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$\mathrm{N}_{\delta^* \mathrm{g}\alpha}$-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT. In this paper, the authors introduced the concept of neutrosophic $\delta^* g\alpha$-closed sets in neutrosophic topological spaces. Some of their properties and relations with other existing neutrosophic closed were established, some of its characterizations were also investigated.

1. INTRODUCTION

Smarandache [7, 8] did his research contribution on neutrosophic set. He researched on the basis of intuitionistic fuzzy sets by Atanassov [11] and fuzzy sets by Zadeh [13]. Smarandache neutrosophic set has its roots and is influenced by the truth membership function, indeterminacy member function and falsity member function. Smarandache identified a glitch that intuistionistic fuzzy and fuzzy set doesn't concentrate with indeterminacy membership functions. Hence, Smarandache inculcated his neutrosophic concept in fields such as probability, algebra, control theory, topology, etc. Suceeding Smarandache, Alblowi et al [15] contributed neutrosophic set based inclusions to the neutrosophic field. The study made by Smarandache served many researchers towards the research in topology. Salama and Albowi [3] made an extensive study and deliberated a novel concept in netrosophic topological spaces based on Coker [6] and Chang's [5] intuitionistic fuzzy topology and fuzzy topology respectively.

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In the field of neutrosophic topological spaces, the research work of Salama et al., [1, 2, 4] paved towards a generalized fusion of neutrosophic sets, neutrosophic crisp sets and neutrosophic closed sets. the concept of closed sets in neutrosophic topology is well formulated and explained by several contributors in the field of research and named as neutrosophic α-closed sets [8], neutrosophic αg-closed sets [6] and generalized neutrosophic closed sets [14]. Recently Damodharan and Vigneshwaran [12] introduced the concept of neutrosophic δ-closure and δ-interior in neutrosophic topological spaces.

In this article, the concept of neutrosophic $\delta^* g\alpha$-interior, neutrosophic $\delta^* g\alpha$-closure are introduced, using that notion neutrosophic $\delta^* g\alpha$-closed sets in neutrosophic topological spaces are introduced and relation with other existing neutrosophic closed sets were studied and investigated some special properties.

2. Preliminaries

In this section, we recollect some relevant basic preliminaries about neutrosophic sets and its operations.

Definition 2.1. *[3] Let X be a non empty fixed set. A neutrosophic set P is an object having the form* $P = \{\langle x, \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x))\rangle \, \forall x \in X\}$*, where* $\mu_m(P(x))$ *represents the degree of membership,* $\sigma_i(P(x))$ *represents the degree of indeterminacy and* $\nu_{nm}(P(x))$ *represents the degree of nonmembership functions of each element* $x \in X$ *to the set P.*

Remark 2.1. *[3] A neutrosophic set*

$$P = \{\langle x, \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x))\rangle \, \forall x \in X\}$$

can be identified to an ordered triple $\langle \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x))\rangle$ *in* $]^-0, 1^+[$ *on X.*

Definition 2.2. *[3] In neutrosophic topological spaces*

$\forall x \in X,\ 0_N$ *may be defined as*	$\forall x \in X,\ 1_N$ *may be defined as*
$0_N = \langle x, 0, 0, 1\rangle$	$1_N = \langle x, 1, 0, 0\rangle$
$0_N = \langle x, 0, 1, 1\rangle$	$1_N = \langle x, 1, 0, 1\rangle$
$0_N = \langle x, 0, 1, 0\rangle$	$1_N = \langle x, 1, 1, 0\rangle$
$0_N = \langle x, 0, 0, 0\rangle$	$1_N = \langle x, 1, 1, 1\rangle$

Definition 2.3. *[3] Let P be two neutrosophic sets of the form:*
$P = \{\langle x, \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x))\rangle \, \forall x \in X\}$*, Then the compliment of P* $[P^c]$ *may be defined as:*

$$P^c = \{\langle x, 1-\mu_m(P(x)), 1-\sigma_i(P(x)), 1-\nu_{nm}(P(x))\rangle \, \forall x \in X\} \, .$$

