



Pythagorean neutrosophic Subring of a ring

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Abstract

In this paper, we study some of the properties of Pythagorean neutrosophic subring of a ring and prove some results on these. Using some basic definitions, we derive some important theorems. Intersection is applied into the Pythagorean neutrosophic subring of a ring.

Key words: Pythagorean neutrosophic set, Pythagorean neutrosophic subring.

1. Introduction

Fuzzy sets were introduced by Zadeh [19] and he discussed only membership function. After the extensions of fuzzy set theory Atanassov [6] generalized this concept and introduced a new concept called intuitionistic fuzzy set (IFS). Yager [16] familiarized the model of Pythagorean fuzzy set. IFS has its greatest use in practical multiple attribute decision making (MADM) problems, and the academic research have achieved great development [16,17,18]. However, in some practical problems, the sum of membership degree and non-membership degree to which an alternative satisfying attribute provided by decision maker (DM) may be bigger than 1, but their square sum is less than or equal to 1. Azriel Rosenfeld [7] was studied about fuzzy rings.

In 2006, F. Smarandache introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponents [13]. There are three special cases in neutrosophic set. First, truth membership, falsity membership, indeterminacy are independent. Second, truth membership and falsity membership are dependent and indeterminacy is independent. Third, truth membership, falsity

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membership, indeterminacy are independent. We studied about second case. Jansi, Mohana and F.Smarandache[11] was introduced the concept of Pythagorean neutrosophic set [PN-set]. That is, if truth membership and falsity membership are dependent and indeterminacy is independent under the restriction that the sum of truth membership, falsity membership and indeterminacy does not exceed 2. Sometimes, we face many problems which cannot be handled by using this set, for example when $T = 0.8, I = 0.9, F = 0.4$. T and F are dependent that condition was $T + F \leq 1$. Here $T + F \geq 1$. Totally, the sum of truth membership, falsity membership and indeterminacy does exceed 2. We cannot use that set. At that time we use PN-set. Also PN-set includes truth membership, falsity membership and indeterminacy but under the restriction their square sum of truth membership, falsity membership and indeterminacy does not exceed 2. That is, $T^2 + I^2 + F^2 \leq 2$. We introduce the concept of Pythagorean neutrosophic subring and established some results.

2. Preliminaries

Definition 3.1 [19] Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{(x : \mu_A(x)) : x \in X\}$, where $\mu_A : X \rightarrow [0, 1]$ is the membership function of the fuzzy set A .

Definition 3.2 (Pythagorean Fuzzy Set)[16] Let X be a non-empty set and I the unit interval $[0, 1]$. A PF set S is an object having the form $P = \{(x, \mu_P(x), \nu_P(x)) : x \in X\}$ where the function $\mu_P : X \rightarrow [0, 1]$ and $\nu_P : X \rightarrow [0, 1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P , and $0 \leq (\mu_P(x))^2 + (\nu_P(x))^2 \leq 1$ for each $x \in X$.

Definition 3.3 (Pythagorean neutrosophic set [PN]-sets)[11] Let X be a non-empty set (universe). A PN sets $A = \{(x, P_A(x), Q_A(x), R_A(x)) / x \in X\}$ where $P_A : X \rightarrow [0, 1], Q_A : X \rightarrow [0, 1]$ and $R_A : X \rightarrow [1, 0]$ are the mappings such that $0 \leq (P_A(x))^2 + (Q_A(x))^2 + (R_A(x))^2 \leq 2$ and $P_A(x)$ denote the membership degree, $Q_A(x)$ denote the Indeterminacy and $R_A(x)$ denote the non-membership degree. Here T and F are dependent neutrosophic components and I is an independent neutrosophic components.

