Malaysian Journal of Mathematical Sciences 13(S) December: 153–168 (2019)
Special Issue: Conference on Mathematics, Informatics and Statistics (CMIS2018)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Multi-Valued Neutrosophic Soft Set

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ABSTRACT

In this paper, we introduce the new concept of multi-valued neutrosophic soft set. This concept combines the beneficial properties of both multi-valued neutrosophic set introduced by Wang and Li (2015) and soft set introduced by Molodtsov (1999). We also revise some of its properties. The basic operations of complement, intersection and union are defined. Lastly, an application of multi-valued neutrosophic soft set in solving decision making problem is presented in the last section of this research.

Keywords: Decision making, multi-valued neutrosophic set, multi-valued neutrosophic soft set, neutrosophic set.

1. Introduction

In order to solve problems with fuzzy information, Zadeh (1965) introduced fuzzy sets which are now became popular tools in the purpose of decision making problems. However, the membership degree alone cannot describe the information in some cases of decision making problems. Thus, intuitionistic fuzzy sets are introduced by Atanassov (1986) to measure both membership degree and non-membership degree. Then, some researchers extended the intuitionistic fuzzy sets and applied them to decision making problems. But the intuitionistic fuzzy sets can only deal with inadequate information considering both the membership degree and non-membership degree values. It cannot deal with the uncertainty and unpredictable information which exist in belief system. In order to overcome the problem, theory of neutrosophic set was proposed by Smarandache (1995).

The theory of neutrosophic set is a mathematical tool which handles the problems with inconsistent and imprecise data. Smarandache (2005) also proved that the neutrosophic set is a continuation of the intuitionistic fuzzy sets. Neutrosophic set is provided by the triplet (T, I, F) which representing the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively, where $]^{-}0, 1^{+}[$ represents the non-standard interval. Clearly, it represents $[0, 1]$ which refers to the continuation of the standard interval in the intuitionistic fuzzy sets Zhang et al. (2014). The ambiguity which refers to the uncertsinty factor is independent of truth and falsity values, while the integrated ambiguity is dependent of the degree of belongingness and the degree of non-belongingness in intuitionistic fuzzy sets. Some researchers studied about the theory of neutrosophic, such as Aydo (2015), Majumdar (2015), and Sahin and Kucuk (2015). Moreover, Molodtsov (1999) who is a Russian mathematician, had solved the difficult problem involving uncertainty by proposing a new mathematical tool called *soft set theory*. This theory is easily used to set the function of membership in a certain case and inadequacy of parameterization tool of theory. After the research done by Molodtsov, there were many researchers who integrated soft set with fuzzy set, which combines the beneficial properties of both soft set and fuzzy set method, such as Cagman et al. (2007), Kong et al. (2009) and Wang and Li (2015), and Roy and Maji (2007). So in this paper, we present a new model which combine two concepts which are multi-valued neutrosophic set introduced by Wang and Li (2015) and soft set introduced by Molodtsov (1999), together by proposing a new concept called multi-valued neutrosophic soft set. Thus, we propose its operations which are union, intersection and complement operations. Lastly, we show an application of multi-valued neutrosophic soft set in solving decision making problem.

2. Preliminary

In this section, we present some definitions and properties which are used in this paper related to neutrosophic set theory, soft set theory and neutrosophic soft set.

Definition 2.1. *Smarandache (1999)*
A neutrosophic set A on the universe U is defined as

$$A = \{x; T_A(x); I_A(x); F_A(x) >; x \in U\}$$

where $T; I; F : U \to]^{-}0; 1^{+}[$ *and* $^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2.2. *(Molodtsov, 1999)*
A pair (F, A) is called a soft set over U and the function f is a mapping defined by

$$F : A \to P(U)$$

where $F(e)(x) = \emptyset$ *if* $x \notin U$.

Here, $F(e)$ is called approximate function of the soft set (F, A) and the value is a set called $x-$element of the soft set for all $x \in U$. The sets may be arbitrary, empty, or have non-empty intersection.

Example 2.1. *A soft set (F, A) represents the characteristics of the gadgets which Mrs. N plans to buy. Let $U = \{x_1, x_2, x_3, x_4\}$ be the set of gadgets under consideration. A is the set of parameters where $A = \{e_1, e_2, e_3, e_4\}$. Each parameter refers to a word or a sentence, which represents the parameters branded (e_1), specification (e_2), cheap (e_3) and originality (e_4). Let the mapping F be a mapping of A into the set of all subsets of the set U. Now consider a soft set (F, A) which represents the "characteristics of the gadgets for purchase". According to the data collection, the soft set (F, A) is given by*

$$(F, A) = \{(e_1, \{x_1, x_3, x_4\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_2, x_3\}), (e_4, \{x_1, x_4\})\}$$

The tabular representation of the soft set (F, A) is shown in Table 1.

Table 1: The tabular representation of the soft set (F, A)

U	e_1	e_2	e_3	e_4
x_1	1	0	1	1
x_2	0	1	1	0
x_3	1	1	1	0
x_4	1	0	0	1

Proposition 2.1. *(Maji, 2013) The complement of a neutrosophic soft set (F, A) is denoted by $(F, A)_c$ where $F_c : A \rightarrow P(U)$ is a mapping given by $F_c(\alpha) =$ neutrosophic soft complement with $T_{F_c(x)} = F_{F(x)}, I_{F_c(x)} = I_{F(x)}$ and $F_{F_c(x)} = T_{F(x)}$.*

Proposition 2.2. *(Maji, 2013)*
Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the union of (F, A) and G, B is denoted by $'(F, A) \cup (G, B)'$ and is defined by $(F, A) \cup (G, B) = (H, C)$ where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (H, C) are defined in Table 2.

