Pentapartitioned Neutrosophic Pythagorean Topological Spaces

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Abstract- The aim of this paper is to introduce the new concept of Penta partitioned neutrosophic Pythagorean topological space and discussed some of its properties.

Keywords –Penta partitioned Neutrosophic set, Penta partitioned neutrosophic topological space, Pentapartitioned Neutrosophic Pythagorean topological space

I. INTRODUCTION

The fuzzy set was introduced by Zadeh [13] in 1965. The concept of Neutrosophic set was introduced by F. Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminancy and inconsistent data. Smarandache is proposed neutrosophic set[11]. In neutrosophic sets, the indeterminacy membership function walks along independently of the truth membership or of the falsity membership. Neutrosophic theory has been widely explored by researchers for application purpose in handling real life situations involving uncertainty.

Rama Malik and Surpati Pramanik [7] introduced Pentapartitioned neutrosophic set and its properties. Here indeterminacy is divided into three parts as contradiction, ignorance and unknown membership function.

Also we introduced the concept of Penta partitioned neutrosophic Pythagorean set [4]t and establish some of its properties in our previous work. Now we have extended our work in this Pentapartitioned neutrosophic Pythagorean set as a topological space.

II Preliminaries

2.1 Definition

Let X be a non-empty set. A PNS A over X characterizes each element p in X by a truth-membership function T_A , a contradiction membership function C_A, an ignorance membership functionG_A, unknown membership function $U_A$and a falsity membership function F_A ,such that for each $p \in X$,

$$0 \leq T_A + C_A + U_A + G_A + F_A \leq 5$$

2.2 Definition

Let X be a universe. A Pentapartitioned neutrosophic pythagorean set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ $and (T_A)^2 + (C_A)^2$

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

2.3 Definition

A Pentapartitioned neutrosophic pythagorean set A is contained in another Pentapartitioned neutrosophic pythagorean set B (i.e) $A \subseteq B$ if $T_A \leq T_B, C_A \leq C_B, I_A \geq I_B, U_A \geq U_B$ and $F_A \geq F_B$

2.4 Definition

The complement of a Pentapartitioned neutrosophic pythagorean set (F, A) on X denoted by $(F, A)^c$ and is defined as $F^c(x) = \{< x, F_A, U_A, 1 - I_A, C_A, T_A >: x \in X\}$

2.5 Definition

Let X be a non-empty set, $A = < x, T_A, C_A, I_{gukA}, U_A, F_A >$ and $B = < x, T_B, C_B, I_B, U_B, F_B >$ are two Pentapartitioned neutrosophic pythagorean sets. Then

$A \cup B = < x, \max(T_A, T_B), \max(C_A, C_B), \min(I_A, I_B), \min(U_A, U_B), \min(F_A, F_B) >$

$A \cap B = < x, \min(T_A, T_B), \min(C_A, C_B), \max(I_A, I_B), \max(U_A, U_B), \max(F_A, F_B) >$

2.6 Definition

A Pentapartitioned neutrosophic pythagorean set (F, A) over the universe X is said to be empty Pentapartitioned neutrosophic soft set 0_X with respect to the parameter A if

$T_{F(e)} = 0, C_{F(e)} = 0, I_{F(e)} = 1, U_{F(e)} = 1, F_{F(e)} = 1, \forall x \in X, \forall e \in A$. It is denoted by 0_X

2.7 Definition

A Pentapartitioned neutrosophic pythagorean set (F, A) over the universe X is said to be universe Pentapartitioned neutrosophic pythagorean set with respect to the parameter A if

$T_{F(e)} = 1, C_{F(e)} = 1, I_{F(e)} = 0, U_{F(e)} = 0, F_{F(e)} = 0, \forall x \in X, \forall e \in A$. It is denoted by 1_X

III Penta Partitioned Neutrosophic Pythagorean Topological Space

3.1 Definition

A Pentapartitioned Neutrosophic Pythagorean topology on a non-empty set M is a τ of Pentapartitioned Neutrosophic Pythagorean sets satisfying the following axioms.

i) $0_M, 1_M \in \tau$

ii) The union of the elements of any sub collection of τ is in τ

iii) The intersection of the elements of any finite sub collection τ is in τ

The pair (M, τ) is called an Pentapartitioned Neutrosophic Pythagorean Topological Space over M.

