International Research Journal of Education and Technology
ISSN 2581-7795

NEUTROSOPHIC GRILL TOPOLOGICAL SPACES

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Abstract. In the present paper, we define a kind of neutrosophic topology obtained as an associated structure on a neutrosophic topological space K induced by a grill on K. Such a neutrosophic topology is studied in certain detail as to some of its basic properties. Also we introduce new definition of neutrosophic grill topological space like neutrosophic grill α-open, neutrosophic grill pre-open, neutrosophic grill semi-open, neutrosophic grill b-open, neutrosophic grill β-open, neutrosophic grill regular-open and neutrosophic grill π-open.

Keywords : neutrosophic set, grill topology, neutrosophic grill topology and $\Phi_{\mathcal{N}}$-open.

1. Introduction

Topology is a classical subject, as a generalization topological spaces many type of topological spaces introduced over the year. The idea of grill on a topological space was first introduced by Choquet [5] in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many a topological concept quite effectively. A number of theories and features has

[0]IRJEdT Volume: 2, Issue: 2, June (2021), Pages: 01 - 16

been handled in [1, 7, 10]. It helps to expand the topological structure which is used to measure the description rather than quantity, such as love, intelligence, beauty, quality of education and etc. In [11], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. C. L. Chang [4] was introduced and developed fuzzy topological space by using L. A. Zadehs [15] fuzzy sets. Coker [6] introduced the concepts of intuitionistic fuzzy topological spaces by using K. T. Atanassovs [3] intuitionistic fuzzy set theory in many fields such as differential equations, computer science and so on. F. Smarandache [13, 14] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. The rest of this article is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, we formulate a condition which when imposed on a grill $\mathcal{G}$, makes the induced neutrosophy topology well behaved and more applicable. Also we introduce new definition of neutrosophic grill topological space like $\Phi_{\mathcal{N}}$-open, neutrosophic grill α-open, neutrosophic grill pre-open, neutrosophic grill semi-open, neutrosophic grill b-open, neutrosophic grill β-open, neutrosophic grill regular-open, neutrosophic grill π-open and discuss some of their properties.

2. Preliminaries

Definition 2.1. [5] *A collection $\mathcal{G}$ of a nonempty subsets of a topological space X is called a grill on X if (i) $A \in \mathcal{G}$ and $A \subseteq B \subseteq X \Rightarrow B \in \mathcal{G}$, (ii) A, $B \subseteq X$ and $A \cup B \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$. For any point x of a topological space (X, τ), we shall let $\tau(x)$ denote the collection of all open neighbourhoods of x.*

Definition 2.2. [11] *$\mathcal{G}$ be a grill on a topological space (X, τ). An operator $\Phi : \wp(X) \rightarrow \wp(X)$ (where, $\wp(X)$ stands for the power set of X) was defined by $\Phi(A) = \{x \in X : U \cap A \in \mathcal{G}, \forall\ U \in \tau(x)\}$. It was also shown in the same paper that the map $\Psi : \wp(X) \rightarrow \wp(X)$, given by $\Psi(A) = A \cup \Phi(A)$ (for $A \in \wp(X)$) is a Kuratowski closure operator determining $\tau_{\mathcal{G}}$ (say) on X, strictly finer than τ. Thus a subset A of X is $\tau_{\mathcal{G}}$-closed (resp. $\tau_{\mathcal{G}}$-dense in itself) if $\Psi(A) = A$ or equivalently if $\Phi(A) \subseteq A$ (resp. $A \subseteq \Phi(A)$).*

Definition 2.3. [13, 14] *A neutrosophic set (in short ns) K on a set $X \neq \emptyset$ is defined by $K = \{\prec a,\ P_K(a),\ Q_K(a),\ R_K(a) \succ\ : a \in X\}$ where $P_K : X \rightarrow [0,1]$, $Q_K : X \rightarrow [0,1]$ and $R_K : X \rightarrow [0,1]$ denotes the membership of an object, indeterminacy and non-membership of an object, for each a X to K, respectively and $0 \subseteq P_K(a) + Q_K(a) + R_K(a) \subseteq 3$ for each $a \in X$.*

Definition 2.4. [12] *Let $K = \{\prec a,\ P_K(a),\ Q_K(a),\ R_K(a) \succ\ : a \in X\}$ be a ns. We must introduce the ns 0_N and 1_N in X as follows:*

0_N may be defined as:

(1) $0_N = \{\prec x,\ 0,\ 0,\ 1 \succ\ : x \in X\}$
(2) $0_N = \{\prec x,\ 0,\ 1,\ 1 \succ\ : x \in X\}$
(3) $0_N = \{\prec x,\ 0,\ 1,\ 0 \succ\ : x \in X\}$
(4) $0_N = \{\prec x,\ 0,\ 0,\ 0 \succ\ : x \in X\}$

1_N may be defined as:

(1) $1_N = \{\prec x,\ 1,\ 0,\ 0 \succ\ : x \in X\}$
(2) $1_N = \{\prec x,\ 1,\ 0,\ 1 \succ\ : x \in X\}$

(3) $1_N = \{\prec x,\ 1,\ 1,\ 0 \succ\ :\ x \in X\}$

(4) $1_N = \{\prec x,\ 1,\ 1,\ 1 \succ\ :\ x \in X\}$

Proposition 2.5. [12] *For any ns S, then the following conditions are holds:*

(1) $0_N \subseteq S$, $0_N \subseteq 0_N$.

(2) $S \subseteq 1_N$, $1_N \subseteq 1_N$.

