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$\mathcal{N}_{NC}$ β-OPEN SETS

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ABSTRACT. In this article, we study a new types of β-open sets and β-closed sets in $\mathcal{N}_{nc}ts$ and their properties are evaluated with different forms of near sets.

1. INTRODUCTION

The concepts of neutrosophy and neutrosophic set was first presented by Smarandache [6, 7]. In 2014, the concept of neutrosophic crisp topological space presented by Salama, Smarandache and Kroumov [5]. Al-Omeri [2] also investigated neutrosophic crisp sets in the build of neutrosophic crisp topological Spaces. Lellis Thivagar et al. [8] introduced the concept of N_n-open (closed) sets in N-neutrosophic topological spaces. Al-Hamido [4] explore the possibilities in idea of neutrosophic crisp topological spaces into N_{nc}-topological spaces. In 1983, Abd EL Monsef et al. [1] presented β- open sets in topology. Also, the equivalent notion of semi-pre open sets was independently developed by Andrijevic [3] in 1986.

2. PRELIMINARIES

Throughout this article, the preliminaries are as mentioned in the paper [9] and other undefined symbols and definitions are also from [9].

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3. β-OPEN SETS IN $\mathcal{N}_{nc}ts$

Throughout the sections 3 & 4, let $(Y, \mathcal{N}_{nc}\Gamma)$ be any $\mathcal{N}_{nc}ts$. Let K and M be an $\mathcal{N}_{nc}s$'s in $(Y, \mathcal{N}_{nc}\Gamma)$.

Definition 3.1. *A set K is said to be a $\mathcal{N}_{nc}$-β-open (briefly, $\mathcal{N}_{nc}\beta o$) set if $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. The $\mathcal{N}_{nc}$-β-closed set (briefly, $\mathcal{N}_{nc}\beta c$) set is the complement of an $\mathcal{N}_{nc}\beta o$ set in Y. The family of all $\mathcal{N}_{nc}\beta o$ (resp. $\mathcal{N}_{nc}\beta c$) set of Y is denoted by $\mathcal{N}_{nc}\beta OS(Y)$ (resp. $\mathcal{N}_{nc}\beta CS(Y)$).*

Definition 3.2. *The $N_{nc}\beta$ interior of K (briefly, $N_{nc}\beta int(K)$) and $N_{nc}\beta$ closure of K (briefly, $N_{nc}\beta cl(K)$) are defined as*

(i) $\mathcal{N}_{nc}\beta int(K) = \cup\{A : A \subseteq K \,\&\, A$ *is a* $\mathcal{N}_{nc}\beta o$ *set in* $Y\}$.
(ii) $\mathcal{N}_{nc}\beta cl(K) = \cap\{C : K \subseteq C \,\&\, C$ *is a* $\mathcal{N}_{nc}\beta c$ *set in* $Y\}$.

Proposition 3.1. *The $\mathcal{N}_{nc}\beta$-closure and $\mathcal{N}_{nc}\beta$-interior operator satisfies*

(i) $K \subseteq \mathcal{N}_{nc}\beta cl(K)$.
(ii) $\mathcal{N}_{nc}\beta int(K) \subseteq K$.
(iii) $K \subseteq M \Rightarrow \mathcal{N}_{nc}\beta cl(K) \subseteq \mathcal{N}_{nc}\beta cl(M)$.
(iv) $K \subseteq M \Rightarrow \mathcal{N}_{nc}\beta int(K) \subseteq \mathcal{N}_{nc}\beta int(M)$.
(v) $\mathcal{N}_{nc}\beta cl(K \cup M) = \mathcal{N}_{nc}\beta cl(K) \cup \mathcal{N}_{nc}\beta cl(M)$.
(vi) $\mathcal{N}_{nc}\beta int(K \cap M) = \mathcal{N}_{nc}\beta int(K) \cap \mathcal{N}_{nc}\beta int(M)$.
(vii) $(\mathcal{N}_{nc}\beta cl(K))^c = \mathcal{N}_{nc}\beta int(K^c)$.
(viii) $(\mathcal{N}_{nc}\beta int(K))^c = \mathcal{N}_{nc}\beta cl(K^c)$.
(ix) $\mathcal{N}_{nc}\beta cl(K) = K$ *iff* K *is an* $\mathcal{N}_{nc}\beta c$ *set.*
(x) $\mathcal{N}_{nc}\beta int(K) = K$ *iff* K *is an* $\mathcal{N}_{nc}\beta o$ *set.*
(xi) $\mathcal{N}_{nc}\beta cl(K)$ *is the smallest* $\mathcal{N}_{nc}\beta c$ *set containing* K.
(xii) $\mathcal{N}_{nc}\beta int(K)$ *is the largest* $\mathcal{N}_{nc}\beta o$ *set containing* K.