Table 2: The truth-membership, indeterminacy-membership and falsity-membership of (H, C)

$T_{H(e)(x)}$	$= T_{F(e)(x)};$	if	$e \in A - B$
	$= T_{G(e)(x)};$	if	$e \in B - A$
	$= max(T_{F(e)(x)}, T_{G(e)(x)});$	if	$e \in A \cap B$
$I_{H(e)(x)}$	$= I_{F(e)(x)};$	if	$e \in A - B$
	$= I_{G(e)(x)};$	if	$e \in B - A$
	$= \frac{I_{F(e)(x)} + I_{G(e)(x)}}{2}$	if	$e \in A \cap B$
$F_{H(e)(x)}$	$= F_{F(e)(x)};$	if	$e \in A - B$
	$= F_{G(e)(x)};$	if	$e \in B - A$
	$= min(F_{F(e)(x)}, F_{G(e)(x)});$	if	$e \in A \cap B$

Proposition 2.3. *(Maji, 2013)*
Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the intersection of (F, A) and G, B is denoted by $'(F, A) \cap (G, B)'$ and is defined by $(F, A) \cap (G, B) = (H, C)$ where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (H, C) are defined as follows:

$$\begin{aligned} T_{H(e)(x)} &= \min(T_{F(e)(x)}, T_{G(e)(x)}); && \forall e \in A \cap B \\ I_{H(e)(x)} &= \frac{I_{F(e)(x)} + I_{G(e)(x)}}{2}; && \forall e \in A \cap B \\ F_{H(e)(x)} &= \max(F_{F(e)(x)}, F_{G(e)(x)}); && \forall e \in A \cap B \end{aligned}$$

Proposition 2.4. *(Maji, 2013)*
Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the AND operation on them is denoted by $'(F, A) \wedge (G, B)'$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$ where the truth-membership,

indeterminacy-membership and falsity-membership of $(H, A \times B)$ are defined as follows:

$$\begin{array}{lll} T_{H(\alpha,\beta)(x)} & = \min(T_{F(\alpha)(x)}, T_{G(\beta)(x)}); & \forall \alpha \in A, \beta \in B \\ I_{H(\alpha,\beta)(x)} & = \frac{I_{F(\alpha)(x)} + I_{G(\beta)(x)}}{2}; & \forall \alpha \in A, \beta \in B \\ F_{H(\alpha,\beta)(x)} & = \max(F_{F(\alpha)(x)}, F_{G(\beta)(x)}); & \forall \alpha \in A, \beta \in B \end{array}$$

Proposition 2.5. *(Maji, 2013)*
Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. Then the OR operation on them is denoted by $'(F, A) \vee (G, B)'$ and is defined by $(F, A) \vee (G, B) = (H, A \times B)$ where the truth-membership, indeterminacy-membership and falsity-membership of $(H, A \times B)$ are defined as follows:

$$\begin{array}{lll} T_{H(\alpha,\beta)(x)} & = \max(T_{F(\alpha)(x)}, T_{G(\beta)(x)}); & \forall \alpha \in A, \beta \in B \\ I_{H(\alpha,\beta)(x)} & = \frac{I_{F(\alpha)(x)} + I_{G(\beta)(x)}}{2}; & \forall \alpha \in A, \beta \in B \\ F_{H(\alpha,\beta)(x)} & = \min(F_{F(\alpha)(x)}, F_{G(\beta)(x)}); & \forall \alpha \in A, \beta \in B \end{array}$$

Definition 2.3. *(Wang and Li, 2015) Let U be a space of points with a generic element in U denoted by x. Then, a multi-valued neutrosophic set A in U is characterized by $\tilde{T}_A(x), \tilde{I}_A(x)$ and $\tilde{F}_A(x)$ in the form of subset of [0,1] and can be defined as follows:*

$$A = \{< x; \tilde{T}_A(x); \tilde{I}_A(x); \tilde{F}_A(x) >; x \in U\}$$

where $\tilde{T}_A(x), \tilde{I}_A(x)$ and $\tilde{F}_A(x)$ are three sets of discrete real numbers in [0,1], showing the truth-membership, indeterminacy-membership and falsity-membership respectively, satisfying $0 \leq \mu, \omega, \varepsilon \leq 1$ and $0 \leq \mu^+ + \omega^+ + \varepsilon^+ \leq 3$ in which $\mu \in \tilde{T}_A(x)$,$\omega \in \tilde{I}_A(x)$,$\varepsilon \in \tilde{F}_A(x)$ and $\mu^+ = \sup \tilde{T}_A(x)$,$\omega^+ = \sup \tilde{I}_A(x)$,$\varepsilon^+ = \sup \tilde{F}_A(x)$. If U has only one element, then A is called as multi-valued neutrosophic number (MVNN) denoted by $A = \{< \tilde{T}_A(x); \tilde{I}_A(x); \tilde{F}_A(x) >\}$ The set of all MVNNs is presented as MVNS. Clearly, MVNSs are usually considered as a generalization of neutrosophic sets. If each of $\tilde{T}_A(x), \tilde{I}_A(x)$ and $\tilde{F}_A(x)$ for any x has only one value, i.e μ, ω and ε, and $0 \leq \mu + \omega + \varepsilon \leq 3$ then MVNSs are reduced to single neutrosophic sets.

