV(.)$
$\widetilde{PC}_{**,*}$	$(PC_{**,*}^{(1)}, PC_{**,*}^{(2)}, PC_{**,*}^{(3)})$	$\left[\frac{PC_{**,*}^{(1)} + PC_{**,*}^{(2)}}{2}, \frac{PC_{**,*}^{(2)} + PC_{**,*}^{(3)}}{2} \right]$	$\frac{PC_{**,*}^{(1)} + 2PC_{**,*}^{(2)} + PC_{**,*}^{(3)}}{4}$
$\widetilde{TC}_{**,*}$	$(TC_{**,*}^{(1)}, TC_{**,*}^{(2)}, TC_{**,*}^{(3)})$	$\left[\frac{TC_{**,*}^{(1)} + TC_{**,*}^{(2)}}{2}, \frac{TC_{**,*}^{(2)} + TC_{**,*}^{(3)}}{2} \right]$	$\frac{TC_{**,*}^{(1)} + 2TC_{**,*}^{(2)} + TC_{**,*}^{(3)}}{4}$
$\widetilde{T}_{**,*}$	$(T_{**,*}^{(1)}, T_{**,*}^{(2)}, T_{**,*}^{(3)})$	$\left[\frac{T_{**,*}^{(1)} + T_{**,*}^{(2)}}{2}, \frac{T_{**,*}^{(2)} + T_{**,*}^{(3)}}{2} \right]$	$\frac{T_{**,*}^{(1)} + 2T_{**,*}^{(2)} + T_{**,*}^{(3)}}{4}$
$\widetilde{PU}_{**,*}$	$(PU_{**,*}^{(1)}, PU_{**,*}^{(2)}, PU_{**,*}^{(3)})$	$\left[\frac{PU_{**,*}^{(1)} + PU_{**,*}^{(2)}}{2}, \frac{PU_{**,*}^{(2)} + PU_{**,*}^{(3)}}{2} \right]$	$\frac{PU_{**,*}^{(1)} + 2PU_{**,*}^{(2)} + PU_{**,*}^{(3)}}{4}$
$\widetilde{SP}_{**,*}$	$(SP_{**,*}^{(1)}, SP_{**,*}^{(2)}, SP_{**,*}^{(3)})$	$\left[\frac{SP_{**,*}^{(1)} + SP_{**,*}^{(2)}}{2}, \frac{SP_{**,*}^{(2)} + SP_{**,*}^{(3)}}{2} \right]$	$\frac{SP_{**,*}^{(1)} + 2SP_{**,*}^{(2)} + SP_{**,*}^{(3)}}{4}$
$\widetilde{MC}_{**,*}$	$(MC_{**,*}^{(1)}, MC_{**,*}^{(2)}, MC_{**,*}^{(3)}, MC_{**,*}^{(4)})$	$\left[\frac{MC_{**,*}^{(1)} + MC_{**,*}^{(2)}}{2}, \frac{MC_{**,*}^{(3)} + MC_{**,*}^{(4)}}{2} \right]$	$\frac{MC_{**,*}^{(1)} + MC_{**,*}^{(2)} + MC_{**,*}^{(3)} + MC_{**,*}^{(4)}}{4}$

Notes: * represents the different numbers 1, 2, 3, ... used in parameters.
 (*, *) and (*, *, *) in suffixes represent the different indices set.

subject to

$$\sum_{b=1}^B X1_{m,a,b} \leq (1 - \beta)E_2^{MC1_{m,a}} + \beta E_1^{MC1_{m,a}}, \tag{15.40}$$

$$\sum_{c=1}^C X2_{m,b,c} \leq (1 - \beta)E_2^{MC2_{m,b}} + \beta E_1^{MC2_{m,b}}, \tag{15.41}$$

$$\sum_{b=1}^B X2_{m,b,c} + \sum_{i=1}^I r m_m X10_{m,i,c} \geq \beta E_2^{MC3_{m,c}} + (1 - \beta)E_1^{MC3_{m,c}}, \tag{15.42}$$

$$\sum_{c=1}^C X3_{l,c,d} + \sum_{g=1}^G r f_l X7_{l,g,d} \leq (1 - \beta)E_2^{MC4_{l,d}} + \beta E_1^{MC4_{l,d}}, \tag{15.43}$$

$$\sum_{e=1}^E r c_l X5_{l,e,f} \leq (1 - \beta)E_2^{MC6_{l,f}} + \beta E_1^{MC6_{l,f}}, \tag{15.44}$$

$$\sum_{g=1}^G X6_{l,f,g} \leq (1 - \beta)E_2^{MC7_{l,g}} + \beta E_1^{MC7_{l,g}}, \tag{15.45}$$

$$\sum_{h=1}^H X8_{l,f,h} \leq (1 - \beta)E_2^{MC8_{l,h}} + \beta E_1^{MC8_{l,h}}, \tag{15.46}$$

$$\sum_{i=1}^I r r_m X11_{m,i,j} \leq (1 - \beta)E_2^{MC10_{m,j}} + \beta E_1^{MC10_{m,j}}, \tag{15.47}$$

$$\sum_{j=1}^J X13_{m,j,a} \geq \beta E_2^{MC1_{m,a}} + (1 - \beta)E_1^{MC1_{m,a}}, \tag{15.48}$$

$$\sum_{i=1}^I r d_m X12_{m,i,k} \geq \frac{\beta}{2} E_2^{MC11_{m,k}} + \left(1 - \frac{\beta}{2}\right) E_1^{MC11_{m,k}}, \tag{15.49}$$

$$\sum_{i=1}^I r d_m X12_{m,i,k} \leq \left(1 - \frac{\beta}{2}\right) E_2^{MC11_{m,k}} + \frac{\beta}{2} E_1^{MC11_{m,k}}, \tag{15.50}$$

$$\sum_{e=1}^E X4_{l,d,e} \geq \frac{\beta}{2} E_2^{MC5_{l,e}} + \left(1 - \frac{\beta}{2}\right) E_1^{MC5_{l,e}}, \tag{15.51}$$

$$\sum_{e=1}^E X4_{l,d,e} \leq \left(1 - \frac{\beta}{2}\right) E_2^{MC5_{l,e}} + \frac{\beta}{2} E_1^{MC5_{l,e}}, \tag{15.52}$$

$$\sum_{c=1}^C r t_m X10_{m,i,c} + \sum_{j=1}^J r t_m X11_{m,i,j} + \sum_{k=1}^K r t_m X12_{m,i,k} \geq \frac{\beta}{2} E_2^{MC9_{m,i}} + \left(1 - \frac{\beta}{2}\right) E_1^{MC9_{m,i}}, \tag{15.53}$$

$$\sum_{c=1}^C r t_m X10_{m,i,c} + \sum_{j=1}^J r t_m X11_{m,i,j} + \sum_{k=1}^K r t_m X12_{m,i,k} \leq \left(1 - \frac{\beta}{2}\right) E_2^{MC9_{m,i}} + \frac{\beta}{2} E_1^{MC9_{m,i}}. \tag{15.54}$$

15.5.2 Neutrosophic fuzzy programming approach

The multiobjective optimization problems are prevalent in real-life scenarios. Due to the existence of complex and conflicting multiple goals or objectives, the task of obtaining optimal solutions is a vital issue. The different conventional optimization techniques for obtaining the compromise solution of multiobjective programming problems are based on the marginal evaluation (degree of validity) for each objective (say Z_o) in the feasible solution set. By marginal evaluation, we mean a transformation function (say $\mu(Z_o) \rightarrow [0, 1] | \alpha \in [0, 1]$) that assigned the values between 0 and 1 to each objective function which shows that the decision makers' preferences have been fulfilled up to α level of satisfaction. Therefore, the quantification of marginal evaluation is based on the different decision set theory. Initially, Zadeh [28] proposed the FS theory, which explicitly contains the membership function (degree of belongingness) of the element into the feasible solution set. Later on, Zimmermann [29] introduced the fuzzy programming approach to solve multiobjective optimization problems. In a fuzzy programming approach, the quantification of marginal evaluation is represented by a membership function, which only maximizes the degree of belongingness under the fuzzy decision set. The extended version of the fuzzy optimization technique has been applied in a wide range of real-life applications. Furthermore, the generalizations or extensions of the FS were initially proposed by Atanassov [30] and named the intuitionistic fuzzy set (IFS). The analytical coverage spectrum of IFS is versatile and flexible compared to FS as it deals with the membership (degree of belongingness) as well as nonmembership (degree of nonbelongingness) functions of the element into the feasible set. Based on IFS, first Angelov [31] proposed the intuitionistic fuzzy programming approach to obtain the compromise solution of the multiobjective optimization problems. The quantification of marginal evaluation of each objective function under the IF decision set depends on the membership and nonmembership functions, which are to be achieved by maximizing the membership function and minimizing the nonmembership functions simultaneously. The intuitionistic fuzzy programming approach has been extensively studied with various real-life problems.

In the past few decades, it has been observed that the situation may arise in real-life decision-making problems where the indeterminacy or neutral thoughts about an element into the feasible set exists. Indeterminacy/neutral is the region of the negligence of a proposition's value and lies between a truth and falsity degree. Therefore, the further generalization of FS and IFS has been presented by introducing a new member into the feasible decision set. First, Smarandache [32] investigated the neutrosophic set (NS) which comprises three membership functions: truth (degree of belongingness), indeterminacy (degree of belongingness up to some extent), and falsity (degree of nonbelongingness) functions of the element into the NS. The word *neutrosophic* is the hybrid mixture of two different words, *neutre*, taken from the French, meaning neutral, and *sophia*, derived from the Greek, meaning skill/wisdom, which literally gives the meaning *knowledge of neutral thoughts* (see [32]). The independent indeterminacy degree is sufficient to differentiate itself from FS and IFS. Recent literature on the NS reveals that many researchers have taken an interest in the neutrosophic

domain (see [33–36]) and this is likely to be a prominent emerging research area in the future. This study has also taken advantage of the versatile and effective texture of a neutrosophic fuzzy decision set to develop the NFPA. The NFPA has been designed to solve the proposed CLSC model with multiple objectives under the set of constraints. The NFPA quantifies the marginal evaluation of each objective function under three different membership functions: truth, indeterminacy, and falsity membership functions. Thus the NFPA optimization techniques for the multiobjective optimization problem has a significant role in the implementation and execution of the neutral thoughts in decision-making processes.