3. PN Subring of a Ring

Definition 4.1 Let $A = \{(X, P_A(x), Q_A(x), R_A(x)) / x \in X\}$ and

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$B = \{(X, P_B(x), Q_B(x), R_B(x))/x \in X\}$ be two PN sets, then their operations are defined as follows:

(1) Type 1:

$$A \cup B = \{(x, \max(P_A(x), P_B(x)), \min(Q_A(x), Q_B(x)), \min(R_A(x), R_B(x))) : x \in X\}$$

Type 2:

$$A \cup B = \{(x, \max(P_A(x), P_B(x)), \max(Q_A(x), Q_B(x)), \min(R_A(x), R_B(x))) : x \in X\}$$

(2) Type 1

$$:A \cap B = \{(x, \min(P_A(x), P_B(x)), \max(Q_A(x), Q_B(x)), \max(R_A(x), R_B(x))) : x \in X\}$$

Type 2

$$:A \cap B = \{(x, \min(P_A(x), P_B(x)), \min(Q_A(x), Q_B(x)), \max(R_A(x), R_B(x))) : x \in X\}.$$

Definition 4.2 Let $A = \{(X, P_A(x), Q_A(x), R_A(x))/x \in X\}$ be PN set, then the complement of A is

$$\text{Type 1: } A^c = \{(x, R_A(x), 1 - Q_A(x), P_A(x)) : x \in X\}.$$

$$\text{Type 2: } A^c = \{(x, R_A(x), Q_A(x), P_A(x)) : x \in X\}.$$

Definition 4.3 Let $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in X\}$ and

$B = \{(x, P_B(x), Q_B(x), R_B(x))/x \in X\}$ be two PN sets, then $A \subseteq B$ if and only if $P_A(x) \leq P_B(x), Q_A(x) \geq Q_B(x), R_A(x) \geq R_B(x)$.

and $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Definition 4.4 Let $(R, +, \cdot)$ be a ring. A PN subset A of R is said to be a PN subring of R if the following conditions are satisfied

$$(i) P_A(x + y) \geq \min \{P_A(x), P_A(y)\}$$

$$(ii) P_A(xy) \geq \min \{P_A(x), P_A(y)\}$$

$$(iii) Q_A(x + y) \leq \max \{Q_A(x), Q_A(y)\}$$

$$(iv) Q_A(xy) \leq \max \{Q_A(x), Q_A(y)\}$$

$$(v) R_A(x + y) \leq \max \{R_A(x), R_A(y)\}$$

$$(vi) R_A(xy) \leq \max \{R_A(x), R_A(y)\}$$

for all x and y in R.

Theorem 4.5 Let $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of a

ring R. Then $P_A(-x) = P_A(x), Q_A(-x) = Q_A(x), R_A(-x) = R_A(x)$

, $P_A(x) \leq P_A(e), Q_A(x) \geq Q_A(e), R_A(x) \geq R_A(e)$ for all x in R and the identity element e in R.

Proof: Let x be in R. Now $P_A(x) = P_A(-(-x)) \geq P_A(-x) \geq P_A(x)$.

Therefore $P_A(x) = P_A(-x)$ for all x in R .
 $Q_A(x) = Q_A(-(-x)) \leq Q_A(-x) \leq Q_A(x)$.
Therefore $Q_A(x) = Q_A(-x)$ for all x in R .
 $R_A(x) = R_A(-(-x)) \leq R_A(-x) \leq R_A(x)$.
Therefore $R_A(x) = R_A(-x)$ for all x in R .
Also, $P_A(e) = P_A(x - x) \geq \min \{P_A(x), P_A(x)\} = P_A(x)$.
Therefore $P_A(e) \geq P_A(x)$ for all x in R .
 $Q_A(e) = Q_A(x - x) \leq \max \{Q_A(x), Q_A(x)\} = Q_A(x)$.
Therefore $Q_A(e) \leq Q_A(x)$ for all x in R .
 $R_A(e) = R_A(x - x) \leq \max \{R_A(x), R_A(x)\} = R_A(x)$.
Therefore $R_A(e) \leq R_A(x)$ for all x in R .

Theorem 4.6 Let $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of a ring R . Then

- (i) $P_A(x + y) = P_A(e)$ implies that $P_A(x) = P_A(y)$ for x and y in R
- (ii) $Q_A(x + y) = Q_A(e)$ implies that $Q_A(x) = Q_A(y)$ for x and y in R .
- (iii) $R_A(x + y) = R_A(e)$ implies that $R_A(x) = R_A(y)$ for x and y in R .