Definition 2.6. [12] *Let* $K = \{\prec a,\ P_K(a),\ Q_K(a),\ R_K(a) \succ\ :\ a \in X\}$ *be a ns.*

(1) *A ns K is an empty set i.e.,* $K = 0_N$ *if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e.,* $0_N = \{x,\ (0,\ 0,\ 1)\ :\ x \in X\}$

(2) *A ns K is a universal set i.e.,* $K = 1_N$ *if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively.* $1_N = \{x,\ (1,\ 1,\ 0)\ :\ x \in X\}$

(3) $K_1 \cup K_2 = \{a,\ max\,\{P_{K_1}(a),\ P_{K_2}(a)\},\ max\,\{Q_{K_1}(a),\ Q_{K_2}(a)\},\ min\,\{R_{K_1}(a),\ R_{K_2}(a)\}\ :\ a \in X\}$

(4) $K_1 \cap K_2 = \{a,\ min\,\{P_{K_1}(a),\ P_{K_2}(a)\},\ min\,\{Q_{K_1}(a),\ Q_{K_2}(a)\},\ max\,\{R_{K_1}(a),\ R_{K_2}(a)\}\ :\ a \in X\}$

(5) $K^C = \{\prec a,\ R_K(a),\ 1 -\ Q_K(a),\ P_K(a) \succ\ :\ a \in X\}$

Definition 2.7. [12] *A neutrosophic topology (nt) in Salamas sense on a nonempty set X is a family* $\tau_{\mathcal{N}}$ *of ns in X satisfying three axioms:*

(1) *Empty set* (0_N) *and universal set* (1_N) *are members of* $\tau_{\mathcal{N}}$.

(2) $K_1 \cap K_2 \in \tau_{\mathcal{N}}$ *where* $K_1,\ K_2 \in \tau_{\mathcal{N}}$.

(3) $\cup K_\delta \in \tau_{\mathcal{N}}$ *for every* $\{K_\delta\ :\ \delta \in \Delta\} \subseteq \tau_{\mathcal{N}}$.

Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.8. [12] *Let* $(X, \tau_{\mathcal{N}})$ *be neutrosophic topological spaces and* $K = \{\prec x, P_K(x), Q_K(x), R_K(x) \succ : x \in X\}$ *be a ns in X. Then*

(1) *neutrosophic closure of* $A = \cap \{F : F$ *is a neutrosophic closed set and* $A \subseteq F\}$ *and it is denoted by N-cl(A).*

(2) *neutrosophic interior of* $A = \cup \{S : S$ *is a neutrosophic open set and* $S \subseteq A\}$ *and it is denoted by N-int(A).*

Definition 2.9. *Let* $(X, \tau_{\mathcal{N}})$ *be neutrosophic topological spaces. Then,*

(1) *A is called neutrosophic* α*-open if* $A \subseteq$ *N-int(N-cl(N-int(A))).* [2]

(2) *A is called neutrosophic semi open set if* $A \subseteq$ *N-cl(N-int(A))* [2, 8]

(3) *A is called neutrosophic pre-open if* $A \subseteq$ *N-int(N-cl(A)).* [2]

(4) *A is called neutrosophic b-open if* $A \subseteq$ *N-int(N-cl(A))* $\vee$ *N-cl(N-int(A)).* [9]

(5) *A is called neutrosophic* β*-open if* $A \subseteq$ *N-cl(N-int(N-cl(A))).* [2]

(6) *A is called neutrosophic regular-open if* $A =$ *N-int(N-cl(A)).* [2]

The complement of above mentioned neutrosophic open sets are called their respective neutrosophic closed sets.

3. Neutrosophic Grill Topological spaces

The idea of grill on a topological space was first introduced by Choquet [5] in 1947. In [11], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space.

In this section we shall introduce a similar type with the operator associated in neutrosophic grill topological spaces. Before starting the discussion we shall consider the following concepts.

Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and $\mathcal{G}$ be a grill on K is called a neutrosophic grill topological space.

Let a space K we shall mean a neutrosophic topological spaces. Also, the power set of K will be written as $\wp$(K), we shall let $\tau_{\mathcal{N}(k_1,k_2,k_3)}$ to stand for the collection of all neutrosophic open neighbourhoods of (k_1, k_2, k_3). If we take, (k_1, k_2, k_3) = k.

Definition 3.1. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and $\mathcal{G}$ be a grill on K. We define a mapping $\Phi_{\mathcal{N}} : \wp(K) \to \wp(K)$, denoted by $\Phi_{\mathcal{N}\mathcal{G}}(A, \tau_{\mathcal{N}})$ (for $A \in \wp(K)$) or $\Phi_{\mathcal{N}\mathcal{G}}(A)$ or simply by $\Phi_{\mathcal{N}}(A)$(when it is known which neutrosophic topology and grill on K we are talking about), called the operator associated with the grill $\mathcal{G}$ and the neutrosophic topology $\tau_{\mathcal{N}}$, and is defined by $\Phi_{\mathcal{N}}(A) = \Phi_{\mathcal{N}\mathcal{G}}(A, \tau_{\mathcal{N}}) = \{k \in K : U \cap A \in \mathcal{G}, \forall\ U \in \tau_{\mathcal{N}(k_1,k_2,k_3)}\}$.*

Note 3.2. *(K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space with a grill $\mathcal{G}$ on K and for every A , B be subsets of K. Then*

(1) *$A \subseteq B\ (\subseteq K) \Rightarrow \Phi_{\mathcal{N}}(A) \subseteq \Phi_{\mathcal{N}}(B)$,*
(2) *$\mathcal{G}_1 \subseteq \mathcal{G}_2 \Rightarrow \Phi_{\mathcal{N}\mathcal{G}1}(A) \subseteq \Phi_{\mathcal{N}\mathcal{G}2}(A)$ (if $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on K),*
(3) *If $A \notin \mathcal{G}$ then $\Phi_{\mathcal{N}}(A) = 0_N$.*

Proposition 3.3. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and a grill $\mathcal{G}$ on K. Then for all A, B $\subseteq$ K.*