Example 2.2. *Assume that $U = \{x_1, x_2, x_3\}$, where x_1 is the technology, x_2 is the risk, and x_3 is the market potential, is the universal set. The values of x_1, x_2 and x_3 are in $[0,1]$. The options for the decision makers could be a degree of 'high', a degree of indeterminacy and a degree of 'low'. A is a multi-valued neutrosophic set of U defined as follows:*

$$A = \left\{ \frac{\langle\{0.1, 0.2\}, \{0.3\}, \{0.4\}\rangle}{x_1}, \frac{\langle\{0.5\}, \{0.3\}, \{0.4\}\rangle}{x_2}, \frac{\langle\{0.4\}, \{0.2, 0.3\}, \{0.5\}\rangle}{x_3} \right\}$$

Proposition 2.6. *(Peng and Wang, 2015)*
The complement of a multi-valued neutrosophic set $A = \{\langle x; \tilde{T}_A(x); \tilde{I}_A(x); \tilde{F}_A(x)\rangle; x \in U\}$ is denoted by A^c and is defined as

$$A^c = \langle \cup_{\mu \in \tilde{T}_A(x)}\{1-\mu\}; \cup_{\omega \in \tilde{I}_A(x)}\{1-\omega\}; \cup_{\varepsilon \in \tilde{F}_A(x)}\{1-\varepsilon\}\rangle$$

Proposition 2.7. *(Peng and Wang, 2015) The multi-valued neutrosophic set $A = \{\langle x; \tilde{T}_A(x); \tilde{I}_A(x); \tilde{F}_A(x)\rangle; x \in U\}$ is contained in other multi-valued neutrosophic set $B = \{\langle x; \tilde{T}_B(x); \tilde{I}_B(x); \tilde{F}_B(x)\rangle; x \in U\}$, $A \subseteq B$ if and only if $\tilde{T}_A(x) \leq \tilde{T}_B(x), \tilde{I}_A(x) \geq \tilde{I}_B(x), \tilde{F}_A(x) \geq \tilde{F}_B(x)$.*

3. Multi-Valued Neutrosophic Soft Set

In this section, the concept of multi-valued neutrosophic soft set is presented based on the integration of multi-valued neutrosophic sets (Peng and Wang, 2015) and soft sets Molodtsov (1999). Multi-valued neutrosophic soft set is a collection of multi-valued neutrosophic sets. Refer to the definition given by Peng and Wang (2015), multi-valued neutrosophic soft set can be defined as follows.

Definition 3.1. *(Peng and Wang, 2015)*
The pair $(\tilde{F}, A)$ is called a multi-valued neutrosophic soft set over $\tilde{P}(U)$, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \to \tilde{P}(U)$. Let $MVNSS$ be the collection of multi-valued neutrosophic soft sets on U with parameters from A. $(\tilde{F}, A)$ is characterized by $\tilde{T}_{\tilde{F}(e)}(x)$,$\tilde{I}_{\tilde{F}(e)}(x)$ and $\tilde{F}_{\tilde{F}(e)}(x)$ in the form of subset of $[0,1]$ and can be defined as follows:

$$(\tilde{F}, A) = \{\langle x; \tilde{T}_{\tilde{F}(e)}(x); \tilde{I}_{\tilde{F}(e)}(x); \tilde{F}_{\tilde{F}(e)}(x)\rangle; \forall e \in A; x \in U\}$$

where $\tilde{T}_{\tilde{F}(e)}(x)$, $\tilde{I}_{\tilde{F}(e)}(x)$ and $\tilde{F}_{\tilde{F}(e)}(x)$ are the truth-membership, indeterminacy-membership and falsity-membership respectively that object x holds on parameter e.

An example of a multi-valued neutrosophic soft set is given in Example 3.1.

Example 3.1. *Let $U = \{x_1, x_2, x_3\}$ be the set of gadgets under consideration and A is a set of parameter which is a neutrosophic word. Consider $A = \{e_1 = branded, e_2 = specification, e_3 = cheap, e_4 = originality\}$. Consider that the multi-valued neutrosophic soft set $(\tilde{F}, A)$ given as follows:*

$$(\tilde{F}, A) = \Big(e_1, \Big\{\frac{\langle\{0.2,0.3\},\{0.4\},\{0.8\}\rangle}{x_1}, \frac{\langle\{0.3\},\{0.5\},\{0.6,0.7\}\rangle}{x_2}, \frac{\langle\{0.6\},\{0.3,0.5\},\{0.1,0.3\}\rangle}{x_3}\Big\}\Big),$$
$$\Big(e_2, \Big\{\frac{\langle\{0.6\},\{0.8\},\{0.1,0.4\}\rangle}{x_1}, \frac{\langle\{0.5\},\{0.7,0.8\},\{0.9\}\rangle}{x_2}, \frac{\langle\{0.2,0.4\},\{0.7\},\{0.5,0.9\}\rangle}{x_3}\Big\}\Big),$$
$$\Big(e_3, \Big\{\frac{\langle\{0.6,0.9\},\{0.8\},\{0.6\}\rangle}{x_1}, \frac{\langle\{0.1,0.3\},\{0.7,0.9\},\{0.6\}\rangle}{x_2}, \frac{\langle\{0.5\},\{0.3,0.6\},\{0.8\}\rangle}{x_3}\Big\}\Big),$$
$$\Big(e_4, \Big\{\frac{\langle\{0.3\},\{0.4,0.6\},\{0.2,0.5\}\rangle}{x_1}, \frac{\langle\{0.2\},\{0.5\},\{0.6,0.7\}\rangle}{x_2}, \frac{\langle\{0.6\},\{0.3\},\{0.,0.3,0.5\}\rangle}{x_3}\Big\}\Big)$$

The tabular representation of multi-valued neutrosophic soft set $(\tilde{F}, A)$ is shown in Table 3.

