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P-union and P-intersection of neutrosophic cubic sets

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Abstract

Conditions for the P-intersection and P-intersection of falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic sets to be an falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic set are provided. Conditions for the P-union and the P-intersection of two truth-external (resp. indeterminacy-external and falsity-external) neutrosophic cubic sets to be a truth-internal (resp. indeterminacy-internal and falsity-internal) neutrosophic cubic set are discussed.

1 Introduction

The concept of neutrosophic set (NS) developed by Smarandache ([3, 4]) is a more general platform which extends the concepts of the classic set and fuzzy set, intuitionistic fuzzy set and interval valued intuitionistic fuzzy set. Neutrosophic set theory is applied to various part (refer to the site http://fs.gallup.unm.edu/ neutrosophy.htm). Jun et al. [2] extended the concept of cubic sets to the neutrosophic sets. They introduced the notions of truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic sets and truth-external (indeterminacy-external, falsity-external) neutrosophic cubic sets, and investigate related properties. Generally, the P-intersection

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of falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic sets may not be a falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic set (see [2]). As a continuation of the paper [2], we provide a condition for the P-intersection of falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic sets to be a falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic set. We provide examples to show that the P-union of falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic sets may not be a falsity-external (resp. indeterminacy-external and truth-external) neutrosophic cubic set. We consider a condition for the P-union of truth-external (resp. indeterminacy-external and falsity-external) neutrosophic cubic sets to be a truth-external (resp. indeterminacy-external and falsity-external) neutrosophic cubic set. We also give a condition for the P-intersection of two neutrosophic cubic sets to be both a truth-internal (resp. indeterminacy-internal and falsity-internal) neutrosophic cubic set and a truth-external (resp. indeterminacy-external and falsity-external) neutrosophic cubic set. Generally, the P-union of two truth-external (resp. indeterminacy-external and falsity-external) neutrosophic cubic sets may not be a truth-internal (resp. indeterminacy-internal and falsity-internal) neutrosophic cubic set. We provide conditions for the P-union and the P-intersection of two truth-external (resp. indeterminacy-external and falsity-external) neutrosophic cubic sets to be a truth-internal (resp. indeterminacy-internal and falsity-internal) neutrosophic cubic set.

2 Preliminaries

Jun et al. [1] have defined the cubic set as follows:

Let X be a non-empty set. A cubic set in X is a structure of the form:

$$\mathbf{C} = \{(x, A(x), \lambda(x)) \mid x \in X\}$$

where A is an interval-valued fuzzy set in X and λ is a fuzzy set in X.

Let X be a non-empty set. A neutrosophic set (NS) in X (see [3]) is a structure of the form:

$$\Lambda := \{\langle x; \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle \mid x \in X\}$$

where $\lambda_T : X \to [0,1]$ is a truth membership function, $\lambda_I : X \to [0,1]$ is an indeterminate membership function, and $\lambda_F : X \to [0,1]$ is a false membership function.

Let X be a non-empty set. An interval neutrosophic set (INS) in X (see [5]) is a structure of the form:

$$\mathbf{A} := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}$$

where A_T, A_I and A_F are interval-valued fuzzy sets in X, which are called an interval truth membership function, an interval indeterminacy membership function and an interval falsity membership function, respectively.

Jun et al. [2] considered the notion of neutrosophic cubic sets as an extension of cubic sets.

Let X be a non-empty set. A neutrosophic cubic set (NCS) in X is a pair $\mathscr{A} = (\mathbf{A}, \Lambda)$ where $\mathbf{A} := \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\}$ is an interval neutrosophic set in X and $\Lambda := \{\langle x; \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle \mid x \in X\}$ is a neutrosophic set in X.

Definition 2.1 ([2])**.** Let X be a non-empty set. A neutrosophic cubic set $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is said to be

- truth-internal (briefly, T-internal) if the following inequality is valid

$$(\forall x \in X) \left(A_T^-(x) \leq \lambda_T(x) \leq A_T^+(x)\right), \tag{2.1}$$

- indeterminacy-internal (briefly, I-internal) if the following inequality is valid

$$(\forall x \in X) \left(A_I^-(x) \leq \lambda_I(x) \leq A_I^+(x)\right), \tag{2.2}$$

- falsity-internal (briefly, F-internal) if the following inequality is valid

$$(\forall x \in X) \left(A_F^-(x) \leq \lambda_F(x) \leq A_F^+(x)\right). \tag{2.3}$$

Definition 2.2 ([2])**.** Let X be a non-empty set. A neutrosophic cubic set $\mathscr{A} = (\mathbf{A}, \Lambda)$ in X is said to be

- truth-external (briefly, T-external) if the following inequality is valid

$$(\forall x \in X) \left(\lambda_T(x) \notin (A_T^-(x), A_T^+(x))\right), \tag{2.4}$$