Proof: Now

$$P_A(x) = P_A(x + y - y) \geq \min \{P_A(x + y), P_A(y)\} = \min \{P_A(e), P_A(y)\} = P_A(y).$$
$$P_A(y) = P_A(y + x - x) \geq \min \{P_A(y + x), P_A(x)\} = \min \{P_A(e), P_A(x)\} = P_A(x).$$

Therefore $P_A(x) = P_A(y)$ for x and y in R .

$$Q_A(x) = Q_A(x + y - y) \leq \max \{Q_A(x + y), Q_A(y)\} = \max \{Q_A(e), Q_A(y)\}$$
$$= Q_A(y).$$

$$Q_A(y) = Q_A(y + x - x) \leq \max \{Q_A(y + x), Q_A(x)\} = \max \{Q_A(e), Q_A(x)\}$$
$$= Q_A(x).$$

Therefore $Q_A(x) = Q_A(y)$ for x and y in R .

$$R_A(x) = R_A(x + y - y) \leq \max \{R_A(x + y), R_A(y)\} = \max \{R_A(e), R_A(y)\}$$
$$= R_A(y).$$

$$R_A(y) = R_A(y + x - x) \leq \max \{R_A(y + x), R_A(x)\} = \max \{R_A(e), R_A(x)\}$$
$$= R_A(x).$$

Therefore $R_A(x) = R_A(y)$ for x and y in R .

Theorem 4.7 Let $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of a ring R .

- (i) If $P_A(x + y) = 1$, then $P_A(x) = P_A(y)$ for x and y in R .
- (ii) If $Q_A(x + y) = 0$, then $Q_A(x) = Q_A(y)$ for x and y in R .
- (iii) If $R_A(x + y) = 0$, then $R_A(x) = R_A(y)$ for x and y in R .

Proof: Now $P_A(x) = P_A(x + y - y) \geq \min \{P_A(x + y), P_A(y)\} = \min \{1, P_A(y)\} = P_A(y) = P_A(-y) = P_A(x - x - y) \geq \min \{P_A(x), P_A(x + y)\} = \min \{P_A(x), 1\} = P_A(x)$.

Therefore $P_A(x) = P_A(y)$ for x and y in R .

Hence (i) is proved.

$Q_A(x) = Q_A(x + y - y) \leq \max \{Q_A(x + y), Q_A(y)\} = \max \{0, Q_A(y)\} = Q_A(y) = Q_A(-y) = Q_A(x - x - y) \leq \max \{Q_A(x), Q_A(x + y)\} = \max \{Q_A(x), 0\} = Q_A(x)$.

Therefore $Q_A(x) = Q_A(y)$ for x and y in R .

Hence (ii) is proved.

$R_A(x) = R_A(x + y - y) \leq \max \{R_A(x + y), R_A(y)\} = \max \{0, R_A(y)\} = R_A(y) = R_A(-y) = R_A(x - x - y) \leq \max \{R_A(x), R_A(x + y)\} = \max \{R_A(x), 0\} = R_A(x)$.

Therefore $R_A(x) = R_A(y)$ for x and y in R .

Hence (iii) is proved.

Hence (iv) is proved.

Theorem 4.8 Let $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of a ring R .

- (i) $P_A(xy^{-1}) = 0$, then either $P_A(x) = 0$ or $P_A(y) = 0$, for x and y in R .
- (ii) $Q_A(xy^{-1}) = 0$, then either $Q_A(x) = 0$ or $Q_A(y) = 0$, for x and y in R .
- (iii) $R_A(xy^{-1}) = 0$, then either $R_A(x) = 0$ or $R_A(y) = 0$, for x and y in R .
- (iv) $(xy^{-1}) = 0$, then either $(x) = 0$ or $(y) = 0$, for x and y in R .

Proof: Let x and y in R .

(i) By the definition $P_A(xy^{-1}) \geq \min \{P_A(x), P_A(y)\}$, which implies that $0 \geq \min \{P_A(x), P_A(y)\}$.

Therefore, either $P_A(x) = 0$ or $P_A(y) = 0$.

(ii) By the definition $Q_A(xy^{-1}) \leq \max \{Q_A(x), Q_A(y)\}$, which implies that $0 \leq \max \{Q_A(x), Q_A(y)\}$.

Therefore, either $Q_A(x) = 0$ or $Q_A(y) = 0$.