Table 3: The tabular representation of the multi-valued neutrosophic soft set $(\tilde{F}, A)$

U	x_1	x_2	x_3
e_1	$\langle\{0.2, 0.3\}, \{0.4\}, \{0.8\}\rangle$	$\langle\{0.3\}, \{0.5\}, \{0.6, 0.7\}\rangle$	$\langle\{0.6\}, \{0.3, 0.5\}, \{0., 0.3\}\rangle$
e_2	$\langle\{0.6\}, \{0.8\}, \{0.1, 0.4\}\rangle$	$\langle\{0.5\}, \{0.7, 0.8\}, \{0.9\}\rangle$	$\langle\{0.2, 0.4\}, \{0.7\}, \{0.5, 0.9\}\rangle$
e_3	$\langle\{0.6, 0.9\}, \{0.8\}, \{0.6\}\rangle$	$\langle\{0.1, 0.3\}, \{0.7, 0.9\}, \{0.6\}\rangle$	$\langle\{0.5\}, \{0.3, 0.6\}, \{0.8\}\rangle$
e_4	$\langle\{0.3\}, \{0.4, 0.6\}, \{0.2, 0.5\}\rangle$	$\langle\{0.2\}, \{0.5\}, \{0.6, 0.7\}\rangle$	$\langle\{0.6\}, \{0.3\}, \{0., 0.3, 0.5\}\rangle$

Suppose $(\tilde{F}, A)$ is a multi-valued neutrosophic soft set in $MVNSS(U)$ where $U = \{x_1, x_2, ..., x_n\}$. The basic operations on multi-valued neutrosophic soft sets are given as follows:

Proposition 3.1. *The complement of a multi-valued neutrosophic soft set $(\tilde{F}, A)$ is denoted by $(\tilde{F}, A)_c$ and is defined as $\tilde{T}_{\tilde{F}_c(e)}(x) = \tilde{F}_{\tilde{F}(e)}(x)$, $\tilde{I}_{\tilde{F}_c(e)}(x) = \tilde{I}_{\tilde{F}(e)}(x)$ and $\tilde{F}_{\tilde{T}_c(e)}(x) = \tilde{T}_{\tilde{F}(e)}(x)$ for every x in U.*

Proposition 3.2. *Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-valued neutrosophic soft sets over the common universe U. Then the union of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ which is denoted by $'(\tilde{F}, A) \cup (\tilde{G}, B)'$ is defined by $(\tilde{F}, A) \cup (\tilde{G}, B) = (\tilde{H}, C)$ where*

$C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of $(\tilde{H}, C)$ are defined as follows:

$$
\begin{aligned}
(\tilde{H}, C) &= \langle \tilde{T}_{\tilde{F}(e)}(x); \tilde{I}_{\tilde{F}(e)}(x); \tilde{F}_{\tilde{F}(e)}(x) \rangle; if e \in A - B; \\
(\tilde{H}, C) &= \langle \tilde{T}_{\tilde{G}(e)}(x); \tilde{I}_{\tilde{G}(e)}(x); \tilde{F}_{\tilde{G}(e)}(x) \rangle; if e \in B - A; \\
(\tilde{H}, C) &= \langle \max\{\tilde{T}_{\tilde{F}(e)}(x), \tilde{T}_{\tilde{G}(e)}(x)\}; \frac{\tilde{I}_{\tilde{F}(e)}(x) + \tilde{I}_{\tilde{G}(e)}(x)}{2}, \min\{\tilde{T}_{\tilde{F}(e)}(x); \tilde{T}_{\tilde{G}(e)}(x)\} \rangle; \\
& if e \in A \cap B.
\end{aligned}
$$

Proposition 3.3. *Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-valued neutrosophic soft sets over the common universe U. Then, the intersection of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is denoted by $'(\tilde{F}, A) \cap (\tilde{G}, B)'$ and is defined by $(\tilde{F}, A) \cap (\tilde{G}, B) = (\tilde{H}, C)$ where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of $(\tilde{H}, C)$ are defined as follows:*

$$
(\tilde{H}, C) = \langle \min\{\tilde{T}_{\tilde{F}(e)}(x), \tilde{T}_{\tilde{G}(e)}(x)\}; \frac{\tilde{I}_{\tilde{F}(e)}(x) + \tilde{I}_{\tilde{G}(e)}(x)}{2}; \max\{\tilde{T}_{\tilde{F}(e)}(x), \tilde{T}_{\tilde{G}(e)}(x)\} \rangle;
$$

if $e \in A \cap B$.

Proposition 3.4. *Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two multi-valued neutrosophic soft sets over the common universe U. Then,*

1. $(\tilde{F}, A) \cup (\tilde{F}, A) = (\tilde{F}, A)$
2. $(\tilde{F}, A) \cap (\tilde{F}, A) = (\tilde{F}, A)$
3. $(\tilde{F}, A) \cup (\tilde{G}, B) = (\tilde{G}, B) \cup (\tilde{F}, A)$
4. $(\tilde{F}, A) \cap (\tilde{G}, B) = (\tilde{G}, B) \cap (\tilde{F}, A)$

Proof. Let e be an arbitrary element of $(\tilde{F}, A) \cup (\tilde{F}, A)$. Then, $e \in (\tilde{F}, A)$ or $e \in (\tilde{F}, A)$. Hence $e \in (\tilde{F}, A)$. Thus, $(\tilde{F}, A) \cup (\tilde{F}, A) \subseteq (\tilde{F}, A)$. Conversely, if e is an arbitrary element of $(\tilde{F}, A)$, then $e \in (\tilde{F}, A) \cup (\tilde{F}, A)$ since it is in $(\tilde{F}, A)$. Therefore $(\tilde{F}, A) \subseteq (\tilde{F}, A) \cup (\tilde{F}, A)$.
$\therefore (\tilde{F}, A) \cup (\tilde{F}, A) = (\tilde{F}, A)$ □