Definition 15.5. Neutrosophic set [32]

Let there be a universe discourse Y such that $y \in Y$, then an NS W in Y is defined by three membership functions, truth $p_W(y)$, indeterminacy $q_W(y)$, and falsity $r_W(y)$, and denoted by the following form:

$$W = \{ \langle y, p_W(y), q_W(y), r_W(y) \rangle | y \in Y \},$$

where $p_W(y)$, $q_W(y)$, and $r_W(y)$ are real standard or nonstandard subsets belonging to $]0^-, 1^+[$, also given as $p_W(y) : Y \rightarrow]0^-, 1^+[$, $q^+[$, $r_W(y) : Y \rightarrow]0^-, 1^+[$, and $r_W(y) : Y \rightarrow]0^-, 1^+[$. There is no restriction on the sum of $p_W(y)$, $q_W(y)$, and $r_W(y)$, so we have

$$0^- \leq \sup p_W(y) + q_W(y) + \sup r_W(y) \leq 3^+.$$

Definition 15.6. Smarandache [32]

Let there be two single-valued NSs A and B , then $C = (A \cup B)$ with truth $p_C(y)$, indeterminacy $q_C(y)$, and falsity $r_C(y)$ membership functions are given by

$$\begin{aligned} p_C(y) &= \max(p_A(y), p_B(y)), \\ q_C(y) &= \min(q_A(y), q_B(y)), \text{ and} \\ r_C(y) &= \min(r_A(y), r_B(y)) \text{ for each } y \in Y. \end{aligned}$$

Definition 15.7. Smarandache [32]

Let there be two single-valued NSs A and B , then $C = (A \cap B)$ with truth $p_C(y)$, indeterminacy $q_C(y)$, and falsity $r_C(y)$ membership functions are given by

$$\begin{aligned} p_C(y) &= \min(p_A(y), p_B(y)), \\ q_C(y) &= \max(q_A(y), q_B(y)), \text{ and} \\ r_C(y) &= \max(r_A(y), r_B(y)) \text{ for each } y \in Y. \end{aligned}$$

First, Bellman and Zadeh [37] introduced the idea of the fuzzy decision set (D) which contains a set of fuzzy goals (G) and fuzzy constraints (C). Later on, it was widely used in many real-life decision-making problems. Thus, a fuzzy decision set (D) can be stated as follows:

$$D = G \cap C.$$

Equivalently, the neutrosophic decision set $D_{Neutrosophic}$, with a set of neutrosophic goals and constraints, can be given as follows:

$$D_{Neutrosophic} = (\cap_{o=1}^O G_o) (\cap_{n=1}^N C_n) = (y, p_D(y), q_D(y), r_D(y)),$$

where

$$p_D(y) = \min \left\{ \begin{array}{l} p_{G_1}(y), p_{G_2}(y), \dots, p_{G_O}(y) \\ p_{C_1}(y), p_{C_2}(y), \dots, p_{C_N}(y) \end{array} \right\} \quad \forall y \in Y,$$

$$q_D(y) = \max \left\{ \begin{array}{l} q_{G_1}(y), q_{G_2}(y), \dots, q_{G_O}(y) \\ q_{C_1}(y), q_{C_2}(y), \dots, q_{C_N}(y) \end{array} \right\} \quad \forall y \in Y,$$

$$r_D(y) = \max \left\{ \begin{array}{l} r_{G_1}(y), r_{G_2}(y), \dots, r_{G_O}(y) \\ r_{C_1}(y), r_{C_2}(y), \dots, r_{C_N}(y) \end{array} \right\} \quad \forall y \in Y,$$

where the truth, indeterminacy, and falsity membership functions have been represented by $p_W(y)$, $q_W(y)$, and $r_W(y)$ under neutrosophic decision set $D_{Neutrosophic}$, respectively.

The marginal evaluation for each objective function by using the transformation functions of truth $p_W(y)$, indeterminacy $q_W(y)$, and falsity $r_W(y)$ membership functions can be derived with the help of the upper and lower bounds of each objective function. The solution of each single objective under the given set of constraints provides the upper and lower bounds for each objective function and can be denoted as U_o and L_o with a set of decision variables X^1, X^2, \dots, X^o , respectively.

Mathematically, it can be shown as follows:

$$U_o = \max [Z_o(X^o)] \text{ and } L_o = \min [Z_o(X^o)] \quad \forall o = 1, 2, 3, \dots, O. \tag{15.55}$$

The upper and lower bounds for o objective function under the neutrosophic environment can be obtained as follows:

$$\begin{aligned} U_o^p &= U_o, L_o^p = L_o \text{ for truth membership,} \\ U_o^q &= L_o^p + s_o, L_o^q = L_o^p \text{ for indeterminacy membership,} \\ U_o^r &= U_o^p, L_o^r = L_o^p + t_o \text{ for falsity membership,} \end{aligned}$$

where s_o and $t_o \in (0, 1)$ are predetermined real numbers assigned by the decision maker(s). With the help of upper and lower bounds for each of the three membership

functions, we have presented the linear membership function under a neutrosophic decision-making framework.

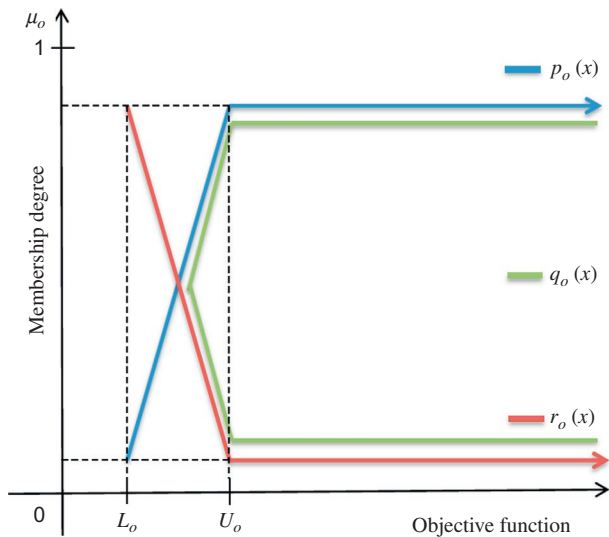
$$p_o(Z_o(x)) = \begin{cases} 1 & \text{if } Z_o(x) < L_o^p \\ \frac{U_o^p - Z_o(x)}{U_o^p - L_o^p} & \text{if } L_o^p \leq Z_o(x) \leq U_o^p, \\ 0 & \text{if } Z_o(x) > U_o^p \end{cases} \tag{15.56}$$

$$q_o(Z_o(x)) = \begin{cases} 1 & \text{if } Z_o(x) < L_o^q \\ \frac{U_o^q - Z_o(x)}{U_o^q - L_o^q} & \text{if } L_o^q \leq Z_o(x) \leq U_o^q, \\ 0 & \text{if } Z_o(x) > U_o^q \end{cases} \tag{15.57}$$

$$r_o(Z_o(x)) = \begin{cases} 1 & \text{if } Z_o(x) > U_o^r \\ \frac{Z_o(x) - L_o^r}{U_o^r - L_o^r} & \text{if } L_o^r \leq Z_o(x) \leq U_o^r. \\ 0 & \text{if } Z_o(x) < L_o^r \end{cases} \tag{15.58}$$

In the above-discussed membership functions, $L_o^{(\cdot)} \neq U_o^{(\cdot)}$ for all o objective functions. The value of these membership will be equal to 1, if for any membership $L_o^{(\cdot)} = U_o^{(\cdot)}$. The diagrammatic representation of the objective function with different components of membership functions under a neutrosophic decision set is shown in Fig. 15.2.

Fig. 15.2 Diagrammatic representation of truth, indeterminacy, and falsity membership degrees for the objective function.



Logically, the aim of developing the different achievement function is to achieve the maximum satisfaction degree or level according to the preference of the decision maker(s). Therefore, here also we have defined the individual achievement variables for each membership function, such as by maximization of truth membership, maximization of indeterminacy degree, and minimization of a falsity degree of each objective function efficiently. With the aid of linear truth, indeterminacy, and falsity membership functions under a neutrosophic environment, the neutrosophic fuzzy mathematical programming model can be presented as follows:

$$\begin{aligned}
 M_3: & \text{Max } \min_{o=1,2,3,\dots,o} p_o(Z_o(x)) \\
 & \text{Max } \min_{o=1,2,3,\dots,o} q_o(Z_o(x)) \\
 & \text{Min } \max_{o=1,2,3,\dots,o} r_o(Z_o(x)) \\
 & \text{subject to} \\
 & p_o(Z_o(x)) \geq q_o(Z_o(x)), p_o(Z_o(x)) \geq r_o(Z_o(x)) \\
 & 0 \leq p_o(Z_o(x)) + q_o(Z_o(x)) + r_o(Z_o(x)) \leq 3. \\
 & \text{Eqs. (15.40) – (15.54)}
 \end{aligned}$$

With the help of auxiliary parameters, model M_3 can be transformed into the following form M_4 .

$$\begin{aligned}
 M_4: & \text{Max } \lambda_o \\
 & \text{Max } \theta_o \\
 & \text{Min } \eta_o \\
 & \text{subject to} \\
 & p_o(Z_o(x)) \geq \lambda_o \\
 & q_o(Z_o(x)) \geq \theta_o \\
 & r_o(Z_o(x)) \leq \eta_o \\
 & \lambda_o \geq \theta_o, \lambda_o \geq \eta_o, \quad 0 \leq \lambda_o + \theta_o + \eta_o \leq 3 \\
 & \lambda_o, \theta_o, \eta_o \in (0, 1). \\
 & \text{Eqs. (15.40) – (15.54)}
 \end{aligned}$$

Without loss of generality, the model M_4 can be rewritten as in M_5 .

$$\begin{aligned}
 M_5: & \text{Max } \sum_{o=1}^o (\lambda_o + \theta_o - \eta_o) \\
 & \text{subject to} \\
 & Z_o(x) + (U_o^p - L_o^p)\lambda_o \leq U_o^p \\
 & Z_o(x) + (U_o^q - L_o^q)\theta_o \leq U_o^q \\
 & Z_o(x) - (U_o^r - L_o^r)\eta_o \leq L_o^r \\
 & \lambda_o \geq \theta_o, \lambda_o \geq \eta_o, \quad 0 \leq \lambda_o + \theta_o + \eta_o \leq 3 \\
 & \lambda_o, \theta_o, \eta_o \in (0, 1), \\
 & \text{Eqs. (15.40) – (15.54)}
 \end{aligned}$$

where λ_o , θ_o , and η_o are auxiliary achievement variables for truth, indeterminacy, and falsity membership functions, respectively. Therefore, the proposed NFPA is a convenient conventional optimization technique that is only preferred over others due to the existence of its independent indeterminacy degree.

15.5.3 Modified neutrosophic fuzzy programming with intuitionistic fuzzy preference relations

The effective modeling and optimization framework of multiobjective optimization problems explicitly results in the best possible compromise solution under adverse circumstances, since, while dealing with multiple objectives or goals, most often, DM(s) intends to provide priorities among the different objectives over each other. Generally, the preferences among the objective function have been defined by assigning the maximum crisp weight parameter (say $w_o = 0.1, 0.2, \dots, 1 | \sum_o^O w_o = 1$) to the preferred objective function. In the past few decades, Aköz and Petrovic [38] proposed a new methodology to assign the preference among different objectives or goals based on the linguistic importance relation and investigated three different fuzzy linguistic importance relationship such as *slightly more important than*, *moderately more important than*, and *significantly more important than* for different conflicting objectives. These linguistic terms have taken the advantages of membership functions associated with corresponding objectives or goals between which the important relation has been defined. Later on, this linguistic preference scheme was adopted by several researchers (see [27, 39–46]) in various real-life applications and decision-making processes. The appropriate selection of membership functions is always a crucial task for decision makers. Since the quantification of preference, the membership function has been done for the three linguistic fuzzy preference relations, but it would be more convenient and realistic to consider the nonmembership function as well as the similar linguistic fuzzy preference relations.

