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Selection of Alternative under the Framework of Single-Valued Neutrosophic Sets

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Abstract. The multiple criteria decision making (MCDM) problems indicate the alternatives which have more or less resemblance to each other. An important mathematical tool used by decision-makers (DMs) to quantify these resemblances is the similarity measure (SM). SM is a powerful tool that measures the resemblance more accurately. Mostly, fuzzy sets (FSs) and its extensions handle the vague and uncertain information by considering the membership, non-membership, and indeterminacy degrees whose sum always lies in the interval $[0, 1]$. However, single-valued neutrosophic sets (SVNSs) and interval-valued neutrosophic sets (IVNSs) have information whose sum is bounded in $[0, 3]$. In the present work, we extended the SM presented by William and Steel for SVNSs and IVNSs by using the concept of Euclidean distance. The weights of criteria indicate much influence for the selection of the best alternative, sometimes DMs feel hesitation to allocate the weights to the criteria. We applied the linear programming (LP) model to evaluate the weights of the criteria to reduce the hesitancy. Later on, SM is utilized to establish an MCDM model for the selection of the best option. Moreover, the Spearman's rank correlation coefficient is implemented to analyze the ranking order. Finally, a medical diagnosis example is illustrated for the feasibility and effectiveness of the proposed model.

Keywords: picture fuzzy sets; fuzzy sets; similarity measure; neutrosophic sets; linear programming model.

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1. Introduction

Most of the information provided to the experts or decision makers (DMs) are ambiguous and uncertain. DMs handle such information precisely by using the fuzzy sets (FSs) theory presented by Zadeh [31] in 1965. FSs contain a single value in its specification, called a membership degree (MDg) which is always bounded in the closed interval $[0, 1]$. FSs have been broadly used in different fields, for example, medical diagnosis, image processing, etc. [12, 17].

In various ambiguous decision making problems, the MD_g is assumed not exactly as a numerical value but as an interval. Therefore, Zadeh [32] introduced the interval-valued fuzzy sets (IVFSs), an augmentation of FSs. Though, the FSs and IVFSs only have the MD_g, and they cannot designate the non membership degree (NMDg) of the element belonging to the set. Consider that in a competition of university's postgraduate students, a board of seven experts evaluate the efficiency of a student. According to three experts a student can be accepted for admission, according to two experts he or she is rejected and the remaining two experts remained impartial. In such circumstances, FSs and IVFSs could not handle the vagueness and uncertainty precisely. Atanassov [6] further extended the notion of FSs into intuitionistic fuzzy sets (IFSs) to cope such problems which comprise both MD_g and NMD_g in its structure so that, $0 \leq MD_g + NMD_g \leq 1$. Most rapidly, IFSs become an important device to deal with the imprecise and ambiguous information than the FSs and IVFSs.

In spite of the fact that, IFSs have been successfully implemented in distinct fields, however, IFSs were not covering the human's attitude perfectly. Casting of vote is an excellent example of such type of attitude, we may divide the voters into four groups: vote for, vote against, neutral and refusal of voting. When a person refuses to vote, we can say that the person is not anxious about the general election. Cuong [11] focused such types of human's attitude by presenting the idea of picture fuzzy sets P_cFSs, the generalized form of IFSs. P_cFSs have three components in its formation called, MD_g, NMD_g and of degree refusal (D_gR) such that, $0 \leq MD_g + NMD_g + D_gR \leq 1$. But P_cFSs also have some limitations to express the decision information. For instance, three groups of decision makers (DMs) assess the advantages of a new business. First group predicts that the business will be profitable is 0.7, according to second group the possibility of loss is 0.2 and the third group is not sure whether the business will be profitable is 0.4. In this scenario, P_cFSs cannot handle the information because, $0.7 + 0.2 + 0.3 = 1.2 > 1$.

Therefore, to handle such situations Wang et al. [22] introduced an amazing concept of single-valued neutrosophic sets (SVNSs) that consists of three degrees, the truth-membership ($T_n(x)$) degree, indeterminacy-membership ($I_n(x)$) degree, and falsity membership ($F_n(x)$) degree in the closed interval $[0, 1]$ so that it satisfy the condition, $0 \leq T_n(x) + I_n(x) + F_n(x) \leq 3$. Later on, Wang [23] described these three degrees in the form of an interval, called an interval-valued neutrosophic sets (IVNSs). Nowadays, NSs have become the center of the eye of the researcher due to its innovation. Many researchers are trying to print it for example, Abdel-Basset et.al [1–4] used the score and accuracy functions of trapezoidal neutrosophic numbers to minimize the cost of projects under uncertain environmental conditions, in order to tackle the ambiguity and uncertainty present in the data for MCDM problems, utilized the plithogenic set, a generalization of NSs, a novel hybrid neutrosophic MCDM model is presented on the basis of TOPSIS by using bipolar neutrosophic numbers and resolve the supply chain issues with the help of best-worst method (evaluating weights) and plithogenic set, respectively.

SM is one of the vital and powerful tools that measures the level of resemblance among the objects. In order to show the preference strength among the alternatives, the similarity measures have achieved more attention from the DMs since the previous few decades. Various DMs have presented a number of similarity measures for MCDM problems to select the most favorable alternative from the various options having identical features under the certain criteria. For example, Beg and Ashraf discussed the various characteristic of similarity measures under the framework of FSs [7]. Ye [28–30] introduced the cosine similarity measures (vector similarity) and implemented it to pattern recognition and medical diagnosis under the environments of simplified neutrosophic sets, interval neutrosophic sets and IFSs. Intarapaiboon [14] applied two new similarity measures to pattern recognition in IFSs situations. Moreover, Song and Hu [20] established two measures of similarity between hesitant fuzzy linguistic term sets and used it for MCDM problems. Recently, Wei and Gao [26] developed the generalized Dice similarity measures for P_cFSs and implemented for pattern recognition. Consequently, Wang et al. [24] presented the generalized Dice similarity measures for Pythagorean fuzzy sets and used it in multiple attribute group decision making.

The linear programming (LP) model introduced by Vanderbei [21], permits some target function to be minimized or maximized inside the system of given situational limitations. LP is a computational technique that enables DMs to solve the problems which they face in decision-making model. It encourages the DMs to deal with constrained ideal conditions which they need to make the best of their resources. Various experts utilized LP model in MCDM for different extensions of FSs [5, 10, 13, 18, 25]. Recently, Sindhu et al. [19] implemented the LP methodology with extended TOPSIS (technique for order of preference by similarity to ideal solution) for picture fuzzy sets. The weights of criteria appear to specify that the DMs identify the significance of people views and its influence on attaining the objective. Sometimes DMs hesitate or confused to allocate the weights to criteria. Thereby, we applied TOPSIS to get the objective function and then find out the weights of criteria under some constraints by using LP model. The novelty of this article is concerned about proposing the SM to overcome the shortcoming present in the existing technique. The following are the major contributions of this study:

- William and Steel SM is extended on the basis of novel distance measure.
- Evaluate the objective function by using TOPSIS.