- indeterminacy-external (briefly, I-external) if the following inequality is valid

$$(\forall x \in X) \left(\lambda_I(x) \notin (A_I^-(x), A_I^+(x))\right), \tag{2.5}$$

- falsity-external (briefly, F-external) if the following inequality is valid

$$(\forall x \in X) \left(\lambda_F(x) \notin (A_F^-(x), A_F^+(x))\right). \tag{2.6}$$

3 P-union and P-intersection of neutrosophic cubic sets

Note that P-intersection of F-external (resp. I-external and T-external) neutrosophic cubic sets may not be an F-external (resp. I-external and T-external) neutrosophic cubic set (see [2]). We provide a condition for the P-intersection of F-external (resp. I-external and T-external) neutrosophic cubic sets to be an F-external (resp. I-external and T-external) neutrosophic cubic set.

Theorem 3.1. *Let* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *be T-external neutrosophic cubic sets in* X *such that*

$$\begin{aligned} &\max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\} < (\lambda_T \wedge \psi_T)(x) \\ &\leq \min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\} \end{aligned} \tag{3.1}$$

for all $x \in X$. *Then the P-intersection* $\mathscr{A} \cap_P \mathscr{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \wedge \Psi)$ *is a T-external neutrosophic cubic set in* X.

Proof. For any $x \in X$, let

$$a_x := \min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\}$$

and

$$b_x := \max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\}.$$

Then $a_x = A_T^-(x)$, $a_x = B_T^-(x)$, $a_x = A_T^+(x)$, or $a_x = B_T^+(x)$. It is possible to consider the cases $a_x = A_T^-(x)$ and $a_x = A_T^+(x)$ only because the remaining cases are similar to these cases. If $a_x = A_T^-(x)$, then

$$B_T^-(x) \leq B_T^+(x) \leq A_T^-(x) \leq A_T^+(x).$$

Thus $b_x = B_T^+(x)$, and so

$$\begin{aligned} B_T^-(x) = (A_T \cap B_T)^-(x) &\leq (A_T \cap B_T)^+(x) \\ &= B_T^+(x) = b_x < (\lambda_T \wedge \psi_T)(x). \end{aligned}$$

Hence $(\lambda_T \wedge \psi_T)(x) \notin ((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x))$. If $a_x = A_T^+(x)$, then $B_T^-(x) \leq A_T^+(x) \leq B_T^+(x)$ and thus $b_x = \max\{A_T^-(x), B_T^-(x)\}$. Suppose that $b_x = A_T^-(x)$. Then

$$B_T^-(x) \leq A_T^-(x) < (\lambda_T \wedge \psi_T)(x) \leq A_T^+(x) \leq B_T^+(x). \tag{3.2}$$

It follows that

$$B_T^-(x) \leq A_T^-(x) < (\lambda_T \wedge \psi_T)(x) < A_T^+(x) \leq B_T^+(x) \tag{3.3}$$

or

$$B_T^-(x) \le A_T^-(x) < (\lambda_T \wedge \psi_T)(x) = A_T^+(x) \le B_T^+(x). \tag{3.4}$$

The case (3.3) induces a contradiction. The case (3.4) implies that

$$(\lambda_T \wedge \psi_T)(x) \notin \left((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x)\right)$$

since $(\lambda_T \wedge \psi_T)(x) = A_T^+(x) = (A_T \cap B_T)^+(x)$. Now, if $b_x = B_T^-(x)$, then

$$A_T^-(x) \le B_T^-(x) < (\lambda_T \wedge \psi_T)(x) \le A_T^+(x) \le B_T^+(x). \tag{3.5}$$

Hence we have

$$A_T^-(x) \le B_T^-(x) < (\lambda_T \wedge \psi_T)(x) < A_T^+(x) \le B_T^+(x) \tag{3.6}$$

or

$$A_T^-(x) \le B_T^-(x) < (\lambda_T \wedge \psi_T)(x) = A_T^+(x) \le B_T^+(x). \tag{3.7}$$

The case (3.6) induces a contradiction. The case (3.7) induces

$$(\lambda_T \wedge \psi_T)(x) \notin \left((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x)\right).$$

Therefore the P-intersection $\mathscr{A} \cap_P \mathscr{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \wedge \Psi)$ is a T-external neutrosophic cubic set in X. □

Similarly, we have the following theorems.

Theorem 3.2. *Let* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *be I-external neutrosophic cubic sets in* X *such that*

$$\begin{aligned} &\max\left\{\min\{A_I^+(x), B_I^-(x)\}, \min\{A_I^-(x), B_I^+(x)\}\right\} < (\lambda_I \wedge \psi_I)(x) \\ &\le \min\left\{\max\{A_I^+(x), B_I^-(x)\}, \max\{A_I^-(x), B_I^+(x)\}\right\} \end{aligned} \tag{3.8}$$

for all $x \in X$. *Then the P-intersection* $\mathscr{A} \cap_P \mathscr{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \wedge \Psi)$ *is an I-external neutrosophic cubic set in* X.