3.2 Note

1. Every member of τ is called a PNP open set in M.
2. The set A_M is called a PNP closed set in M if $A_M \in \tau^c$, where $\tau^c = \{{A_M}^c : A_M \in \tau\}$.

3.3 Example

Let M = {b_1, b_2} and Let A_M, B_M, C_M be Penta Partitioned Neutrosophic Pythagorean sets where

$A_M = \{< b_1, 0.5, 0.1, 0.5, 0.7, 0.2 >< b_2, 0.7, 0.5, 0.6, 0.2, 0.1 >< b_3, 0.6, 0.5, 0.8, 0.4, 0.3 >\}$

$B_M = \{< b_1, 0.6, 0.7, 0.6, 0.1, 0.2 >< b_2, 0.2, 0.3, 0.6, 0.4, 0.7 >< b_3, 0.5, 0.6, 0.7, 0.1, 0.3 >\}$

$C_M = \{< b_1, 0.6, 0.7, 0.5, 0.1, 0.2 >< b_2, 0.7, 0.5, 0.6, 0.2, 0.1 >< b_3, 0.6, 0.6, 0.7, 0.1, 0.3 >\}$

$\tau = \{A_M, B_M, C_M, 0_M, 1_M\}$ is an Penta Partitioned Neutrosophic Pythagorean topology on M.

3.4 Preposition

Let (M, τ_1) and (M, τ_2) be two Penta Partitioned Neutrosophic Pythagorean topological space on M,Then$\tau_1 \cap \tau_2$ is an Penta Partitioned Neutrosophic Pythagorean topology on M where $\tau_1 \cap \tau_2 = \{A_M : A_M \in \tau_1$ and $A_M \in \tau_2\}$

Proof :

Obviously $0_M, 1_M \in \tau$.

Let $A_M, B_M \in \tau_1 \cap \tau_2$

Then $A_M, B_M \in \tau_1$ and $A_M, B_M \in \tau_2$

We know that $\tau_1$and τ_2 are two Pentapartitioned Neutrosophic Pythagorean topological space M.

Then $A_M \cap B_M \in \tau_1$ and $A_M \cap B_M \in \tau_2$

Hence $A_M \cap B_M \in \tau_1 \cap \tau_2$.

Let τ_1 and $\tau_2$are two Penta Partitioned Neutrosophic Pythagorean topological spaces on M.

Denote $\tau_1 \vee \tau_2 = \{A_M \cup B_M : A_M \in \tau_1$ and $A_M \in \tau_2\}$

$\tau_1 \wedge \tau_2 = \{A_M \cap B_M : A_M \in \tau_1$ and $A_M \in \tau_2\}$

3.5 Example

Let A_M *and* $B_M$be two Penta Partitioned Neutrosophic Pythagorean topological space on M.

Define $\tau_1 = \{0_M, 1_M, A_M\}$

$$\tau_2 = \{0_M, 1_M, B_M\}$$

Then $\tau_1 \cap \tau_2 = \{0_M, 1_M\}$ is a Penta Partitioned Neutrosophic Pythagorean topological space on M.

But $\tau_1 \cup \tau_2 = \{0_M, A_M, B_M, 1_M\}$,

$$\tau_1 \vee \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cup B_M\} \textit{ and}$$

$\tau_1 \wedge \tau_2 = \{0_M, A_M, B_M, 1_M, A_M \cap B_M\}$ are not Penta Partitioned Neutrosophic Pythagorean topological space on M.