Therefore, to incorporate the membership and nonmembership function for linguistic preference relations among the objective, we have designed the structure of our proposed linguistic preference relations among different objectives or goals. Again, we have developed the linear membership and nonmembership function for each linguistic preference relation among the different objectives in the intuitionistic fuzzy environment. The transformation function has been defined with the help of truth membership functions of each objective. The information regarding linguistic preference relations under the intuitionistic fuzzy environment is shown in Table 15.2. The membership and nonmembership function for intuitionistic fuzzy linguistic preference relations is shown in Fig. 15.3.

The linear membership function for each linguistic preference relation can be defined as follows and achieved by maximizing it [38].

$$\mu_{R_{1(o,u)}}^{\sim} = \begin{cases} (p_o - p_u + 1) & \text{if } -1 \leq p_o - p_u \leq 0 \\ 1 & \text{if } 0 \leq p_o - p_u \leq 1 \end{cases}, \quad (15.59)$$

Table 15.2 Linguistic relative preferences of objective o over u .

Linguistic term	Intuitionistic fuzzy relation	Membership and nonmembership functions	Transform function
Slightly more important than	\tilde{R}_1	$\mu_{\tilde{R}_1}$ and $\nu_{\tilde{R}_1}$	$p_o(X) - p_u(X) \forall o, u \in (1 \dots O)$
Moderately more important than	\tilde{R}_2	$\mu_{\tilde{R}_2}$ and $\nu_{\tilde{R}_2}$	
Significantly more important than	\tilde{R}_3	$\mu_{\tilde{R}_3}$ and $\nu_{\tilde{R}_3}$	

$$\mu_{\tilde{R}_2(o,u)} = \left\{ \left(\frac{p_o - p_u + 1}{2} \right) \text{ if } -1 \leq p_o - p_u \leq 1, \right. \tag{15.60}$$

$$\mu_{\tilde{R}_3(o,u)} = \begin{cases} 0 & \text{if } -1 \leq p_o - p_u \leq 0 \\ (p_o - p_u) & \text{if } 0 \leq p_o - p_u \leq 1 \end{cases} \tag{15.61}$$

The linear nonmembership function for the linguistic preference relations can be given as follows and achieved by minimizing it.

$$\nu_{\tilde{R}_1(o,u)} = \begin{cases} -(p_o - p_u) & \text{if } -1 \leq p_o - p_u \leq 0 \\ 0 & \text{if } 0 \leq p_o - p_u \leq 1 \end{cases} \tag{15.62}$$

$$\nu_{\tilde{R}_2(o,u)} = \left\{ \frac{1 - (p_o - p_u)}{2} \text{ if } -1 \leq p_o - p_u \leq 1, \right. \tag{15.63}$$

$$\nu_{\tilde{R}_3(o,u)} = \begin{cases} 1 & \text{if } -1 \leq p_o - p_u \leq 0 \\ 1 - (p_o - p_u) & \text{if } 0 \leq p_o - p_u \leq 1 \end{cases} \tag{15.64}$$

where \tilde{R}_1 , \tilde{R}_2 , and \tilde{R}_3 are the importance relations defined by the linguistic term *slightly more important than*, *moderately more important than*, and *significantly more important than*, respectively.

The new achievement function for satisfaction degrees of the imprecise linguistic importance relations can be defined with the aid of the membership and nonmembership function for intuitionistic fuzzy linguistic preference relations. We have defined a score function $S_{\tilde{R}(o,u)} = (\mu_{\tilde{R}(o,u)} - \nu_{\tilde{R}(o,u)})$, which has been used to express the satisfactory degree of decision makers' linguistic importance relations. Let us define a binary variable $BI_{(o, u)}$; $o, u = 1, 2, \dots, O$, where $o \neq u$ such that

$$BI_{o,u} = \begin{cases} 1 & \text{if a linguistic preference relation is defined between the objective } Z_o \text{ and } Z_u \\ 0 & \text{otherwise} \end{cases} \tag{15.65}$$

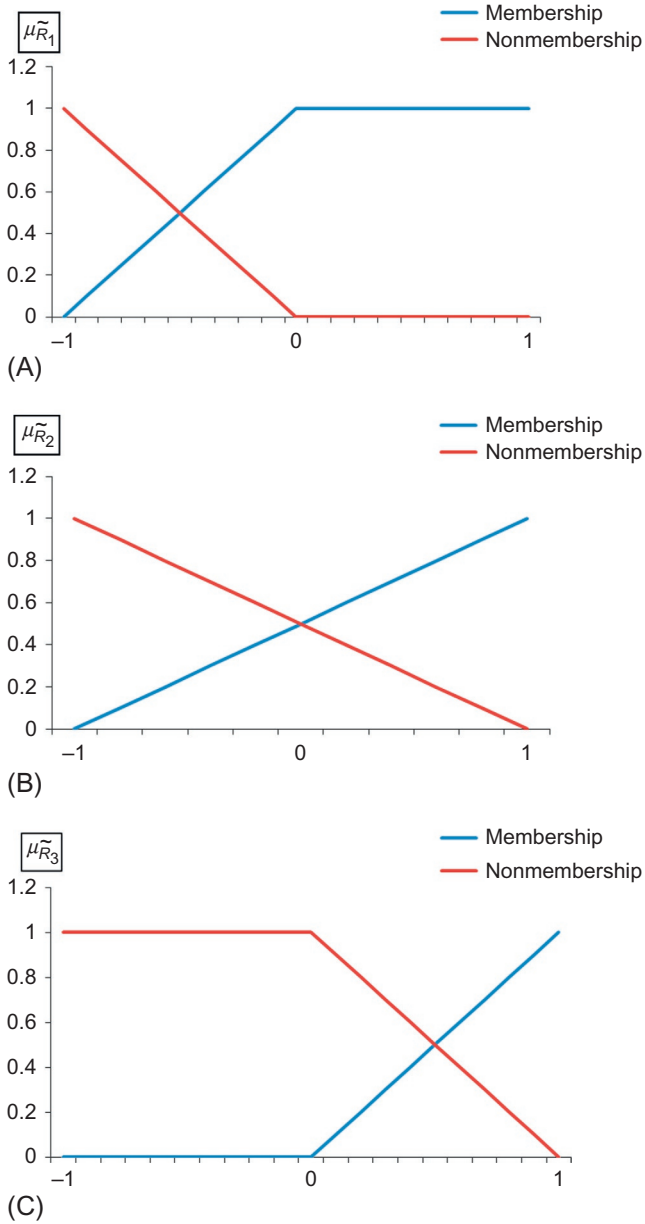


Fig. 15.3 Linear membership and nonmembership functions for intuitionistic fuzzy linguistic preference relations. (A) $R_1(o, u) = \tilde{R}_1$. (B) $R_2(o, u) = \tilde{R}_2$. (C) $R_3(o, u) = \tilde{R}_3$.

The modified NFPA with intuitionistic fuzzy linguistic preference relations has been designed with the hybrid integration of the achievement function under the NFPA model and score functions for the satisfaction degree of decision makers. The achievement function for the modified NFPA can be defined as the convex combination of the sum of individual truth membership, indeterminacy function, and falsity membership function of each objective or goals and the sum of score functions of the imprecise linguistic importance relations. Thus the proposed modified NFPA can be given as follows:

$$\begin{aligned}
 M_6: \quad & \text{Max } \alpha \sum_{o=1}^O (\lambda_o + \theta_o - \eta_o) + (1 - \alpha) \sum_{o=1}^O \sum_{u=1}^O BI_{o,u} S_{R(o,u)}^{\sim} \\
 & \text{subject to} \\
 & Z_o(x) + (U_o^p - L_o^p) \lambda_o \leq U_o^p \\
 & Z_o(x) + (U_o^q - L_o^q) \theta_o \leq U_o^q \\
 & Z_o(x) - (U_o^r - L_o^r) \eta_o \leq L_o^r \\
 & (p_o - p_u + 1) \geq \mu_{R_1(o,u)}^{\sim} \\
 & \left(\frac{p_o - p_u + 1}{2} \right) \geq \mu_{R_2(o,u)}^{\sim} \\
 & (p_o - p_u) \geq \mu_{R_3(o,u)}^{\sim} \\
 & -(p_o - p_u) \leq \nu_{R_1(o,u)}^{\sim} \\
 & \frac{1 - (p_o - p_u)}{2} \leq \nu_{R_2(o,u)}^{\sim} \\
 & 1 - (p_o - p_u) \leq \nu_{R_3(o,u)}^{\sim} \\
 & S_{R(o,u)}^{\sim} = (\mu_{R(o,u)}^{\sim} - \nu_{R(o,u)}^{\sim}) \\
 & \mu_{R(o,u)}^{\sim} \geq \nu_{R(o,u)}^{\sim} \\
 & 0 \leq \mu_{R(o,u)}^{\sim} + \nu_{R(o,u)}^{\sim} \leq 1 \\
 & 0 \leq \mu_{R(o,u)}^{\sim}, \nu_{R(o,u)}^{\sim} \leq 1 \quad \forall BI_{o,u} = 1 \\
 & \lambda_o \geq \theta_o, \lambda_o \geq \eta_o, \quad 0 \leq \lambda_o + \theta_o + \eta_o \leq 3 \\
 & \lambda_o, \theta_o, \eta_o \in (0, 1), \\
 & \text{Eqs. (15.40) - (15.54)}
 \end{aligned}$$

where α is a nonzero parameter taking values between 0 and 1 and can be assigned by tuning it for either the membership function of objectives or linguistic preference relations.

The proposed modified NFPA modeling approach considers the degree of belongingness and nonbelongingness simultaneously, which is a better representation of uncertain importance relations among objectives because it enhances the membership degree as well as efficiently reducing the nonmembership degree. In spite of all this, while dealing with a large number of goals at a time, assigning the different crisp weight to all objectives according to the decision-makers' priority level is not feasible, because it may be time-consuming. To avoid the weight assignment complexity, it would be the best technique to assign linguistic priorities among different objectives.

15.5.3.1 Stepwise solution algorithm

The stepwise solution procedures for the proposed modified NFPA with intuitionistic fuzzy preference relations can be represented as follows:

Step 1. Design the proposed CLSC planning problem under uncertainty as given in model M_1 .

Step 2. Convert each fuzzy parameter involved in model M_1 into its crisp form by using the expected intervals and values method as given in Eqs. (15.28)–(15.31) or presented in Table 15.1. Transform fuzzy constraints into their crisp versions by using Eqs. (15.38)–(15.39).