Theorem 3.3. *Let* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *be F-external neutrosophic cubic sets in* X *such that*

$$\begin{aligned} &\max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\} < (\lambda_F \wedge \psi_F)(x) \\ &\le \min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\} \end{aligned} \tag{3.9}$$

for all $x \in X$. *Then the P-intersection* $\mathscr{A} \cap_P \mathscr{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \wedge \Psi)$ *is an F-external neutrosophic cubic set in* X.

Corollary 3.4. *Let* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *be external neutrosophic cubic sets in* X*. Then the P-intersection of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is an external neutrosophic cubic set in* X *when the conditions* (3.1)*,* (3.8) *and* (3.9) *are valid.*

The following example shows that the P-union of F-external (resp. I-external and T-external) neutrosophic cubic sets may not be an F-external (resp. I-external and T-external) neutrosophic cubic set.

Example 3.5. (1) Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in $X = [0, 1]$ with the tabular representations in Tables 1 and 2, respectively.

Table 1: Tabular representation of $\mathscr{A} = (\mathbf{A}, \Lambda)$

X	$\mathbf{A}(x)$	$\Lambda(x)$
$0 \leq x < 0.5$	$([0.25, 0.26], [0.2, 0.3], [0.15, 0.25])$	$(0.25, 0.15, 0.5x + 0.5)$
$0.5 \leq x \leq 1$	$([0.5, 0.7], [0.5, 0.6], [0.6, 0.7])$	$(0.55, 0.75, 0.30)$

Table 2: Tabular representation of $\mathscr{B} = (\mathbf{B}, \Psi)$

X	$\mathbf{B}(x)$	$\Psi(x)$
$0 \leq x < 0.5$	$([0.25, 0.26], [0.2, 0.3], [0.8, 0.9])$	$(0.25, 0.15, 0.40)$
$0.5 \leq x \leq 1$	$([0.5, 0.7], [0.5, 0.6], [0.1, 0.2])$	$(0.55, 0.75, x)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ are F-external neutrosophic cubic sets in $X = [0, 1]$, and the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ of $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ is given by Table 3.

Table 3: Tabular representation of $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$

X	$(\mathbf{A} \cup \mathbf{B})(x)$	$(\Lambda \vee \Psi)(x)$
$0 \leq x < 0.5$	$([0.25, 0.26], [0.2, 0.3], [0.8, 0.9])$	$(0.25, 0.15, 0.5x + 0.5)$
$0.5 \leq x \leq 1$	$([0.5, 0.7], [0.5, 0.6], [0.6, 0.7])$	$(0.55, 0.75, x)$

Then

$$\begin{aligned}(\lambda_F \vee \psi_F)(0.67) &= 0.67 \in (0.6, 0.7)\\ &= \big((A_F \cup B_F)^-(0.67), (A_F \cup B_F)^+(0.67)\big),\end{aligned}$$

and so the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ is not an F-external neutrosophic cubic set in $X = [0, 1]$.

(2) Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in $X = [0, 1]$ with the tabular representations in Tables 4 and 5, respectively.

Table 4: Tabular representation of $\mathscr{A} = (\mathbf{A}, \Lambda)$

X	$\mathbf{A}(x)$	$\Lambda(x)$
$0 \leq x \leq 0.3$	$([0.3, 0.6], [0.3, 0.5], [0.6, 1])$	$(x + 0.6, 0.15, \frac{1}{2}x + \frac{1}{2})$
$0.3 < x \leq 1$	$([0.4, 0.9], [0.5, 0.6], [0.6, 0.7])$	$(-\frac{2}{5}x + 0.4, 0.75, 0.30)$

Table 5: Tabular representation of $\mathscr{B} = (\mathbf{B}, \Psi)$

X	$\mathbf{B}(x)$	$\Psi(x)$
$0 \leq x \leq 0.3$	$([0.4, 0.8], [0.2, 0.3], [0.8, 0.9])$	$(\frac{1}{2}x + 0.8, 0.15, 0.40)$
$0.3 < x \leq 1$	$([0.3, 0.5], [0.5, 0.6], [0.1, 0.2])$	$(\frac{1}{3}x + 0.5, 0.75, x)$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ are T-external neutrosophic cubic sets in $X = [0, 1]$. Note that

$$(A_T \cup B_T)^-(x) = \begin{cases} [0.4,\ 0.8] & \text{if } 0 \leq x \leq 0.3, \\ [0.4, 0.9] & \text{if } 0.3 < x \leq 1, \end{cases}$$

$$(\lambda_T \vee \psi_T)(x) = \begin{cases} \frac{1}{2}x + 0.8 & \text{if } 0 \leq x \leq 0.3, \\ \frac{1}{3}x + 0.5 & \text{if } 0.3 < x \leq 1, \end{cases}$$

and so the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ is not a T-external neutrosophic cubic set in $X = [0, 1]$ since

$$(\lambda_T \vee \psi_T)(0.6) = 0.7 \in (0.4, 0.9) = \big((A_T \cup B_T)^-(0.6), (A_T \cup B_T)^+(0.6)\big).$$

(3) Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in $X = [0, 1]$ with the tabular representations in Tables 6 and 7, respectively.