Definition 2.4. *[3] Let P and Q be two neutrosophic sets of the form,*
$P = \{\langle x, \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x))\rangle \, \forall x \in X\}$ *and*
$Q = \{\langle x, \mu_m(Q(x)), \sigma_i(Q(x)), \nu_{nm}(Q(x))\rangle \, \forall x \in X\}$*. Then,*

(i) *Subsets* $(P \subseteq Q)$ *may be defined as*
$P \subseteq Q$ *if and only if* $\mu_m(P(x)) \leq \mu_m(Q(x)), \sigma_i(P(x)) \geq \sigma_i(Q(x)), \nu_{nm}(P(x)) \geq \nu_{nm}(Q(x))$

(ii) *Subsets* $P = Q$ *if and only if* $P \subseteq Q$ *and* $Q \subseteq P$*.*

(iii) *Union of subsets* $P \cup Q$ *may be defined as*
$P \cup Q = \{x, max[\mu_m(P(x), \mu_m(Q(x)))], max[\sigma_i(P(x)), \sigma_i(Q(x))], min[\nu_{nm}(P(x)), \nu_{nm}(Q(x))] \, \forall x \in X\}$*.*

(iv) *Intersection of subsets* $P \cap Q$ *may be defined as*
$P \cap Q = \{x, min[\mu_m(P(x), \mu_m(Q(x)))], min[\sigma_i(P(x)), \sigma_i(Q(x))], max[\nu_{nm}(P(x)), \nu_{nm}(Q(x))] \, \forall x \in X\}$*.*

Definition 2.5. *[3] A neutrosophic topolgy on a non empty set X is a family* τ *of neutrosophic subsets in X satisfying the following axioms:*

(i) $0_N, 1_N \in \tau$*,*

(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$*,*

(iii) $\cup G_i \in \tau \forall \{G_i : i \in J\} \subseteq \tau$.

Then the pair (X, τ) *or simply X is called a neutroscopic topological space.*

Definition 2.6. *[3] Let P be a neutrosophic set in a neutrosophic topological space* (X, τ)*. Then*

(i) $Nint(P) = \bigcup\{Q/Q$ *is a neutrosophic open set in* (X, τ) *and* $Q \subseteq P\}$ *is called the neutrosophic interior of P;*

(ii) $Ncl(P) = \bigcap\{Q/Q$ *is a neutrosophic closed set in* (X, τ) *and* $Q \supseteq P\}$ *is called the neutrosophic closure of P.*

Definition 2.7. *[9] A subset* A *of* (X, τ) *is said to be*

(i) *neutrosophic semi-open set if* $P \subseteq Ncl(Nint(P))$*.*

(ii) *neutrosophic pre-open set if* $P \subseteq Nint(Ncl(P))$*.*

(iii) *neutrosophic semi-preopenset if* $P \subseteq Ncl(Nint(Ncl(P)))$.
(iv) *neutrosophic* α*-open set if* $P \subseteq Nint(Ncl(Nint(P)))$.
(v) *neutrosophic regular open set if* $P = Nint(Ncl(P))$.

The complement of a neutrosophic semi-open (resp. neutrosophic pre-open, neutrosophic α*-open, neutrosophic regular open)set is called neutrosophic semi-closed (resp. neutrosophic pre-closed, neutrosophic* α*-closed, neutrosophic regular closed).*

Definition 2.8. *[10] Let* $\alpha, \beta, \lambda \in [0,1]$ *and* $\alpha + \beta + \lambda \leq 3$*. A neutrosophic point* $x_{(\alpha,\beta,\lambda)}$*of X is a neutrosophic point of X which is defined by*

$$x_{(\alpha,\beta,\lambda)}(y) = \begin{cases} (\alpha,\beta,\lambda), & \text{if } y = x \\ (0,0,1), & \text{if } y \neq x. \end{cases} .$$

