N_β-Closed Sets In Neutrosophic Topological Spaces

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Abstract-----The aim of this paper to introduced the new concept of β-closed sets in Neutrosophic topological spaces. We also analyze the properties and characterize the Neutrosophic β-closed sets.

Keywords—**Neutrosophic Closed Sets,Neutrosophic Topological Spaces,Neutrosophic β-closed sets,N-semi open,N-preopen.**

I. INTRODUCTION

In 1965, Zadeh[9] introduced fuzzy set theory which deals with uncertainties where each element has a degree of membership. In 1983, Atanassov[1] introduced the Intuitionistic fuzzy set where each element has a degree of membership and degree of non-membership values.In 2005, Florentin Smarandache[7] introduced the Neutrosophic set and explained that the generalization of intuitionistic fuzzy set is the Neutrosophic set. In 2012, A.A.Salama and S.A.Alblowi[6] introduced the concept of Neutrosophic topological spaces besides the degree of membership, degree of indeterminacy and the degree of non-membership for each element. In 2014 A.A.Salama, Smarandache and Veleri[5] introduced the concept of Neutrosophic closed sets and continuous functions. In this paper, we introduce the concept N_β closed set and characterized some of its properties in Neutrosophic topological spaces.

II. PRELIMINARIES

In this paper the Neutrosophic topological space is denoted by (X,τ).Alsoneutrosophic interior of A is denoted by NInt(A) andneutrosophic closure of A is denoted by NCl(A).The complement of neutrosophic A is denoted by A^c.

Definition 2.1:

Let X be a nonempty fixed set A is an object having the form A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x\in X\}$ where $\mu_A(x)$,$\sigma_A(x)$,$\nu_A(x)$ represents the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x∈X to the set A.

Definition 2.2:

Let A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x\in X\}$ and B=$\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x\in X\}$ are the two neutrosophic sets on X, then the complements become,

C(A)=$\{\langle x, 1-\mu_A(x), 1-\sigma_A(x), 1-\nu_A(x)\rangle : x\in X\}$

C(A)=$\{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle : x\in X\}$

C(A)= $\{\langle x, \nu_A(x), 1-\sigma_A(x), \mu_A(x)\rangle : x\in X\}$

C(A∪B)=C(A)∩C(B)

C(A∩B)=C(A)∪C(B).

Definition 2.3:

Let A={⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x∈X} and B={⟨x, μ_B(x), σ_B(x), ν_B(x)⟩: x∈X} are the two neutrosophic sets on X,thenA⊆B⇔ $\mu_A(x) \leq \mu_B(x)$,$\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$

A=B⇔A⊆B and B⊆A

A∪B=<x, $\mu_A(x) \vee \mu_B(x)$,$\sigma_A(x) \vee \sigma_B(x)$, $\nu_A(x) \wedge \nu_B(x)$>

A∩B=<x, $\mu_A(x) \wedge \mu_B(x)$,$\sigma_A(x) \wedge \sigma_B(x)$, $\nu_A(x) \vee \nu_B(x)$>

Definition 2.4

A neutrosophic topological space on a nonempty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

i) $0_N, 1_N \in \tau$,

ii) $G_1 \cap G_2 \in \tau$,for every G_1 and $G_2 \in \tau$,

iii) $\cup G_i \in \tau$,for every G_i:I ∈ J ⊆τ

The pair (X,τ) is a neutrosophic topological space (NTS) and the element τ is called neutrosophic open sets(NOS) in X.A neutrosophic set A is called the neutrosophic closed set A if and only if its complement C(A) is a neutrosophic open set in X.

The empty set (0_N) and the whole set (1_N) may be defined as follows:

(0_1) 0_N = {⟨x, 0,0,1⟩:x∈X}

(0_2) 0_N ={⟨x, 0,1,1⟩:x∈X}

(0_3) 0_N = {⟨x, 0,1,0⟩:x∈X}

(0_4) 0_N = {⟨x, 0,0,0⟩:x∈X}

(1_1) 1_N = {⟨x, 1,0,0⟩:x∈X}

(1_2) 1_N = {⟨x, 1,0,1⟩:x∈X}

(1_3) 1_N = {⟨x, 1,1,0⟩:x∈X}

(1_4) 1_N = {⟨x, 1,1,1⟩:x∈X}

Definition 2.5: Let (X,τ) be the neutrosophic topological spaces and A={⟨x, μ_A(x), σ_A(x), ν_A(x)⟩: x∈X} be the neutrosophic set in X. Then the neutrosophic interior and closure becomes,NInt(A)= ∪ {G:G is an NOS in X and G ⊆ A}NCl(A)= ∩ {K:K is an NCS in X and A ⊆ K}