(1) *$\Phi_{\mathcal{N}}(A \cup B) = \Phi_{\mathcal{N}}(A) \cup \Phi_{\mathcal{N}}(B)$,*
(2) *$\Phi_{\mathcal{N}}(\Phi_{\mathcal{N}}(A)) \subseteq \Phi_{\mathcal{N}}(A) = \mathcal{N}\text{-}cl(\Phi_{\mathcal{N}}(A)) \subseteq \mathcal{N}\text{-}cl(A)$.*

Proof. (1) In view of Note 3.2 (1) it suffices to show that $\Phi_{\mathcal{N}}$(A ∪ B) ⊆ $\Phi_{\mathcal{N}}$(A) ∪ $\Phi_{\mathcal{N}}$(B). Suppose k ∉ $\Phi_{\mathcal{N}}$(A) ∪ $\Phi_{\mathcal{N}}$(B). Then there are U_1, $U_2 \in \tau_{\mathcal{N}(k)}$ such that A ∩ U_1, B ∩ $U_2 \notin \mathcal{G}$ and hence (A ∩ U_1) ∪ (B ∩ U_2) $\notin \mathcal{G}$. Now $U_1 \cap U_2 \in \tau_{\mathcal{N}(k)}$ and (A ∪ B) ∩ (U_1 ∩ U_2) ⊆ (A ∩ U_1) ∪ (B ∩ U_2) $\notin \mathcal{G}$, proving that k ∉ $\Phi_{\mathcal{N}}$(A ∪ B).
(2) k ∉ $\mathcal{N}$-cl(A) ⇒ ∃ U ∈ $\tau_{\mathcal{N}(k)}$ such that U ∩ A = $0_N \notin \mathcal{G}$ ⇒ k ∈ $\Phi_{\mathcal{N}}$(A). Thus

$\Phi_{\mathcal{N}}$(A) $\subseteq$ $\mathcal{N}$-cl(A). Now we shall show that $\mathcal{N}$-cl($\Phi_{\mathcal{N}}$(A)) $\subseteq$ $\Phi_{\mathcal{N}}$(A). Indeed, k $\in$ $\mathcal{N}$-cl($\Phi_{\mathcal{N}}$(A)) and U $\in \tau_{\mathcal{N}(k)} \Rightarrow$ U $\cap$ $\Phi_{\mathcal{N}}$(A) $\neq 0_N$. Let m $\in$ U $\cap$ $\Phi_{\mathcal{N}}$(A), i.e., m $\in$ U and m $\in$ $\Phi_{\mathcal{N}}$(A). Then U $\cap$ A $\in \mathcal{G}$ and so k $\in$ $\Phi_{\mathcal{N}}$(A). Thus $\mathcal{N}$-cl($\Phi_{\mathcal{N}}$(A)) = $\Phi_{\mathcal{N}}$(A). Now, $\Phi_{\mathcal{N}}$($\Phi_{\mathcal{N}}$(A)) $\subseteq$ $\mathcal{N}$-cl($\Phi_{\mathcal{N}}$(A)) = $\Phi_{\mathcal{N}}$(A) $\subseteq$ $\mathcal{N}$-cl(A). □

Theorem 3.4. *Let $\mathcal{G}$ be a grill on a neutrosophic topological spaces (K, $\tau_{\mathcal{N}}$). If U $\in$ $\tau_{\mathcal{N}}$ then U $\cap$ $\Phi_{\mathcal{N}}$(A) = U $\cap$ $\Phi_{\mathcal{N}}$(U $\cap$ A), for any A $\subseteq$ K.*

Proof. By Note 3.2 (1), we have U $\cap$ $\Phi_{\mathcal{N}}$(A) $\supseteq$ U $\cap$ $\Phi_{\mathcal{N}}$(U $\cap$ A). Let k $\in$ U $\cap$ $\Phi_{\mathcal{N}}$(A) and V $\in \tau_{\mathcal{N}(k)}$. Then U $\cap$ V $\in \tau_{\mathcal{N}(k)}$ and k $\in$ $\Phi_{\mathcal{N}}$(A) $\Rightarrow$ (U $\cap$ V) $\cap$ A $\in \mathcal{G}$, i.e., (U $\cap$ A) $\cap$ V $\in \mathcal{G} \Rightarrow$ k $\in$ $\Phi_{\mathcal{N}}$(U $\cap$ A) $\Rightarrow$ k $\in$ U $\cap$ $\Phi_{\mathcal{N}}$(U $\cap$ A). Thus U $\cap$ $\Phi_{\mathcal{N}}$(A) = U $\cap$ $\Phi_{\mathcal{N}}$(U $\cap$ A). □

Theorem 3.5. *If $\mathcal{G}$ is a grill on a space (K, $\tau_{\mathcal{N}}$) with $\tau_{\mathcal{N}} \setminus \{0_N\} \subseteq \mathcal{G}$, then for all U $\in \tau_{\mathcal{N}}$, U $\subseteq$ $\Phi_{\mathcal{N}}$(U).*

Proof. In case U = 0_N, we obviously have $\Phi_{\mathcal{N}}$(U) = 0_N = U. Now note that if $\tau_{\mathcal{N}} \setminus \{0_N\} \subseteq \mathcal{G}$, then $\Phi_{\mathcal{N}}$(K) = K. In fact, k $\notin$ $\Phi_{\mathcal{N}}$(K) $\Rightarrow \exists$ V $\in \tau_{\mathcal{N}(k)}$ such that V $\cap$ K $\notin \mathcal{G} \Rightarrow$ V $\notin \mathcal{G}$, a contradiction. Now, by using Theorem 3.4, we have for any U $\in \tau_{\mathcal{N}} \setminus \{0_N\}$, U $\cap$ $\Phi_{\mathcal{N}}$(K) = U $\cap$ $\Phi_{\mathcal{N}}$(U $\cap$ K), and hence by above, U = U $\cap$ K = U $\cap$ $\Phi_{\mathcal{N}}$(U). Thus $\Phi_{\mathcal{N}}$(U) $\supseteq$ U. □