- Weights of criteria are calculated with the help of LP model.
- An MCDM model is developed on the basis of SM and implemented it for medical diagnosis under the framework of SVNSs and IVNSs.
- Spearman's rank-correlation coefficient and the critical value are applied to strength the proposed MCDM model.

Rest of the article is organized as: Section 2 encloses some preliminaries regarding SVNSs and IVNSs. Various pre-existing similarity measures of SVNSs, IVNSs and their shortcoming are elaborated in Section 3. The modified similarity measures for SVNSs and IVNSs are described in Section 4. An MCDM model is proposed in Section 5 and the developed model is then applied on an example of medical diagnosis in Section 6 to elaborate the validity and effectiveness. A comprehensive comparative analysis based on Spearman's rank correlation coefficient is penned in Section 7. Conclusions and future work are highlighted in Section 8.

2. Preliminaries

A brief introduction of the notions FSs, P_cFSs, $SVNS$ and $IVNS$ and the LP model is presented in this section.

Definition 2.1. [31] Let $X = \{x_1, x_2, ..., x_n\}$ be a discourse set. A fuzzy set (FS) A on X is represented in terms of a functions $m : X \rightarrow [0, 1]$ such that

$$A = \{\langle x_i, m_A(x_i)\rangle \,|x_i \in X\}.$$

Definition 2.2. [11] Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. A picture fuzzy set P_c on X is defined as:

$$P_c = \{\langle x_i, \alpha_{P_c}(x_i), \gamma_{P_c}(x_i), \beta_{P_c}(x_i)\rangle \,|x_i \in X, i = 1, 2, ..., n\},$$

where $\alpha_{P_c}(x_i)$, $\beta_{P_c}(x_i)$, $\gamma_{P_c}(x_i) \in [0, 1]$ are called the acceptance membership, neutral and rejection membership degrees of $x_i \in X$ to the set P_c, respectively and $\alpha_{P_c}(x_i)$, $\gamma_{P_c}(x_i)$ and $\beta_{P_c}(x_i)$ fulfil the condition: $0 \leq \alpha_{P_c}(x_i) + \gamma_{P_c}(x_i) + \beta_{P_c}(x_i) \leq 1$, for all $x_i \in X$. Also $\zeta_{P_c}(x_i) = 1 - \alpha_{P_c}(x_i) - \gamma_{P_c}(x_i) - \beta_{P_c}(x_i)$, then $\zeta_{P_c}(x_i)$ is said to be a degree of refusal membership of $x_i \in X$ in P_c. For our convenience, we can write $p_i = (\alpha_{P_c}(x_i), \beta_{P_c}(x_i), \gamma_{P_c}(x_i))$ as the picture fuzzy numbers (P_cFNs) over a set P_c, where $i = 1, 2, ..., n$.

Definition 2.3. [22] Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. A SVNS N_s on X is defined as:

$$N_s = \{\langle x_i, \alpha_{N_s}(x_i), \gamma_{N_s}(x_i), \beta_{N_s}(x_i)\rangle \,|x_i \in X, i = 1, 2, ..., n\},$$

where $\alpha_{N_s}(x_i)$, $\gamma_{N_s}(x_i)$, $\beta_{N_s}(x_i) \in [0, 1]$ are called the truth-membership, indeterminacy and falsity- membership degrees of $x_i \in X$ to the set N_s, respectively and $\alpha_{N_s}(x_i)$, $\gamma_{N_s}(x_i)$ and

$\beta_{N_s}(x_i)$ fulfil the condition:
for all $x_i \in X$ then, $0 \leq \alpha_{N_s}(x_i) + \gamma_{N_s}(x_i) + \beta_{N_s}(x_i) \leq 3$. Let N_s^1 and N_s^2 be two SVNS, then following conditions hold:

(1) $N_s^1 \subseteq N_s^2$ iff $\alpha_{N_s^1}(x_i) \leq \alpha_{N_s^2}(x_i)$, $\beta_{N_s^1}(x_i) \geq \beta_{N_s^2}(x_i)$ and $\gamma_{N_s^1}(x_i) \geq \gamma_{N_s^2}(x_i)$,
(2) $N_s^1 = N_s^2$ iff $N_s^1 \subseteq N_s^2$ and $N_s^2 \subseteq N_s^1$.

Definition 2.4. [23] Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. An ISVNS $\tilde{N}_s$ on X is defined as:

$$\tilde{N}_s = \{\left\langle x_i, \alpha_{\tilde{N}_s}(x_i), \gamma_{\tilde{N}_s}(x_i), \beta_{\tilde{N}_s}(x_i)\right\rangle | x_i \in X, i = 1, 2, ..., n\},$$

where $\alpha_{\tilde{N}_s}(x_i) = [\alpha^l_{\tilde{N}_s}(x_i), \alpha^u_{\tilde{N}_s}(x_i)] \subseteq [0,1]$, $\gamma_{\tilde{N}_s}(x_i) = [\gamma^l_{\tilde{N}_s}(x_i), \gamma^u_{\tilde{N}_s}(x_i)] \subseteq [0,1]$, $\beta_{\tilde{N}_s}(x_i) = [\beta^l_{\tilde{N}_s}(x_i), \beta^u_{\tilde{N}_s}(x_i)] \subseteq [0,1]$ are called the truth-membership, indeterminacy and falsity- membership degrees of $x_i \in X$ to the set $\tilde{N}_s$, respectively and satisfy the condition:
for all $x_i \in X$ then, $0 \leq \alpha^u_{\tilde{N}_s}(x_i) + \gamma^u_{\tilde{N}_s}(x_i) + \beta^u_{\tilde{N}_s}(x_i) \leq 3$. Let $\tilde{N}_s^1$ and $\tilde{N}_s^2$ be two SVNS, then following conditions hold:

(1) $\tilde{N}_s^1 \subseteq \tilde{N}_s^2$ iff $\alpha^l_{N_s^1}(x_i) \leq \alpha^l_{N_s^2}(x_i), \alpha^u_{N_s^1}(x_i) \leq \alpha^u_{N_s^2}(x_i)$, $\beta^l_{N_s^1}(x_i) \geq \beta^l_{N_s^2}(x_i), \beta^u_{N_s^1}(x_i) \geq \beta^u_{N_s^2}(x_i)$, $\gamma^l_{N_s^1}(x_i) \geq \gamma^l_{N_s^2}(x_i)$ and $\gamma^u_{N_s^1}(x_i) \geq \gamma^u_{N_s^2}(x_i)$,
(2) $\tilde{N}_s^1 = \tilde{N}_s^2$ iff $\tilde{N}_s^1 \subseteq \tilde{N}_s^2$ and $\tilde{N}_s^2 \subseteq \tilde{N}_s^1$.