(iii) By the definition $R_A(xy^{-1}) \leq \max \{R_A(x), R_A(y)\}$, which implies that $0 \leq \max \{R_A(x), R_A(y)\}$.

Therefore, either $R_A(x) = 0$ or $R_A(y) = 0$.

Theorem 4.9 If $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of R , then

- (i) $P_A(xy) = P_A(yx)$ if and only if $P_A(x) = P_A(y^{-1}xy)$, for x and y in R .
- (ii) $Q_A(xy) = Q_A(yx)$ if and only if $Q_A(x) = Q_A(y^{-1}xy)$, for x and y in R .
- (iii) $R_A(xy) = R_A(yx)$ if and only if $R_A(x) = R_A(y^{-1}xy)$, for x and y in R .

Proof: Let x and y be in R .

Assume that $P_A(xy) = P_A(yx)$, so, $P_A(y^{-1}xy) = P_A(y^{-1}yx) = P_A(x)$.

Therefore $P_A(x) = P_A(y^{-1}xy)$, for x and y in R .

Conversely, assume that $P_A(x) = P_A(y^{-1}xy)$,

we get, $P_A(xy) = P_A(xyxx^{-1}) = P_A(yx)$.

Therefore $P_A(xy) = P_A(yx)$, for x and y in R .

Hence $P_A(xy) = P_A(yx)$ if and only if $P_A(x) = P_A(y^{-1}xy)$, for x and y in R .

Also assume that $Q_A(xy) = Q_A(yx)$, so, $Q_A(y^{-1}xy) = Q_A(y^{-1}yx) = Q_A(x)$.

Therefore $Q_A(x) = Q_A(y^{-1}xy)$, for x and y in R .

Conversely, assume that $Q_A(x) = Q_A(y^{-1}xy)$,

we get, $Q_A(xy) = Q_A(xyxx^{-1}) = Q_A(yx)$.

Therefore $Q_A(xy) = Q_A(yx)$, for x and y in R .

Hence $P_A(xy) = P_A(yx)$ if and only if $Q_A(x) = Q_A(y^{-1}xy)$, for x and y in R .

Also assume that $R_A(xy) = R_A(yx)$, so, $R_A(y^{-1}xy) = R_A(y^{-1}yx) = R_A(x)$.

Therefore $R_A(x) = R_A(y^{-1}xy)$, for x and y in R .

Conversely, assume that $R_A(x) = R_A(y^{-1}xy)$,

we get, $R_A(xy) = R_A(xyxx^{-1}) = R_A(yx)$.

Therefore $R_A(xy) = R_A(yx)$, for x and y in R .

Hence $R_A(xy) = R_A(yx)$ if and only if $R_A(x) = R_A(y^{-1}xy)$, for x and y in R .

Theorem 4.10 If $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of R , then

$R_2 = \{x \in R/P_A(x) = P_A(e), Q_A(x) = Q_A(e), R_A(x) = R_A(e)\}$ is a subring of R .

Proof: Here $R_2 = \{x \in R/P_A(x) = P_A(e), Q_A(x) = Q_A(e), R_A(x) = R_A(e)\}$, by Theorem 2.4.4,

$P_A(x^{-1}) = P_A(x) = P_A(e)$, $Q_A(x^{-1}) = Q_A(x) = Q_A(e)$, $R_A(x^{-1}) = R_A(x) = R_A(e)$ and $(x^{-1})^{-1} = (x) = (e)$.

Therefore $x^{-1} \in R_2$.

Now, $P_A(xy^{-1}) \geq \min \{P_A(x), P_A(y)\} = \min \{P_A(e), P_A(e)\} = P_A(e)$, and

$P_A(e) = P_A((xy^{-1})(xy^{-1})^{-1}) \geq \min \{P_A(xy^{-1}), P_A(xy^{-1})\} = P_A(xy^{-1})$.

Hence $P_A(e) = P_A(xy^{-1})$.

$Q_A(xy^{-1}) \leq \max \{Q_A(x), Q_A(y)\} = \max \{Q_A(e), Q_A(e)\} = Q_A(e)$,

and $Q_A(e) = Q_A((xy^{-1})(xy^{-1})^{-1}) \leq \max \{Q_A(xy^{-1}), Q_A(xy^{-1})\} = Q_A(xy^{-1})$.