Definition 2.6:

A neutrosophic set of a neutrosophic topological space X is said to be i) A neutrosophic pre-open set(NPOS) if A ⊆ NInt(NCl(A))ii) A neutrosophic semi-open set (NSOS) if A⊆ NCl(NInt(A))iii) A neutrosophic α-open set (Nα-OS) if A⊆ NInt(NCl(NInt(A)))iv) A neutrosophic regular open set (NR-OS) if A= NInt(NCl(A))

Definition 2.7: A neutrosophic set of a neutrosophic topological space X is said to be

i) A neutrosohic pre-closed set(NPCS) if NCl(NInt(A)) ⊆ Aii) A neutrosophic semi-closed set(NSCS) if NInt(NCl(A)) ⊆ Aiii) A neutrosophic α-closed set(Nα-CS)if NCl(NInt(NCl(A))) ⊆A

iv) A neutrosophic regular closed set(NRCS) if A=NCl(NInt(A))

III. NEUTROSOPHIC β-CLOSED SETS

Definition 3.1:

Let (X,τ) be an neutrosophic topological space. Then A is said to be an neutrosophic β closed set (N_β-CS) if N_β cl(A) ⊆ U whenever A ⊆ U and U is neutrosophic pre-open set in X(NPOS).The complement C(A) of a N_βCS A is a N_βOS in X.

Theorem 3.2: Every Neutrsophic closed set A is N_β closed set.*Proof:* Let A ⊆ U where U is preopen set in X. Let NInt(A)⊆A. Then NCl(NInt(A))⊆NCl(A). By the definition of NCS, NCl(NInt(A))⊆A. NInt(NCl(NInt(A)))⊆NInt(A).

Which implies NInt(NCl(NInt(A)))⊆A.

Therefore A is N_β closed set in X.

Remark 3.3: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.4:

Let X={a,b,c} and τ={0_N,1_N,G_1,G_2,G_3,G_4} where $G_1=\langle x, (0.5,0.6), (0.4,0.7), (0.3,0.7)\rangle$ $G_2=\langle x, (0.7,0.5), (0.6,0.5), (0.7,0.4)\rangle$ $G_3=\langle x, (0.7,0.5), (0.6,0.7), (0.7,0.7)\rangle$ $G_4=\langle x, (0.5,0.5), (0.4,0.5), (0.3,0.4)\rangle$Let M=$\langle x, (0.7,0.6), (0.3,0.2), (0.8,0.9)\rangle$Then the set M is N_β closed set but M is not Neutrosophic closed set.

Theorem 3.5:

Every Neutrosophic pre-closed set is N_β closed set in (X,τ_N). *Proof:*

Let A⊆U where U is preopen set in X.

Given, Let A be neutrosophic pre-closed set. Let NCl(NInt(A))⊆A. Then NInt(NCl(NInt(A)))⊆NInt(A).

Which implies NInt(NCl(NInt(A)))⊆A.

Therefore A is N_β closed set in X.

Remark 3.6: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.7:

Let X={a,b,c} and $\tau=\{0_N, 1_N, G_1, G_2\}$ where

$G_1=\langle x,(0.7,0.6),(0.5,0.5),(0.5,0.7)\rangle$ $G_2=\langle x,(0.3,0.4),(0.5,0.3),(0.6,0.8)\rangle$ Let

$M=\langle x,(0.4,0.5),(0.5,0.5),(0.5,0.8)\rangle$ Then the set M is N_β closed set but M is not Neutrosophic pre-closed set.

Theorem 3.8: Every Neutrosophic semi-closed set is N_β closed set in (X,τ_N).

Proof:

Let A⊆U where U is preopen set in X.

Given, Let A be neutrosophic semi-closed set.

That is NInt(NCl(A))⊆A.

Consider NInt(A)⊆A, Then

NCl(NInt(A)))⊆NCl(A) implies NInt(NCl(NInt(A)))⊆NInt(NCl(A))

which implies NInt(NCl(NInt(A)))⊆A.