Definition 2.5. [21]. The linear programming model is constructed as:

$$\begin{aligned}
\text{Maximize:} \quad & Z = c_1t_1 + c_2t_2 + c_3t_3 + ... + c_nt_n \\
\text{Subject to:} \quad & a_{11}t_1 + a_{12}t_2 + a_{13}t_3 + ... + a_{1n}t_n \leq b_1 \\
& a_{21}t_1 + a_{22}t_2 + a_{23}t_3 + ... + a_{2n}t_n \leq b_2 \\
& \vdots \\
& a_{m1}t_1 + a_{m2}t_2 + a_{m3}t_3 + ... + a_{mn}t_n \leq b_m \\
& t_1, t_2, ..., t_n \geq 0,
\end{aligned}$$

where m and n denotes the cardinalities of the constraints and decision variables $t_1, t_2, ..., t_n$, respectively. A solution $(t_1, t_2, ..., t_n)$ is called feasible point if it fulfils all of the restrictions. LP model is used to find the optimal solution of the decision variables to maximize or minimize the linear function Z.

3. Some existing similarity measures for SVNSs and IVNSs

Similarity measure is a most widely used tool to evaluate the relationship between two sets. Two sets are said to be perfectly similar if similarity measure between them is exactly 1. The following are the compulsory axioms for the sets (SVNSs or IVNSs) to be perfectly similar:

Definition 3.1. Let $X = \{x_1, x_2, ..., x_n\}$ be a universal set and $N_s^1 = \{< x_i, \alpha_{N_s^1}(x_i), \gamma_{N_s^1}(x_i), \beta_{N_s^1}(x_i)\}$ and $N_s^2 = \{< x_i, \alpha_{N_s^1}(x_i), \gamma_{N_s^2}(x_i), \beta_{N_s^2}(x_i) >\}$ be two SVNS, where, $i = 1, 2, ..., n$. Then,

(1) $0 \leq S(N_s^1, N_s^2) \leq 1$,
(2) $S(N_s^1, N_s^2) = S(N_s^2, N_s^1)$,
(3) $S(N_s^1, N_s^2) = 1$ if and only if $N_s^1 = N_s^2$.

A cosine similarity measure $S(N_s^1, N_s^2)$ of SVNS presented by Ye [29] is given as:
$S(N_s^1, N_s^2) = \frac{(\alpha_{N_s^1}(x_i))(\alpha_{N_s^2}(x_i))+(\gamma_{N_s^1}(x_i))(\gamma_{N_s^2}(x_i))+(\beta_{N_s^1}(x_i))(\beta_{N_s^2}(x_i))}{[\sqrt{(\alpha_{N_s^1}(x_i))^2+(\gamma_{N_s^1}(x_i))^2+(\beta_{N_s^1}(x_i))^2}][\sqrt{(\alpha_{N_s^2}(x_i))^2+(\gamma_{N_s^2}(x_i))^2+(\beta_{N_s^2}(x_i))^2}]}$.
Suppose that $N_s^1 = (x, 0.4, 0.2, 0.6)$ and $N_s^2 = (x, 0.2, 0.1, 0.3)$ are two SVNSs, the Definition 2.3 shows that $N_s^1 \neq N_s^2$. However, by using cosine similarity measure presented by Ye [29], we see that, $S(N_s^1, N_s^2) = 1$, show the contradiction of the property 3 of Definition 3.1 which describe that $S(N_s^1, N_s^2) = 1$ if and only if $N_s^1 = N_s^2$. Similarly, if we take, $\alpha_{N_s^1}(x_i) = (k+1)\alpha_{N_s^2}(x_i)$, $\gamma_{N_s^1}(x_i) = (k+1)\gamma_{N_s^2}(x_i)$ and $\beta_{N_s^1}(x_i) = (k+1)\beta N_s^2(x_i)$, where $k \geq 1$, then according to cosine similarity measure, its value is:
$S(N_s^1, N_s^2) = \frac{(\alpha_{N_s^1}(x_i))(\alpha_{N_s^2}(x_i))+(\gamma_{N_s^1}(x_i))(\gamma_{N_s^2}(x_i))+(\beta_{N_s^1}(x_i))(\beta_{N_s^2}(x_i))}{[\sqrt{(\alpha_{N_s^1}(x_i))^2+(\gamma_{N_s^1}(x_i))^2+(\beta_{N_s^1}(x_i))^2}][\sqrt{(\alpha_{N_s^2}(x_i))^2+(\gamma_{N_s^2}(x_i))^2+(\beta_{N_s^2}(x_i))^2}]}$,
$S(N_s^1, N_s^2) = \frac{((k+1)\alpha_{N_s^2}(x_i))(\alpha_{N_s^2}(x_i))+((k+1)\gamma_{N_s^2}(x_i))(\gamma_{N_s^2}(x_i))+((k+1)\beta_{N_s^2}(x_i))(\beta_{N_s^2}(x_i))}{[\sqrt{((k+1)\alpha_{N_s^2}(x_i))^2+((k+1)\gamma_{N_s^2}(x_i))^2+((k+1)\beta_{N_s^2}(x_i))^2}][\sqrt{(\alpha_{N_s^2}(x_i))^2+(\gamma_{N_s^2}(x_i))^2+(\beta_{N_s^2}(x_i))^2}]}$,
$S(N_s^1, N_s^2) = \frac{(k+1)(\alpha_{N_s^2}(x_i))^2+(\gamma_{N_s^2}(x_i))^2+(\beta_{N_s^2}(x_i))^2)}{(k+1)((\alpha_{N_s^2}(x_i))^2+(\gamma_{N_s^2}(x_i))^2+(\beta_{N_s^2}(x_i))^2)}$ =1, which again opposes the property 3 of Definition 3.1.
Further, if $N_s^1 = (0, 0, 0)$ and $N_s^2 = (0, 0, 0)$ are two SVNS then according to Jaccrd and Dice similarity measures presented in [29] become undefined or meaningless.
Same as, if $\tilde{N}_s^1 = (y, [0.3, 0.4], [0.2, 0.3], [0.4, 0.5])$ and
$\tilde{N}_s^1 = (y, [0.6, 0.8], [0.4, 0.6], [0.8, 1])$ are two IVNSs, then according to Definition 2.4, $\tilde{N}_s^1 \neq \tilde{N}_s^2$, but the similarity measure presented by Ye [30] gives that, $S(\tilde{N}_s^1, \tilde{N}_s^2) = 1$, that is, $\tilde{N}_s^1 = \tilde{N}_s^2$ which again presents a contradiction with property 3 of Definition 3.1. Also for two IVNSs, $\tilde{N}_s^1 = [0, 0]$ and $\tilde{N}_s^2 = [0, 0]$, we get the meaningless or undefined results by using Equation 9 presented in [15]. So the similarity measures presented in [15, 29, 30] have a deficiency.
Hence, from the above discussion, it is clear that the existing similarity measures have some drawbacks and cannot be able to select the best alternative. Consequently, there is a need to improve the similarity measure which satisfy the axiom of Definition 3.1.