Theorem 3.6. *For any grill $\mathcal{G}$ on a space (K, $\tau_{\mathcal{N}}$) and any A, B $\subseteq$ K, $\Phi_{\mathcal{N}}$(A) $\setminus$ $\Phi_{\mathcal{N}}$(B) = $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\setminus$ $\Phi_{\mathcal{N}}$(B)*

Proof. We have by using Proposition 3.3 (1) and Note 3.2 (1) that $\Phi_{\mathcal{N}}$(A) = $\Phi_{\mathcal{N}}$((A $\setminus$ B) $\cup$ (A $\cap$ B)) = $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\cup$ $\Phi_{\mathcal{N}}$(A $\cap$ B) $\subseteq$ $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\cup$ $\Phi_{\mathcal{N}}$(B). Thus $\Phi_{\mathcal{N}}$(A) $\setminus$ $\Phi_{\mathcal{N}}$(B) $\subseteq$ $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\setminus$ $\Phi_{\mathcal{N}}$(B). Again $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\subseteq$ $\Phi_{\mathcal{N}}$(A) (by Note 3.2 (1)) $\Rightarrow$ $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\setminus$ $\Phi_{\mathcal{N}}$(B) $\subseteq$ $\Phi_{\mathcal{N}}$(A) $\setminus$ $\Phi_{\mathcal{N}}$(B). Hence $\Phi_{\mathcal{N}}$(A) $\setminus$ $\Phi_{\mathcal{N}}$(B) = $\Phi_{\mathcal{N}}$(A $\setminus$ B) $\setminus$ $\Phi_{\mathcal{N}}$(B). □

Corollary 3.7. *Let $\mathcal{G}$ be a grill on a space (K, τ_N) and suppose A, $B \subseteq K$, with $B \notin \mathcal{G}$. Then $\Phi_{\mathcal{N}}(A \cup B) = \Phi_{\mathcal{N}}(A) = \Phi_{\mathcal{N}}(A \setminus B)$.*

Proof. $\Phi_{\mathcal{N}}(\mathrm{A} \cup \mathrm{B}) = \Phi_{\mathcal{N}}(\mathrm{A}) \cup \Phi_{\mathcal{N}}(\mathrm{B})$ (by Proposition 3.3 (1) $= \Phi_{\mathcal{N}}(\mathrm{A})$ (using Note 3.2 (3)). Now by Note 3.2 (1), $\Phi_{\mathcal{N}}(\mathrm{A} \setminus \mathrm{B}) \subseteq \Phi_{\mathcal{N}}(\mathrm{A})$. Also by the above Theorem, $\Phi_{\mathcal{N}}(\mathrm{A}) \setminus \Phi_{\mathcal{N}}(\mathrm{B}) \subseteq \Phi_{\mathcal{N}}(\mathrm{A} \setminus \mathrm{B})$, so that $\Phi_{\mathcal{N}}(\mathrm{A}) \subseteq \Phi_{\mathcal{N}}(\mathrm{A} \setminus \mathrm{B})$. Thus $\Phi_{\mathcal{N}}(\mathrm{A}) = \Phi_{\mathcal{N}}(\mathrm{A} \setminus \mathrm{B})$. □

Theorem 3.8. *Let (K, τ_N) be an neutrosophic topological space with a grill $\mathcal{G}$ and $A \subseteq \Phi_{\mathcal{N}}(A)$, then $\Phi_{\mathcal{N}}(A) = \mathcal{N}\text{-}cl(\Phi_{\mathcal{N}}(A)) = \mathcal{N}\text{-}cl(A)$.*

Proof. For every subset A of K, we have $\Phi_{\mathcal{N}}(\mathrm{A}) = \mathcal{N}\text{-cl}(\Phi_{\mathcal{N}}(\mathrm{A})) = \mathcal{N}\text{-cl}(\mathrm{A})$, by Theorem 3.3 (2). $\mathrm{A} \subseteq \Phi_{\mathcal{N}}(\mathrm{A})$ implies that $\mathcal{N}\text{-cl}(\mathrm{A}) \subseteq \mathcal{N}\text{-cl}(\Phi_{\mathcal{N}}(\mathrm{A}))$ and so $\Phi_{\mathcal{N}}(\mathrm{A}) = \mathcal{N}\text{-cl}(\Phi_{\mathcal{N}}(\mathrm{A})) = \mathcal{N}\text{-cl}(\mathrm{A})$. □

Definition 3.9. *Let $\mathcal{G}$ be a grill on a space (K, τ_N). We define a map $\Psi_{\mathcal{N}} : \wp(K) \to \wp(K)$ by $\Psi_{\mathcal{N}}(A) = A \cup \Phi_{\mathcal{N}}(A)$, for all $A \in \wp(K)$.*

Theorem 3.10. *The above map $\Psi_{\mathcal{N}}$ satisfies the following conditions:*

(1) *$A \subseteq \Psi_{\mathcal{N}}(A)$, $\forall$ $A \subseteq K$,*
(2) *$\Psi_{\mathcal{N}}(0_N) = 0_N$ and $\Psi_{\mathcal{N}}(1_N) = 1_N$,*
(3) *If $A \subseteq B$ ($\subseteq K$), then $\Psi_{\mathcal{N}}(A) \subseteq \Psi_{\mathcal{N}}(B)$,*
(4) *$\Psi_{\mathcal{N}}(A \cup B) = \Psi_{\mathcal{N}}(A) \cup \Psi_{\mathcal{N}}(B)$,*
(5) *$\Psi_{\mathcal{N}}(\Psi_{\mathcal{N}}(A)) = \Psi_{\mathcal{N}}(A)$.*

Proof. (1), (2) and (3) are obvious.