Therefore A is N_β closed set in X.

Remark 3.9: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.10:

Let X={a,b,c} and $\tau=\{0_N, 1_N, G_1, G_2\}$ where $G_1=\langle x,(0.2,0.3),(0.4,0.3),(0.5,0.6)\rangle$

$G_2=\langle x,(0.6,0.8),(0.5,0.4),(0.4,0.2)\rangle$ Let $M=\langle x,(0.6,0.5),(0.4,0.4),(0.7,0.8)\rangle$ Then the set M is N_β closed set but M is not Neutrosophic semi-closed set.

Theorem 3.11:

Every Neutrosophic α-closed set is N_β closed set in (X,τ_N).

Proof:

Let A⊆U where U is preopen set in X.

Given, Let A be neutrosophic α-closed set. That is

NCl(NInt(NCl(A)))⊆A. Consider

A⊆NCl(A). Then NInt(A)⊆NInt(NCl(A)). Then

NCl(NInt(A))⊆NCl(NInt(NCl(A)))⊆A. Which

implies NCl(NInt(A))⊆A Then

NInt(NCl(NInt(A)))⊆NInt(A)⊆A. Hence

NInt(Ncl(NInt(A)))⊆A. Therefore A is N_β closed set in X.

Remark 3.12: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.13:

Let X={a,b,c} and τ={0_N, 1_N, G_1, G_2, G_3, G_4} where $G_1=\langle x, (0.4,0.3), (0.5,0.8), (0.4,0.3)\rangle$

$G_2=\langle x, (0.2,0.5), (0.6,0.3), (0.5,0.7)\rangle$

$G_3=\langle x, (0.4,0.5), (0.6,0.8), (0.4,0.3)\rangle$ $G_4=\langle x, (0.2,0.3), (0.5,0.3), (0.5,0.7)\rangle$ Let

$M=\langle x, (0.2,0.4), (0.6,0.8), (0.6,0.7)\rangle$ Then the set M is N_β closed set but M is not Neutrosophic α-closed set.

Theorem 3.14:

Every Neutrosophic regular closed set is N_β closed set in (X,τ_N). *Proof:*

Let A ⊆ U where U is preopen set in X.

Given, Let A be neutrosophic regular closed set.

That is NCl(NInt(A))=A. which implies NInt(NCl(NInt(A)))⊆NInt(A)⊆A.

Which implies NInt(NCl(NInt(A)))⊆A.

Therefore A is N_β closed set in X.

Remark 3.15: The converse of the above theorem need not be true which can be shown by the following example.

Example 3.16:

Let X={a,b,c} and τ={0_N, 1_N, G_1, G_2} where $G_1=\langle x, (0.2,0.3), (0.4,0.3), (0.6,0.7)\rangle$

$G_2=\langle x, (0.6,0.5), (0.5,0.5), (0.4,0.3)\rangle$ Let $M=\langle x, (0.4,0.4), (0.5,0.4), (0.5,0.5)\rangle$ Then the set M is N_β closed set but M is not Neutrosophic regular closed set.

Theorem 3.17: The Union of two N_β closed set in (X,τ_N) is a N_β closed set in (X.τ_N). *Proof :*

Let A and B are N_β closed sets in (X,τ_N). From the definition of N_β closed set, N_β cl(A) ⊆ U whenever A ⊆ U and U is Preopen in (X,τ_N). Similarly, $N_\beta$cl(B) ⊆ U whenever B ⊆ U and U is Preopen in (X,τ_N). Since A and B are the subsets of U then A∪B also the subsets of U and U is the neutrosophic Preopen set,which implies $N_\beta$cl (A∪B) = $N_\beta$cl(A) ∪ $N_\beta$cl(B). Therefore $N_\beta$cl(A∪B) ⊆ U.

Therefore A∪B is N_β closed set in (X,τ_N).