Table 6: Tabular representation of $\mathscr{A} = (\mathbf{A}, \Lambda)$

X	$\mathbf{A}(x)$	$\Lambda(x)$
$0 \leq x \leq 0.5$	$([0.3, 0.6], [0.2, 0.7], [0.6, 1.0])$	$(0.4, \frac{1}{5}x + 0.7, \frac{1}{2}x + \frac{1}{2})$
$0.5 < x \leq 1$	$([0.4, 0.9], [0.3, 1.0], [0.6, 0.7])$	$(0.3, -\frac{1}{10}x + 0.3, 0.30)$

Table 7: Tabular representation of $\mathscr{B} = (\mathbf{B}, \Psi)$

X	$\mathbf{B}(x)$	$\Psi(x)$
$0 \leq x \leq 0.5$	$([0.4, 0.8], [0.3, 0.8], [0.8, 0.9])$	$(0.3, -\frac{1}{5}x + 0.3, 0.40)$
$0.5 < x \leq 1$	$([0.3, 0.5], [0.5, 0.9], [0.1, 0.2])$	$(0.5, -\frac{1}{10}x + 1.0, x)$

It is routine to verify that $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ are I-external neutrosophic cubic sets in $X = [0, 1]$, but their P-union is not an I-external neutrosophic cubic sets in $X = [0, 1]$ since

$$(\lambda_I \vee \psi_I)(0.7) = 0.93 \in (0.5, 1.0) = \big((A_I \cup B_I)^-(0.7), (A_I \cup B_I)^+(0.7)\big).$$

We consider a condition for the P-union of T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-external (resp. I-external and F-external) neutrosophic cubic set.

Theorem 3.6. *Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ be F-external neutrosophic cubic sets in X such that*

$$\begin{aligned} &\max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\} \leq (\lambda_F \vee \psi_F)(x) \\ &< \min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\} \end{aligned} \tag{3.10}$$

for all $x \in X$. Then the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ is an F-external neutrosophic cubic set in X.

Proof. For any $x \in X$, let

$$a_x := \min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\}$$

and

$$b_x := \max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\}.$$

Then $a_x = A_F^-(x)$, $a_x = B_F^-(x)$, $a_x = A_F^+(x)$, or $a_x = B_F^+(x)$. It is possible to consider the cases $a_x = A_F^-(x)$ and $a_x = A_F^+(x)$ only because the remaining cases are similar to these cases. If $a_x = A_F^-(x)$, then

$$B_F^-(x) \leq B_F^+(x) \leq A_F^-(x) \leq A_F^+(x),$$

and so $b_x = B_F^+(x)$. Thus

$$(A_F \cup B_F)^-(x) = A_F^-(x) = a_x > (\lambda_F \vee \psi_F)(x),$$

and hence $(\lambda_F \vee \psi_F)(x) \notin ((A_F \cup B_F)^-(x), (A_F \cup B_F)^+(x))$. If $a_x = A_F^+(x)$, then $B_F^-(x) \leq A_F^+(x) \leq B_F^+(x)$ and thus $b_x = \max\{A_F^-(x), B_F^-(x)\}$. Suppose that $b_x = A_F^-(x)$. Then

$$B_F^-(x) \leq A_F^-(x) \leq (\lambda_F \vee \psi_F)(x) < A_F^+(x) \leq B_F^+(x), \tag{3.11}$$

and so

$$B_F^-(x) \leq A_F^-(x) < (\lambda_F \vee \psi_F)(x) < A_F^+(x) \leq B_F^+(x) \tag{3.12}$$

or

$$B_F^-(x) \leq A_F^-(x) = (\lambda_F \vee \psi_F)(x) < A_F^+(x) \leq B_F^+(x). \tag{3.13}$$

The case (3.12) induces a contradiction. The case (3.13) implies that

$$(\lambda_F \vee \psi_F)(x) \notin \big((A_F \cup B_F)^-(x), (A_F \cup B_F)^+(x)\big)$$

since $(\lambda_F \vee \psi_F)(x) = A_F^-(x) = (A_F \cup B_F)^-(x)$. Now, if $b_x = B_F^-(x)$, then

$$A_F^-(x) \leq B_F^-(x) \leq (\lambda_F \vee \psi_F)(x) \leq A_F^+(x) \leq B_F^+(x). \tag{3.14}$$