Proposition 3.2. *The union (resp. intersection) of any family of $\mathcal{N}_{nc}\beta OS(Y)$ (resp. $\mathcal{N}_{nc}\beta CS(Y)$) is a $\mathcal{N}_{nc}\beta OS(Y)$ (resp. $\mathcal{N}_{nc}\beta CS(Y)$).*

Remark 3.1. *The intersection of two $\mathcal{N}_{nc}\beta os$'s need not be $\mathcal{N}_{nc}\beta os$.*

Example 1. *Let $Y = \{l_1, m_1, n_1, o_1\}$, ${}_{nc}\Gamma_1 = \{\phi_N, X_N, L, M, N\}$, ${}_{nc}\Gamma_2 = \{\phi_N, X_N\}$. $L = \langle\{l_1\}, \{\phi\}, \{m_1, n_1, o_1\}\rangle$, $M = \langle\{m_1, o_1\}, \{\phi\}, \{l_1, n_1\}\rangle$, $N = \langle\{l_1, m_1, o_1\}, \{\phi\}, \{n_1\}\rangle$, then we have $2_{nc}\Gamma = \{\phi_N, X_N, L, M, N\}$. The sets $\langle\{m_1, n_1\}, \{\phi\}, \{l_1, o_1\}\rangle$ & $\langle\{n_1, o_1\}, \{\phi\}, \{l_1, m_1\}\rangle$ are $\mathcal{N}_{nc}\beta os$ but the intersection $\langle\{n_1\}, \{\phi\}, \{l_1, m_1, o_1\}\rangle$ is not $\mathcal{N}_{nc}\beta os$.*

Proposition 3.3. *The statements are hold but the equality does not true.*

(i) *Every $\mathcal{N}_{nc}ros$ (resp. $\mathcal{N}_{nc}rcs$) is a $\mathcal{N}_{nc}\beta os$ (resp. $\mathcal{N}_{nc}\beta cs$).*
(ii) *Every $\mathcal{N}_{nc}os$ (resp. $\mathcal{N}_{nc}cs$) is a $\mathcal{N}_{nc}\beta os$ (resp. $\mathcal{N}_{nc}\beta cs$).*
(iii) *Every $\mathcal{N}_{nc}\alpha os$ (resp. $\mathcal{N}_{nc}\alpha cs$) is a $\mathcal{N}_{nc}\beta os$ (resp. $\mathcal{N}_{nc}\beta cs$).*
(iv) *Every $\mathcal{N}_{nc}\mathcal{S}os$ (resp. $\mathcal{N}_{nc}\mathcal{S}cs$) is a $\mathcal{N}_{nc}\beta os$ (resp. $\mathcal{N}_{nc}\beta cs$).*
(v) *Every $\mathcal{N}_{nc}\mathcal{P}os$ (resp. $\mathcal{N}_{nc}\mathcal{P}cs$) is a $\mathcal{N}_{nc}\beta os$ (resp. $\mathcal{N}_{nc}\beta cs$).*
(vi) *Every $\mathcal{N}_{nc}\gamma os$ (resp. $\mathcal{N}_{nc}\gamma cs$) is a $\mathcal{N}_{nc}\beta os$ (resp. $\mathcal{N}_{nc}\beta cs$).*

Proof. (i) K is a $\mathcal{N}_{nc}ros$, then $K = \mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. K is a $\mathcal{N}_{nc}\beta os$.

(ii) K is a $\mathcal{N}_{nc}os$, then $K = \mathcal{N}_{nc}int(K)$ and so $K \subseteq \mathcal{N}_{nc}cl(K)$. Then $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}\ cl(K)))$. K is a $\mathcal{N}_{nc}\beta os$.

(iii) Since $\mathcal{N}_{nc}int(K) \subseteq K$, $\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)) \subseteq \mathcal{N}_{nc}cl(K)$. Then $\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))) \subseteq \mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. K is a $\mathcal{N}_{nc}\beta os$.

(iv) Suppose that K is a $\mathcal{N}_{nc}\mathcal{S}os$, then $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. K is a $\mathcal{N}_{nc}\beta os$.

(v) Suppose that K is a $\mathcal{N}_{nc}\mathcal{P}os$, then $K \subseteq \mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. K is a $\mathcal{N}_{nc}\beta os$.

(vi) Suppose that K is a $\mathcal{N}_{nc}\gamma os$ then by Proposition 3.2, (iv) & (v), $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. K is a $\mathcal{N}_{nc}\beta os$.