(4) $\Psi_{\mathcal{N}}(\mathrm{A} \cup \mathrm{B}) = \mathrm{A} \cup \mathrm{B} \cup \Phi_{\mathcal{N}}(\mathrm{A} \cup \mathrm{B}) = \mathrm{A} \cup \mathrm{B} \cup (\Phi_{\mathcal{N}}(\mathrm{A}) \cup \Phi_{\mathcal{N}}(\mathrm{B})$ (by Proposition 3.3 (1) $= \Psi_{\mathcal{N}}(\mathrm{A}) \cup \Psi_{\mathcal{N}}(\mathrm{B})$.

(5) $\Psi_{\mathcal{N}}(\Psi_{\mathcal{N}}(\mathrm{A})) = \Psi_{\mathcal{N}}(\mathrm{A} \cup \Phi_{\mathcal{N}}(\mathrm{A})) = \mathrm{A} \cup \Phi_{\mathcal{N}}(\mathrm{A}) \cup \Phi_{\mathcal{N}}(\mathrm{A} \cup \Phi_{\mathcal{N}}(\mathrm{A})) = \mathrm{A} \cup \Phi_{\mathcal{N}}(\mathrm{A}) \cup \Phi_{\mathcal{N}}(\Phi_{\mathcal{N}}(\mathrm{A}))$ (by Proposition 3.3 (1)) $= \mathrm{A} \cup \Phi_{\mathcal{N}}(\mathrm{A}) \cup \Phi_{\mathcal{N}}(\mathrm{A}) = \mathrm{A} \cup \Phi_{\mathcal{N}}(\mathrm{A})$ (by Proposition 3.3 (2)) $= \Psi_{\mathcal{N}}(\mathrm{A})$. □

Definition 3.11. *Corresponding to a grill $\mathcal{G}$ on a neutrosophic topological spaces (K, $\tau_{\mathcal{N}}$), there exists a unique topology $\tau_{\mathcal{N}\mathcal{G}}$ on K given by $\tau_{\mathcal{N}\mathcal{G}} = \{\ U \subset K\ ;\ \Psi_{\mathcal{N}}\ (K \setminus U) = K \setminus U\}$, where for any $A \subseteq K$, $\Psi_{\mathcal{N}}(A) = A \cup \Phi_{\mathcal{N}}(A) = \tau_{\mathcal{N}\mathcal{G}}$-cl(A).*

Definition 3.12. *A basis $\beta(\mathcal{G}, \tau_{\mathcal{N}})$ for $\tau_{\mathcal{N}\mathcal{G}}$ can be described as follows: $\beta(\mathcal{G}, \tau_{\mathcal{N}}) = \{V \setminus F : V \in \tau_{\mathcal{N}},\ F \notin \mathcal{G}\}$.*

Theorem 3.13. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and $\mathcal{G}$ be a grill on K. Then $\beta(\mathcal{G}, \tau_{\mathcal{N}})$ is a basis for $\tau_{\mathcal{N}\mathcal{G}}$.*

Proof. Straight forward. □

Corollary 3.14. *For any grill $\mathcal{G}$ on a neutrosophic topological space (K, $\tau_{\mathcal{N}}$), $\tau_{\mathcal{N}} \subseteq \beta(\mathcal{G}, \tau_{\mathcal{N}}) \subseteq \tau_{\mathcal{N}\mathcal{G}}$.*

Example 3.15. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space. If $\mathcal{G} = \wp(K) \setminus \{0_N\}$, then $\tau_{\mathcal{N}\mathcal{G}} = \tau_{\mathcal{N}}$. In fact, for any $\tau_{\mathcal{N}\mathcal{G}}$ -basic neutrosophic open set $V = U \setminus A$ with $U \in \tau_{\mathcal{N}}$ and $A \notin \mathcal{G}$, we have $A = 0_N$, so that $V = U \in \tau_{\mathcal{N}}$. Hence in view of Corollary 3.14, we have in this case $\tau_{\mathcal{N}} = \beta(\mathcal{G}, \tau_{\mathcal{N}}) = \tau_{\mathcal{N}\mathcal{G}}$.*

Theorem 3.16. *(1) If $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on a space K with $\tau_{\mathcal{N}\mathcal{G}1} \subseteq \tau_{\mathcal{N}\mathcal{G}2}$, then $\tau_{\mathcal{N}\mathcal{G}2} \subseteq \tau_{\mathcal{N}\mathcal{G}1}$,*
(2) If $\mathcal{G}$ is a grill on a space K and $B \notin \mathcal{G}$, then B is $\tau_{\mathcal{N}\mathcal{G}}$-closed in (K, $\tau_{\mathcal{N}\mathcal{G}}$,
(3) For any subset A of a space K and any grill $\mathcal{G}$ on K, $\Phi_{\mathcal{N}}(A)$ is $\tau_{\mathcal{N}\mathcal{G}}$-closed.

Proof. (1) $\mathrm{U} \in \tau_{\mathcal{N}\mathcal{G}2} \Rightarrow \tau_{\mathcal{N}\mathcal{G}2}$ -cl$(\mathrm{K} \setminus \mathrm{U}) = \Psi_{\mathcal{N}}(\mathrm{K} \setminus \mathrm{U}) \Rightarrow \mathrm{K} \setminus \mathrm{U} = (\mathrm{K} \setminus \mathrm{U}) \cup \Phi_{\mathcal{N}\mathcal{G}2}(\mathrm{K} \setminus \mathrm{U}) \Rightarrow \Phi_{\mathcal{N}\mathcal{G}2}(\mathrm{K} \setminus \mathrm{U}) \subseteq \mathrm{K} \setminus \mathrm{U} \Rightarrow \Phi_{\mathcal{N}\mathcal{G}1}(\mathrm{K} \setminus \mathrm{U}) \subseteq (\mathrm{K} \setminus \mathrm{U})$ (by Note 3.2(2)) $\Rightarrow \mathrm{K} \setminus$

U = $\tau_{\mathcal{N}\mathcal{G}1}$ -cl(K \ U) ⇒ U ∈ $\tau_{\mathcal{N}\mathcal{G}1}$.