Hence we have

$$A_F^-(x) \leq B_F^-(x) < (\lambda_F \vee \psi_F)(x) \leq A_F^+(x) \leq B_F^+(x) \tag{3.15}$$

or

$$A_F^-(x) \leq B_F^-(x) = (\lambda_F \vee \psi_F)(x) \leq A_F^+(x) \leq B_F^+(x). \tag{3.16}$$

The case (3.15) induces a contradiction. The case (3.16) induces

$$(\lambda_F \vee \psi_F)(x) \notin \big((A_F \cup B_F)^-(x), (A_F \cup B_F)^+(x)\big)\,.$$

Therefore the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ is an F-external neutrosophic cubic set in X. $\square$

Similarly, we have the following theorems.

Theorem 3.7. *Let* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *be T-external neutrosophic cubic sets in* X *such that*

$$\begin{aligned} &\max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\} \leq (\lambda_T \vee \psi_T)(x) \\ &< \min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\} \end{aligned} \tag{3.17}$$

for all $x \in X$. *Then the P-union* $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ *is a T-external neutrosophic cubic set in* X.

Theorem 3.8. *Let* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *be I-external neutrosophic cubic sets in* X *such that*

$$\begin{aligned} &\max\left\{\min\{A_I^+(x), B_I^-(x)\}, \min\{A_I^-(x), B_I^+(x)\}\right\} \leq (\lambda_I \vee \psi_I)(x) \\ &< \min\left\{\max\{A_I^+(x), B_I^-(x)\}, \max\{A_I^-(x), B_I^+(x)\}\right\} \end{aligned} \tag{3.18}$$

for all $x \in X$. *Then the P-union* $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ *is an I-external neutrosophic cubic set in* X.

We give a condition for the P-intersection of two neutrosophic cubic sets to be both a T-internal (resp. I-internal and F-internal) neutrosophic cubic set and a T-external (resp. I-external and F-external) neutrosophic cubic set.

Theorem 3.9. *If neutrosophic cubic sets* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ X *satisfy the following condition*

$$\begin{aligned} &\min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\} = (\lambda_F \wedge \psi_F)(x) \\ &= \max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\} \end{aligned} \tag{3.19}$$

for all $x \in X$, *then the P-intersection of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is both an F-internal neutrosophic cubic set and an F-external neutrosophic cubic set in* X.

Proof. For any $x \in X$, take

$$a_x := \min\left\{\max\{A_F^+(x), B_F^-(x)\}, \max\{A_F^-(x), B_F^+(x)\}\right\}$$

and

$$b_x := \max\left\{\min\{A_F^+(x), B_F^-(x)\}, \min\{A_F^-(x), B_F^+(x)\}\right\}.$$

Then a_x is one of $A_F^-(x)$, $B_F^-(x)$, $A_F^+(x)$ and $B_F^+(x)$. We consider $a_x = A_F^-(x)$ or $a_x = A_F^+(x)$ only. For remaining cases, it is similar to these cases. If $a_x = A_F^-(x)$, then

$$B_F^-(x) \leq B_F^+(x) \leq A_F^-(x) \leq A_F^+(x)$$

and thus $b_x = B_F^+(x)$. This implies that

$$A_F^-(x) = a_x = (\lambda_F \wedge \psi_F)(x) = b_x = B_F^+(x).$$

Hence $B_F^-(x) \leq B_F^+(x) = (\lambda_F \wedge \psi_F)(x) = A_F^-(x) \leq A_F^+(x)$, which implies that

$$(\lambda_F \wedge \psi_F)(x) = B_F^+(x) = (A_F \cap B_F)^+(x).$$

Hence $(\lambda_F \wedge \psi_F)(x) \notin ((A_F \cap B_F)^-(x), (A_F \cap B_F)^+(x))$ and

$$(A_F \cap B_F)^-(x) \leq (\lambda_F \wedge \psi_F)(x) \leq (A_F \cap B_F)^+(x).$$

If $a_x = A_F^+(x)$, then $B_F^-(x) \leq A_F^+(x) \leq B_F^+(x)$ and thus

$$(\lambda_F \wedge \psi_F)(x) = A_F^+(x) = (A_F \cap B_F)^+(x).$$

Hence $(\lambda_F \wedge \psi_F)(x) \notin ((A_F \cap B_F)^-(x), (A_F \cap B_F)^+(x))$ and

$$(A_F \cap B_F)^-(x) \leq (\lambda_F \wedge \psi_F)(x) \leq (A_F \cap B_F)^+(x).$$

Consequently, the P-intersection of $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ is both an F-internal neutrosophic cubic set and an F-external neutrosophic cubic set in X. □

Similarly, we get the following theorems.