Hence $Q_A(e) = Q_A(xy^{-1})$.

$R_A(xy^{-1}) \leq \max \{R_A(x), R_A(y)\} = \max \{R_A(e), R_A(e)\} = R_A(e)$, and

$R_A(e) = R_A((xy^{-1})(xy^{-1})^{-1}) \leq \max \{R_A(xy^{-1}), R_A(xy^{-1})\}$

$= R_A(xy^{-1})$.

Hence $R_A(e) = R_A(xy^{-1})$.

$$(xy^{-1}) \geq \min \{(x), (y)\} = \min \{(e), (e)\} = (e).$$

Theorem 4.11 Let R be a ring, If $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ be a PN subring of R , then $P_A(xy) = \min \{P_A(x), P_A(y)\}$, $Q_A(xy) = \max \{Q_A(x), Q_A(y)\}$, $R_A(xy) = \max \{R_A(x), R_A(y)\}$ for each x, y in R with $P_A(x) \neq P_A(y)$, $Q_A(x) \neq Q_A(y)$, $R_A(x) \neq R_A(y)$.

Proof: Assume that $P_A(x) > P_A(y)$, $Q_A(x) > Q_A(y)$, $R_A(x) > R_A(y)$ and $Q_A(x) > Q_A(y)$.

Then

$$P_A(y) = P_A(x^{-1}xy) \geq \min \{P_A(x^{-1}), P_A(xy)\} = \min \{P_A(x), P_A(xy)\} \\ = P_A(xy) \geq \min \{P_A(x), P_A(y)\} = P_A(y).$$

Therefore $P_A(xy) = P_A(y) = \min \{P_A(x), P_A(y)\}$.

$$Q_A(y) = Q_A(x^{-1}xy) \leq \max \{Q_A(x^{-1}), Q_A(xy)\} = \max \{Q_A(x), Q_A(xy)\} \\ = Q_A(xy) \leq \max \{Q_A(x), Q_A(y)\} = Q_A(y).$$

Therefore $Q_A(xy) = Q_A(y) = \max \{Q_A(x), Q_A(y)\}$.

$$R_A(y) = R_A(x^{-1}xy) \leq \max \{R_A(x^{-1}), R_A(xy)\} = \max \{R_A(x), R_A(xy)\} \\ = R_A(xy) \leq \max \{R_A(x), R_A(y)\} = R_A(y).$$

Therefore $R_A(xy) = R_A(y) = \max \{R_A(x), R_A(y)\}$.

Theorem 4.12 If $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ and $B = \{(x, P_B(x), Q_B(x), R_B(x))/x \in R\}$ are two PN subring of a ring R , then their intersection $A \cap B$ is a PN subring of R .

Proof: Let $A = \{(x, P_A(x), Q_A(x), R_A(x))/x \in R\}$ and

$$B = \{(x, P_B(x), Q_B(x), R_B(x)) : x \in R\}.$$

Let $C = A \cap B$ and $C = \{(x, P_C(x), Q_C(x), R_C(x))/x \in R\}$.

Now,

$$P_C(xy^{-1}) = \min \{P_A(xy^{-1}), P_B(xy^{-1})\} \geq \\ \min \{\min \{P_A(x), P_A(y)\}, \min \{P_B(x), P_B(y)\}\} \\ \geq \min \{\min \{P_A(x), P_B(x)\}, \min \{P_A(y), P_B(y)\}\} = \min \{P_C(x), P_C(y)\}.$$

Also,

$$Q_C(xy^{-1}) = \max \{Q_A(xy^{-1}), Q_B(xy^{-1})\} \leq \\ \max \{\max \{Q_A(x), Q_A(y)\}, \max \{Q_B(x), Q_B(y)\}\} \\ \leq \max \{\max \{Q_A(x), Q_B(x)\}, \max \{Q_A(y), Q_B(y)\}\} = \max \{Q_C(x), Q_C(y)\}.$$

$$R_C(xy^{-1}) = \max \{R_A(xy^{-1}), R_B(xy^{-1})\} \leq \\ \max \{\max \{R_A(x), R_A(y)\}, \max \{R_B(x), R_B(y)\}\} \\ \leq \max \{\max \{R_A(x), R_B(x)\}, \max \{R_A(y), R_B(y)\}\} = \max \{R_C(x), R_C(y)\}.$$

Hence $A \cap B$ is a PN subring of R .