(2) By Note 3.2 (3), B $\notin \mathcal{G}$ ⇒ $\Phi_{\mathcal{N}}$(B) = 0_N, and then $\tau_{\mathcal{N}\mathcal{G}}$-cl(B) = $\Psi_{\mathcal{N}}$(B) = B ∪ $\Phi_{\mathcal{N}}$(B) = B proving B to be $\tau_{\mathcal{N}\mathcal{G}}$-closed.

(3) We have, $\Psi_{\mathcal{N}}(\Phi_{\mathcal{N}}$(A)) = $\Phi_{\mathcal{N}}$(A) ∪ $\Phi_{\mathcal{N}}(\Phi_{\mathcal{N}}$(A)) = $\Phi_{\mathcal{N}}$(A) ∪ $\Phi_{\mathcal{N}}$(A) = $\Phi_{\mathcal{N}}$(A), (by Proposition 3.3 (2)) ⇒ $\Phi_{\mathcal{N}}$(A) is $\tau_{\mathcal{N}\mathcal{G}}$-closed. □

Theorem 3.17. *Let (K, τ_N) be an neutrosophic topological space with a grill $\mathcal{G}$ on K and for every A ⊆ K. A ⊆ $\Phi_{\mathcal{N}}$(A), then*

(1) *N-cl(A) = $\tau_{\mathcal{N}\mathcal{G}}$-cl(A),*
(2) *N-int(K \ A) = $\tau_{\mathcal{N}\mathcal{G}}$-int(K \ A),*
(3) *N-cl(K \ A) = $\tau_{\mathcal{N}\mathcal{G}}$-cl(K \ A),*
(4) *N-int(A) = $\tau_{\mathcal{N}\mathcal{G}}$-int(A).*

Proof. (1) Follows immediately from Theorem 3.8.

(2) If A ⊆ $\Phi_{\mathcal{N}}$(A) then N-cl(A) = $\tau_{\mathcal{N}\mathcal{G}}$-cl(A) by (1) and so K \ N-cl(A) = K \ $\tau_{\mathcal{N}\mathcal{G}}$-cl(A). Therefore, N-int(K \ A) = $\tau_{\mathcal{N}\mathcal{G}}$-int(K \ A).

(3) Follows by replacing A by K \ A in(1).

(4) If A ⊆ $\Phi_{\mathcal{N}}$(A) then N-cl(K \ A) = $\tau_{\mathcal{N}\mathcal{G}}$-cl(K \ A) by (3) and so K \ N-cl(K \ A) = K \ $\tau_{\mathcal{N}\mathcal{G}}$-cl(K \ A). Therefore, N-int(A) = $\tau_{\mathcal{N}\mathcal{G}}$-int(A). □

Theorem 3.18. *Let $\mathcal{G}$ be a grill on a space (K, τ_N) and for every A ⊆ K such that A ⊆ $\Phi_{\mathcal{N}}$(A). Then $\Phi_{\mathcal{N}}$(A) = $\mathcal{N}$-cl($\Phi_{\mathcal{N}}$(A)) = $\mathcal{N}$-cl(A) = $\tau_{\mathcal{N}\mathcal{G}}$ -cl(A).*

Proof. Follows from Thoerem 3.8 and Theorem 3.17 (1). □

Definition 3.19. *A subset A of a neutrosophic grill topological space (K, τ_N) is $\tau_{\mathcal{N}\mathcal{G}}$ dense in itself (resp. $\tau_{\mathcal{N}\mathcal{G}}$-perfect, $\tau_{\mathcal{N}\mathcal{G}}$-closed) if A ⊆ $\Phi_{\mathcal{N}}$(A) (resp. A = $\Phi_{\mathcal{N}}$(A), $\Phi_{\mathcal{N}}$(A) ⊆ A).*

Remark 3.20. *we have the following diagram*

$\tau_{\mathcal{N}\mathcal{G}}$ *dense in itself* $\Leftarrow$ $\tau_{\mathcal{N}\mathcal{G}}$*-perfect* $\Rightarrow$ $\tau_{\mathcal{N}\mathcal{G}}$*-closed*

Theorem 3.21. *Let $\mathcal{G}$ be a grill on a neutrosophic topological spaces $(K, \tau_{\mathcal{N}})$. Then the following are equivalent*

(1) *for every $A \in \tau_{\mathcal{N}}$, $A \subseteq \Phi_{\mathcal{N}}(A)$,*
(2) *For any neutrosophic semi closed set A in K, $A \subseteq \Phi_{\mathcal{N}}(A)$.*

Proof. (2) $\Rightarrow$ (1) is obvious.

(1) $\Rightarrow$ (2) Suppose neutrosophic semi closed set A in K. Then there exists an neutrosophic open set P such that $P \subseteq A \subseteq \mathcal{N}$-cl(P). Since P is neutrosophic open, P $\subseteq \Phi_{\mathcal{N}}(P)$ and so by Theorem 3.8, $A \subseteq \mathcal{N}\text{-cl}(P) \subseteq \mathcal{N}\text{-cl}(\Phi_{\mathcal{N}}(P)) = \Phi_{\mathcal{N}}(P) \subseteq \Phi_{\mathcal{N}}(A)$. Hence $A \subseteq \Phi_{\mathcal{N}}(A)$. □

We introduce definitions of neutrosophic grill topological space as follows.