IV Properties of Pentapartitioned Neutrosophic Pythagorean Topological Spaces

4.1 Definition

Let (M, τ) be a Penta Partitioned Neutrosophic Pythagorean topological space on M and let $A_M$belongs to Penta Partitioned Neutrosophic Pythagorean set on M. Then the interior of A_M is denoted as PNPInt (A_M). It is defined by PNPInt $(A_M) = \cup \{B_M \in \tau : B_M \subseteq A_m\}$

4.2 Definition

Let (M, τ) be a Penta Partitioned Neutrosophic Pythagorean topological space on M and let $A_M$belongs to Penta Partitioned Neutrosophic Pythagorean set M. Then the closure of A_M is denoted as PNPCl (A_M). It is defined by PNPCl $(A_M) = \cap \{B_M \in \tau^C : A_M \subseteq B_m$

4.3 Theorem

Let (M, τ) be a Penta Partitioned Neutrosophic Pythagorean topological space over M. Then the following properties are hold.

i) 0_M *and* 1_M are Penta Partitioned Neutrosophic Pythagorean closed sets over M

ii) The intersection of any number of Penta Partitioned Neutrosophic Pythagorean closed set is a Penta

Partitioned Neutrosophic Pythagorean closed set over M.

iii) The union of any two Penta Partitioned Neutrosophic Pythagorean closed set is an Penta Partitioned Neutrosophic Pythagorean closed set over M.

Proof

It is obviously true.

4.4 Theorem

Let (M, τ) be a be a Penta Partitioned Neutrosophic Pythagorean topological space over M and Let A_M ∈Penta Partitioned Neutrosophic Pythagorean topological space.Then the following properties hold.

(i) PNPInt $(A_M) \subseteq A_M$

(ii) $A_M \subseteq B_M$ implies PNPInt $(A_M) \subseteq$ PNPInt (B_M).

(iii) PNPInt$(A_M) \in \tau$.

(iv) A_M is a PNP open set implies PNPInt $(A_M) = A_M$.

(v) PNPInt (PNPInt (A_M)) = PNPInt(A_M)

(vi) PNPInt $(0_M) = 0_M$, PNPInt $(1_M) = 1_M$.

Proof:

(i) and (ii) are obviously true.

(iii) obviously $\cup \{B_M \in \tau : B_M \subseteq A_m\} \in \tau$

Note that $\cup \{B_M \in \tau : B_M \subseteq A_m\}$= PNPInt (A_M)

∴ PNPInt $(A_M) \in \tau$

(iv) Necessity: Let A_M be a PNP open set. ie.,$A_M \in \tau$. By (i) and (ii) PNPInt $(A_M) \subseteq A_m$.

Since $A_M \in \tau$ and $A_M \subseteq A_m$

Then $A_M \subseteq \cup \{B_M \in \tau : B_M \subseteq A_m\}$= QNSInt (A_M)

$A_M \subseteq$PNPInt (A_M)

Thus PNPInt = A_m.

Sufficiency: Let PNPInt $(A_m) = A_m$

By (iii) PNPInt $(A_m) \in \tau$, ie., A_m is a PNP open set.

(v) To prove PNPInt (PNPInt (A_m)) = PNPInt(A_m)

By (iii) PNPInt $(A_m) \in \tau$.

By (iv) PNPInt (PNPInt (A_m)) = PNPInt (A_m).

(vi) We know that $0_M$and 1_M are in τ

By (iv) PNPInt $(0_M) = 0_M$, PNPInt $(1_M) = 1_M$.Hence the result.

4.5 Theorem

Let (M, τ) be a be a Penta Partitioned Neutrosophic Pythagorean topological space over M and Let A_M is in the Penta Partitioned Neutrosophic Pythagorean topological space.Then the following properties hold.

(i) $A_M \subseteq$ PNPCl(A_M)

(ii) $A_M \subseteq B_M$ implies PNPCl (A_M) $\subseteq$ QNSCl (B_M).

