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Steam turbine fault diagnosis based on single-valued neutrosophic multigranulation rough sets over two universes

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Abstract. Steam turbine fault diagnosis is a significant issue in fault diagnosis technology, which has made remarkable progress in the area of electromechanical engineering. Although many studies based on fuzzy approaches are developed on this topic, they can only cope with incomplete and uncertain information, but have limitations in processing indeterminate and inconsistent information in practical decision-making procedures. In addition, it is beneficial to solve problems under group decision-making background that aims to aggregate each expert's preference to reach a final conclusion by consensus and unanimity. To deal with these difficulties in steam turbine fault diagnosis, by combining multigranulation rough sets over two universes with single-valued neutrosophic sets theories, a single-valued neutrosophic multigranulation rough set over two universes is investigated in this paper. Then, we construct a general decision-making rule through using single-valued neutrosophic multigranulation rough sets over two universes within the background of steam turbine fault diagnosis. Finally, the validity of the decision-making method is verified by an illustrative case.

Keywords: Steam turbine fault diagnosis, single-valued neutrosophic sets, multigranulation rough sets over two universes, group decision-making

1. Introduction

In the area of electric power production and management, the steam turbine generator unit serves as a significant equipment in power plant industry. By studying the steam turbine generator unit, many researchers have constructed relationships between the fault pattern and the fault phenomenon and have made several studies based on fuzzy approaches [10, 13, 14]. According to the fuzzy set theory [17] introduced by Zadeh, fuzzy approaches play an important place and have been widely used in various fault diagnosis techniques. However, due to time pressure and knowledge limitation, the description of vagueness has been widely discussed and several extended fuzzy sets have been introduced. Among them, by adding the concept of non-membership

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function in fuzzy sets, Atanassov [16] generalized this idea to the intuitionistic fuzzy set (IFS), which is expressed by a membership function and a non-membership function. Thus, it provides a flexible framework for handling fuzziness and uncertainty. However, due to the complexity of objective things, the IFS can not cope with all kinds of uncertainties perfectly like problems including inconsistent or indeterminate information. Hence, the IFS have been further expanded.

Smarandache [8] founded the theory of neutrosophic sets. A neutrosophic set is a set that each element in neutrosophic sets is characterized by a degree of truth, indeterminacy, and falsity, respectively. However, neutrosophic sets can be only applied in philosophical problems. To use neutrosophic sets easily in practical applications, Wang et al. [9] further developed the definition and some operational laws of single-valued neutrosophic sets. Since then, many scholars have extended single-valued neutrosophic set theory from different viewpoints and obtained an increasing number of achievements [11, 12, 15].

In addition, group decision-making using multigranulation rough sets over two universes [3] constitutes another approach to aid steam turbine fault diagnosis. The multigranulation rough set over two universes takes full advantages of rough set over two universes [7] and multigranulation rough set [21, 22] theories. These two theories are both generalization forms of classical rough sets [23].

In rough set theory, many existing studies are focusing on the same universe and few researches have been conducted about the approximation operators over different universes. By introducing rough sets model over two universes, it is advantageous for experts to express their preferences about intrinsic relationships involving two different objections, such as the relation between the fault pattern and the fault phenomenon for a steam turbine generator unit. More details about latest achievements of rough sets on two universes model can be found in the literatures [2, 4, 7, 18].

For multigranulation rough sets, according to problem solving targets and different user's requirements, it is more reasonable to depict the target concept by considering multiple relations on the universe. According to the different risk decision-making strategies, Qian and Liang [21, 22] developed multigranulation rough sets model by employing SCRD strategy (seeking common ground while reserving differences) and SCED strategy (seeking common ground while eliminating differences). Later, since multigranulation rough sets model has demonstrated its superior performances via rule acquisition and information fusion, there have been many studies on this topic [3, 5, 6, 19, 20].

In this paper, to handle issues of single-valued neutrosophic data analysis and group decision-makings in steam turbine fault diagnosis, it is necessary to develop single-valued neutrosophic (SVN) multigranulation rough sets over two universes model by combining single-valued neutrosophic sets with multigranulation rough sets over two universes. Then, we will explore a novel decision-making approach to steam turbine fault diagnosis problems by utilizing the proposed model. Moreover, we give an illustrative example to interpret the fundamental steps and a practical application to steam turbine fault diagnosis.

Comparing previous studies of approaches to fault diagnosis techniques, this paper makes the following main contributions: (1) By combining multigranulation rough sets over two universes with single-valued neutrosophic sets theories, a single-valued neutrosophic multigranulation rough set over two universes is developed in this paper; (2) The model of SVN multigranulation rough sets over two universes provides engineers with a flexible way to analyze the relation between fault patterns and fault characteristics and integrate several experts' preferences for steam turbines; (3) This study develops a reasonable fault diagnosis approach under uncertain environment.