Theorem 3.10. *If neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ X satisfy the following condition*

$$\begin{aligned} &\min\left\{\max\{A_I^+(x), B_I^-(x)\}, \max\{A_I^-(x), B_I^+(x)\}\right\} = (\lambda_I \wedge \psi_I)(x) \\ &= \max\left\{\min\{A_I^+(x), B_I^-(x)\}, \min\{A_I^-(x), B_I^+(x)\}\right\} \end{aligned} \tag{3.20}$$

for all $x \in X$, then the P-intersection of $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ is both an I-internal neutrosophic cubic set and an I-external neutrosophic cubic set in X.

Theorem 3.11. *If neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ X satisfy the following condition*

$$\begin{aligned} &\min\left\{\max\{A_T^+(x), B_T^-(x)\}, \max\{A_T^-(x), B_T^+(x)\}\right\} = (\lambda_T \wedge \psi_T)(x) \\ &= \max\left\{\min\{A_T^+(x), B_T^-(x)\}, \min\{A_T^-(x), B_T^+(x)\}\right\} \end{aligned} \tag{3.21}$$

for all $x \in X$, then the P-intersection of $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ is both a T-internal neutrosophic cubic set and a T-external neutrosophic cubic set in X.

Corollary 3.12. *If neutrosophic cubic sets* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ X *satisfy conditions 3.19, 3.20 and 3.21, then the P-intersection of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is both an internal neutrosophic cubic set and an external neutrosophic cubic set in* X*.*

Given two neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ in X where

$$\begin{aligned}
\mathbf{A} &:= \{\langle x; A_T(x), A_I(x), A_F(x)\rangle \mid x \in X\},\\
\Lambda &:= \{\langle x; \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle \mid x \in X\},\\
\mathbf{B} &:= \{\langle x; B_T(x), B_I(x), B_F(x)\rangle \mid x \in X\},\\
\Psi &:= \{\langle x; \psi_T(x), \psi_I(x), \psi_F(x)\rangle \mid x \in X\},
\end{aligned}$$

we try to exchange Λ and Ψ, and make new neutrosophic cubic sets $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ in X. Under these circumstances, we have questions.

Question. 1. If two neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ in X are T-external (resp., I-external and F-external), then are the new neutrosophic cubic sets $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ T-internal (resp., I-internal and F-internal) neutrosophic cubic sets in X?

2. If two neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ in X are T-external (resp., I-external and F-external), then are the new neutrosophic cubic sets $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ T-external (resp., I-external and F-external) neutrosophic cubic sets in X?

The answer to the question above is negative as seen in the following example.

Example 3.13. (1) Let $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in $[0, 1]$ where

$$\begin{aligned}
\mathbf{A} &= \{\langle x; [0.6, 0.7], [0.5, 0.7], [0.3, 0.5]\rangle \mid x \in [0, 1]\},\\
\Lambda &= \{\langle x; 0.8, 0.4, 0.8\rangle \mid x \in [0, 1]\},\\
\mathbf{B} &= \{\langle x; [0.3, 0.4], [0.4, 0.7], [0.7, 0.9]\rangle \mid x \in [0, 1]\},\\
\Psi &= \{\langle x; 0.2, 0.3, 0.4\rangle \mid x \in [0, 1]\}.
\end{aligned}$$

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$, and $\mathscr{B} = (\mathbf{B}, \Psi)$ are both T-external and F-external neutrosophic cubic sets in $[0, 1]$. It is easy to verify that $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ are F-internal neutrosophic cubic sets in $[0, 1]$, but not T-internal neutrosophic cubic sets in $[0, 1]$.

(2) For $X = \{a, b\}$, let $\mathscr{A} = (\mathbf{A}, \Lambda)$, and $\mathscr{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in X with the tabular representations in Tables 8 and 9, respectively.
Then $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ are I-external neutrosophic cubic sets in X, and $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ are represented as Tables 10 and 11, respectively,

Table 8: Tabular representation of $\mathscr{A} = (\mathbf{A}, \Lambda)$

X	$\mathbf{A}(x)$	$\Lambda(x)$
a	$([0.3, 0.6], [0.2, 0.3], [0.2, 0.5])$	$(0.25, 0.15, 0.40)$
b	$([0.5, 0.7], [0.5, 0.6], [0.3, 0.4])$	$(0.55, 0.75, 0.35)$

Table 9: Tabular representation of $\mathscr{B} = (\mathbf{B}, \Psi)$

X	$\mathbf{B}(x)$	$\Psi(x)$
a	$([0.3, 0.7], [0.4, 0.5], [0.1, 0.5])$	$(0.35, 0.95, 0.60)$
b	$([0.5, 0.8], [0.7, 0.9], [0.2, 0.5])$	$(0.45, 0.35, 0.30)$