Proof. Let e be an arbitrary element of $(\tilde{F}, A) \cap (\tilde{F}, A)$. Then, $e \in (\tilde{F}, A)$ and $e \in (\tilde{F}, A)$. Hence $e \in (\tilde{F}, A)$. Thus, $(\tilde{F}, A) \cap (\tilde{F}, A) \subseteq (\tilde{F}, A)$. Conversely,

if $e \in (\tilde{F}, A)$ is an arbitrary, then $e \in (\tilde{F}, A)$ and $e \in (\tilde{F}, A)$. Therefore $(\tilde{F}, A) \cap (\tilde{F}, A) \subseteq (\tilde{F}, A)$.
$\therefore (\tilde{F}, A) \cap (\tilde{F}, A) = (\tilde{F}, A)$ □

Proof. Let e is any element in $(\tilde{F}, A) \cup (\tilde{G}, B)$, then, by definition of union, $e \in (\tilde{F}, A)$ or $e \in (\tilde{G}, B)$. But, if e is in $(\tilde{F}, A)$ or $(\tilde{G}, B)$, then it is in $(\tilde{G}, B)$ or $(\tilde{F}, A)$, and by definition of union, this means $e \in (\tilde{G}, B) \cup (\tilde{F}, A)$. Therefore, $(\tilde{F}, A) \cup (\tilde{G}, B) \subseteq (\tilde{G}, B) \cup (\tilde{F}, A)$.

The other inclusion is identical. If e is any element of $(\tilde{G}, B) \cup (\tilde{F}, A)$, then, $e \in (\tilde{G}, B)$ or $e \in (\tilde{F}, A)$. But, $e \in (\tilde{G}, B)$ or $e \in (\tilde{F}, A)$ implies that e is in $(\tilde{F}, A)$ or $(\tilde{G}, B)$. Hence, $e \in (\tilde{G}, B) \cup (\tilde{F}, A)$. Therefore $(\tilde{G}, B) \cup (\tilde{F}, A) \subseteq (\tilde{F}, A) \cup (\tilde{G}, B)$
$\therefore (\tilde{F}, A) \cup (\tilde{G}, B) = (\tilde{G}, B) \cup (\tilde{F}, A)$. □

Proof. Let e is any element in $(\tilde{F}, A) \cap (\tilde{G}, B)$, then, by definition of intersection, $e \in (\tilde{F}, A)$ and $e \in (\tilde{G}, B)$. Hence, $e \in (\tilde{G}, B)$ and $e \in (\tilde{F}, A)$, and so, $e \in (\tilde{G}, B) \cap (\tilde{F}, A)$. Therefore, $(\tilde{F}, A) \cap (\tilde{G}, B) \subseteq (\tilde{G}, B) \cap (\tilde{F}, A)$.

The reverse inclusion is again identical. If e is any element of $(\tilde{G}, B) \cap (\tilde{F}, A)$, then, $e \in (\tilde{G}, B)$ and $e \in (\tilde{F}, A)$. Hence, $e \in (\tilde{F}, A)$ and $e \in (\tilde{G}, B)$. This implies $e \in (\tilde{F}, A) \cap (\tilde{G}, B)$. Therefore $(\tilde{G}, B) \cap (\tilde{F}, A) \subseteq (\tilde{F}, A) \cap (\tilde{G}, B)$
$\therefore (\tilde{F}, A) \cap (\tilde{G}, B) = (\tilde{G}, B) \cap (\tilde{F}, A)$. □

Proposition 3.5. *Let $(\tilde{F}, A), (\tilde{G}, B)$ and $(\tilde{K}, C)$ be three multi-valued neutrosophic soft sets over the common universe U. Then,*

1. $(\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)] = [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$
2. $(\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)] = [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$
3. $(\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)] = [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$
4. $(\tilde{F}, A) \cap [(\tilde{G}, B) \cup (\tilde{K}, C)] = [(\tilde{F}, A) \cap (\tilde{G}, B)] \cup [(\tilde{F}, A) \cap (\tilde{K}, C)]$

Proof. Let $e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)]$. If $e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)]$, then $e \in (\tilde{F}, A)$ or $e \in [(\tilde{G}, B) \cup (\tilde{K}, C)]$
$e \in (\tilde{F}, A)$ or $e \in [(\tilde{G}, B)$ or $(\tilde{K}, C)]$
$e \in [(\tilde{G}, B)$ or $(\tilde{K}, C)]$ implies $e \in (\tilde{G}, B)$ or $e \in (\tilde{K}, C)$

So, we have $e \in (\tilde{F}, A)$, $e \in (\tilde{G}, B)$ or $e \in (\tilde{K}, C)$
$e \in [(\tilde{F}, A)$ or $(\tilde{G}, B)]$ or $e \in (\tilde{K}, C)$
$e \in [(\tilde{F}, A)$ or $e \in (\tilde{G}, B)]$ or $(\tilde{K}, C)$
$e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$
$e \in [(F,\tilde{} A) \cup [(\tilde{G}, B)] \cup (\tilde{K}, C)] \Rightarrow e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$

Therefore $[(\tilde{F}, A) \cup [(\tilde{G}, B)] \cup (\tilde{K}, C)] \subseteq [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$.