Step 3. Modify model M_1 into M_2 and solve for each objective function individually in order to obtain the best and worst solution set.

Step 4. Determine the upper and lower bounds for each objective function by using Eq. (15.55). With the aid of U_o and L_o , define the upper and lower bounds for truth, indeterminacy, and falsity memberships as given in Eqs. (15.56)–(15.58).

Step 5. Develop the neutrosophic optimization model M_5 with the aid of auxiliary variables.

Step 6. Assign linguistic importance relations among different objectives under an intuitionistic fuzzy environment (see Eqs. 15.59–15.64). Integrate the preference relation into model M_5 and transform into model M_6 , which includes constraints of CLSC given in Eqs. (15.40)–(15.54).

Step 7. Model M_6 represents the modified neutrosophic fuzzy optimization model with intuitionistic fuzzy importance relations. Solve the model in order to obtain the compromise solution using suitable techniques or some optimizing software packages.

15.6 Computational study

The city of Nizam (Deccan), currently known as Hyderabad, is one of the leading IT hubs of India. It is well known for its IT hub service-oriented firms. A Hyderabad-based ABC (name changed) reputed multinational laptop manufacturing company has intended to model the production, transportation, distribution, and collection problems, due to the existence of a testing center facility in the proposed CLSC designed network. The prominent features of the CLSC design made it possible for the modeling and optimization approach under uncertainty. Regardless, unique, potentially functional components of the proposed CLSC design model have attracted the attention of decision makers. Less opportunity for the disposal of scrap parts/components is also a leading factor to adopt the model which ensures less accountability toward governmental managerial laws. The ecofriendly environmental nature of the modeling approach is a beneficial factor and guarantees freedom from the different governmental legislative traps. The interference of uncertainty among the various parameters reveals the realistic modeling approach. Ample scope for generating different solutions set by tuning the weight parameter and feasibility degree is the crucial promising factor for modeling choice by decision makers. To maintain sustainability in the competitive market, it would be more effective and efficient to develop the proposed CLSC design network.

The company has a fully functional multiechelon facility location and a well-organized decision policy scheme. In the forward chain, five multiechelon facilities

are the main constituent part of the forward process. Three raw material storage centers, three supplier points, three hybrid manufacturing/remanufacturing plants, three distribution centers, and six customer/market zones explicitly represent the forward flow chain. In the reverse flow chain, six multiechelon facilities are taken into consideration, which signifies more emphasis on the opposite chain. The reverse flow chain consists of three collection centers of used laptops, three refurbishing or repair centers, three disassembling centers, three testing points, three recycling centers, and three disposal sites at which the end-of-life parts/components are removed from the designed CLSC network.

Every new and refurbished laptop is a hybrid combination of three different types of raw materials and parts/components. Refurbished laptops are also usable and acceptable in the market. Manufacturing plants provide a new laptop whereas the refurbishing center is responsible for renovated or refurbished laptops. The forward chain starts from the shipment of raw parts from the raw materials storage center to three supplier points. All three suppliers are responsible for the delivery of raw materials to hybrid manufacturing plants. Afterward, the newly manufactured laptops are shipped to three distribution centers. The demand quantity of the laptops must be fulfilled by the distribution center only. There is no scope for direct shipment from the manufacturing plant to the hybrid facility center. The collection center is accountable for the accumulation of end-of-use products from customers/market zones. The used products are disassembled into three parts or components. The testing facility carefully inspects the various parts/components and decides to implement a particular service to make it usable. From the testing center, three different destinations—the manufacturing plant, recycling point, and disposal center—have been postulated. Recyclable products are sent to the recycling center, whereas scrap or end-of-life parts/components are dumped at the disposal center. Parts/components that can constitute raw materials are entered into the forward chain through manufacturing plants. The recycling process turns the pieces into new raw materials, which ensures the procurement of raw materials and initiates the forward chain. Hence to implement the proposed CLSC model efficiently, the triangular fuzzy input data for transportation cost, purchasing cost, revenues, and time have been summarized in Table 15.3. Various capacities at each echelon in the CLSC chain network have been represented by trapezoidal fuzzy data, whereas processing cost parameters have been considered as triangular fuzzy input data. Since numerous objective functions have been developed in the proposed CLSC model, the following preference relations have been decided among different objective functions. However, the preference scheme has been randomly assigned, and there are no hard and fast rules. It solely depends upon the decision maker's choices. The type of preference relations between the objectives have been defined as follows:

- Objective Z_2 is moderately more important than objective Z_1 (i.e., $\tilde{R}_2(2,1)$).
- Objective Z_4 is slightly more important than objective Z_3 (i.e., $\tilde{R}_1(4,3)$).
- Objective Z_3 is significantly more important than objective Z_5 (i.e., $\tilde{R}_3(3,5)$).
- Objective Z_4 is slightly more important than objective Z_5 (i.e., $\tilde{R}_1(4,5)$).

Table 15.3 Input fuzzy data for the parameters.

Transportation cost from sources to destinations (*, *)	Types of raw materials (<i>m</i>) or products (<i>l</i>)		
	1	2	3
$\widetilde{TC1}_{m,a,b}$	(14, 24, 34)	(22, 32, 44)	(34, 36, 38)
$\widetilde{TC2}_{m,b,c}$	(30, 32, 34)	(34, 36, 38)	(38, 40, 42)
$\widetilde{TC3}_{l,c,d}$	(52, 56, 60)	(60, 63, 66)	(66, 67, 68)
$\widetilde{TC4}_{l,d,e}$	(60, 65, 70)	(66, 69, 72)	(71, 74, 77)
$\widetilde{TC5}_{l,e,f}$	(28, 29, 30)	(35, 37, 39)	(42, 44, 46)
$\widetilde{TC6}_{l,f,g}$	(32, 34, 36)	(35, 39, 43)	(41, 42, 43)
$\widetilde{TC7}_{l,g,d}$	(40, 42, 44)	(45, 48, 51)	(50, 55, 60)
$\widetilde{TC8}_{l,f,h}$	(44, 48, 52)	(50, 53, 56)	(55, 59, 63)
$\widetilde{TC9}_{m,h,i}$	(50, 51, 52)	(50, 55, 60)	(60, 63, 66)
$\widetilde{TC10}_{m,i,c}$	(25, 27, 29)	(30, 32, 34)	(35, 39, 41)
$\widetilde{TC11}_{m,i,j}$	(55, 60, 65)	(65, 67, 69)	(71, 73, 75)
$\widetilde{TC12}_{m,i,k}$	(33, 36, 39)	(40, 43, 46)	(44, 49, 54)
$\widetilde{TC13}_{m,j,a}$	(68, 71, 74)	(73, 75, 77)	(60, 62, 64)
Time			
$\widetilde{T}_{l,d,e}$	(05, 07, 09)	(04, 06, 08)	(01, 03, 06)
Purchasing cost			
$\widetilde{PU1}_m$	(36, 38, 40)	(45, 47, 49)	(24, 26, 28)
$\widetilde{PU2}_l$	(25, 27, 29)	(15, 17, 19)	(15, 17, 19)
Selling price			
$\widetilde{SP1}_m$	(42, 46, 50)	(40, 45, 50)	(21, 23, 26)
$\widetilde{SP2}_l$	(36, 38, 40)	(42, 45, 48)	(24, 26, 28)
rf_i	0.71	0.53	0.58
$rc_{l,e}$	0.82	0.76	0.38
rt_m	0.23	0.49	0.73
rm_m	0.81	0.67	0.35
rr_m	0.32	0.43	0.61
rd_m	0.12	0.19	0.23
Processing cost at each echelon			
$\widetilde{PC1}_{m,a}$	(14, 24, 34)	(22, 32, 44)	(34, 36, 38)
$\widetilde{PC2}_{m,b}$	(30, 32, 34)	(34, 36, 38)	(38, 40, 42)
$\widetilde{PC3}_{l,c}$	(52, 56, 60)	(60, 63, 66)	(66, 67, 68)
$\widetilde{PC4}_{l,d}$	(60, 65, 70)	(66, 69, 72)	(71, 74, 77)
$\widetilde{PC5}_{l,f}$	(28, 29, 30)	(35, 37, 39)	(42, 44, 46)
$\widetilde{PC6}_{l,g}$	(32, 34, 36)	(35, 39, 43)	(41, 42, 43)

Table 15.3 Continued

Transportation cost from sources to destinations (*, *)	Types of raw materials (<i>m</i>) or products (<i>l</i>)		
	1	2	3
$\widetilde{PC7}_{l,h}$	(40, 42, 44)	(45, 48, 51)	(50, 55, 60)
$\widetilde{PC8}_{m,i}$	(44, 48, 52)	(50, 53, 56)	(55, 59, 63)
$\widetilde{PC9}_{m,j}$	(50, 51, 52)	(50, 55, 60)	(60, 63, 66)
$\widetilde{PC10}_{m,k}$	(25, 27, 29)	(30, 32, 34)	(35, 39, 41)
Capacity/demand at each echelon			
$\widetilde{MC1}_{m,a}$	(512, 514, 516, 518)	(622, 624, 626, 628)	(718, 724, 726, 728)
$\widetilde{MC2}_{m,b}$	(613, 614, 615, 616)	(514, 516, 518, 520)	(512, 514, 516, 518)
$\widetilde{MC3}_{m,c}$	(724, 725, 726, 727)	(812, 813, 814, 815)	(914, 916, 918, 920)
$\widetilde{MC4}_{l,d}$	(212, 214, 216, 218)	(221, 222, 223, 224)	(217, 218, 219, 220)
$\widetilde{MC5}_{l,e}$	(314, 318, 322, 326)	(312, 314, 316, 318)	(329, 339, 349, 359)
$\widetilde{MC6}_{l,f}$	(115, 116, 117, 118)	(119, 120, 121, 122)	(114, 116, 118, 120)
$\widetilde{MC7}_{l,g}$	(124, 125, 126, 127)	(113, 114, 115, 116)	(117, 119, 121, 123)
$\widetilde{MC8}_{m,h}$	(110, 111, 112, 113)	(114, 116, 118, 120)	(119, 120, 121, 122)
$\widetilde{MC9}_{m,i}$	(224, 225, 226, 227)	(212, 214, 216, 218)	(314, 316, 318, 320)
$\widetilde{MC10}_{m,j}$	(324, 325, 326, 327)	(212, 213, 214, 215)	(214, 216, 218, 220)
$\widetilde{MC11}_{m,k}$	(212, 214, 216, 218)	(221, 222, 223, 224)	(317, 318, 319, 320)