(iii) PNPCl$(A_M)^c \in \tau$.

(iv) A_M is a PNP closed set implies PNPCl (A_M) =A_M.

(v) PNPCl (PNPCl (A_M)) = PNPCl(A_M)

(vi) PNPCl (0_M) = 0_M, PNPCl (1_M) =1_M.

Proof:

(i) and (ii) are obviously true.

(iii) By theorem, PNPInt ($A_M{}^c$) $\in \tau$.

Therefore PNPCl (A_M)]^c = ($\cap$ {$B_M \in \tau^c$: $B_M \subseteq A_m$})^c

=$\cup$ {$B_M \in \tau$: $B_M \subseteq A_m{}^c$}= PNPInt ($A_M{}^c$)

$\therefore$ [PNPCl (A_M)]$^c \in \tau$

(iv) Necessity:

By theorem, $A_M \subseteq$ PNPCl(A_M)

Let A_M be a PNP closed set. ie.,$A_M \in \tau^c$.

Since $A_M \in \tau and A_M \subseteq A_m$

PNPCl (A_M)=$\cap$ {$B_M \in \tau^c$: $A_M \subseteq B_m$} $\subseteq$ {$B_M \in \tau^c$: $A_M \subseteq A_m$}

PNPCl (A_M) $\subseteq A_m$

Thus A_m =PNPCl(A_m)

Sufficiency: This is obviously true by (iii)

(v) and (vi) can be proved by (iii) and (iv)

4.6 Theorem

Let (M, τ) be a be a Penta Partitioned Neutrosophic Pythagorean topological space over M and Let A_M, B_M are in Penta Partitioned Neutrosophic Pythagorean topological space M.Then the following properties hold.

(i) PNPInt (A_M) $\cap$ PNPInt (B_M) = PNPInt ($A_M \cap B_M$)

(ii) PNPInt (A_M) $\cup$ QNSInt (B_M) $\subseteq$ PNPInt ($A_M \cup B_M$)

(iii) PNPCl (A_M) $\cup$ QNSCl (B_M) $\subseteq$ PNPCl ($A_M \cup B_M$)

(iv) PNPCl ($A_M \cup B_M$) $\subseteq$ PNPCl (A_M) $\cap$ PNPCl (B_M)

(v) (PNPInt (A_M))^c = PNPCl ($A_M{}^c$)

(vi) (PNPCl (A_M))^c = PNPInt ($A_M{}^c$)

Proof:

(i) Since $A_M \cap B_M \subseteq A_m$ for any m in M

By theorem, PNPInt ($A_M \cap B_M$) $\subseteq$ PNPInt (A_M)

Similarly, PNPInt ($A_M \cap B_M$) $\subseteq$ PNPInt (B_M)

PNPInt ($A_M \cap B_M$) $\subseteq$ PNPInt (A_M) $\cap$ PNPInt(B_M)

By theorem, PNPInt (A_M) $\subseteq A_M$ and PNPInt (B_M) $\subseteq B_M$

Thus PNPInt ($A_M \cap B_M$) $\subseteq A_M \cap B_M$

Therefore, PNPInt (A_M) $\cap$ PNPInt (B_M) = PNPInt ($A_M \cap B_M$)

Similarly we can prove (ii),(iii) and (iv).

v) (PNPInt (A_M))^c = ($\cap$ $\{B_M \in \tau : B_M \subseteq A_m\}$)^c

=$\cap$ $\{B_M \in \tau^c : A_M{}^c \subseteq B_m\}$

= PNPCl ($A_M{}^c$)

Similarly we can prove (vi)

4.7 Example

Let M = {b_1, b_2} and Let A_M, B_M, C_M be Penta Partitioned Neutrosophic Pythagorean where