Let $e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$. If $e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$, then
$e \in [(\tilde{F}, A)$ or $(\tilde{G}, B)]$ or $e \in [(\tilde{K}, C)$
$e \in [(\tilde{F}, A)$ or $(\tilde{G}, B)]$ implies $e \in (\tilde{F}, A)$ or $e \in (\tilde{G}, B)$

So, we have $e \in (\tilde{F}, A)$, $e \in (\tilde{G}, B)$ or $e \in (\tilde{K}, C)$
$e \in (\tilde{F}, A)$ or $[e \in (\tilde{G}, B)$ or $(\tilde{K}, C)]$
$e \in (\tilde{F}, A)$ or $[(\tilde{G}, B)$ or $(\tilde{K}, C)]$
$e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)]$
$e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C) \Rightarrow e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)]$

Therefore, $[(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C) \subseteq (\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)]$.

$\therefore (\tilde{F}, A) \cup [(\tilde{G}, B) \cup (\tilde{K}, C)] = [(\tilde{F}, A) \cup (\tilde{G}, B)] \cup (\tilde{K}, C)$. □

Proof. Let $e \in (\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)]$.
If $e \in (\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)]$, then $e \in (\tilde{F}, A)$ and $e \in [(\tilde{G}, B) \cap (\tilde{K}, C)]$
$e \in (\tilde{F}, A)$ and $e \in [(\tilde{G}, B)$ and $(\tilde{K}, C)]$
$e \in [(\tilde{G}, B)$ and $(\tilde{K}, C)]$ implies $e \in (\tilde{G}, B)$ and $e \in (\tilde{K}, C)$

So, we have $e \in (\tilde{F}, A)$, $e \in (\tilde{G}, B)$ and $e \in (\tilde{K}, C)$
$e \in [(\tilde{F}, A)$ and $(\tilde{G}, B)]$ and $e \in (\tilde{K}, C)$
$e \in [(\tilde{F}, A)$ and $e \in (\tilde{G}, B)]$ and $(\tilde{K}, C)$
$e \in [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$
$e \in [(F,\tilde{} A) \cap [(\tilde{G}, B)] \cap (\tilde{K}, C)] \Rightarrow e \in [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$

Therefore, $[(\tilde{F}, A) \cap [(\tilde{G}, B)] \cap (\tilde{K}, C)] \subseteq [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$.

Let $e \in [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$.
If $e \in [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$, then $e \in [(\tilde{F}, A)$ and $(\tilde{G}, B)]$ and $e \in [(\tilde{K}, C)$
$e \in [(\tilde{F}, A)$ and $(\tilde{G}, B)]$ implies $e \in (\tilde{F}, A)$ and $e \in (\tilde{G}, B)$

So, we have $e \in (\tilde{F}, A)$, $e \in (\tilde{G}, B)$ and $e \in (\tilde{K}, C)$
$e \in (\tilde{F}, A)$ and $[e \in (\tilde{G}, B)$ and $(\tilde{K}, C)]$
$e \in (\tilde{F}, A)$ and $[(\tilde{G}, B)$ and $(\tilde{K}, C)]$
$e \in (\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)]$
$e \in [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C) \Rightarrow e \in (\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)]$

Therefore, $[(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C) \subseteq (\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)]$.

$\therefore (\tilde{F}, A) \cap [(\tilde{G}, B) \cap (\tilde{K}, C)] = [(\tilde{F}, A) \cap (\tilde{G}, B)] \cap (\tilde{K}, C)$. □

Proof. Let $e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)]$
If $e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)]$, then e is either in $(\tilde{F}, A)$ or in $[(\tilde{G}, B)$ and $(\tilde{K}, C)]$.
$\Rightarrow e \in (\tilde{F}, A)$ or $e \in [(\tilde{G}, B)$ and $(\tilde{K}, C)]$
$\Rightarrow e \in (\tilde{F}, A)$ or $\{e \in (\tilde{G}, B)$ and $e \in (\tilde{K}, C)\}$
$\Rightarrow \{e \in (\tilde{F}, A)$ or $e \in (\tilde{G}, B)\}$ and $\{e \in (\tilde{F}, A)$ or $e \in (\tilde{K}, C)\}$
$\Rightarrow e \in [(\tilde{F}, A)$ or $(\tilde{G}, B)]$ and $e \in [(\tilde{F}, A)$ or $(\tilde{K}, C)]$
$\Rightarrow e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$

Since $\exists e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)]$ such that $e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$

Therefore, $(\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)] \subseteq [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$.

Let $e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$.

If $e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$, then e is in $[(\tilde{F}, A)$ or $(\tilde{G}, B)]$ and e is in $[(\tilde{F}, A)$ or $(\tilde{K}, C)]$.
$\Rightarrow e \in [(\tilde{F}, A)$ or $(\tilde{G}, B)]$ and $e \in [(\tilde{F}, A)$ or $(\tilde{K}, C)]$
$\Rightarrow \{e \in (\tilde{F}, A)$ or $e \in (\tilde{G}, B)\}$ and $\{e \in (\tilde{F}, A)$ or $e \in (\tilde{K}, C)\}$
$\Rightarrow e \in (\tilde{F}, A)$ or $\{e \in [(\tilde{G}, B)$ and $e \in (\tilde{K}, C)]\}$
$\Rightarrow e \in (\tilde{F}, A)$ or $\{e \in [(\tilde{G}, B)$ and $(\tilde{K}, C)]\}$
$\Rightarrow e \in (\tilde{F}, A) \cup \{e \in [(\tilde{G}, B) \cap (\tilde{K}, C)]\}$
$\Rightarrow e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)]$

Since $\exists e \in [(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)]$ such that $e \in (\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)]$