4. **Proposed similarity measures for SVNSs and IVNSs**

In order to overcome the deficiencies present in the above discussed similarity measures, we extend a similarity measure presented by William and Steel [27] for the SVNSs (IVNSs) based on the novel distance measure as:

$$D(N_s^1, N_s^2) = \frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c} \left[\left|\alpha_{N_s^1}(x_i) - \alpha_{N_s^2}(x_i)\right| + \left|\gamma_{N_s^1}(x_i) - \gamma_{N_s^2}(x_i)\right| + \left|\beta_{N_s^1}(x_i) - \beta_{N_s^2}(x_i)\right|\right] + \\ \max\left[\left|\alpha_{N_s^1}(x_i) - \alpha_{N_s^2}(x_i)\right|, \left|\gamma_{N_s^1}(x_i) - \gamma_{N_s^2}(x_i)\right|, \left|\beta_{N_s^1}(x_i) - \beta_{N_s^2}(x_i)\right|\right] \end{array}\right), \tag{1}$$

$$S_m^i(N_s^1, N_s^2) = e^{-\frac{1}{n}D(N_s^1, N_s^2)}, \tag{2}$$

where n is the number of alternatives and $1 \leq i \leq n$.

Similarly for the IVNSs the distance and similarity measures are:

$$\tilde{D}(\tilde{N}_s^1, \tilde{N}_s^2) = \frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{c} [|\alpha^l_{\tilde{N}_s^1}(x_i) - \alpha^l_{\tilde{N}_s^2}(x_i)| + |\alpha^u_{\tilde{N}_s^1}(x_i) - \alpha^u_{\tilde{N}_s^2}(x_i)| + \\ |\gamma^l_{\tilde{N}_s^1}(x_i) - \gamma^l_{\tilde{N}_s^2}(x_i)| + |\gamma^u_{\tilde{N}_s^1}(x_i) - \gamma^u_{\tilde{N}_s^2}(x_i)| + \\ |\beta^l_{\tilde{N}_s^1}(x_i) - \beta^l_{\tilde{N}_s^2}(x_i)| + |\beta^u_{\tilde{N}_s^1}(x_i) - \beta^u_{\tilde{N}_s^2}(x_i)|] + \\ \max[|\alpha^l_{\tilde{N}_s^1}(x_i) - \alpha^l_{\tilde{N}_s^2}(x_i)|, |\alpha^u_{\tilde{N}_s^1}(x_i) - \alpha^u_{\tilde{N}_s^2}(x_i)|, \\ |\gamma^l_{\tilde{N}_s^1}(x_i) - \gamma^l_{\tilde{N}_s^2}(x_i)|, |\gamma^u_{\tilde{N}_s^1}(x_i) - \gamma^u_{\tilde{N}_s^2}(x_i)| \\ , |\beta^l_{\tilde{N}_s^1}(x_i) - \beta^l_{\tilde{N}_s^2}(x_i)|, |\beta^u_{\tilde{N}_s^1}(x_i) - \beta^u_{\tilde{N}_s^2}(x_i)|] \end{array}\right), \tag{3}$$

$$\tilde{S}_m^i(\tilde{N}_s^1, \tilde{N}_s^2) = e^{-\frac{1}{n}\tilde{D}(\tilde{N}_s^1, \tilde{N}_s^2)}. \tag{4}$$

Theorem 4.1. *The SM $S_m^i(N_s^1, N_s^2)$ defined in Equation (2) amongst $N_s^1 = \{\langle x_i, \alpha_{N_s^1}(x_i), \gamma_{N_s^1}(x_i), \beta_{N_s^1}(x_i)\rangle\}$ and $N_s^2 = \{\langle x_i, \alpha_{N_s^2}(x_i), \gamma_{N_s^2}(x_i), \beta_{N_s^2}(x_i)\rangle\}$ satisfies the given properties:*

(1) $S_m^i(N_s^1, N_s^2) = 1$ *if and only if* $N_s^1 = N_s^2$,
(2) $S_m^i(N_s^1, N_s^2) = S_m^i(N_s^2, N_s^1)$,
(3) $0 \leq S_m^i(N_s^1, N_s^2) \leq 1$.

Proof

(1) Suppose that, $N_s^1 = N_s^2$ that is, $\alpha_{N_s^1}(x_i) = \alpha_{N_s^2}(x_i)$, $\gamma_{N_s^1}(x_i) = \gamma_{N_s^2}(x_i)$ and $\beta_{N_s^1}(x_i) = \beta_{N_s^2}(x_i)$, then by using Equation (2), we have

$$S_m^i(N_s^1, N_s^2) = e^0 = 1.$$

(2) Consider $S_m^i(N_s^1, N_s^2) = e^{-\frac{1}{n}D(N_s^1, N_s^2)}$

$$= e^{-\frac{1}{3n^2}\sum_{i=1}^{n}\left(\begin{array}{c} \left[\left|\alpha_{N_s^1}(x_i) - \alpha_{N_s^2}(x_i)\right| + \left|\gamma_{N_s^1}(x_i) - \gamma_{N_s^2}(x_i)\right| + \left|\beta_{N_s^1}(x_i) - \beta_{N_s^2}(x_i)\right|\right] + \\ \max\left[\left|\alpha_{N_s^1}(x_i) - \alpha_{N_s^2}(x_i)\right|, \left|\gamma_{N_s^1}(x_i) - \gamma_{N_s^2}(x_i)\right|, \left|\beta_{N_s^1}(x_i) - \beta_{N_s^2}(x_i)\right|\right] \end{array}\right)},$$

$$= e^{-\frac{1}{3n^2}\sum_{i=1}^{n}\left(\begin{array}{c} \left[\left|\alpha_{N_s^2}(x_i) - \alpha_{N_s^1}(x_i)\right| + \left|\gamma_{N_s^2}(x_i) - \gamma_{N_s^1}(x_i)\right| + \left|\beta_{N_s^2}(x_i) - \beta_{N_s^1}(x_i)\right|\right] + \\ \max\left[\left|\alpha_{N_s^2}(x_i) - \alpha_{N_s^1}(x_i)\right|, \left|\gamma_{N_s^2}(x_i) - \gamma_{N_s^1}(x_i)\right|, \left|\beta_{N_s^2}(x_i) - \beta_{N_s^1}(x_i)\right|\right] \end{array}\right)},$$

$$= e^{-\frac{1}{n}D(N_s^2, N_s^1)} = S_m^i(N_s^2, N_s^1),$$

(3) From Equations (1) and (2), it is obvious that, $S^i_m(N^1_s, N^2_s) \leq 1$ and it become zero i.e., $S^i_m(N^1_s, N^2_s) = 0$ only when the distance between N^1_s and N^2_s is very large.