The paper is organized as follows. Section 2 gives a brief introduction to single-valued neutrosophic sets. Section 3 develops the model of SVN multigranulation rough sets over two universes. Then, we construct a general decision-making rule to steam turbine fault diagnosis problems through using the proposed rough set model in Section 4. An illustrative case is provided in Section 5 to show the effectiveness of our approach. Concluding remarks are given in Section 6.

2. Preliminaries

Neutrosophic sets [8] were introduced by Smarandache based on philosophical viewpoints, since it is difficult to apply neutrosophic sets to various practical problems, Wang et al. [9] developed the concept of single-valued neutrosophic set, which can be regarded as a subclass of the neutrosophic set and a powerful structure in reflecting an expert's indeterminate and inconsistent preferences in real-life decision-making procedures. In what follows, we review the concept of single-valued neutrosophic set.

Definition 2.1. Let U be the universe of discourse, a single-valued neutrosophic set A on U is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. Then, a single-valued neutrosophic set A can be expressed as the following mathematical symbol:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in U\},$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ for all $x \in U$ to the set A. Hence, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition: $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Let U be the universe of discourse, then the set of all single-valued neutrosophic sets on U is denoted by $SVN(U)$ in this paper. Moreover, $\forall A \in SVN(U)$. Based on the above definition, Wang et al. [9] defined the following operational laws on single-valued neutrosophic sets.

Definition 2.2. Let U be the universe of discourse, $\forall A, B \in SVN(U)$, then

(1) The complement of A is denoted by A^c such that for any $x \in U$,
$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in U\}$;
(2) the intersection of A and B is denoted by $A \cap B$ such that for any $x \in U$,
$A \cap B = \{\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)\rangle \mid x \in U\}$;
(3) the union of A and B is denoted by $A \cup B$ such that for any $x \in U$,
$A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)\rangle \mid x \in U\}$;
(4) $A \oplus B = \{\langle x, T_A(x) + T_B(x) - T_A(x) T_B(x), I_A(x) + I_B(x) - I_A(x) I_B(x), F_A(x) + F_B(x) - F_A(x) F_B(x)\rangle \mid x \in U\}$;
(5) $\lambda \cdot A = \{\langle x, 1 - (1 - T_A(x))^\lambda, 1 - (1 - I_A(x))^\lambda, 1 - (1 - F_A(x))^\lambda\rangle \mid x \in U\}$;
(6) the normalized Euclidian distance between A and B is denoted by $d(A, B)$,

$$d(A, B) = \sqrt{\frac{1}{3n} \sum_{i=1}^{n} \left\{ \begin{array}{l} (T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) \\ -I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2 \end{array} \right\}}.$$

To compare the magnitude of different single-valued neutrosophic sets, Zhang et al. [11] introduced the following comparison laws.

Definition 2.3. Let the single-valued neutrosophic set $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in U\}$, the score function set of A is defined as: $s(A) = \{\langle x, T_A(x) + 1 - I_A(x) + 1 - F_A(x)\rangle \mid x \in U\}$.

3. SVN multigranulation rough sets over two universes

This section will establish the concept of the SVN relation over two universes as well as SVN multigranulation rough sets over two universes from optimistic and pessimistic views.

Definition 3.1. Suppose U and V are two universes of discourse, a single-valued neutrosophic relation R from U to V is defined by:

$$R = \{\langle (x, y), \mu_R(x, y), \nu_R(x, y), \omega_R(x, y)\rangle \mid (x, y) \in U \times V\},$$

where $\mu_R(x, y), \nu_R(x, y), \omega_R(x, y) \in [0, 1]$, denoting the truth-membership, the indeterminacy-membership and the falsity-membership for all $(x, y) \in U \times V$, respectively. And the family of all SVN relations over $U \times V$ is represented by $SVNR(U \times V)$.

Definition 3.2. Suppose U and V are two universes of discourse, $R_i \in SVNR(U \times V)$ are the single-valued neutrosophic relations over $U \times V$, the pair (U, V, R_i) is called a single-valued neutrosophic multigranulation approximation space over two universes. For any $A \in SVN(V)$, the optimistic lower and upper approximations of A in terms of (U, V, R_i) are defined by:

$$\underline{\sum_{i=1}^{m} R_i}^{O}(A) = \{\langle x, \mu_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \nu_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \omega_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\rangle \mid x \in U\},$$

$$\overline{\sum_{i=1}^{m} R_i}^{O}(A) = \{\langle x, \mu_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \nu_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \omega_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\rangle \mid x \in U\},$$

where:

$$\mu_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \overset{m}{\underset{i=1}{\vee}} \wedge_{y\in V} \left[\omega_R(x, y) \vee \mu_A(y)\right],$$