Therefore, $[(\tilde{F}, A) \cup (\tilde{G}, B)] \cap [(\tilde{F}, A) \cup (\tilde{K}, C)] \subseteq (\tilde{F}, A) \cup [(\tilde{G}, B) \cap (\tilde{K}, C)]$

$\therefore (\tilde{F},A)\cup[(\tilde{G},B)\cap(\tilde{K},C)] = [(\tilde{F},A)\cup[(\tilde{G},B)]\cap[(\tilde{F},A)\cup(\tilde{K},C)]$. □

Proof. Let $e\in(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]$

If $e\in(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]$,

then e is either in $(\tilde{F},A)$ and in $[(\tilde{G},B)$ or $(\tilde{K},C)]$.
$\Rightarrow e\in(\tilde{F},A)$ and e$\in[(\tilde{G},B)$ or $(\tilde{K},C)]$
$\Rightarrow e\in(\tilde{F},A)$ and $\{e\in(\tilde{G},B)$ or $e\in(\tilde{K},C)\}$
$\Rightarrow \{e\in(\tilde{F},A)$ and $e\in(\tilde{G},B)\}$ or $\{e\in(\tilde{F},A)$ and $e\in(\tilde{K},C)\}$
$\Rightarrow e\in[(\tilde{F},A)$ and $(\tilde{G},B)]$ or $e\in[(\tilde{F},A)$ and $(\tilde{K},C)]$
$\Rightarrow e\in[(\tilde{F},A)\cap(\tilde{G},B)]\cup e\in[(\tilde{F},A)\cap(\tilde{K},C)]$
$\Rightarrow e\in[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$

Since $\exists e\in(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]$ such that $e\in[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$

Therefore, $(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]\subseteq[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$.

Let $e\in[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$.

If $e\in[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$, then e is in $\Big[(\tilde{F},A)$ and $(\tilde{G},B)\Big]$ or e is in $\Big[(\tilde{F},A)$ and $(\tilde{K},C)\Big]$.
$\Rightarrow e\in[(\tilde{F},A)$ and $(\tilde{G},B)]$ or $e\in[(\tilde{F},A)$ and $(\tilde{K},C)]$
$\Rightarrow \{e\in(\tilde{F},A)$ and $e\in(\tilde{G},B)\}$ or $\{e\in(\tilde{F},A)$ and $e\in(\tilde{K},C)\}$
$\Rightarrow e\in(\tilde{F},A)$ and $\{e\in[(\tilde{G},B)$ or $e\in(\tilde{K},C)]\}$
$\Rightarrow e\in(\tilde{F},A)$ and $\{e\in[(\tilde{G},B)$ or $(\tilde{K},C)]\}$
$\Rightarrow e\in(\tilde{F},A)\cap\{e\in[(\tilde{G},B)\cup(\tilde{K},C)]\}$
$\Rightarrow e\in(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]$

Since $\exists e\in[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$ such that $e\in(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]$

Therefore, $[(\tilde{F},A)\cap(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]\subseteq(\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)]$

$\therefore (\tilde{F},A)\cap[(\tilde{G},B)\cup(\tilde{K},C)] = [(\tilde{F},A)\cap[(\tilde{G},B)]\cup[(\tilde{F},A)\cap(\tilde{K},C)]$. □

Proposition 3.6. *Let $(\tilde{F},A),(\tilde{G},B)$ and $(\tilde{K},C)$ be three multi-valued neutrosophic soft sets over the common universe U. Then,*

1. $[(\tilde{F}, A) \vee (\tilde{G}, B)]^c = (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$
2. $[(\tilde{F}, A) \wedge (\tilde{G}, B)]^c = (\tilde{F}, A)^c \vee (\tilde{G}, B)^c$
3. $[(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c] = [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$
4. $[(\tilde{F}, A)^c \wedge (\tilde{G}, B)^c \wedge (\tilde{K}, C)^c] = [(\tilde{F}, A) \vee (\tilde{G}, B) \vee (\tilde{K}, C)]^c$

Proof. Let $e \in [(\tilde{F}, A) \vee (\tilde{G}, B)]^c$
$\Rightarrow e \notin (\tilde{F}, A) \vee (\tilde{G}, B)$
$\Rightarrow e \notin (\tilde{F}, A)$ and $e \notin (\tilde{G}, B)$
$\Rightarrow e \in (\tilde{F}, A)^c$ and $e \in (\tilde{G}, B)^c$
$\Rightarrow e \in (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$

Since $\exists e \in [(\tilde{F}, A) \vee (\tilde{G}, B)]^c$ such that $e \in (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$

Therefore, $[(\tilde{F}, A) \vee (\tilde{G}, B)]^c \subseteq (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$.

Then consider $e \in (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$
$\Rightarrow e \in (\tilde{F}, A)^c$ and $e \in (\tilde{G}, B)^c$
$\Rightarrow e \notin (\tilde{F}, A)$ and $e \notin (\tilde{G}, B)$
$\Rightarrow e \notin (\tilde{F}, A) \vee (\tilde{G}, B)$
$\Rightarrow e \in [(\tilde{F}, A) \vee (\tilde{G}, B)]^c$

Since $\exists e \in (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$ such that $e \in [(\tilde{F}, A) \vee (\tilde{G}, B)]^c$

Therefore, $(\tilde{F}, A)^c \wedge (\tilde{G}, B)^c \subseteq [(\tilde{F}, A) \vee (\tilde{G}, B)]^c$.