$A_M = \{< b_1, 0.3,0.3,0.2,0.1,0.3 >< b_2, 0.6,0.4,0.2,0.3,0.1 >\}$

$B_M = \{< b_1, 0.2,0.3,0.5,0.1,0.5 >< b_2, 0.6,0.5,0.2,0.3,0.2 >\}$

$C_M = \{< b_1, 0.3,0.3,0.2,0.3,0.3 >< b_2, 0.6,0.5,0.2,0.3,0.1 >\}$

$\tau = \{A_M, B_M, C_M, 0_M, 1_M\}$ is an Penta Partitioned Neutrosophic Pythagorean topology on M.

i) PNPInt (A_M) = 0_M= PNPInt(B_M)

Then $A_M \cup B_M = C_M$

PNPInt (A_M) $\cup$ PNPInt (B_M) =$0_M \cup 0_M$=0_M

And PNPInt ($A_M \cup B_M$) = PNPInt (C_M)=C_M

PNPInt (A_M) $\cup$ PNPInt (B_M) $\neq$ PNPInt ($A_M \cup B_M$)

ii) PNPCl (B_M)^c = (PNPCl (B_M))^c = $0_M{}^c = 1_M$

Similarly, PNPCl (C_M)^c= X_M

PNPCl (A_M)^c$\cap$ PNPCl (B_M)^c= $1_M \cap 1_M = 1_M$

Similarly, PNPCl ($A_M{}^c \cap B_M{}^c$) = PNPCl ($A_M \cap B_M$)^c

= PNPInt ($A_M \cup B_M$)^c

= $C_M{}^c$

PNPCl ($A_M{}^c \cap B_M{}^c$) $\neq$PNPCl (A_M)^c$\cap$ (PNPCl (B_M)]^c

V.CONCLUSION

In this paper, we have studied the properties of Pentapartitioned Neutrosophic Pythagorean topological space and in future I have extended the concept to heptapartitioned neutrosophic topological space.

REFERENCES

[1] I. Arockiarani, R. Dhavaseelan, S. Jafari, M. Parimala, On some notations and functions in neutrosophic topological spaces, Neutrosophic sets and systems

[2] I. Arockiarani, I.R. Sumathi and J, Martina Jency, Fuzzy neutrosophic soft topological spaces, IJMA-4[10], oct-2013.

[3] R. Radha, A. Stanis Arul Mary, F. Smarandache, Quadripartitioned Neutrosophic pythagorean soft set with T, C, U and F as dependent components (submitted)

[4] R. Radha, A. Stanis Arul Mary, Pentapartitioned Neutrosophic pythagorean set with T, C, U and F as dependent components (Accepted)

[5] R. Radha, A. Stanis Arul Mary, Heptapartitioned neutrosophic sets, IRJCT, volume 2,222-230

[6] R. Radha, A. Stanis Arul Mary, Quadripartitioned Neutrosophic pythagorean soft set with dependent components (communicated)

[7] Rama Malik, Surapati Pramanik, Pentapartitioned Neutrosophic set and its properties, Neutrosophic Sets and Systems, Vol 36,2020

[8] A. Salama and S. A. Al – Blowi, Neutrosophic Set and Neutrosophic topological spaces, IOSR Journal of Math., Vol. (3) ISSUE4 (2012), 31 – 35

[9] F. Smarandache, Degree of Dependence and independence of the sub components of fuzzy set and neutrosophic set, Neutrosophic sets and systems, vol 11,2016 95-97

[10] F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA (2002).

[11] F. Smarandache, Neutrosophic set, A generialization of the intuituionistics fuzzy sets, Inter. J. Pure Appl. Math., 24 (2005), 287 – 297.

[12] Xindong Peng, Yong Yang, Some results for pythagorean fuzzy sets, International Journal of Intelligent systems,30(2015).1133-1160.

[13] L. A. Zadeh, Fuzzy Sets, Inform and Control 8(1965) 338 – 353.

[14] Zhaowen Li, Rongchen cui, On the topological structure of intuitionistic fuzzy soft sets, Annals of Fuzzy Mathematics and Informatics, Volume 5, No.1, (2013),229-239.