Theorem 3.18: Suppose A and B are N_β closed set in (X,τ_N) then $N_\beta cl(A\cap B) \subseteq N_\beta cl(A)\cap N_\beta l(B)$. *Proof:* Let A be N_β closed set in (X,τ_N),Then N_β cl(A)⊆U whenever A ⊆U and U is neutrosophic preopen in (X,τ_N).Moreover, Since A and B are subsets of U, then A∩B is a subset of U where U is a neutrosophic preopen set .From the result, If A⊆B, then N_β cl(A) ⊆ N_β cl(B),

which implies N_β cl(A∩B) ⊆ N_β cl(A) and N_β cl(A∩B) ⊆ N_β cl(B). Therefore N_β cl(A∩B) ⊆ N_β(A) ∩ $N_\beta$cl(B).

Example 3.19: The intersection of two N_β closed sets are need not be N_β closed set.

Let X={a,b,c} and $\tau=\{0_N,1_N,G_1,G_2,G_3,G_4\}$ where $G_1=\langle x,(0.8,0.6),(0.5,0.4),(0.5,0.7)\rangle$ $G_2=\langle x,(0.7,0.3),(0.7,0.1),(0.4,0.9)\rangle$ $G_3=\langle x,(0.8,0.6),(0.7,0.4),(0.4,0.7)\rangle$ $G_4=\langle x,(0.7,0.3),(0.5,0.1),(0.5,0.9)\rangle$Let $M=\langle x,(0.7,0.9),(0.6,0.3),(0.5,0.5)\rangle$ and $N=\langle x,(0.7,0.6),(0.7,0.4),(0.4,0.9)\rangle$. Then M and N are N_β closed set, And the union $M\cup N=\langle x,(0.7,0.9),(0.7,0.4),(0.4,0.5)\rangle$ is also an N_β closed set. But the intersection $M\cap N=\langle x,(0.7,0.6),(0.6,0.3),(0.5,0.9)\rangle$ is not an N_β closed set.

Theorem 3.20:

Suppose M is N_β closed set in (X,τ_N) and M ⊆ N ⊆ N_β cl(A),then N is also N_β closed set in (X,τ_N).

Proof: Let N ⊆ U and U is neutrosophic preopen in (X,τ_N).Then M ⊆ N which implies M ⊆ U, Since M is N_β closed set, N_β cl(M) ⊆ U also M ⊆ N_β cl(N) ⊆ U and N_β cl(N) ⊆ N_β cl(M). Which implies M ⊆ N_β cl(N) ⊆ N_β cl(M) ⊆ U. Therefore N_β cl(N) ⊆ U. and so N is also N_β closed set in (X,τ_N).

Theorem 3.21: If M be a N_β closed subset of (X,τ), then N_β cl(M)-M does not contain any nonempty N_β closed set.

Proof:

Assume that M is N_β closed set.

Let F be a nonempty N_β closed set, such that F ⊆ N_β cl(M)-M = N_β cl(M) ∩ $\overline{M}$.

That is, F⊆N_β cl(M) and F ⊆$\overline{M}$. Therefore M⊆$\overline{F}$. Since $\overline{F}$ is N_β open set, N_β cl(M)⊆$\overline{F}$⇒ F⊆((N_β cl(M)-M)∩($\overline{N_\beta\, cl(M)}$)⊆$N_\beta$ cl(M)∩($\overline{N_\beta\, cl(M)}$).

That is $F \subseteq \phi \Rightarrow F$ is empty.

Therefore N_β cl(M)-M does not contain any nonempty N_β closed set.

Theorem 3.22: If M is neutrosophic open also N_β closed set, then M is neutrosophic closed set. *Proof:*

Since M is both neutrosophic open and N_β closed set in (X,τ_N).

Then $N_\beta cl(M) \subseteq M$ and $M \subseteq N_\beta\, cl(M)$

$\Rightarrow N_\beta\, cl(M) = M$.

Therefore M is neutrosophic closed set in (X,τ_N).

Theorem 3.23:

If A is both N_β closed set and neutrosophic closed set if and only if N cl(A)-A is neutrosophic closed. *Proof:*

From the hypothesis, Let A be N_β closed set.

If A is neutrosophic closed set then N cl(A)=A.

$\Rightarrow$N cl(A)-A = ϕ.

Therefore N cl(A)-A is neutrosophic closed set.

Conversely,

Assume that N cl(A)-A is neutrosophic closed,

But A is N_β closed set and N cl(A)-A $\subseteq$ neutrosophic closed.

$\Rightarrow$N cl(A)-A = ϕ. which implies N cl(A)=A. Therefore A is Neutrosophic set.

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