In this case, x *is called the support of* $x_{(\alpha,\beta,\lambda)}$ *and* α, β *and* λ*, respectively. A neutrosophic point* $x_{(\alpha,\beta,\lambda)}$ *is said to belong to a neutrosophic set*

$$P = \langle \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x)) \rangle \text{ in } X ,$$

denoted by $x_{(\alpha,\beta,\lambda)} \in P$ *if* $\alpha \leq \mu_m(P(x)), \beta \geq \sigma_i(P(x))$ *and* $\lambda \geq \nu_{nm}(P(x))$. *Clearly a neutrosophic point can be represented by an ordered triple of neutrosophic point as follows :* $x_{(\alpha,\beta,\lambda)} = \left(x_\alpha, x_\beta, C\left(x_{C(\lambda)}\right)\right)$.

Definition 2.9. *[12] Let* (X,τ) *be a neutrosophic topological space. A neutrosophic point* $x_{(\alpha,\beta,\lambda)}$ *is said to be quasi-coincident, with the neutrosophic set* $P = \langle \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x)) \rangle$*, denoted by* $x_{(\alpha,\beta,\lambda)}qP$*, if it satisfies* $\alpha > \sigma_i(P(x))$ *and* $\nu_{nm}(P(x))$ *or* $\beta, \lambda < \mu_m(P(x))$.

Definition 2.10. *[12] Let* (X,τ) *be a neutrosophic topological space were*
$P = \{\langle x, \mu_m(P(x)), \sigma_i(P(x)), \nu_{nm}(P(x)) \rangle \, \forall x \in X\}$ *and*
$Q = \{\langle x, \mu_m(Q(x)), \sigma_i(Q(x)), \nu_{nm}(Q(x)) \rangle \, \forall x \in X\}$
be two neutrosophic sets in X. then P and Q are said to be quasi-coincident, denoted by PqQ, if there exists an element $x \in X$ *such that if it satisfies:* $\mu_m(P(x)) > \sigma_i(Q(x)), \nu_{nm}(Q(x))$ *or* $\sigma_i(P(x)), \nu_{nm}(P(x)) < \mu_m(Q(x))$

The word 'not quasi-coincident' will be abbreviated as $\tilde{q}$.

Definition 2.11. *[12] Let* (X,τ) *be a neutrosophic topological space. A neutrosophic point* $x_{(\alpha,\beta,\lambda)}$ *is said to be a neutroscopic* δ*-cluster point of a neutrosophic set P if* AqP *for each neutrosophic regular open q-neighborhood A of* $x_{(\alpha,\beta,\lambda)}$*. The set of all neutroscopic* δ*-cluster points of P is called the neutrosophic* N_δ*-closure* *of P and denoted by $Ncl_\delta(P)$. A neutrosophic set P is said to be a neutrosophic δ-closed set if $P = Ncl_\delta(P)$. The complement of a neutrosophic N_δ-closed set is said to be a neutrosophic N_δ-open set.*

Example 1. *Let $X = \{p, q\}$ and the neutrosophic sets A and B are defined as*
$A = \left\{\left\langle x, \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.6}, \frac{q}{0.5}\right), \left(\frac{p}{0.7}, \frac{q}{0.5}\right)\right\rangle, \forall x \in X\right\}$;
$B = \left\{\left\langle x, \left(\frac{p}{0.2}, \frac{q}{0.6}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}\right), \left(\frac{p}{0.4}, \frac{q}{0.6}\right)\right\rangle, \forall x \in X\right\}$;
$A \cup B = \left\{\left\langle x, \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}\right)\right\rangle, \forall x \in X\right\}$;
$A \cap B = \left\{\left\langle x, \left(\frac{p}{0.2}, \frac{q}{0.3}\right), \left(\frac{p}{0.6}, \frac{q}{0.6}\right), \left(\frac{p}{0.7}, \frac{q}{0.6}\right)\right\rangle, \forall x \in X\right\}$.
represents a neutrosophic open sets of neutrosophic topological space (X, τ) with neutrosophic topology $\tau = \{0_N, A, B, A \cup B, A \cap B\}$. Hence
$C_1 = \left\{\left\langle x, \left(\frac{p}{0.5}, \frac{q}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right)\right\rangle, \forall x \in X\right\}$ *is $N_{*g\alpha}$-open set,*
$C_2^c = \left\{\left\langle x, \left(\frac{p}{0.3}, \frac{q}{0.2}\right), \left(\frac{p}{0.7}, \frac{q}{0.8}\right), \left(\frac{p}{0.8}, \frac{q}{0.7}\right)\right\rangle, \forall x \in X\right\}$ *is $N_{\delta^*g\alpha}$-closed set of a neutrosophic topological space (X, τ). Since $Ncl_\delta(C_2^c) = (A \cap B)^c$ which is contained in C_1. That is $Ncl_\delta(C_2^c) \subseteq C_1$. Also, $C_2^c \subseteq C_1$.*