15.6.1 Results and discussions

The modified neutrosophic fuzzy optimization model for the proposed CLSC network has been written in AMPL language and solved using the solver Kintro 10.3.0 through the NEOS server version 5.0 online facility provided by Wisconsin Institutes for Discovery at the University of Wisconsin in Madison for solving optimization problems; see Refs. [47,48]. The characteristic description of the problem is presented as follows: The final multiobjective optimization model along with a set of well-defined multiple objectives comprises 459 variables including 42 binary variables and 417 linear variables, 530 constraints including 498 linear one-sided inequalities constraints and 32 linear equality constraints, respectively. The total computational time for

obtaining the final solution was 0.113 seconds (CPU time). Due to space limitations, only the final solution results of all decision variables obtained at a feasibility degree ($\beta = 0.5$) with weight parameter ($\alpha = 0.5$) have been discussed in detail. The optimum allocation of raw materials, new products, and used parts/components among different echelons has been depicted in Tables 15.4 and 15.5. In the forward chain, procurement of raw materials initiates from a raw material storage center (RMS) to a supplier point (SP). The total allocation of raw materials from RMS 1 to all three SPs is found to be 504.17, 592.38, and 681.51, whereas from RMS 2 and 3 to all three SPs have been obtained as 572.14, 553.84, and 703.15, and 497.57, 457.32, and 646.87, respectively. The maximum shipment quantity has been observed from RMS 2 to SP 3 due to the lowest transportation and processing cost incurred over the raw materials. Suppliers are responsible for fulfilling the requirement for starting the manufacturing processes at the hybrid manufacturing plant (MP). The optimum shipment quantity from SP 1 to all three MPs is 706.25, 630.15, and 625.18, respectively. Similarly, from SP 2 and 3 to all three MPs have been obtained as 630.25, 630.21, and 656.51, and 563.70, 498.34, and 533.18, respectively. The highest shipment amount of raw materials has been allocated to MP 1 whereas the least amount of raw materials has been delivered to MP 2 bearing in mind the fact that the outbound capacity of manufacturing plant receives the maximum raw materials and parts from the SPs and testing points (TPs). SP 3 also provides the maximum amount of raw materials to all three MPs and are obtained as 563.7, 488.34, and 533.18 bearing in mind the fact that outbound restrictions on manufacturing plants have been satisfied, and tested and approved parts/components are sent back to the manufacturing plant for further utilization. Newly built products are transferred to the distribution center (DC) so that the demand from customers (Cs) could be met. The optimal distribution scheme among different customers has been obtained. From DC 1 to all six Cs, the total shipment of products is found to be 332.23, 400.85, 350.61, 297.21, 274.95, and 266.61, respectively. However, DC 1 has a negligible contribution to meet the demand of C 2, 3, 4, and 5, with other types of products to avoid the maximum transportation cost and late expected delivery time. Similarly, the total quantities of each product distributed from DC 2 and 3 to all six Cs have been depicted, which ensures the minimum transportation costs along with the timely shipment of products. It has been observed that no product has been shipped from DC 2 to C 1, 2, and 3 due to the maximum chances for late delivery of the products. Hence a minimum transportation cost and shipment time have been achieved without significantly affecting the demand constraint. Overall, DC 2 outsourced the maximum shipment of products to all six Cs and revealed a significant contribution to fulfilling the demand. Since refurbished products are also acceptable in the market, approximately 13.32% of total used products are renovated and shipped to DCs for the fulfillment of further needs.

The significant role of the collection center (CC) starts when end-of-use and end-of-life products come into existence. The potential accumulation framework for used products from the customer zone is much needed. The designed CLSC model inherently involves the CC, which is the first echelon of the reverse supply chain network. The exclusive collection of the end-of-use product from customers is found to be a significant percentage, that is, approximately 91.34% of the total fulfilled demand,

Table 15.4 Optimal quantities of raw materials and products shipped from different sources to various destinations.

Raw material storage center (a)	Supplier point (b)	Types of raw material (m)		
		1	2	3
Storage center 1	1	127.83	232.78	143.56
Storage center 1	2	248.23	151.62	192.53
Storage center 1	3	201.32	312.28	167.91
Storage center 2	1	164.21	264.67	143.26
Storage center 2	2	321.34	109.23	123.27
Storage center 2	3	221.63	368.83	112.69
Storage center 3	1	116.94	219.43	161.20
Storage center 3	2	213.52	142.20	101.60
Storage center 3	3	329.53	127.64	189.70
Supplier point (b)	Manufacturing plant (c)	Types of raw materials (m)		
		1	2	3
Supplier point 1	1	261.24	243.12	201.89
Supplier point 1	2	291.64	124.15	214.36
Supplier point 1	3	236.39	213.56	175.23
Supplier point 2	1	218.95	189.67	221.63
Supplier point 2	2	253.68	128.63	247.90
Supplier point 2	3	287.25	112.46	256.80
Supplier point 3	1	212.54	187.62	163.54
Supplier point 3	2	202.35	142.37	143.62
Supplier point 3	3	298.34	116.52	118.32
Manufacturing plant (c)	Distribution center (d)	Types of products (l)		
		1	2	3
Manufacturing plant 1	1	127.83	132.78	143.56
Manufacturing plant 1	2	148.23	151.62	192.53
Manufacturing plant 1	3	121.32	112.28	67.91
Manufacturing plant 2	1	164.21	64.67	163.26
Manufacturing plant 2	2	171.34	119.23	143.27
Manufacturing plant 2	3	181.63	68.83	152.69
Manufacturing plant 3	1	196.94	89.43	61.20
Manufacturing plant 3	2	113.52	42.20	101.60
Manufacturing plant 3	3	129.53	127.64	189.70
Distribution center (d)	Customers (e)	Types of products (l)		
		1	2	3
Distribution center 1	1	112.34	145.26	74.63
Distribution center 1	2	85.26	163.23	152.36
Distribution center 1	3	152.36	–	198.35
Distribution center 1	4	163.98	–	115.23

Continued

Table 15.4 Continued

Distribution center (<i>d</i>)	Customers (<i>e</i>)	Types of products (<i>l</i>)		
		1	2	3
Distribution center 1	5	165.32	–	109.63
Distribution center 1	6	154.23	–	112.38
Distribution center 2	1	198.43	167.23	–
Distribution center 2	2	165.24	144.23	–
Distribution center 2	3	180.50	143.20	–
Distribution center 2	4	155.96	124.27	127.52
Distribution center 2	5	169.58	153.65	65.87
Distribution center 2	6	187.65	84.59	154.23
Distribution center 3	1	169.75	–	159.86
Distribution center 3	2	–	173.89	168.27
Distribution center 3	3	–	196.43	149.26
Distribution center 3	4	–	142.35	149.37
Distribution center 3	5	184.26	73.68	163.87
Distribution center 3	6	179.35	97.36	135.98
Customers (<i>e</i>)	Collection center (<i>f</i>)	Types of products (<i>l</i>)		
		1	2	3
Customer 1	1	61.32	53.68	94.38
Customer 1	2	85.23	78.56	145.80
Customer 1	3	84.32	58.50	145.23
Customer 2	1	52.31	16.78	61.83
Customer 2	2	47.50	51.32	134.62
Customer 2	3	79.68	45.23	84.23
Customer 3	1	52.63	89.45	79.56
Customer 3	2	74.96	98.74	112.34
Customer 3	3	47.89	114.90	78.46
Customer 4	1	89.56	89.45	74.68
Customer 4	2	76.34	94.68	52.60
Customer 4	3	58.35	78.89	53.46
Customer 5	1	61.23	106.83	45.3
Customer 5	2	63.85	117.40	47.6
Customer 5	3	86.34	127.63	76.85
Customer 6	1	74.68	121.69	44.62
Customer 6	2	85.90	153.45	57.67
Customer 6	3	84.32	173.65	79.85
Collection center (<i>f</i>)	Refurbishing center (<i>g</i>)	Types of products (<i>l</i>)		
		1	2	3
Collection center 1	1	36.24	45.32	32.65
Collection center 1	2	41.58	31.25	21.32
Collection center 1	3	16.23	74.32	24.12
Collection center 2	1	14.23	61.32	42.37

Table 15.4 Continued

Collection center (<i>f</i>)	Refurbishing center (<i>g</i>)	Types of products (<i>l</i>)		
		1	2	3
Collection center 2	2	71.20	41.23	54.64
Collection center 2	3	24.53	85.93	27.65
Collection center 3	1	34.53	22.38	68.53
Collection center 3	2	74.30	72.30	23.60
Collection center 3	3	25.90	33.56	67.84

Table 15.5 Optimal quantities of used products and parts shipped from different sources to various destinations.

Refurbishing plant (<i>g</i>)	Distribution center (<i>d</i>)	Types of products (<i>l</i>)		
		1	2	3
Refurbishing plant 1	1	34.28	24.89	52.37
Refurbishing plant 1	2	25.36	41.98	56.35
Refurbishing plant 1	3	27.85	39.38	49.35
Refurbishing plant 2	1	23.89	54.23	63.45
Refurbishing plant 2	2	31.45	47.68	54.38
Refurbishing plant 2	3	43.56	42.89	47.86
Refurbishing plant 3	1	44.87	57.98	53.78
Refurbishing plant 3	2	38.45	47.56	63.45
Refurbishing plant 3	3	37.84	49.63	57.68

Collection center (<i>f</i>)	Disassembling center (<i>h</i>)	Types of products (<i>l</i>)		
		1	2	3
Collection center 1	1	146.23	98.29	154.78
Collection center 1	2	131.26	157.23	74.39
Collection center 1	3	157.89	158.96	84.97
Collection center 2	1	98.46	143.69	87.56
Collection center 2	2	87.60	89.63	178.87
Collection center 2	3	89.68	63.84	187.20
Collection center 3	1	107.35	84.96	172.86
Collection center 3	2	118.35	97.63	166.34
Collection center 3	3	112.57	98.68	136.94

Disassembling center (<i>h</i>)	Testing center (<i>i</i>)	Types of products (<i>m</i>)		
		1	2	3
Disassembling center 1	1	98.86	47.52	112.36
Disassembling center 1	2	187.34	145.26	75.40
Disassembling center 1	3	85.32	143.26	146.37