$\therefore [(\tilde{F}, A) \vee (\tilde{G}, B)]^c = (\tilde{F}, A)^c \wedge (\tilde{G}, B)^c$. □

Proof. Let $e \in [(\tilde{F}, A) \wedge (\tilde{G}, B)]^c$
$\Rightarrow e \notin (\tilde{F}, A) \wedge (\tilde{G}, B)$
$\Rightarrow e \notin (\tilde{F}, A)$ or $e \notin (\tilde{G}, B)$
$\Rightarrow e \in (\tilde{F}, A)^c$ or $e \in (\tilde{G}, B)^c$
$\Rightarrow e \in (\tilde{F}, A)^c \vee (\tilde{G}, B)^c$

Since $\exists e \in [(\tilde{F}, A) \wedge (\tilde{G}, B)]^c$ such that $e \in (\tilde{F}, A)^c \vee (\tilde{G}, B)^c$ Therefore, $[(\tilde{F}, A) \wedge (\tilde{G}, B)]^c \subseteq (\tilde{F}, A)^c \vee (\tilde{G}, B)^c$.

Then consider $e \in (\tilde{F}, A)^c \vee (\tilde{G}, B)^c$
$\Rightarrow e \in (\tilde{F}, A)^c$ or $e \in (\tilde{G}, B)^c$
$\Rightarrow e \notin (\tilde{F}, A)$ or $e \notin (\tilde{G}, B)$

$\Rightarrow e \notin (\tilde{F}, A) \wedge (\tilde{G}, B)$
$\Rightarrow e \in [(\tilde{F}, A) \wedge (\tilde{G}, B)]^c$

Since $\exists e \in (\tilde{F}, A)^c \vee (\tilde{G}, B)^c$ such that $e \in [(\tilde{F}, A) \wedge (\tilde{G}, B)]^c$ Therefore, $(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \subseteq [(\tilde{F}, A) \wedge (\tilde{G}, B)]^c$.

$$\therefore [(\tilde{F}, A) \wedge (\tilde{G}, B)]^c = (\tilde{F}, A)^c \vee (\tilde{G}, B)^c. \qquad \square$$

Proof. Let $e \in [(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c]$
$\Rightarrow e \in (\tilde{F}, A)^c \vee e \in (\tilde{G}, B)^c \vee e \in (\tilde{K}, C)^c$
$\Rightarrow e \notin (\tilde{F}, A) \vee e \notin (\tilde{G}, B) \vee e \notin (\tilde{K}, C)$
$\Rightarrow e \notin [(\tilde{F}, A) \wedge (\tilde{G}, B)] \vee e \notin (\tilde{K}, C)$
$\Rightarrow e \notin [(\tilde{F}, A) \wedge (\tilde{G}, B)] \wedge (\tilde{K}, C)$
$\Rightarrow e \notin [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]$
$\Rightarrow e \in [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$

Since $\exists e \in [(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c]$ such that $e \in [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$

Therefore, $[(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c] \subseteq [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$

Then consider $e \in [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$
$\Rightarrow e \notin [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]$
$\Rightarrow e \notin [(\tilde{F}, A) \wedge (\tilde{G}, B)] \wedge (\tilde{K}, C)$
$\Rightarrow e \notin [(\tilde{F}, A) \wedge (\tilde{G}, B)] \vee e \notin (\tilde{K}, C)$
$\Rightarrow e \notin (\tilde{F}, A) \vee e \notin (\tilde{G}, B) \vee e \notin (\tilde{K}, C)$
$\Rightarrow e \in (\tilde{F}, A)^c \vee e \in (\tilde{G}, B)^c \vee e \in (\tilde{K}, C)^c$
$\Rightarrow e \in [(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c]$

Since $\exists e \in [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$ such that $e \in [(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c]$

Therefore, $(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)^c \subseteq [(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c]$.
$[(\tilde{F}, A)^c \vee (\tilde{G}, B)^c \vee (\tilde{K}, C)^c] = [(\tilde{F}, A) \wedge (\tilde{G}, B) \wedge (\tilde{K}, C)]^c$. $\square$

4. An Application of Multi-Valued Neutrosophic Soft Set in a Decision Making Problem

The problem of finding the most suitable gadget is considered in which Mrs. N is going to select on the basis of his parameters out of a number of gadgets. She may construct a multi-valued neutrosophic soft set which represents the attribute of gadgets according to her interests. Assume that $U = \{x_1, x_2, x_3\}$ be the universe contains three gadgets under consideration and A be the parameter where $A = \{e_1 = branded, e_2 = specification, e_3 = cheap, e_4 = originality\}$. Now, the method can be applied as follows:

1. Input the multi-valued neutrosophic soft set
2. By using avg-level decision rule for decision making, input a threshold multi-valued neutrosophic soft set $((\tilde{F}, A) : \langle \mu, \omega, \varepsilon \rangle^{avg}_{(\tilde{F},A)})$
3. Compute avg-level soft set $((\tilde{F}, A) : \langle \mu, \omega, \varepsilon \rangle^{avg}_{(\tilde{F},A)})$
4. Present the level soft set $((\tilde{F}, A) : \langle \mu, \omega, \varepsilon \rangle^{avg}_{(\tilde{F},A)})$ in tabular form
5. Compute th choice value c_i of u_i for any $u_i \in U$
6. The optimal decision is to select u_3 since $c_3 = max_{u_i} \in U^{c_i}$

5. Conclusions

In this paper, we firstly propose the concept of multi-valued neutrosophic soft set and studied some of its properties. We redefine the notion of multi-valued neutrosophic soft set operations. Finally, we present an application of multi-valued neutrosophic soft sets in decision making problem.

Acknowledgement

We would like to acknowledge Universiti Malaysia Terengganu and Ministry of Higher Education Malaysia for the financial support received in the preparation of the paper.

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