3. $\mathbf{N}_{\delta^*\mathbf{g}\alpha}$-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

Definition 3.1. *A subset P of (X, τ) is called*

(i) *a neutrosophic generalized closed (briefly N_g-closed) set if Ncl(P)⊆U whenever P⊆U and U is $Nopen$ set in (X, τ).*
(ii) *a neutrosophic generalized semi-closed (briefly N_{gs}-closed) set if Nscl(P)⊆U whenever P⊆U and U is $Nopen$ set in (X, τ).*
(iii) *a neutrosophic α-generalized closed (briefly $N_{\alpha g}$-closed) set if $N\alpha$cl(P)⊆U whenever P⊆U and U is $Nopen$ set in (X, τ).*
(iv) *a neutrosophic generalized preclosed (briefly N_{gp}-closed) set if $Npcl(P)$ ⊆U whenever P⊆U and U is $Nopen$ set in (X, τ).*
(v) *a neutrosophic generalized semi-preclosed (briefly N_{gsp}-closed) set if $Nspcl(P)$ ⊆U whenever P⊆U and U is $Nopen$ set in (X, τ).*
(vi) *a neutrosophic *generalized α-closed (briefly $N_{*g\alpha}$-closed) set if $Ncl(P)$ ⊆U whenever P⊆U and U is $Ng\alpha$-open set in (X, τ).*

The complement of a N_g-closed (resp. N_{gs}-closed, $N_{\alpha g}$-closed, N_{gp}-closed, N_{gsp}-closed and $N_{*g\alpha}$-closed) set is called N_g-open (resp. N_{gs}-open, $N_{\alpha g}$-open, N_{gp}-open, N_{gsp}-open and $N_{*g\alpha}$-open).

The authors introduce the following definition.

Definition 3.2. *A subset P of a space (X,τ) is called $N_{\delta^*g\alpha}$-closed if Ncl_δ(P)⊆U whenever P⊆U and U is a $N_{*g\alpha}$-open set in (X,τ).*

Theorem 3.1. *In the neutrosophic topological space (X,τ), every N_δ-closed set is $N_{*g\alpha}$-closed.*

Proof. Let P be N_δ-closed and U be any $N_{g\alpha}$-open set containing P. Since P is N_δ-closed, Ncl_δ(P) = P. Therefore $Ncl_\delta(P) \subseteq P \subseteq U$. We know that $Ncl(P) \subseteq Ncl_\delta\,(P)$ ⊆U. Hence P is $N_{*g\alpha}$-closed. □

Theorem 3.2. *In the neutrosophic topological space (X,τ), every N_δ-closed set is $N_{\delta^*g\alpha}$-closed set.*

Proof. Let P ⊆ U and U is $_{N^*g\alpha}$-open set. Since P is N_δ-closed Ncl_δ(P) = P, then Ncl_δ(P)⊆U therefore P is $N_{\delta^*g\alpha}$-closed set. □

Note that the converse part of the theorem is not true is shown through an example.