Definition 3.22. *Let $(K, \tau_{\mathcal{N}})$ be a neutrosophic topological space and a grill $\mathcal{G}$ on K. Then for all $A \subseteq K$. Then,*

(1) *$\Phi_{\mathcal{N}}$-open if $A \subset N\text{-}int(\Phi_{\mathcal{N}}(A))$,*
(2) *$g_{\mathcal{N}}$-set if $N\text{-}int(\Psi_{\mathcal{N}}(A)) = N\text{-}int(A)$,*
(3) *$g\Phi_{\mathcal{N}}$-set if $N\text{-}int(\Phi_{\mathcal{N}}(A)) = N\text{-}int(A)$.*

Definition 3.23. *Let $(K, \tau_{\mathcal{N}})$ be a neutrosophic topological space and a grill $\mathcal{G}$ on K. Then for all $A \subseteq K$. Then,*

(1) *A is called neutrosophic grill α-open (short, $\mathcal{N}\mathcal{G}$-α-open) if $A \subseteq N\text{-}int(\Psi_{\mathcal{N}}(N\text{-}int(A)))$.*

(2) *A is called neutrosophic grill pre-open (short, $\mathcal{NG}$-pre-open) if A ⊆ N-int($\Psi_{\mathcal{N}}(A)$).*

(3) *A is called neutrosophic grill semi-open (short, $\mathcal{NG}$-semi-open) if A ⊆ $\Psi_{\mathcal{N}}$(N-int(A)).*

(4) *A is called neutrosophic grill b-open (short, $\mathcal{NG}$-b-open) if A ⊆ N-int($\Psi_{\mathcal{N}}(A)$) ∪ $\Psi_{\mathcal{N}}$(N-int(A)).*

(5) *A is called neutrosophic grill β-open (short, $\mathcal{NG}$-β-open) if A ⊆ $\Psi_{\mathcal{N}}$(N-int($\Psi_{\mathcal{N}}(A)$)).*

(6) *A is called neutrosophic grill regular-open (short, $\mathcal{NG}$-regular-open) if A = N-int($\Psi_{\mathcal{N}}(A)$).*

(7) *A is called neutrosophic grill π-open if A is the finite union of neutrosophic grill regular-open.*

The complement of above mentioned neutrosophic grill open sets are called their respective neutrosophic grill closed sets.

Theorem 3.24. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and a grill $\mathcal{G}$ on K. Then the following hold:*

(1) *Every $\mathcal{NG}$-α-open set is neutrosophic α-open.*

(2) *Every $\mathcal{NG}$-semi-open set is neutrosophic semi-open.*

(3) *Every $\mathcal{NG}$-β-open set is neutrosophic β-open.*

(4) *Every $\mathcal{NG}$-pre-open set is neutrosophic pre-open.*

(5) *Every $\mathcal{NG}$-b-open set is neutrosophic b-open.*

Proof. (1) Let A be an $\mathcal{NG}$-α-open set. Then we have A ⊆ N-int($\Psi_{\mathcal{N}}$(N-int(A))) = N-int($\Phi_{\mathcal{N}}$(N-int(A)) ∪ N-int(A)) ⊆ N-int($\Phi_{\mathcal{N}}$(N-int(A)) ∪ N-int(N-int(A)) ⊆ N-int($\Phi_{\mathcal{N}}$(N-int(A)) ∪ (N-int(A)) ⊆ N-int($\Phi_{\mathcal{N}}$(N-int(A))) ⊆ N-int(N-cl(N-int(A))) = N-int(N-cl(N-int(A))). Hence A is neutrosophic α-open.

(2) Let A be an $\mathcal{NG}$-semi-open set. Then we have A $\subseteq \Psi_{\mathcal{N}}$(N-int(A)) $\subseteq \Phi_{\mathcal{N}}$((N-int(A))) $\cup$ (N-int(A)) $\subseteq$ N-cl(N-int(A)) $\cup$ N-cl(N-int(A)) $\subseteq$ N-cl(N-int(A)) = N-cl(N-int(A)). Hence A is neutrosophic semi-open.

(3) Let A be an $\mathcal{NG}$-β-open set. Then we have A $\subseteq \Psi_{\mathcal{N}}$(N-int$\Psi_{\mathcal{N}}$(A))) $\subseteq \Psi_{\mathcal{N}}$(N-int(A $\cup \Phi_{\mathcal{N}}$(A)) $\subseteq \Psi_{\mathcal{N}}$(N-int(A $\cup$ N-cl(A))) $\subseteq \Psi_{\mathcal{N}}$(N-int(N-cl(A) $\cup$ N-cl(A)))$\subseteq \Psi_{\mathcal{N}}$(N-int(N-cl(A))) $\subseteq$ ($\Phi_{\mathcal{N}}$(N-int(N-cl(A))) $\cup$ (N-int(N-cl(A))) $\subseteq$ N-cl(N-int(N-cl(A))) = N-cl(N-int(N-cl(A))). Hence A is neutrosophic β-open.

(4) Let A be a $\mathcal{NG}$-pre-open. Then A $\subset$ N-int$\Psi_{\mathcal{N}}$(A)) = N-int(A $\cup \Phi_{\mathcal{N}}$(A)) $\subset$ N-int(A $\cup$ N-cl(A)) = N-int(N-cl(A)). Therefore, A is a neutrosophic pre-open set.

(5) Let A be an $\mathcal{NG}$-b-open set. Then we have A $\subseteq$ N-int$\Psi_{\mathcal{N}}$(A)) $\cup \Psi_{\mathcal{N}}$(N-int(A)) $\subseteq$ N-int($\Phi_{\mathcal{N}}$(A) $\cup$ A) $\cup$ ($\Phi_{\mathcal{N}}$(N-int(A)) $\cup$ N-int(A)) $\subseteq$ N-int(N-cl(A) $\cup$ A) $\cup \Phi_{\mathcal{N}}$(N-int(A)) $\cup$ N-int(A) $\subseteq$ N-int(N-cl(A)) $\cup$ (N-cl(N-int(A)) $\cup$ N-cl(N-int(A)) $\subseteq$ N-int(N-cl(A)) $\cup$ (N-cl(N-int(A)) $\subseteq$ N-int(N-cl(A)) $\cup$ N-cl(N-int(A)). Hence A is neutrosophic b-open. □