Continued

Table 15.5 Continued

Disassembling center (<i>h</i>)	Testing center (<i>i</i>)	Types of products (<i>m</i>)		
		1	2	3
Disassembling center 2	1	55.85	121.35	141.23
Disassembling center 2	2	65.36	185.98	124.36
Disassembling center 2	3	80.45	178.90	142.58
Disassembling center 3	1	60.85	42.38	173.45
Disassembling center 3	2	75.03	63.57	156.89
Disassembling center 3	3	86.08	53.76	154.36
Recycling center (<i>j</i>)	Raw material storage facility (<i>a</i>)	Types of parts (<i>m</i>)		
		1	2	3
Recycling center 1	1	24.23	227.35	16.35
Recycling center 1	2	12.54	14.80	22.35
Recycling center 1	3	28.34	14.25	25.36
Recycling center 2	1	21.50	22.24	16.80
Recycling center 2	2	17.35	12.36	13.52
Recycling center 2	3	18.53	11.98	16.39
Recycling center 3	1	24.37	22.35	14.35
Recycling center 3	2	17.98	27.85	17.68
Recycling center 3	3	19.63	14.32	13.84
Testing center (<i>i</i>)	Manufacturing plant (<i>c</i>)	Types of tested parts (<i>m</i>)		
		1	2	3
Testing center 1	1	42.35	17.43	11.75
Testing center 1	2	13.40	16.98	27.06
Testing center 1	3	17.31	12.37	18.08
Testing center 2	1	08.32	29.56	21.07
Testing center 2	2	21.43	39.75	19.01
Testing center 2	3	17.35	18.56	12.89
Testing center 3	1	38.96	21.29	28.34
Testing center 3	2	17.51	10.37	12.34
Testing center 3	3	21.27	03.78	22.48
Testing center (<i>i</i>)	Recycling facility (<i>j</i>)	Types of recyclable parts (<i>m</i>)		
		1	2	3
Testing center 1	1	34.53	45.05	48.35
Testing center 1	2	54.27	35.64	53.42
Testing center 1	3	58.34	56.34	24.35
Testing center 2	1	62.78	49.64	31.70
Testing center 2	2	28.34	51.46	41.32
Testing center 2	3	25.32	47.86	105.06

Table 15.5 Continued

Testing center (<i>i</i>)	Recycling facility (<i>j</i>)	Types of recyclable parts (<i>m</i>)		
		1	2	3
Testing center 3	1	78.35	48.36	62.37
Testing center 3	2	48.32	42.36	56.28
Testing center 3	3	51.43	118.36	29.23
Testing center (<i>i</i>)	Disposal facility (<i>k</i>)	Types of scrap parts (<i>m</i>)		
		1	2	3
Testing center 1	1	14.25	–	16.52
Testing center 1	2	17.24	–	13.25
Testing center 1	3	–	–	11.24
Testing center 2	1	–	21.85	18.54
Testing center 2	2	–	19.65	–
Testing center 2	3	16.35	12.35	19.32
Testing center 3	1	12.89	14.22	–
Testing center 3	2	15.45	–	19.34
Testing center 3	3	17.40	–	21.30

which indicates the vast need for the reverse supply chain to tackle used products. The required service at different echelons in the reverse chain has been designed especially for socioenvironmental concerns. An optimal amount of used products has been collected by all three CCs from all six customer zones. The maximum amount of used products has been received by CC 3 from C 6 which is 337.82, and the least quantity 190.70 by CC 3 from C 4 to ensure the least collection and transportation costs levied over each type of product. At CCs, complete inspection of the collected, used products has been performed and a decision taken to ship either to the disassembling center (DS) or refurbishing center (RC) to initiate the required services. The total amounts of used products transported from CC 1 to all three RCs are obtained as 114.21, 94.15, and 114.67, whereas the total shipment quantities from CC 2 and 3 to all three RCs have been allocated as 117.92, 167.07, and 138.11, and 125.44, 170.20, and 127.30, respectively. The maximum quantity of used products has been transported from CC 3 to RC 2 whereas the minimum shipment quantity is found to be shipped from CC 1 to RC 2 because of the lowest transportation cost and availability of the required service for particular types of products. The quantity of used products is approximately 88.21% of the total capacity of the RC, which ensures the significant need for such a functional echelon in CLSC. The disassembling center (DS) only receives those end-of-use products that require reliability tests of each part/component. At the

DS, used products are disassembled into different parts/components for the testing process where all necessary measures would be taken regarding the useful life of parts. From CC 1 to all three DSs, the total shipment amounts of used products have been obtained as 399.30, 362.88, and 401.82, which shows approximately 31.98% of the entire collection of used products. Likewise, the net amount of used products transported from CC 2 and 3 to all three DSs are 329.71, 356.10, and 340.72, and 365.17, 382.32, and 348.19, respectively. The shipment of end-of-use products at DS 2 and 3 are found to be 14.29% and 39.47% of the net used products collected at all three CCs, which shows that approximately 94% of the total raised used products have been completely dealt with at the CC and signify that the design of the reverse chain is much needed to avoid environmental issues. The total disassembled parts that have been shipped from DS 1 to all three TPs are found to be 258.74, 408, and 374.95, which comprise 31.35% of the disassembled parts/components and ensures that transportation and inspection costs incurred over these parts would be minimal. Similarly, from DS 2 and 3 to all TPs the optimal amounts of pieces have been shipped, which are found to be 33.84%, and 29.27% of the total disassembled parts at DSs to minimize the total cost of inspection by ensuring the capacity restrictions at TPs, respectively.

The testing point (TP) inevitably inspects the reliability or usefulness of parts/components and provides the best decision to deal with tested parts. TPs are interconnected with three echelons: manufacturing plants, recycling centers, and disposal sites. The TP is also a promising source for the procurement of raw materials to hybrid manufacturing/remanufacturing plants. Approximately 9.84% of the total requirement for raw materials has been met by different TPs with the aid of dissembled parts of used products. However, the recycling point (RP) receives a significant amount of tested parts that ensures green practice with recyclable components. The net quantity of recyclable parts that have been transported from all three TPs to RP 1 is found to be 127.93, 143.33, and 139.03, which is 93.66% of the total recyclable capacity of tested parts at RP 1. The maximum quantity of recyclable parts has been received by RP 3 whereas the least amount of certified parts has been shipped to RP 2 bearing in mind the fact that transportation and recycling costs levied over each component are minimal at these facilities. Finished recycled products have been sent back to raw material storage centers and recycled for the smooth running of the production processes. After the inspection procedure, the declared disposable parts have been shipped for disposal. The optimal quantity of disposable parts has been obtained with the satisfaction of disposal capacity constraints of each disposal facility center (DF). The obtained results showing that at some DFs, there is no amount of tested parts for disposal. However, the total shipped amounts from all three TPs to DF 1 are found to be 30.77, 40.39, and 27.11, which is 47.32% of the full capacity of DF 1. Moreover, from all three TPs to DF 2 and 3, the net amounts of disposable parts that have been transported are found to be 30.49, 19.65, and 34.79, and 11.24, 48.02, and 38.70, respectively. At these DFs, approximately 53.57% and 69.38% shares of the total disposal capacity have been disposed of, which strictly ensures that there is still an abundant opportunity for incineration.

Multi-echelon CLSC design networks require potential capital investment to the flow of products throughout the supply chain processes. Processing cost,

transportation cost, and purchasing cost have been depicted as different objectives that inherently require capital. The obtained results of these three objectives show a remarkable contribution to the total capital investment. At each feasibility degree β and weight parameter α , the average share of processing cost has been obtained as approximately 83.39%, the total ordinary dividends of the transportation cost is found to be approximately 14.78% and that of the average purchasing cost is approximately 1.83% of the total investment in the proposed CLSC network. The maximum shares have been exhausted by the processing charge with the fact that multiple different echelons have been associated with specific functional services to raw materials, new products, and used parts in the proposed CLSC network. Transportation costs hold a slightly smaller portion of the total investment, which shows the reduced to and fro movement of products among different echelons. Due to the interconnected systematic facility centers, the optimal shipment strategy turns into fewer transportation costs. The purchasing of raw materials in bulk from raw material storage centers and used products from customers has comparatively very low in the total capital investment. The expected whole delivery time and revenues from sales have also been included as conflicting potential objectives in the proposed CLSC model, which sufficiently reflects the effective exogenous solution results. The flow of new products in the forward chain and end-of-use products in reverse chain much depends on keen managerial insight and decision-making strategy. The potential performances of each echelon would be recognized in the context of allocation and required service to the different products and parts. The solution results have been presented only for $\alpha = 0.5$ and $\beta = 0.5$, but more information could be extracted by obtaining the solution results at different values of α and β regarding the optimum allocation of products and parts, respectively.

15.6.2 Sensitivity analyses

Sensitivity analyses have been performed for all the objective functions by tuning the feasibility degree (β) and weight parameter (α) simultaneously. The feasibility degree (β) referred to the preference or acceptance level of decision makers. The higher value of (β) ensures the maximum satisfaction level of decision makers. The feasibility degree among parameters reflects the satisfaction level by offering different choices. Hence more substantial feasibility degree generally gives the worse solution of objectives. The weight parameter (α) provides the weight to either the membership function of all the objectives or the score functions of the intuitionistic fuzzy preference relations among different objectives. Therefore, a higher value of (α) signifies a higher weight to either the corresponding membership functions or the score function of linguistic preference relations. The priority structure has been designed as the convex combination between the membership function of the objectives and the score function of the linguistic preference relations. The weight parameter (α) is directly assigned to the membership functions of each goal whereas $(1 - \alpha)$ has been assigned to the score function of linguistic preference relations. The solution results of all the objective functions and preference relations are shown in [Fig. 15.4](#).

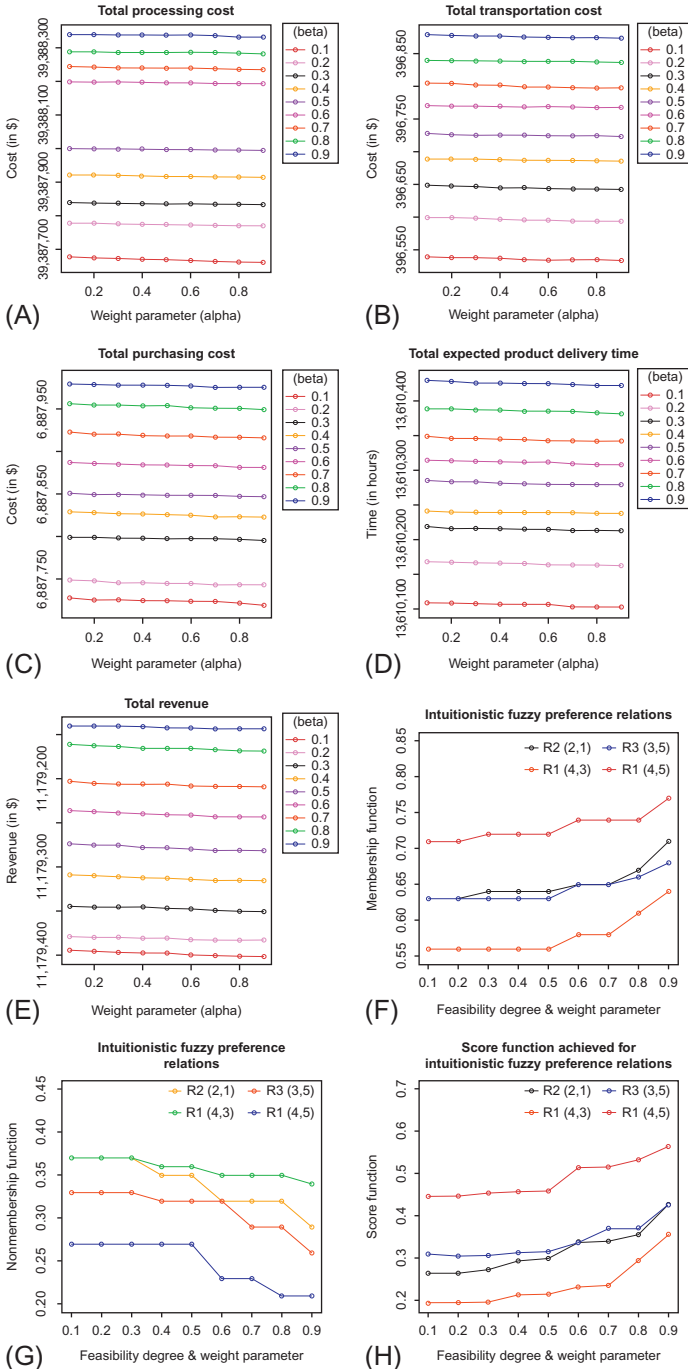


Fig. 15.4 Graphical representation of obtained results. (A) First objective (Z_1). (B) Second objective (Z_2). (C) Third objective (Z_3). (D) Fourth objective (Z_4). (E) Fifth objective (Z_5). (F) Membership degree. (G) Nonmembership degree. (H) Score function.