Example 4.2. Let $N^1_s = (x, 0.4, 0.2, 0.6)$ and $N^2_s = (x, 0.2, 0.1, 0.3)$ be two SVNSs, then by using Equations (1) and (2), the similarity measure is, $S^i_m(N^1_s, N^2_s) = 0.7408$.

Example 4.3. Let $\tilde{N}^1_s = (x, [0.3, 0.4], [0.2, 0.3], [0.4, 0.5])$ and $\tilde{N}^2_s = (x, [0.6, 0.8], [0.4, 0.6], [0.8, 1])$ be two IVNSs, then by using Equations (3) and (4), the similarity measure is, $S^i_m(\tilde{N}^1_s, \tilde{N}^2_s) = 0.3679$.

Theorem 4.4. *The SM $\tilde{S}^i_m(\tilde{N}^1_s, \tilde{N}^2_s)$ defined in Equation (4) amongst $\tilde{N}^1_s = \{\left\langle x_i, \alpha_{\tilde{N}^1_s}(x_i), \gamma_{\tilde{N}^1_s}(x_i), \beta_{\tilde{N}^1_s}(x_i)\right\rangle\}$ and $\tilde{N}^2_s = \{\left\langle x_i, \alpha_{\tilde{N}^2_s}(x_i), \gamma_{\tilde{N}^2_s}(x_i), \beta_{\tilde{N}^2_s}(x_i)\right\rangle\}$ satisfies the given properties:*

(1) $\tilde{S}^i_m(\tilde{N}^1_s, \tilde{N}^2_s) = 1$ *if and only if* $\tilde{N}^1_s = \tilde{N}^2_s$,

(2) $\tilde{S}^i_m(\tilde{N}^1_s, \tilde{N}^2_s) = \tilde{S}^i_m(\tilde{N}^2_s, \tilde{N}^1_s)$,

(3) $0 \leq \tilde{S}^i_m(\tilde{N}^1_s, \tilde{N}^2_s) \leq 1$.

Proof The proof of this Theorem is obvious.

4.1. *Proposed weighted similarity measures (WSM) for SVNSs and IVNSs*

Since the weights of the criteria have a great impact in making decision process therefore we can further extend the proposed similarity measures into the WSM. Let $w = (w_1, w_2, ..., w_m)^T$ be a weight vector of the m criteria with $\sum_{j=1}^m w_j = 1$. In order to get WSM $S^{iw}_m(N^1_s, N^2_s)$ for SVNSs, we first define the weighted distance as:

$$D^w(N^1_s, N^2_s) = \sum_{i=1}^{n}\sum_{j=1}^{m} w_j \begin{pmatrix} \left[\left|\alpha_{N^1_s}(x_i) - \alpha_{N^2_s}(x_i)\right| + \left|\gamma_{N^1_s}(x_i) - \gamma_{N^2_s}(x_i)\right| + \left|\beta_{N^1_s}(x_i) - \beta_{N^2_s}(x_i)\right|\right] + \\ \max\left[\left|\alpha_{N^1_s}(x_i) - \alpha_{N^2_s}(x_i)\right|, \left|\gamma_{N^1_s}(x_i) - \gamma_{N^2_s}(x_i)\right|, \left|\beta_{N^1_s}(x_i) - \beta_{N^2_s}(x_i)\right|\right] \end{pmatrix}, \tag{5}$$

and

$$S^{iw}_m(N^1_s, N^2_s) = e^{-\frac{1}{n}D^w(N^1_s, N^2_s)}. \tag{6}$$

In the similar way, a WSM $\tilde{S}^{iw}_m(\tilde{N}^1_s, \tilde{N}^2_s)$ on the basis of weighted distance $\tilde{D}^w(\tilde{N}^1_s, \tilde{N}^2_s)$ for IVNSs is obtained as:

$$\tilde{D}^w(\tilde{N}^1_s, \tilde{N}^2_s) = \sum_{i=1}^{n}\sum_{j=1}^{m} w_j \begin{pmatrix} [|\alpha^l_{\tilde{N}^1_s}(x_i) - \alpha^l_{\tilde{N}^2_s}(x_i)| + |\alpha^u_{\tilde{N}^1_s}(x_i) - \alpha^u_{\tilde{N}^2_s}(x_i)| + \\ |\gamma^l_{\tilde{N}^1_s}(x_i) - \gamma^l_{\tilde{N}^2_s}(x_i)| + |\gamma^u_{\tilde{N}^1_s}(x_i) - \gamma^u_{\tilde{N}^2_s}(x_i)| + \\ |\beta^l_{\tilde{N}^1_s}(x_i) - \beta^l_{\tilde{N}^2_s}(x_i)| + |\beta^u_{\tilde{N}^1_s}(x_i) - \beta^u_{\tilde{N}^2_s}(x_i)|] + \\ \max[|\alpha^l_{\tilde{N}^1_s}(x_i) - \alpha^l_{\tilde{N}^2_s}(x_i)|, |\alpha^u_{\tilde{N}^1_s}(x_i) - \alpha^u_{\tilde{N}^2_s}(x_i)|, \\ |\gamma^l_{\tilde{N}^1_s}(x_i) - \gamma^l_{\tilde{N}^2_s}(x_i)|, |\gamma^u_{\tilde{N}^1_s}(x_i) - \gamma^u_{\tilde{N}^2_s}(x_i)| \\ , |\beta^l_{\tilde{N}^1_s}(x_i) - \beta^l_{\tilde{N}^2_s}(x_i)|, |\beta^u_{\tilde{N}^1_s}(x_i) - \beta^u_{\tilde{N}^2_s}(x_i)|] \end{pmatrix}, \tag{7}$$

and

$$\tilde{S}_m^{iw}(\tilde{N}_s^1, \tilde{N}_s^2) = e^{-\frac{1}{n}\tilde{D}^w(\tilde{N}_s^1, \tilde{N}_s^2)}. \quad (8)$$

Theorem 4.5. *Let $N_s^1 = \{< x_i,\ \alpha_{N_s^1}(x_i),\ \gamma_{N_s^1}(x_i),\ \beta_{N_s^1}(x_i) >\}$ and $N_s^2 = \{< x_i,\ \alpha_{N_s^2}(x_i),\ \gamma_{N_s^2}(x_i),\ \beta_{N_s^2}(x_i) >\}$ be two SVNSs (IVNSs) , then the WSM presented in Equation (6) (Equation (8)) between two SVNSs (IVNSs) satisfies the following properties:*

(1) $0 \leq S_m^{iw}(N_s^1, N_s^2) \leq 1$,

(2) $S_m^{iw}(N_s^1, N_s^2) = S_m^{iw}(N_s^2, N_s^1)$,

(3) $S_m^{iw}(N_s^1, N_s^2) = 1$ *if and only if* $N_s^1 = N_s^2$.

Proof It is obvious as Theorem 4.1.