Theorem 3.25. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and a grill $\mathcal{G}$ on K. Then the following hold:*

(1) *Every $\mathcal{NG}$-α-open set is $\mathcal{NG}$-pre-open.*
(2) *Every $\mathcal{NG}$-α-open set is $\mathcal{NG}$-semi-open.*
(3) *Every $\mathcal{NG}$-pre-open set is $\mathcal{NG}$-b-open.*
(4) *Every $\mathcal{NG}$-pre-open set is $\mathcal{NG}$-β-open.*
(5) *Every $\mathcal{NG}$-b-open set is $\mathcal{NG}$-β-open.*
(6) *$\mathcal{NG}$-semi-open set is $\mathcal{NG}$-b-open.*

Proof. The proof is straightforward from the definitions. □

Proposition 3.26. *If A is a $\mathcal{NG}$-pre-open, then N-cl(N-int($\Psi_{\mathcal{N}}$(A))) = N-cl(A).*

Proof. N-cl(A) $\subset$ N-cl(N-int($\Psi_{\mathcal{N}}$(A))) $\subset$ N-cl($\Psi_{\mathcal{N}}$(A)) = N-cl(A $\cup$ $\Phi_{\mathcal{N}}$(A)) = N-cl(A) $\cup$ N-cl($\Phi_{\mathcal{N}}$(A)) = N-cl(A) $\cup$ $\Phi_{\mathcal{N}}$(A) $\subset$ N-cl(A). □

Theorem 3.27. *Every $\Phi_{\mathcal{N}}$-open set A is $\mathcal{N}\mathcal{G}$-pre-open.*

Proof. Let A be a $\Phi_{\mathcal{N}}$-open. Then A $\subset$ N-int($\Phi_{\mathcal{N}}$(A)) $\subset$ N-int(A $\cup$ $\Phi_{\mathcal{N}}$(A)) = N-int($\Psi_{\mathcal{N}}$(A)). Therefore A is $\mathcal{N}\mathcal{G}$-pre-open. □

Proposition 3.28. *Every $\mathcal{N}\mathcal{G}$-regular-open set is a $g_{\mathcal{N}}$-set*

Proof. Obvious. □

Definition 3.29. *Let (K, $\tau_{\mathcal{N}}$) be a neutrosophic topological space and a grill $\mathcal{G}$ on K. A subset A in K is said to be*

(1) *$\mathcal{N}G$-set if $H = S \cap G$ where S is $\tau_{\mathcal{N}}$-open and G is a $g_{\mathcal{N}}$-set.*
(2) *$G\Phi_{\mathcal{N}}$-set if $H = S \cap G$ where S is $\tau_{\mathcal{N}}$-open and G is a $g\Phi_{\mathcal{N}}$-set.*

Proposition 3.30. *(1) Every $g_{\mathcal{N}}$-set is a $\mathcal{N}G$-set but not conversely.*
(2) Every $g\Phi_{\mathcal{N}}$-set is a $G\Phi_{\mathcal{N}}$-set but not conversely.

Proof. Obvious. □.

Proposition 3.31. *An $\tau_{\mathcal{N}}$-open set U is a $\mathcal{N}G$-set (resp. $G\Phi_{\mathcal{N}}$-set).*

Proof. U = U $\cap$ K, N-int($\Psi_{\mathcal{N}}$(K)) = N-int(K).

Conclusion

We presented several definitions, properties, explanations and examples inspired from the concept of neutrosophic grill topological space. The results of this study may be help in many researches.

Acknowledgement

It is our pleasant duty to thank referees for their useful suggestions which helped us to improve our manuscript.

References

[1] A. Al-Omari, T. Noiri, Decomposition of continuity via grilles, Jordan J. Math. Stat, 4(1), (2011), 33-46.

[2] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala1, On some new notions and functions in neutrosophic topollogical spaces, Neutrosophic Sets and Systems, 16, (2017), 16-19.

[3] K. T. Atanassov, Intuitionstic fuzzy sets. Fuzzy sets and systems, 20, (1986), 87-96.

[4] C. L. Chang, Fuzzy topological spaces, J. Math.Anal. Appl, 24, (1968), 182-190.

[5] G. Choquet, Sur les notions de filtre et grille, Comptes Rendus Acad. Sci. Paris, 224, (1947), 171-173.

[6] D. Coker, An introduction to intuitionistic fuzzy topological spaces fuzzy sets and systems, 88, (1997), 81-89.

[7] E. Hater, S. Jafari, On some new classes of sets and a new decomposition of continuity via grills, J. Adv. Math. Studies, 3(1), (2010), 33-40.

[8] P. Iswarya and K. Bageerathi, On neutrosophic semi-open sets in neutrosophic topological spaces, International Journal of Mathematics Trends and Technology(IJMTT), 37(3), (2016), 214-223.

[9] C. Maheswari, M. Sathya bama, S. Chandrasekar, Neutrosophic generalized b-closed sets in neutrosophic topological spaces, Journal of physics Conf. Series 1139 (2018) 012065. doi:10.1088/1742-6596/1139/1/012065

[10] O. Ravi and S. Ganesan, On new types of grill sets and a decomposition of continuity. Journal of Advanced Studies in Topology, 2(1), (2011), 21-29.

[11] B. Roy and M. N. Mukherjee, On a typical topology induced by a grill, Soochow J. Math., 33(4), (2007), 771-786.

[12] A. A. Salama and S. A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces. IOSR J. Math, 3(4), (2012), 31-35.

[13] F. Smarandache, Neutrosophy and Neutrosophic Logic. First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics, University of New Mexico, Gallup, NM, USA, (2002).

[14] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutro-sophic Probability, American Research Press: Rehoboth, NM, USA, (1999).

[15] L. A. Zadeh, Fuzzy Sets, Information and Control, 18, (1965), 338-353.