15.6.2.1 Sensitivity analyses of objective functions

The first objective (Z_1) is to minimize the total processing cost (TPC) supply chain. At $\beta = 0.1$ and $\alpha = 0.9$, the minimum (best) value of (Z_1) has been attained, which is \$39,387,662. As (α) decreases, the values of (Z_1) either increase or remain the same for some (α). With an increase in the feasibility degree (β), a significant increment in the objective function (Z_1) has been observed. The maximum (worst) value of objective (Z_1) has been obtained as \$39,388,338 at $\beta = 0.9$ and $\alpha = 0.1$. Hence it has been concluded that with the increase in feasibility degree (β) and the decrease in weight parameter (α), the value of objective (Z_1) reaches its worst values. The different solution results of (Z_1) ranging between \$39,387,662 and \$39,388,338 are summarized in Table 15.6, and Fig. 15.4A shows the trending behavior of (Z_1) at different feasibility degree (β) and weight parameters (α), respectively. Furthermore, the effects of feasibility degrees (β) are severe, as the marginal increment in the value of (Z_1) rapidly approaches the worst solutions, whereas the effect of the weight parameter (α) on the objective (Z_1) is almost negligible. The TPC has been obtained that solely occurred over four echelons in the forwarding chain. Hence, the obtained results for TPC are due to the high processing cost at the raw material storage center, supplier point, and manufacturing plants. Inbound capacity restrictions at these echelons are also a key factor for increment in TPC. The maximum numbers of raw materials and new products require different processing costs, which turn into more capital investment in the material and product processing purposes.

The minimization of total transportation costs in the CLSC has been represented by the second objective (Z_2). At $\beta = 0.1$ and $\alpha = 0.9$, the minimum (best) value of transportation cost is \$396,534. As a feasibility degree (β) increases, there is a significant marginal increment in the objective (Z_2) that has been found. The values of (Z_2) either increase or remain stable for different values of (α) with the decrease in the weight parameter (α). The maximum (worst) value of (Z_2) has been attained as \$396,879 at $\beta = 0.9$ and $\alpha = 0.1$, respectively. Thus it has emerged that with the increase in the feasibility degree (β) and the decrease in the weight parameter (α), the value of the objective (Z_2) approaches its worst outcomes. The different solution results of (Z_2) have been generated, which lie between \$396,534 and \$396,879, and are presented in Table 15.7. The fluctuating behavior of (Z_2) has also been shown in Fig. 15.4B at a different feasibility degree (β) and weight parameter (α). The utmost influencing capability of the feasibility degree (β) has been reflected by the significant increase in the objective (Z_2) and which lead (Z_2) toward its worst values. The weight parameter (α) has fewer effects on the objective (Z_2) compared to the feasibility degree (β) among all the solution choices. Due to the low processing charges at each echelon in the reverse chain, the total TPC has been obtained much less compared to TPC in the forwarding chain. Each echelon in the reverse chain dealt with either end-of-use products or end-of-life products. To perform the different required services on such products would not necessarily result in higher costs, because of less complexity in dealing with used and returned products compared to the manufacturing of new parts and products.

Table 15.6 Total processing costs (Z_1) at different feasibility degrees (β) and weight parameters (α).

Feasibility degree (β)	Weight parameter (α)								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.1	39,387,662	39,387,664	39,387,666	39,387,668	39,387,670	39,387,672	39,387,674	39,387,676	39,387,678
0.2	39,387,768	39,387,769	39,387,771	39,387,773	39,387,774	39,387,774	39,387,776	39,387,778	39,387,779
0.3	39,387,834	39,387,834	39,387,835	39,387,836	39,387,836	39,387,836	39,387,838	39,387,838	39,387,839
0.4	39,387,914	39,387,916	39,387,916	39,387,916	39,387,918	39,387,918	39,387,921	39,387,921	39,387,922
0.5	39,387,994	39,387,996	39,387,996	39,387,996	39,387,997	39,387,997	39,387,998	39,387,998	39,387,999
0.6	39,388,194	39,388,194	39,388,194	39,388,196	39,388,196	39,388,197	39,388,197	39,388,197	39,388,198
0.7	39,388,234	39,388,234	39,388,236	39,388,238	39,388,239	39,388,241	39,388,241	39,388,243	39,388,244
0.8	39,388,282	39,388,283	39,388,285	39,388,285	39,388,285	39,388,286	39,388,286	39,388,288	39,388,288
0.9	39,388,331	39,388,331	39,388,334	39,388,336	39,388,336	39,388,336	39,388,337	39,388,338	39,388,338

Table 15.7 Total transportation costs (Z_2) at different feasibility degrees (β) and weight parameters (α).

Feasibility degree (β)	Weight parameter (α)								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.1	396,534	396,535	396,535	396,534	396,535	396,537	396,538	396,538	396,539
0.2	396,593	396,593	396,593	396,595	396,595	396,596	396,598	396,599	396,599
0.3	396,642	396,642	396,642	396,643	396,644	396,644	396,646	396,648	396,649
0.4	396,686	396,686	396,687	396,687	396,687	396,688	396,688	396,689	396,689
0.5	396,723	396,724	396,724	396,724	396,725	396,725	396,725	396,726	396,728
0.6	396,767	396,767	396,768	396,768	396,768	396,769	396,769	396,769	396,770
0.7	396,798	396,798	396,798	396,799	396,799	396,802	396,802	396,805	396,805
0.8	396,837	396,837	396,838	396,838	396,838	396,839	396,839	396,839	396,840
0.9	396,873	396,874	396,874	396,874	396,875	396,876	396,876	396,877	396,879

The minimization of the total purchasing cost has been represented by the third objective (Z_3). The minimum (best) value of the objective (Z_3) has been obtained as \$6,887,719 at $\beta = 0.1$ and $\alpha = 0.9$. At $\beta = 0.9$ and $\alpha = 0.1$, the maximum (worst) value of the total purchasing cost has been obtained, which is \$6,887,980. As (α) decreases, the values of (Z_3) either increase or remain inert for some (α). With an increase in the feasibility degree (β), the significant increment in the objective function (Z_3) has been noticed. Thus it has emerged that with the increase in the feasibility degree (β) and the decrease in the weight parameter (α), the value of the objective (Z_3) approaches its worst outcomes. The different solution results of (Z_3) have been generated, which lie between \$6,887,719 and \$6,887,980 and are represented in [Table 15.8](#). The declining performance of (Z_3) has also been shown in [Fig. 15.4C](#) at different feasibility degrees (β) and weight parameters (α). Furthermore, the effect of the feasibility degree (β) is more influential, as the significant increase in the value of (Z_3) rapidly approaches the worst solutions whereas the effect of the weight parameter (α) on the objective (Z_3) is almost negligible.

The fourth objective (Z_4) is the minimization of total product delivery time to different customers/market zones. At $\beta = 0.1$ and $\alpha = 0.9$, the minimum (best) value of the total products delivery time is 13,610,103 hours. As the feasibility degree (β) increases, a significant marginal increment in the objective (Z_4) is observed. The values of (Z_4) either increase or remain inactive for different values of (α) with the decrease in the weight parameter (α). The maximum (worst) value of (Z_4) has been attained as 13,610,429 hours at $\beta = 0.9$ and $\alpha = 0.1$, respectively. Thus it has been concluded that with the increase in the feasibility degree (β) and the decrease in the relative weight parameter (α), the value of the objective (Z_4) approaches its worst results. The various solution outcomes of (Z_4) have been generated, which lie between 13,610,103 and 13,610,429 hours, and are presented in [Table 15.9](#). The trending feature of (Z_4) is also shown in [Fig. 15.4D](#) at different feasibility degrees (β) and weight parameters (α). The powerful performance of the feasibility degree (β) has been observed by the significant increase in the objective (Z_4) and which leads (Z_4) toward its worst values. The weight parameter (α) has fewer effects on the objective (Z_4) compared to the feasibility degree (β) among all the solution sets.

The maximization of revenues earned from the selling of new products has been represented by the fifth objective (Z_5). The maximum (best) value of the objective (Z_5) has been obtained as \$11,179,402 at $\beta = 0.1$ and $\alpha = 0.9$. At $\beta = 0.9$ and $\alpha = 0.1$, the minimum (worst) value of revenues has been obtained, which is \$11,179,140. As (α) decreases, the values of (Z_5) either decrease or remain stable for some (α). With an increase in the feasibility degree (β), the significant decrease in the objective function (Z_5) has been found. Thus it has been elicited that with the increase in the feasibility degree (β) and the decrease in the weight parameter (α), the value of the objective (Z_5) approaches its worst outcomes. The different solution results of (Z_5) ranging between \$6,887,719 and \$6,887,980, and are summarized

Table 15.8 Total purchasing costs (Z_3) at different feasibility degrees (β) and weight parameters (α).

Feasibility degree (β)	Weight parameter (α)								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.1	6,887,719	6,887,721	6,887,723	6,887,723	6,887,724	6,887,724	6,887,725	6,887,725	6,887,727
0.2	6,887,743	6,887,743	6,887,743	6,887,744	6,887,744	6,887,745	6,887,745	6,887,747	6,887,748
0.3	6,887,795	6,887,796	6,887,797	6,887,797	6,887,797	6,887,798	6,887,798	6,887,799	6,887,799
0.4	6,887,823	6,887,823	6,887,823	6,887,825	6,887,826	6,887,827	6,887,827	6,887,828	6,887,829
0.5	6,887,847	6,887,847	6,887,848	6,887,848	6,887,848	6,887,848	6,887,849	6,887,849	6,887,851
0.6	6,887,881	6,887,881	6,887,883	6,887,883	6,887,884	6,887,884	6,887,885	6,887,886	6,887,887
0.7	6,887,916	6,887,917	6,887,917	6,887,918	6,887,918	6,887,919	6,887,921	6,887,921	6,887,923
0.8	6,887,949	6,887,951	6,887,951	6,887,952	6,887,954	6,887,954	6,887,955	6,887,955	6,887,956
0.9	6,887,976	6,887,976	6,887,976	6,887,977	6,887,978	6,887,978	6,887,978	6,887,979	6,887,980

Table 15.9 Total expected product delivery times (Z_4) at different feasibility degrees (β) and weight parameters (α).