4. Conclusion

In this paper, we define the Pythagorean neutrosophic subring of a ring and investigate the relationship among these Pythagorean neutrosophic subring of a ring. Some characterization theorems of Pythagorean neutrosophic subring of a ring are obtained.

References

- [1] Ajay D, Said Broumi and Aldring J, An MCDM Method under Neutrosophic Cubic Fuzzy Sets with Geometric Bonferroni Mean Operator, *Neutrosophic Sets and Systems*, 32, 2020, 187-202. DOI: <http://dx.doi.org/10.5281/zenodo.3723621>
- [2] Ajay D, Aldring J, Nivetha S, Neutrosophic Cubic Fuzzy Dombi Hamy Mean Operators with Application to Multi-Criteria Decision Making, *Neutrosophic Sets and Systems*, 38, 2020, 293-316. DOI: <http://dx.doi.org/10.5281/zenodo.4300516>
- [3] Ajay D, Aldring J, A Decision Making Technique Based on Similarity Measure and Entropy of Bipolar Neutrosophic Sets, *The International journal of analytical and experimental modal analysis*, XI(IX), 2019, 520-529.
- [4] Ajay D, Aldring J, Seles Martina DJ, Abirami S, A SVTrN-number approach of multi objective optimisation on the basis of simple ratio analysis based on MCDM method, *International Journal of Neutrosophic Science*, 5(1), 2020, 16-28. <https://doi.org/10.5281/zenodo.3788321>
- [5] Ajay D, Chellamani P, Pythagorean Neutrosophic Fuzzy Graphs, *International Journal of Neutrosophic Science (IJNS)*, 11(2), 2020, 108-114. DOI: [10.5281/zenodo.4172124](https://doi.org/10.5281/zenodo.4172124)
- [6] Atanassov KT, Intuitionistic fuzzy sets, *Fuzzy Sets Syst*, 20, 1986, 87-96.
- [7] Azriel Rosenfeld, Fuzzy Rings, *Journal of Mathematical analysis and applications*, 35 (1971), 512-517.
- [8] Bosc P, Pivert O, On a fuzzy bipolar relational algebra, *Information Sciences*, 219 (2013), 1-16.
- [9] Carlos Granados, Pythagorean Neutrosophic Pre-Open Sets. *MathLAB Journal*, 6 (2020), 65-74.
- [10] Choudhury FP, Chakraborty AB, A note on fuzzy subrings and fuzzy homomorphism, *Journal of Mathematical analysis and applications*, 131 (1988), 537-553.

- [11] Jansi R, Mohana K, Florentin Smarandache, "Correlation Measure for Pythagorean Neutrosophic Fuzzy Sets with T and F as Dependent Neutrosophic Components" ,Neutrosophic Sets and Systems, 30 (2019), 202-212.
- [12] Smarandache F, A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability; American Research Press: Rehoboth, DE, USA, 1999.
- [13] Smarandache F, Degree of dependence and independence of the (sub)components of fuzzy set and Neutrosophic set. Neutrosophic Sets Syst., 11 (2016), 95-97.
- [14] Wang Z, Wang Z, An approach to multiattribute interval-valued intuitionistic fuzzy decision and Knowledge Discovery, 3 (2008), 346-350.
- [15] Xindong peng, Yong Yan, Some Results for Pythagorean Fuzzy Sets, International Journal of Intelligent Systems, 30 (2015), 1133-1160.
- [16] Yager RR, Pythagorean fuzzy subsets, In: Proc Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada, 2013, 57-61.
- [17] Yager RR, Abbasov AM, Pythagorean membership grades, complex numbers and decision making, Int J Intell Syst, 28, (2013), 436-452.
- [18] Yager RR, Pythagorean membership grades in multicriteria decision making, IEEE Trans Fuzzy Syst, 22, (2014), 958-965.
- [19] Zadeh LA, Fuzzy sets, Inf Control 8, (1965), 338-353.