Remark 3.1. *From example 1, the neutrosophic set C_2^c is $N_{\delta^*g\alpha}$-closed set but it is not N_δ-closed set of a neutrosophic topological space (X,τ). Since $Ncl_\delta\,(C_2^c) = (A\cap B)^c \neq C_2^c$.*

Theorem 3.3. *In the neutrosophic topological space (X,τ), every $N_{\delta^*g\alpha}$ -closed set is N_{gs}-closed.*

Proof. Let P⊆U and U is N-open set. Since every N-open set is $N_{*g\alpha}$-open[9], then U is $N_{*g\alpha}$-open set. Since P is $N_{\delta^*g\alpha}$-closed, then Ncl_δ(P)⊆U. But $Nscl(P) \subseteq Ncl_\delta$(P), then $Nscl(P)$ ⊆U, Therefore P is N_{gs}-closed set. □

Note that the converse part of the theorem is not true is shown through an example.

Remark 3.2. *The neutrosophic set*

$$C_3^c = \left\{\left\langle x, \left(\frac{p}{0.5}, \frac{q}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}\right), \left(\frac{p}{0.3}, \frac{q}{0.5}\right)\right\rangle, \forall x \in X\right\}$$

is N_{gs}-closed set but it is not a $N_{\delta^*g\alpha}$-closed set of a neutrosophic topological space (X,τ). Since $Ncl_\delta\,(C_3^c) = A^c$ does not contain in C_1.

Theorem 3.4. *In the neutrosophic topological space (X,τ), every $N_{\delta^*g\alpha}$ -closed set is $N_{\alpha g}$-closed.*

Proof. Let P⊆U and U is N-open set. Since every N-open set is $N_{*g\alpha}$-open, then U is $N_{*g\alpha}$-open set. Since P is $N_{\delta^*g\alpha}$-closed, then Ncl_δ(P)⊆U. But $N\alpha$cl(P)⊆ Ncl_δ(P), then $N\alpha$cl(P)⊆U, Therefore P is $N_{\alpha g}$-closed set. □

Note that the converse part of the theorem is not true is shown through an example.

Remark 3.3. *The neutrosophic set*

$$C_4^c = \left\{\left\langle x, \left(\frac{p}{0.5}, \frac{q}{0.4}\right), \left(\frac{p}{0.6}, \frac{q}{0.4}\right), \left(\frac{p}{0.6}, \frac{q}{0.4}\right)\right\rangle, \forall x \in X\right\}$$

is $N_{\alpha g}$-closed set but it is not a $N_{\delta^*g\alpha}$-closed set of a neutrosophic topological space (X,τ). Since C_4^c does not contain in A^c, C_4^c does not contain in $(A \cup B)^c$.

Theorem 3.5. *In the neutrosophic topological space (X,τ), every $N_{\delta^*g\alpha}$ -closed set is N_{gsp}-closed.*

Proof. Let P⊆U and U is N-open set. Since every N-open set is $N_{*g\alpha}$-open, then U is $N_{*g\alpha}$-open set. Since P is $N_{\delta^*g\alpha}$-closed, then Ncl_δ(P)⊆U. But $Nspcl(P) \subseteq$ Ncl_δ(P), then $Nspcl(P)$ ⊆U, Therefore P is N_{gsp}-closed set. □

Note that the converse part of the theorem is not true is shown through an example.

Remark 3.4. *The neutrosophic set*

$$C_5^c = \left\{\left\langle x, \left(\frac{p}{0.7}, \frac{q}{0.5}\right), \left(\frac{p}{0.4}, \frac{q}{0.5}\right), \left(\frac{p}{0.5}, \frac{q}{0.3}\right)\right\rangle, \forall x \in X\right\}$$

is N_{gsp}-closed set but it is not a $N_{\delta^*g\alpha}$-closed set of a neutrosophic topological space (X,τ). Since C_5^c does not contain in A^c, C_5^c does not contain in $(A \cup B)^c$

Theorem 3.6. *In the neutrosophic topological space (X,τ), every $N_{\delta^*g\alpha}$ -closed set is N_{gp}-closed.*

Proof. Let P⊆U and U is N-open set. Since every N-open set is $N_{*g\alpha}$-open, then U is $N_{*g\alpha}$-open set. Since P is $N_{\delta^*g\alpha}$-closed, then Ncl_δ(P)⊆U. But $Npcl(P) \subseteq$ Ncl_δ(P), then $Npcl(P)$ ⊆U, Therefore P is N_{gp}-closed. □

Note that the converse part of the theorem is not true is shown through an example.