$$\nu_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \overset{m}{\underset{i=1}{\wedge}} \vee_{y\in V} \left[(1 - \nu_R(x, y)) \wedge \nu_A(y)\right],$$

$$\omega_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \overset{m}{\underset{i=1}{\wedge}} \vee_{y\in V} \left[\mu_R(x, y) \wedge \omega_A(y)\right],$$

$$\mu_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \overset{m}{\underset{i=1}{\wedge}} \vee_{y\in V} \left[\mu_R(x, y) \wedge \mu_A(y)\right],$$

$$\nu_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \overset{m}{\underset{i=1}{\vee}} \wedge_{y\in V} \left[\nu_R(x, y) \vee \nu_A(y)\right],$$

$$\omega_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \overset{m}{\underset{i=1}{\vee}} \wedge_{y\in V} \left[\omega_R(x, y) \vee \omega_A(y)\right].$$

We call $(\underline{\sum_{i=1}^{m} R_i}^{O}(A), \overline{\sum_{i=1}^{m} R_i}^{O}(A))$ an optimistic SVN multigranulation rough set over two universes of A in terms of (U, V, R_i). Similar to the optimistic SVN multigranulation rough set over two universes, the pessimistic version of SVN multigranulation rough set over two universes will be introduced in the following part.

Definition 3.3. Suppose U and V are two universes of discourse, $R_i \in SVNR(U \times V)$ are the single-valued neutrosophic relations over $U \times V$, the pair (U, V, R_i) is called a single-valued neutrosophic multigranulation approximation space over two universes. For any $A \in SVN(V)$, the pessimistic lower and upper approximations of A in terms of (U, V, R_i) are defined by:

$$\underline{\sum_{i=1}^{m} R_i}^{P}(A) = \{\langle x, \mu_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \nu_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \omega_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\rangle \,|\, x \in U\},$$

$$\overline{\sum_{i=1}^{m} R_i}^{P}(A) = \{\langle x, \mu_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \nu_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \omega_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\rangle \,|\, x \in U\},$$

where:

$$\mu_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \overset{m}{\underset{i=1}{\wedge}} \wedge_{y\in V} \left[\omega_R(x, y) \vee \mu_A(y)\right],$$

$$\nu_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \overset{m}{\underset{i=1}{\vee}} \vee_{y\in V} \left[(1 - \nu_R(x, y)) \wedge \nu_A(y)\right],$$

$$\omega_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \overset{m}{\underset{i=1}{\vee}} \vee_{y\in V} \left[\mu_R(x, y) \wedge \omega_A(y)\right],$$

$$\mu_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \overset{m}{\underset{i=1}{\vee}} \vee_{y\in V} \left[\mu_R(x, y) \wedge \mu_A(y)\right],$$

$$\nu_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \overset{m}{\underset{i=1}{\wedge}} \wedge_{y\in V} \left[\nu_R(x, y) \vee \nu_A(y)\right],$$

$$\omega_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \overset{m}{\underset{i=1}{\wedge}} \wedge_{y\in V} \left[\omega_R(x, y) \vee \omega_A(y)\right].$$

We call $(\underline{\sum_{i=1}^{m} R_i}^{P}(A), \overline{\sum_{i=1}^{m} R_i}^{P}(A))$ a pessimistic SVN multigranulation rough set over two universes of A in terms of (U, V, R_i).

4. Steam turbine fault diagnosis using SVN multigranulation rough sets over two universes

Let $U = \{x_1, x_2, \ldots, x_j\}$ be a set of different fault patterns of steam turbine and $V = \{y_1, y_2, \ldots, y_k\}$ be a set of frequency ranges. Let $R_i \in SVNR(U \times V)$ $(i = 1, 2, \ldots, m)$ be m single-valued neutrosophic relations over $U \times V$, indicating various system fault knowledge base in the form of the single-valued neutrosophic set data provided by m mechanical engineers. In what follows, we also let $A \in SVN(V)$ be the fault testing sample. Thus, we obtain a single-valued neutrosophic information system which is denoted as (U, V, R_i, A).

Based on above descriptions, we can calculate optimistic and pessimistic lower and upper approximations of A. And by virtue of above definitions, we can further calculate the sets: $\underline{\sum_{i=1}^{m} R_i}^{O}$

$(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ and $\frac{1}{2}(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)) \oplus \frac{1}{2}(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A))$, which indicate the optimistic decision criterion, the pessimistic decision criterion, and the weighted decision criterion of the previous two criteria with the weighted value 0.5. In practical decision-making situations, according the principle of risk decision-making in classical operational research, these sets represent SVN multigranulation rough sets over two universes with maximum, medium and minimum risk in aggregating expert's decision-making preferences. Thus, it is necessary to choose an ideal SVN multigranulation rough sets over two universes from optimistic and pessimistic ones as the decision-making basis. In what follows, based on prospect theory [1], we introduce decision-making rules for steam turbine fault diagnosis procedures.