Feasibility degree (β)	Weight parameter (α)								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.1	13,610,103	13,610,103	13,610,103	13,610,106	13,610,106	13,610,106	13,610,107	13,610,108	13,610,108
0.2	13,610,162	13,610,163	13,610,163	13,610,163	13,610,165	13,610,166	13,610,166	13,610,167	13,610,168
0.3	13,610,213	13,610,213	13,610,213	13,610,215	13,610,215	13,610,216	13,610,216	13,610,216	13,610,218
0.4	13,610,237	13,610,237	13,610,238	13,610,238	13,610,238	13,610,239	13,610,239	13,610,239	13,610,241
0.5	13,610,279	13,610,279	13,610,279	13,610,279	13,610,280	13,610,281	13,610,283	13,610,283	13,610,285
0.6	13,610,308	13,610,308	13,610,309	13,610,311	13,610,311	13,610,311	13,610,312	13,610,313	13,610,314
0.7	13,610,342	13,610,342	13,610,342	13,610,342	13,610,345	13,610,345	13,610,346	13,610,346	13,610,348
0.8	13,610,381	13,610,383	13,610,385	13,610,385	13,610,385	13,610,386	13,610,386	13,610,388	13,610,388
0.9	13,610,422	13,610,422	13,610,423	13,610,424	13,610,424	13,610,425	13,610,425	13,610,428	13,610,429

Table 15.10 Total revenues (Z_5) at different feasibility degrees (β) and weight parameters (α).

Feasibility degree (β)	Weight parameter (α)								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.1	11,179,402	11,179,402	11,179,401	11,179,400	11,179,398	11,179,398	11,179,397	11,179,396	11,179,395
0.2	11,179,383	11,179,383	11,179,383	11,179,383	11,179,381	11,179,381	11,179,380	11,179,380	11,179,379
0.3	11,179,351	11,179,351	11,179,349	11,179,348	11,179,347	11,179,346	11,179,346	11,179,346	11,179,345
0.4	11,179,316	11,179,316	11,179,316	11,179,314	11,179,313	11,179,313	11,179,311	11,179,310	11,179,309
0.5	11,179,281	11,179,281	11,179,281	11,179,280	11,179,278	11,179,278	11,179,275	11,179,275	11,179,274
0.6	11,179,243	11,179,243	11,179,243	11,179,241	11,179,241	11,179,240	11,179,239	11,179,237	11,179,236
0.7	11,179,209	11,179,209	11,179,209	11,179,208	11,179,206	11,179,206	11,179,206	11,179,205	11,179,203
0.8	11,179,168	11,179,168	11,179,167	11,179,165	11,179,165	11,179,165	11,179,163	11,179,162	11,179,161
0.9	11,179,143	11,179,143	11,179,143	11,179,142	11,179,142	11,179,141	11,179,140	11,179,140	11,179,140

in Table 15.10, and Fig. 15.4E shows the inclining behavior of (Z_5) at different feasibility degrees (β) and weight parameters (α) , respectively. Furthermore, the effect of the feasibility degree (β) is more influential, as the significant increase in the value of (Z_5) rapidly approaches the worst solutions whereas the weight parameter (α) affects the objective (Z_5) almost trivially.

15.6.2.2 Sensitivity analyses of intuitionistic fuzzy linguistic preference relations

Imprecise importance relations have been represented by an intuitionistic fuzzy preference hierarchy for three different linguistic terms. The membership functions for importance relations $\tilde{R}_2(2,1)$, $\tilde{R}_1(4,3)$, $\tilde{R}_3(3,5)$, and $\tilde{R}_1(4,5)$ have been obtained and shown in Table 15.11 and Fig. 15.4F. With the increase in the feasibility degree (β) and the weight parameter (α) , the preference membership function for $\tilde{R}_2(2,1)$ also increases and reaches its maximum, that is, 0.71 at $\beta = 0.9$ and $\alpha = 0.9$. Similarly, the preference membership functions for $\tilde{R}_1(4,3)$, $\tilde{R}_3(3,5)$, and $\tilde{R}_1(4,5)$ also reveal increasing behavior with the increase in the feasibility degree (β) and the weight parameter (α) , and reaches their maximum attainment, that is, 0.64, 0.68, and 0.77 at $\beta = 0.9$ and $\alpha = 0.9$, respectively. Moreover, the nonmembership functions for different linguistic preferences are summarized in Table 15.11 and are shown in Fig. 15.4G. The motive is to minimize the nonmembership functions of each linguistic preference relation. Hence the minimum attainment degrees of nonmembership functions for $\tilde{R}_2(2,1)$, $\tilde{R}_1(4,3)$, $\tilde{R}_3(3,5)$, and $\tilde{R}_1(4,5)$ have been obtained as 0.29, 0.34, 0.26, and 0.21 at $\beta = 0.9$ and $\alpha = 0.9$, respectively. The overall satisfaction degree of linguistic preference relations has been represented by the score function. The maximization of the score function ensures the maximum satisfaction degree for the intended preferences among different objectives and is shown in Fig. 15.4H. In Table 15.11, with the increase in value of β and α , the score function shows the enhancing trend. At $\beta = 0.9$ and $\alpha = 0.9$, it approaches the maximum satisfactory degree, that is, 0.4256, 0.3557, 0.4253, and 0.5637 for $\tilde{R}_2(2,1)$, $\tilde{R}_1(4,3)$, $\tilde{R}_3(3,5)$, and $\tilde{R}_1(4,5)$, respectively. By tuning the parameters β and α , various sets of score functions for satisfaction level could be obtained effectively. Hence, intuitionistic fuzzy linguistic preference relations would be a good representative of priority structure among objectives according to the interest of decision maker(s). They would also be an effective and promising tool for assigning the preference when large numbers of objectives and goals have been dealt with simultaneously. The assignment of crisp weight (such as $w_o = 0.1, 0.2, \dots, 1 | \sum_o w_o = 1$) to significant number objectives might be time-consuming and would involve more complexity to search for the best combination of crisp weight among different objectives or goals. Hence it would be tricky to assign the linguistic preferences among different objectives, which reduced the time and exempted from the best combination of crisp weight.

Table 15.11 Achievement degree of intuitionistic fuzzy linguistic preference relations at different feasibility degrees (β) and weight parameters (α).

Feasibility degree	Weight parameter	Intuitionistic fuzzy preference relations											
		Membership functions				Nonmembership functions				Score function achieved for intuitionistic fuzzy preference relations			
(β)	($1 - \alpha$)	$\mu_{R_2(2,1)}^-$	$\mu_{R_1(4,3)}^-$	$\mu_{R_3(3,5)}^-$	$\mu_{R_1(4,5)}^-$	$\nu_{R_2(2,1)}^-$	$\nu_{R_1(4,3)}^-$	$\nu_{R_3(3,5)}^-$	$\nu_{R_1(4,5)}^-$	$S_{R_2(2,1)}^-$	$S_{R_1(4,3)}^-$	$S_{R_3(3,5)}^-$	$S_{R_1(4,5)}^-$
0.1	0.1	0.63	0.56	0.63	0.71	0.37	0.37	0.33	0.27	0.2634	0.1924	0.3091	0.4453
0.2	0.2	0.63	0.56	0.63	0.71	0.37	0.37	0.33	0.27	0.2637	0.1938	0.3037	0.4467
0.3	0.3	0.64	0.56	0.63	0.72	0.37	0.37	0.33	0.27	0.2721	0.1957	0.3052	0.4531
0.4	0.4	0.64	0.56	0.63	0.72	0.35	0.36	0.32	0.27	0.2932	0.2122	0.3127	0.4567
0.5	0.5	0.64	0.56	0.63	0.72	0.35	0.36	0.32	0.27	0.2981	0.2143	0.3149	0.4579
0.6	0.6	0.65	0.58	0.65	0.74	0.32	0.35	0.32	0.23	0.3381	0.2311	0.3351	0.5133
0.7	0.7	0.65	0.58	0.65	0.74	0.32	0.35	0.29	0.23	0.3393	0.2341	0.3691	0.5148
0.8	0.8	0.67	0.61	0.66	0.74	0.32	0.35	0.29	0.21	0.3547	0.2934	0.3712	0.5321
0.9	0.9	0.71	0.64	0.68	0.77	0.29	0.34	0.26	0.21	0.4256	0.3557	0.4253	0.5637

15.7 Conclusions

In this study, an effective modeling and optimization framework for the CLSC design has been formulated as a mixed-integer neutrosophic fuzzy programming problem under uncertainty. The proposed CLSC designed model comprises multiproduct, multiechelon, and multiobjective scenarios for the optimum allocation of new and end-of-use products. In the forward chain, five functional echelons have been designed, whereas the reverse chain consists of six potential echelons to deal with end-of-use and end-of-life products. The testing center has been depicted in the CLSC model, which ensures the promising useful life of the product. Multiple-conflicting objectives with a well-defined set of constraints reveal typical complexity under a fuzzy environment. To deal with fuzzy parameters and constraints, a fuzzy robust ranking function technique depending on a feasibility degree has been suggested. Fuzzy inequality constraints have been converted into their crisp forms by using the ranking function, whereas fuzzy equality constraints have been transformed into two equivalent auxiliary crisp inequalities. Then the obtained fresh model has been solved by using a modified NFPA which consists of independent indeterminacy thoughts in decision-making processes. A novel linguistic importance scheme named intuitionistic fuzzy preference relations among different objectives has been investigated. With the aid of the linear preference membership and nonmembership function, the marginal achievement of each linguistic preference has been attained. The overall satisfaction level has been represented by the convex combination of membership functions of each objective and score function of intuitionistic fuzzy preference relations. By tuning the feasibility degree and weight parameter, a different set of optimal solution results has been generated. A sensitivity analysis of the obtained results has been performed. Therefore, the presented CLSC modeling study under uncertainty may be helpful for practitioners and decision makers who are actively dedicated in the decision-making process of procurement, production, distribution, transportation, and management of end-of-use and end-of-life products in the CLSC network.

The propounded CLSC study has some limitations that can be addressed in future research. The CLSC network has been designed for a single period, but modeling with multiple periods is much needed in real-life scenarios. Incorporation of the triple bottom lines concept, which means sustainable development of the CLSC model comprising economic policies, environmental issues, and social concerns, would be a remarkable extension of the proposed model. Uncertainty among parameters due to randomness or other uncertain forms would be a significant enhancement of the discussed CLSC model. Various metaheuristic approaches may be applied to solve the proposed model as a future research scope.

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