Remark 3.5. *The neutrosophic set*

$$C_6^c = \left\{ \left\langle x, \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right) \right\rangle, \forall x \in X \right\}$$

is N_{gp}-closed set but it is not a $N_{\delta^* g\alpha}$-closed set of a neutrosophic topological space (X, τ). Since C_6^c does not contain in A^c, C_6^c does not contain in $(A \cup B)^c$

4. CHARATERIZATION OF $\mathbf{N_{\delta^* g\alpha}}$-CLOSED SETS

Theorem 4.1. *The finite union of $N_{\delta^* g\alpha}$-closed sets is $N_{\delta^* g\alpha}$-closed.*

Proof. Let $\{P_i/i = 1, 2, ...n\}$ be a finite class of $N_{\delta^* g\alpha}$-closed subsets of a space (X, τ). Then for each $N_{*g\alpha}$-open set U_i in X containing $P_i, Ncl_\delta(P_i) \subseteq U_i, i \in \{1, 2, ...n\}$. Hence $\bigcup_i P_i \subseteq \bigcup_i U_i = V$. Since arbitrary union of $N_{*g\alpha}$-open sets in (X, τ) is also $N_{*g\alpha}$-open set in (X, τ), V is $N_{*g\alpha}$-open in (X, τ). Also $\bigcup_i Ncl_\delta(P_i) = Ncl_\delta(\bigcup_i P_i) \subseteq V$. Therefore $U_i P_i$ is $N_{\delta^* g\alpha}$-closed in (X, τ). □

Theorem 4.2. *Let P be a $N_{\delta^* g\alpha}$-closed set of (X, τ), then Ncl_δ(P)-P does not contain a non-empty $N_{*g\alpha}$-closed set.*

Proof. Suppose that P is $N_{\delta^* g\alpha}$-closed, let F be a $N_{*g\alpha}$-closed set contained in Ncl_δ(P)-P. Now F^c is $N_{*g\alpha}$-open set of (X, τ) such that P$\subseteq$ F^c. Since P is $N_{\delta^* g\alpha}$-closed set of (X, τ), then $Ncl_\delta(P) \subseteq F^c$. Thus $F \subseteq (Ncl_\delta((P))^c$. Also $F \subseteq Ncl_\delta(P) - P$. Therefore $F \subseteq (Ncl_\delta(P)) \subset \bigcap(Ncl_\delta(P)) = \phi$. Hence F=$\phi$. □

Theorem 4.3. *If P is $N_{*g\alpha}$-open and $N_{\delta^* g\alpha}$-closed subset of (X, τ) then P is an N_δ-closed subset of (X, τ).*

Proof. Since P is $N_{g\alpha}$-open and $N_{\delta^* g\alpha}$-closed, $Ncl_\delta(P) \subseteq P$. Hence P is N_δ-closed. □

Theorem 4.4. *The intersection of a $N_{\delta^* g\alpha}$-closed set and a N_δ-closed set is always $N_{\delta^* g\alpha}$-Closed.*

Proof. Let P be $N_{\delta^* g\alpha}$-Closed and let F be N_δ-closed. If U is an $N_{*g\alpha}$-open set with $P \cup F \subseteq U$, then $P \subseteq U \cup F^c$ and so $Ncl_\delta(P) \subseteq U \cup F^c$. Now $Ncl_\delta(P \cup F) \subseteq Ncl_\delta(P) \cup F \subseteq U$. Hence $P \cup F$ is $N_{\delta^* g\alpha}$-closed. □