Table 10: Tabular representation of $\mathscr{A}^* := (\mathbf{A}, \Psi)$

X	$\mathbf{A}(x)$	$\Psi(x)$
a	$([0.3, 0.6], [0.2, 0.3], [0.2, 0.5])$	$(0.35, 0.95, 0.60)$
b	$([0.5, 0.7], [0.5, 0.6], [0.3, 0.4])$	$(0.45, 0.35, 0.30)$

Table 11: Tabular representation of $\mathscr{B}^* := (\mathbf{B}, \Lambda)$

X	$\mathbf{B}(x)$	$\Lambda(x)$
a	$([0.3, 0.7], [0.4, 0.5], [0.1, 0.5])$	$(0.25, 0.15, 0.40)$
b	$([0.5, 0.8], [0.7, 0.9], [0.2, 0.5])$	$(0.55, 0.75, 0.35)$

which are not I-internal neutrosophic cubic sets in X.

(3) For $X = \{a, b, c\}$, let $\mathscr{A} = (\mathbf{A}, \Lambda)$, and $\mathscr{B} = (\mathbf{B}, \Psi)$ be neutrosophic cubic sets in X with the tabular representations in Tables 12 and 13, respectively.

Then $\mathscr{A} = (\mathbf{A}, \Lambda)$, and $\mathscr{B} = (\mathbf{B}, \Psi)$ are F-external neutrosophic cubic sets in X. Note that $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ are represented as Tables 14 and 15, respectively,

and they are not F-internal neutrosophic cubic sets in X.

Table 12: Tabular representation of $\mathscr{A} = (\mathbf{A}, \Lambda)$

X	$\mathbf{A}(x)$	$\Lambda(x)$
a	$([0.2, 0.3], [0.3, 0.5], [0.31, 0.51])$	$(0.35, 0.25, 0.75)$
b	$([0.4, 0.7], [0.1, 0.4], [0.22, 0.41])$	$(0.35, 0.50, 0.65)$
c	$([0.6, 0.9], [0.0, 0.2], [0.33, 0.42])$	$(0.50, 0.60, 0.75)$

Table 13: Tabular representation of $\mathscr{B} = (\mathbf{B}, \Psi)$

X	$\mathbf{B}(x)$	$\Psi(x)$
a	$([0.3, 0.7], [0.3, 0.5], [0.61, 0.81])$	$(0.25, 0.25, 0.35)$
b	$([0.5, 0.8], [0.5, 0.6], [0.25, 0.55])$	$(0.45, 0.30, 0.10)$
c	$([0.4, 0.9], [0.4, 0.7], [0.71, 0.85])$	$(0.35, 0.10, 0.40)$

Table 14: Tabular representation of $\mathscr{A}^* := (\mathbf{A}, \Psi)$

X	$\mathbf{A}(x)$	$\Psi(x)$
a	$([0.2, 0.3], [0.3, 0.5], [0.31, 0.51])$	$(0.25, 0.25, 0.35)$
b	$([0.4, 0.7], [0.1, 0.4], [0.22, 0.41])$	$(0.45, 0.30, 0.10)$
c	$([0.6, 0.9], [0.0, 0.2], [0.33, 0.42])$	$(0.35, 0.10, 0.40)$

Table 15: Tabular representation of $\mathscr{B}^* := (\mathbf{B}, \Lambda)$

X	$\mathbf{B}(x)$	$\Lambda(x)$
a	$([0.3, 0.7], [0.3, 0.5], [0.61, 0.81])$	$(0.35, 0.25, 0.75)$
b	$([0.5, 0.8], [0.5, 0.6], [0.25, 0.55])$	$(0.35, 0.50, 0.65)$
c	$([0.4, 0.9], [0.4, 0.7], [0.71, 0.85])$	$(0.50, 0.60, 0.75)$

Generally, the P-union of two T-external (resp. I-external and F-external) neutrosophic cubic sets may not be a T-internal (resp. I-internal and F-internal) neutrosophic cubic set.

Example 3.14. Consider the F-external neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$

and $\mathscr{B} = (\mathbf{B}, \Psi)$ in Example 3.13(3). The P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ of $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ is represented by Table 16, and it is not an F-internal neutrosophic cubic set in X.

Table 16: Tabular representation of $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$

X	$(\mathbf{A} \cup \mathbf{B})(x)$	$(\Lambda \vee \Psi)(x)$
a	$([0.3, 0.7], [0.3, 0.5], [0.61, 0.81])$	$(0.35, 0.25, 0.75)$
b	$([0.5, 0.8], [0.5, 0.6], [0.25, 0.55])$	$(0.45, 0.50, 0.65)$
c	$([0.6, 0.9], [0.4, 0.7], [0.71, 0.85])$	$(0.50, 0.60, 0.75)$

We provide conditions for the P-union of two T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-internal (resp. I-internal and F-internal) neutrosophic cubic set.