It is also true for their respective closed sets. □

Example 2. *In Example 1, the set $\langle\{m_1, n_1\}, \{\phi\}, \{l_1, o_1\}\rangle$ is a $\mathcal{N}_{nc}\beta os$ but not a $\mathcal{N}_{nc}ros$, $\mathcal{N}_{nc}os$, $\mathcal{N}_{nc}\alpha os$, $\mathcal{N}_{nc}\mathcal{P}os$, $\mathcal{N}_{nc}\mathcal{S}os$, $\mathcal{N}_{nc}\gamma os$.*

Remark 3.2. *The diagram shows $\mathcal{N}_{nc}\beta os$'s in $\mathcal{N}_{nc}ts$.*

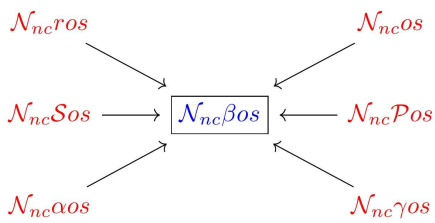

FIGURE 1

4. Properties of $\mathcal{N}_{nc}\beta os$

Proposition 4.1. *If K is a $\mathcal{N}_{nc}os$ and M is a $\mathcal{N}_{nc}\beta os$, then $K \cap M$ is a $\mathcal{N}_{nc}\beta os$.*

Proof. $K \cap M \subseteq K \cap \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))) \subseteq \mathcal{N}_{nc}cl(K \cap \mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K \cap M)))$. Therefore, $K \cap M$ is a $\mathcal{N}_{nc}\beta os$. □

Proposition 4.2. *M is a $\mathcal{N}_{nc}$ subset of Y and K is a $\mathcal{N}_{nc}\mathcal{P}os$ on Y such that $K \subseteq M \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))$. Then M is a $\mathcal{N}_{nc}\beta os$.*

Proof. Since K is a $\mathcal{N}_{nc}\mathcal{P}os$, $K \subseteq \mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))$. Now $M \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))) = \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(M)))$. Hence $M \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(M)))$. Therefore, M is a $\mathcal{N}_{nc}\beta os$. □

Proposition 4.3. *If each K is a $\mathcal{N}_{nc}\beta os$ which is a $\mathcal{N}_{nc}\mathcal{S}cs$ is also a $\mathcal{N}_{nc}\mathcal{S}os$.*

Proof. Let K be a $\mathcal{N}_{nc}\beta os$ and $\mathcal{N}_{nc}\mathcal{S}cs$. Then, $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$ and $\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)) \subseteq K$. Therefore, $\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)) \subseteq \mathcal{N}_{nc}(K)$ and so, $\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))$. Hence, $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))) \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))$. Therefore, K is a $\mathcal{N}_{nc}\mathcal{S}os$. □

Proposition 4.4. *If K is a $\mathcal{N}_{nc}\beta cs$ and $\mathcal{N}_{nc}\mathcal{S}os$, then it is a $\mathcal{N}_{nc}\mathcal{S}cs$.*

Proof. Since K is a $\mathcal{N}_{nc}\beta cs$ and $\mathcal{N}_{nc}\mathcal{S}os$. Then, $Y\backslash K$ is $\mathcal{N}_{nc}\beta os$ and $\mathcal{N}_{nc}\mathcal{S}cs$ and so by Proposition 4.3, $Y\backslash K$ is a $\mathcal{N}_{nc}\mathcal{S}os$. Therefore, K is a $\mathcal{N}_{nc}\mathcal{S}cs$. □

Proposition 4.5. *If K is a $\mathcal{N}_{nc}\beta cs$ iff $\mathcal{N}_{nc}cl(Y\backslash\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))) \backslash(Y\backslash\mathcal{N}_{nc}cl(K)) \supset \mathcal{N}_{nc}cl(K)\backslash K$.*

Proof. $\mathcal{N}_{nc}cl(Y\backslash\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)))\backslash(Y\backslash\mathcal{N}_{nc}cl(K)) \supset \mathcal{N}_{nc}cl(K)\backslash K$ iff $(Y\backslash\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))))\backslash(Y\backslash\mathcal{N}_{nc}cl(K)) \supset \mathcal{N}_{nc}cl(K)\backslash K$ iff $(Y\backslash\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K))))\cap\mathcal{N}_{nc}cl(K) \supset \mathcal{N}_{nc}cl(K)\backslash K$ iff $(Y\cap\mathcal{N}_{nc}cl(K)\backslash(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)))\cap \mathcal{N}_{nc}cl(K)) \supset \mathcal{N}_{nc}cl(K)\backslash K$ iff $\mathcal{N}_{nc}cl(K)\backslash(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)))) \supset \mathcal{N}_{nc}cl(K)\backslash K$ iff $K \supset \mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(K)))$ iff K is a $\mathcal{N}_{nc}\beta cs$. □

Proposition 4.6. *If each K is a $\mathcal{N}_{nc}\beta os$ which is a $\mathcal{N}_{nc}\alpha cs$ is also a $\mathcal{N}_{nc}cs$.*

Proof. Let K be a $\mathcal{N}_{nc}\beta os$ and $\mathcal{N}_{nc}\alpha cs$. Then, $K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$ and $\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))) \subseteq K$. Therefore, $\mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K))) \subseteq K \subseteq \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. So, $K = \mathcal{N}_{nc}cl(\mathcal{N}_{nc}int(\mathcal{N}_{nc}cl(K)))$. Therefore, K is a $\mathcal{N}_{nc}cs$. □

Corollary 4.1. *If each K is a $\mathcal{N}_{nc}\beta cs$ which is $\mathcal{N}_{nc}\alpha os$ is also a $\mathcal{N}_{nc}os$.*

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