Theorem 4.5. *If P is a $N_{\delta^*g\alpha}$-closed set in a space (X,τ) and $P \subseteq Q \subseteq Ncl_\delta(P)$,then Q is also a $N_{\delta^*g\alpha}$-closed set.*

Proof. Let U be a $N_{*g\alpha}$-open set of (X,τ) such that $B \subseteq Ncl_\delta(P)$, Then $P \subseteq U$. Since P is $N_{\delta^*g\alpha}$-closed set, $Ncl_\delta(P) \subseteq U$. Also since $B \subseteq Ncl_\delta(P)$, $Ncl_\delta(Q) \subseteq Ncl_\delta(Ncl_\delta(P)) = Ncl_\delta(P) \subseteq U$. Implies $Ncl_\delta(Q)$ ⊆U. Therefore Q is also a $N_{\delta^*g\alpha}$-closed set. □

Theorem 4.6. *Let P be $N_{\delta^*g\alpha}$-closed of (X,τ), then P is N_δ-closed iff $Ncl_\delta(P) - P$ is $N_{*g\alpha}$-closed.*

Proof. Necessity. Let P be a N_δ-closed subset of X. Then $Ncl_\delta(P) = P$ and so $Ncl_\delta(P) - P = \phi$ which is $N_{*g\alpha}$-closed.
Sufficiency. Since P is $N_{\delta^*g\alpha}$-closed, by proposition, $Ncl_\delta(P) - P$ does not contain a non-empty $N_{*g\alpha}$-closed set. But $Ncl_\delta(P) - P = \phi$. That is $Ncl_\delta(P) = P$. Hence P is N_δ-closed. □

We introduce the following definition.

Definition 4.1. *A space (X,τ) is called $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space if every $N_{\delta^*g\alpha}$-closed set is an N_δ-closed*

Theorem 4.7. *For a topological space (X,τ), the following conditions are equivalent.*

(i) (X,τ) *is a $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space.*
(ii) *Every singleton $\{x\}$ is either $N_{*g\alpha}$-closed or N_δ-open.*

Proof.
$(i) \Rightarrow (ii)$ Let $x \in X$. Suppose $\{x\}$ is not a $N_{*g\alpha}$-closed set of (X,τ). Then $X - \{x\}$ is not a $N_{*g\alpha}$-open set. Thus $X - \{x\}$ is an $N_{\delta^*g\alpha}$-closed set of (X,τ). Since (X,τ) is $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$, $X - \{x\}$ is an N_δ-closed set of (X,τ), i.e. $\{x\}$ is N_δ-open set of (X,τ).
$(ii) \Rightarrow (i)$ Let P be an $N_{\delta^*g\alpha}$-closed set of (X,τ). Let $x \in Ncl_\delta(P)$. By (ii), $\{x\}$ is either $N_{*g\alpha}$-closed or N_δ-open.
Case(i). Let $\{x\}$ be $N_{*g\alpha}$-closed. If we assume that $x \notin P$, then we would have $x \in Ncl_\delta(P) - P$, which cannot happen according to proposition Hence $x \in P$.
Case(ii) Let $\{x\}$ be N_δ-open. Since $x \in Ncl_\delta(P)$, then $\{x\} \bigcap P = \phi$. This shows that $x \in P$. So in both cases we have $Ncl_\delta(P) \subseteq P$. Trivially $P \subseteq$

$Ncl_\delta(P)$. Therefore $P = Ncl_\delta(P)$ or equivalently P is N_δ-closed. Hence (X, τ) is a $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space. □

5. Conclusion

This article defined $N_{\delta^* g\alpha}$-closed set in neutrosophic topological Spaces and relation with other exciting neutrosophic sets in neutrosophic topology were studied. Along with that some of there properties were discussed. Also $N_{\alpha\delta T^{**}_{\frac{3}{4}} g\alpha}$-space of the set were introduced and discussed their properties. This set can be used to derive few more functions such as $N_{\delta^* g\alpha}$-continuous and $N_{\delta^* g\alpha}$-irresolute functions. In addition to that it can be extended to homeomorphsims of neutrosophic topological spaces.

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