Example 4.6. Let $N_s^1 = \{x, (0.3, 0.2, 0.5), (0.4, 0.6, 0.0)\}$ and $N_s^2 = \{x, (0.1, 0.1, 0.8), (0.2, 0.1, 0.7)\}$ be two SVNSs and $w = (0.7, 0.3)^T$ the weight vector, then the WSM for SVNSs is: $S_m^{iw}(N_s^1, N_s^2) = 0.9162$.

Example 4.7. Let $\tilde{N}_s^1 = \{x, ([0.4, 0.6], [0.2, 0.3], [0.3, 0.4]), ([0.5, 0.8], [0.1, 0.4], [0.1, 0.3])\}$ and $\tilde{N}_s^2 = \{x, ([0.7, 0.9], [0.1, 0.2], [0.1, 0.2]), ([0.3, 0.6], [0.1, 0.3], [0.4, 0.7])\}$ be two IVNSs and $w = (0.6, 0.4)^T$ the weight vector, then the weighted similarity measure for IVNSs is: $S_m^{iw}(N_s^1, N_s^2) = 0.8781$.

5. **Decision making model under SVNSs (IVNSs)**

The model for MCDM problems is presented on the basis of proposed weighted similarity measure in this section. Suppose that $Q = \{Q_1, Q_2, ..., Q_n\}$ is a discrete set of alternatives and $G = \{G_1, G_2, ..., G_m\}$ is another discrete set of criteria. If the DMs gave the various values for the alternative $Q_i (i = 1, 2, ..., n)$ under the criteria $G_j (j = 1, 2, ..., m)$, and form a neutrosofic decision matrix $N = [b_{ij}]_{n\times m}$. The concept of optimal solution assists the DMs to identify the best alternative from the decision set in MCDM framework. In spite of the fact that the perfect option does not exist in actual, it provides a valuable paradigm to appraise alternatives. Hence, we can find the ideal options $N^\star$ from the given information as $N^\star = \max([b_{ij}]_{n\times m})$. Since the weights of the criteria have an excessive impact, thereby a weighing vector of criteria is provided as $w = (w_1, w_2, w_3, ..., w_m)^T$, where $\sum_{j=1}^m w_j = 1$ and $w_j > 0$, can be evaluated by using the LP model presented in Definition 2.5. The model based on proposed weighted similarity measure described by Equation (6) (Equation (8)) has the following steps.

Step 1. Based on the information provided by DMs, form a single valued neutrosophic decision matrix (SVNDM) denoted by $N = [b_{ij}]_{n\times m}$.

Step 2. Find the optimal solution $N^\star$ from the SVNDM.

Step 3. On the basis of TOPSIS, an objective function is obtained and then calculate the weights of criteria by using LP model as described in Definition 2.5.

Step 4. With the aid of weights evaluated in Step 3, calculate the similarity measures amongst the alternative $Q_i(i = 1, 2, ..., n)$ and the optimal alternative $N^\star$ by using Equation (6) (Equation (8)).

Step 5. Rank all the alternatives $Q_i(i = 1, 2, ..., n)$ from highest to lowest values of similarity measures obtained in Step 4 and choose the alternative having highest value of the similarity measure.

6. Practical examples

In this section, a medical diagnosis decision problem is considered to see the validity and effectiveness of the proposed MCDM model.

Example 1. For parents, it is significant to be aware of the most updated treatment process so you can be certain about your kids are getting the superlative care possible. According to the child specialist, some common childhood sicknesses and their appropriate symptoms are listed. Suppose a collection of diagnoses, chest infections (C), malaria (M), typhoid (T), sore throat (S) and bronchitis (B) are examined on the basis of some symptoms, fever (S_1), headache (S_2), breathlessness (S_3), cough (S_4) and chest pain (S_5). All the information is given in the form of neutrosophic decision matrix (NDM) $N = [b_{ij}]_{n\times m}$. Assume that patient $K_1 = N^\star$ has all the symptoms in the diagnosis process, all the information collected about the kids $K_i(i = 1, 2, ..., n)$ is provided in the form of SVNS in Table 1.

$$
\begin{aligned}
\text{Maximize:}\quad & Z = 0.2175w_1 + 0.2350w_2 + 0.2200w_3 + 0.1950w_4 + 0.1850w_5\\
\text{Subject to:}\quad & 10w_1 + 8w_2 + 12w_3 + 10w_4 + 15w_5 \geq 10,\\
& 10w_1 + 8w_2 + 12w_3 + 10w_4 + 15w_5 \leq 10.5,\\
& 8w_1 + 11w_2 + 7w_3 + 10w_4 + 10w_5 \geq 8,\\
& 8w_1 + 11w_2 + 7w_3 + 10w_4 + 10w_5 \leq 8.5,\\
& 12w_1 + 15w_2 + 12w_3 + 10w_4 + 6w_5 \geq 12,\\
& 12w_1 + 15w_2 + 12w_3 + 10w_4 + 6w_5 \leq 12.5,\\
& w_1 + w_2 + w_3 + w_4 + w_5 = 1,\\
& w_1, w_2, ..., w_5 \geq 0.
\end{aligned}
$$

TABLE 1. Neutrosophic decision matrix NDM

Daignosis	S_1	S_2	S_3	S_4	S_5
C	$< 0.4, 0.6, 0.0 >$	$< 0.3, 0.2, 0.5 >$	$< 0.1, 0.3, 0.7 >$	$< 0.4, 0.3, 0.3 >$	$< 0.1, 0.2, 0.7 >$
M	$< 0.7, 0.3, 0.0 >$	$< 0.2, 0.2, 0.6 >$	$< 0.0, 0.1, 0.9 >$	$< 0.7, 0.3, 0.0 >$	$< 0.1, 0.1, 0.8 >$
T	$< 0.3, 0.4, 0.3 >$	$< 0.6, 0.3, 0.1 >$	$< 0.2, 0.1, 0.7 >$	$< 0.2, 0.2, 0.6 >$	$< 0.1, 0.0, 0.9 >$
S	$< 0.1, 0.2, 0.7 >$	$< 0.2, 0.4, 0.4 >$	$< 0.8, 0.2, 0.0 >$	$< 0.2, 0.1, 0.7 >$	$< 0.2, 0.1, 0.7 >$
B	$< 0.1, 0.1, 0.8 >$	$< 0.0, 0.2, 0.8 >$	$< 0.2, 0.0, 0.8 >$	$< 0.2, 0.0, 0.8 >$	$< 0.8, 0.1, 0.1 >$

Step 1. Based on the information provided by the professional, form a SVNDM $N = [n_{ij}]_{5\times 5}$.