Theorem 3.15. *For any T-external neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ in X, if $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ are T-internal neutrosophic cubic sets in X, then the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ of $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ is a T-internal neutrosophic cubic set in X.*

Proof. Assume that $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ are T-internal neutrosophic cubic sets in X for any T-external neutrosophic cubic sets $\mathscr{A} = (\mathbf{A}, \Lambda)$ and $\mathscr{B} = (\mathbf{B}, \Psi)$ in X. Then

$$\begin{aligned} &\lambda_T(x) \notin \left(A_T^-(x), A_T^+(x)\right), \ \psi_T(x) \notin \left(B_T^-(x), B_T^+(x)\right), \\ &B_T^-(x) \leq \lambda_T(x) \leq B_T^+(x), \ A_T^-(x) \leq \psi_T(x) \leq A_T^+(x) \end{aligned}$$

for all $x \in X$. We now consider the following cases.

(1) $\lambda_T(x) \leq A_T^-(x) \leq \psi_T(x) \leq A_T^+(x)$ and $\psi_T(x) \leq B_T^-(x) \leq \lambda_T(x) \leq B_T^+(x)$.

(2) $A_T^-(x) \leq \psi_T(x) \leq A_T^+(x) \leq \lambda_T(x)$ and $B_T^-(x) \leq \lambda_T(x) \leq B_T^+(x) \leq \psi_T(x)$.

(3) $\lambda_T(x) \leq A_T^-(x) \leq \psi_T(x) \leq A_T^+(x)$ and $B_T^-(x) \leq \lambda_T(x) \leq B_T^+(x) \leq \psi_T(x)$.

(2) $A_T^-(x) \leq \psi_T(x) \leq A_T^+(x) \leq \lambda_T(x)$ and $\psi_T(x) \leq B_T^-(x) \leq \lambda_T(x) \leq B_T^+(x)$.

First case implies that $\psi_T(x) = A_T^-(x) = B_T^-(x) = \lambda_T(x)$. Since $\mathscr{A}^* := (\mathbf{A}, \Psi)$ and $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ are T-internal neutrosophic cubic sets in X, we have $\psi_T(x) \leq A_T^+(x)$ and $\lambda_T(x) \leq B_T^+(x)$. It follows that

$$\begin{aligned}(A_T \cup B_T)^-(x) &= \max\{A_T^-(x), B_T^-(x)\} = (\lambda_T \vee \psi_T)(x) \\ &\leq \max\{A_T^+(x), B_T^+(x)\} = (A_T \cup B_T)^+(x)\end{aligned}$$

for all $x \in X$. Therefore the P-union $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ is a T-internal neutrosophic cubic set in X. We can prove the other cases by the similar to the first case. □

Similarly, we have the following theorems.

Theorem 3.16. *For any I-external neutrosophic cubic sets* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *in* X, *if* $\mathscr{A}^* := (\mathbf{A}, \Psi)$ *and* $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ *are I-internal neutrosophic cubic sets in* X, *then the P-union* $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ *of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is an I-internal neutrosophic cubic set in* X.

Theorem 3.17. *For any F-external neutrosophic cubic sets* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *in* X, *if* $\mathscr{A}^* := (\mathbf{A}, \Psi)$ *and* $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ *are F-internal neutrosophic cubic sets in* X, *then the P-union* $\mathscr{A} \cup_P \mathscr{B} = (\mathbf{A} \cup \mathbf{B}, \Lambda \vee \Psi)$ *of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is a F-internal neutrosophic cubic set in* X.

We provide conditions for the P-intersection of two T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-internal (resp. I-internal and F-internal) neutrosophic cubic set.

Theorem 3.18. *For any T-external (resp., I-external and F-external) neutrosophic cubic sets* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *in* X, *if* $\mathscr{A}^* := (\mathbf{A}, \Psi)$ *and* $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ *are T-internal (resp., I-internal and F-internal) neutrosophic cubic sets in* X, *then the P-intersection* $\mathscr{A} \cap_P \mathscr{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \wedge \Psi)$ *of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is a T-internal (resp., I-internal and F-internal) neutrosophic cubic set in* X.

Proof. It is similar to the proof of Theorem 3.15. □

Corollary 3.19. *For any external neutrosophic cubic sets* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *in* X, *if* $\mathscr{A}^* := (\mathbf{A}, \Psi)$ *and* $\mathscr{B}^* := (\mathbf{B}, \Lambda)$ *are internal neutrosophic cubic sets in* X, *then the P-intersection* $\mathscr{A} \cap_P \mathscr{B} = (\mathbf{A} \cap \mathbf{B}, \Lambda \wedge \Psi)$ *of* $\mathscr{A} = (\mathbf{A}, \Lambda)$ *and* $\mathscr{B} = (\mathbf{B}, \Psi)$ *is an internal neutrosophic cubic set in* X.

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