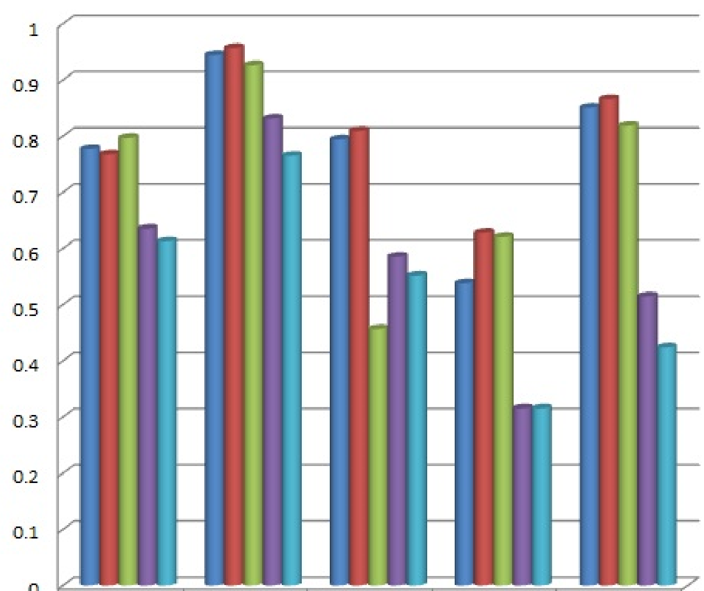
Step 2. Assume that a kid $K_1 = \{(0.8, 0.2, 0.1), (0.9, 0.3, 0.2), (0.2, 0.1, 0.8), (0.6, 0.5, 0.1), (0.1, 0.4, 0.6)\}$ has all the symptoms in the process of diagnosis.

Step 3. By using TOPSIS an objective function is obtained and then calculate the weights of criteria by applying the LP model as described in Definition 2.5.

Step 4. The values of the weighted similarity measure calculated with the help of Equation (6) amongst the diagnoses and the kid K_1 are: $S_m^{1w} = 0.7774$, $S_m^{2w} = 0.7675$, $S_m^{3w} = 0.7969$, $S_m^{4w} = 0.6353$ and $S_m^{5w} = 0.6127$.

Step 5. According to values obtained in Step 4, we get the ranking order as: $T \succ C \succ M \succ B \succ S$. Figure 1 indicates the ranking order presented in [8, 9, 16, 29] and the proposed model graphically.

FIGURE 1. Ranking order of alternatives



	Proposed	[4]	[5]	[12]	[24]
C	0.7774	0.9443	0.7941	0.5383	0.8505
M	0.7675	0.9571	0.8094	0.6282	0.8661
T	0.7969	0.9264	0.4568	0.6206	0.8185
S	0.6353	0.8314	0.5851	0.3154	0.5148
B	0.6127	0.765	0.5517	0.3154	0.4244

Example 2. Consider the same scenario as Example 1 with interval-valued data provided in Table 2. Assume that another Kid K_2 suffers from all the symptoms, which can be expressed by the following IVNS data.

Step 1. Based on the information given by the professional form an interval-valued neutrosofic decision matrix (INDM) denoted by $\tilde{N} = [\tilde{n}_{ij}]_{5\times 5}$.

Step 2. Assume a kid $K_2 = \{([0.3, 0.5], [0.2, 0.3], [0.4, 0.5]), ([0.7, 0.9], [0.1, 0.2], [0.1, 0.2]), ([0.4, 0.6], [0.2, 0.3], [0.3, 0.4]), ([0.3, 0.6], [0.1, 0.3], [0.4, 0.7]), ([0.5, 0.8], [0.1, 0.4], [0.1, 0.3])\}$ has all the symptoms in the process of diagnosis.

Step 3. Use the same weights for the symptoms which are evaluated in Example 1.

Step 4. The values of the weighted similarity measure calculated with the help of Equation

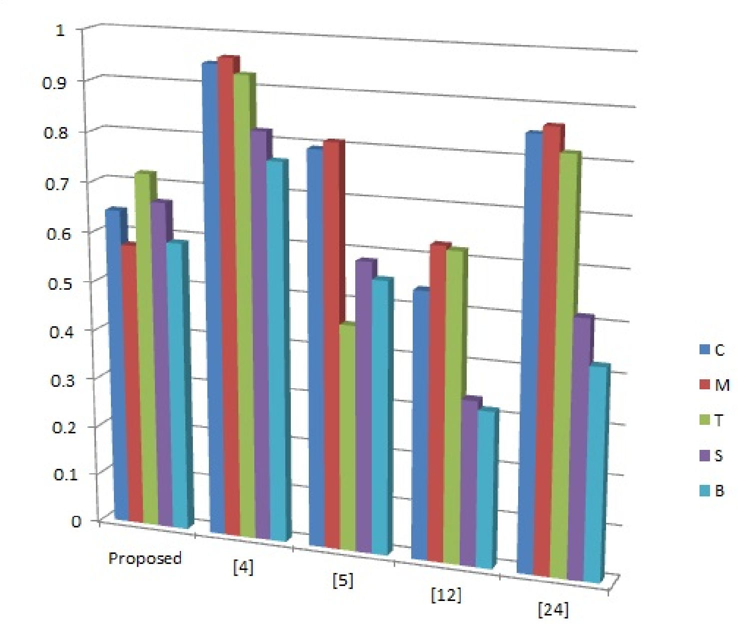
TABLE 2. Neutrosofic decision matrix NDM

Daignosis	S_1	S_2	S_3
C	$([0.4, 0.4], [0.6, 0.6], [0.0, 0.0])$	$([0.3, 0.3], [0.2, 0.2], [0.5, 0.5])$	$([0.1, 0.1], [0.3, 0.3], [0.7, 0.7])$
M	$([0.7, 0.7], [0.3, 0.3], [0.0, 0.0])$	$([0.2, 0.2], [0.2, 0.2], [0.6, 0.6])$	$([0.0, 0.0], [0.1, 0.1], [0.9, 0.9])$
T	$([0.3, 0.3], [0.4, 0.4], [0.3, 0.3])$	$([0.6, 0.6], [0.3, 0.3], [0.1, 0.1])$	$([0.2, 0.2], [0.1, 0.1], [0.7, 0.7])$
S	$([0.1, 0.1], [0.2, 0.2], [0.7, 0.7])$	$([0.2, 0.2], [0.4, 0.4], [0.4, 0.4])$	$([0.8, 0.8], [0.2, 0.2], [0.0, 0.0])$
B	$([0.1, 0.1], [0.1, 0.1], [0.8, 0.8])$	$([0.0, 0.0], [0.2, 0.2], [0.8, 0.8])$	$([0.2, 0.2], [0.0, 0.0], [0.8, 0.8])$
Daignosis	S_4	S_5	
C	$([0.4, 0.4], [0.3, 0.3], [0.3, 0.3])$	$([0.1, 0.1], [0.2, 0.2], [0.7, 0.7])$	
M	$([0.7, 0.7], [0.3, 0.3], [0.0, 0.0])$	$([0.1, 0.1], [0.1, 0.1], [0.8, 0.8])$	
T	$([0.2, 0.2], [0.2, 0.2], [0.6, 0.6])$	$([0.1, 0.1], [0.0, 0.0], [0.9, 0.9])$	
S	$([0.2, 0.2], [0.1, 0.1], [0.7, 0.7])$	$([0.2, 0.2], [0.1, 0.1], [0.7, 0.7])$	
B	$([0.2, 0.2], [0.0, 0.0], [0.8, 0.8])$	$([0.8, 0.8], [0.1, 0.1], [0.1, 0.1])$	

(8) amongst the diagnoses and the kid K_1 are: $\tilde{S}_m^{1w} = 0.6445$, $\tilde{S}_m^{2w} = 0.5760$, $\tilde{S}_m^{3w} = 0.7222$, $\tilde{S}_m^{4w} = 0.6668$ and $\tilde{S}_m^{5w} = 0.5884$.

Step 5. The ranking order obtained by using the values calculated in Step 4 is: $T \succ C \succ M \succ B \succ S$. A graphical representation of ranking order presented in [8, 9, 16, 29] and the proposed model is shown in Figure 2.

FIGURE 2. Ranking order of alternatives



	Proposed	[4]	[5]	[12]	[24]
C	0.6445	0.9443	0.7941	0.5385	0.8505
M	0.576	0.9571	0.8094	0.6282	0.8661
T	0.7222	0.9264	0.4568	0.6206	0.8185
S	0.6668	0.8214	0.5851	0.3336	0.5148
B	0.5884	0.765	0.5517	0.3154	0.4244

7. Comparative analysis with the existing techniques

Various DMs have applied the SMs for medical diagnosis in the environment of SVNSs and IVNSs [8, 9, 16, 29]. In order to portray the usefulness and validation of the proposed SMs, we apply it for the same problem and the results are shown in the Tables 3 and 4. According to the results obtained by applying our proposed MCDM model, we see that the Kids K_1 and K_2 suffered in the disease typhoid (T) under the observations of five symptoms $S_j (j = 1, 2, ..., 5)$. The results obtained by proposed and existing methods are different because of assigning the weights to the criteria, These results are further analyzed by using Spearman's correlation coefficient.

Table 3. Results obtained by proposed SVNS's SM

SMs	C	M	T	S	B	Ranking
Proposed	0.7774	0.7675	0.7969	0.6353	0.6127	$T \succ C \succ M \succ B \succ S$
[8]	0.9443	0.9571	0.9264	0.8214	0.7650	$M \succ C \succ T \succ S \succ B$
[9]	0.7941	0.8094	0.4568	0.5851	0.5517	$M \succ C \succ S \succ B \succ T$
[16]	0.5385	0.6282	0.6206	0.3336	0.3154	$M \succ T \succ C \succ S \succ B$
[28]	0.8505	0.8661	0.8185	0.5148	0.4244	$M \succ C \succ T \succ S \succ B$

Table 4. Results obtained by proposed IVNS's SM

SMs	C	M	T	S	B	Ranking
Proposed	0.6445	0.5760	0.7222	0.6668	0.5884	$T \succ C \succ M \succ B \succ S$
[8]	0.9443	0.9571	0.9264	0.8214	0.7650	$M \succ C \succ T \succ S \succ B$
[9]	0.7941	0.8094	0.4568	0.5851	0.5517	$M \succ C \succ S \succ B \succ T$
[16]	0.5385	0.6282	0.6206	0.3336	0.3154	$M \succ T \succ C \succ S \succ B$
[28]	0.8505	0.8661	0.8185	0.5148	0.4244	$M \succ C \succ T \succ S \succ B$

7.1. *Ranking analysis with Spearman's rank-correlation coefficient*

. The ranking preference of the diagnosis obtained by our and existing techniques are different and presented in Tables 3 and 4. In order to compare the diagnosis further, we use the Spearman's rank-correlation coefficient (ρ_s) and the critical value Z, where, ρ_s and Z can be calculated with the formulae given below:

$$\rho_s = 1 - 6 \sum_{l=1}^{i-1=k} \frac{(\triangle^l)^2}{n(n-1)},$$

and

$$Z = \rho_s \sqrt{n-1}.$$

Here, $\triangle^l$ is the difference between two sets of ranking. The values of ρ_s are always bounded in the closed interval $[-1, 1]$. The values of ρ_s which are nearer to ± 1 show the perfect relationship amongst two ranking orders. Moreover,the critical value Z is compared with a pre-estimated degree of significance value η. The critical value Z corresponding to the degree of significance value $\eta = 0.05$ for the examples $(n = 5)$ is, $Z_{0.05} = 0.9$. If the critical value Z more than 0.9, it indicates that there exist a strong relationship between two rankings. On the other hand, the two rankings can be considered as dissimilar or have weaker relationship.

There are five collections of preference rankings obtained by the proposed method and [8, 9, 16, 28], represented by X, Y, V, T and U, respectively and their ranking order can be seen in Tables 3 and 4. In order to compare these ranking orders, ρ_s and Z evaluated in Table 5. The analysis of the results is summarized in Table 5 as follows:

The results obtained by the proposed model with those obtained in [8] and [28], the critical

Table 5. Comparison with existing methods

Daignosis	X	Y	V	T	U	X-Y	X-V	X-T	X-U
C	2	2	2	3	2	0	0	-1	0
M	3	1	1	1	1	2	2	2	2
T	1	3	5	2	3	-2	-4	-1	-2
S	5	4	3	4	4	1	2	1	1
B	4	5	4	5	5	-1	0	-1	-1
Spearman's rank-correlation coefficient ρ_s						0.5	-0.2	0.6	0.5
Critical value Z						1	-0.4	1.2	1

value $Z = 1 > 0.9$, shows that there is a positive relationship between the ranking of the proposed model (X), the ranking [8] (Y) and [28] (U). Also, the results obtained by the proposed model (X) with those obtained in [16] (T), the critical value $Z = 1.2 > 0.9$ indicates that there is a strongly positive relationship between the ranking X and T. However, the ranking X of the proposed model is significantly dissimilar to the ranking [9] (V) because the critical value $Z = -0.4$ is smaller than 0.9.

8. **Conclusions**

The similarity measures are extensively utilized in MCDM problems from the last few decades. This paper suggested a novel technique to develop the similarity measures on the basis of Euclidean distance measure for SVNSs and IVNSs, respectively. However, the similarity measures presented in [15, 29, 30] have some shortcoming. On the other hand the suggested similarity measures satisfy all the axioms of the similarity measure. Moreover, we used the suggested similarity measures to medical diagnosis decision problems. A practical example is used to exemplify the practicability and efficiency of the proposed similarity measure, which are then compared to other existing similarity measures. We will emphasize to apply the proposed similarity measure in pattern recognition and supply chain problems under the framework of SVNSs and IVNSs in future.

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