In risk decision-making environment, the decision-making behavior is not entirely rational, and existing decision-making approaches are based on expected utility theory that believes decision-makers are completely rational. Thus, the expected utility theory is not valid in complex real-life decision-making situations. On the basis of various experiments, prospect theory pursues the most satisfactory decision result rather than the one with the most largest expected utility. Thus, prospect theory can better reflect subjective risk attitude in decision-making processes. In prospect theory, the key element is the prospect value V, that is determined by decision weight function $w(p_i)$ and value function $v(\Delta x_i)$, which are defined as follows:

$$V = \sum_{i=1}^{n} w(p_i) v(\Delta x_i);$$

$$w(p_i) = \begin{cases} \frac{{p_i}^{\gamma}}{({p_i}^{\gamma} + (1-p_i)^{\gamma})^{\frac{1}{\gamma}}}, \Delta x_i \geq 0 \\ \frac{{p_i}^{\delta}}{\left({p_i}^{\delta} + (1-p_i)^{\delta}\right)^{\frac{1}{\delta}}}, \Delta x_i < 0 \end{cases};$$

$$v(\Delta x_i) = \begin{cases} (\Delta x_i)^{\alpha}, \Delta x_i \geq 0 \\ -\lambda(-\Delta x_i)^{\beta}, \Delta x_i < 0 \end{cases}.$$

where Δx_i represents the difference between outcome x_i with probability p_i and the reference point x_0, we can see there is a loss if x_i is smaller than x_0, while there is a gain if x_i is larger than x_0. The coefficients α and β indicate the curvature of the value function for gains and losses, λ indicate the loss aversion factor and in case of risk aversion $\lambda > 1$, γ and δ denote the curvature of the decision weight function for gains and losses.

Based on above discussion, we present an algorithm for steam turbine fault diagnosis model based on SVN multigranulation rough sets over two universes as follows:

Input: The relation between the universe U and V provided by decision-makers (U, V, R_i) and a fault testing sample A.

Output: The determined fault pattern.

Step 1: Calculate the sets: $\underline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\overline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\underline{\sum_{i=1}^{m} R_i}^{P}(A)$ and $\overline{\sum_{i=1}^{m} R_i}^{P}(A)$, respectively.

Step 2: Calculate the sets: $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ and $\frac{1}{2}(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)) \oplus \frac{1}{2}(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A))$.

Step 3: Let the optimistic decision criterion and the pessimistic decision criterion be two alternatives, the weighted decision criterion be the reference point, the one with larger prospect value can be considered as the best alternative.

Step 4: In the best alternative, the fault pattern with the largest score function value is the determined fault pattern.

5. An illustrative example

In steam turbine fault diagnosis, we suppose a set of ten fault patterns denoted as: $U = \{x_1, x_2, \ldots, x_{10}\}$, where $x_j\,(j = 1, 2, \ldots, 10)$ stands for unbalance, pneumatic force couple, offset center, oil-membrane oscillation, radial impact friction of rotor, symbiosis looseness, damage of antithrust bearing, surge, looseness of bearing block, nonuniform bearing stiffness. Another universe $V = \{y_1, y_2, \ldots, y_9\}$ stands for a set of nine frequency ranges, where $y_k\,(k = 1, 2, \ldots, 9)$ denotes some frequency ranges for different frequency spectrum: $C_1\,(0.01 - 0.39f)$, $C_2\,(0.4 - 0.49f)$, $C_3\,(0.5f)$, $C_4\,(0.51 - 0.99f)$,

$C_5\,(f)$, $C_6\,(2f)$, $C_7\,(3\ \ -5f)$, C_8 (odd times of f), C_9(high frequency > 5f). And the fault testing sample is denoted as A. Then, the system fault knowledge base proposed by Ye [14] can be composed of two kinds of objections related to the steam turbine fault diagnosis. That is, the fault pattern sets and the frequency range sets of the steam turbine. In order to solve problems under group decision-making background, each mechanical engineer could provide their preferences about the system fault knowledge base that is shown as the following Tables 1, 2 and 3. As pointed by Ye [13, 14], the information of the fault knowledge was modeled in [10] firstly, taking an example of steam turbine generator units in a local power plant in China, the authors treated the normalized value of different frequency peak energy amounts on nine bands in vibration signal frequency of each fault pattern as the vague value to form the system fault knowledge base. Then, Ye [13] studied steam turbine fault diagnosis based on fuzzy cross entropy of vague sets according to [10]. Recently, the same author [14] transformed the vague value into the single-valued neutrosophic value according to [13], proposed single-valued neutrosophic similarity measures-based steam turbine fault diagnosis methods. Hence, the data in this paper is originated from [10, 13, 14]. According to the algorithm of the model, we aim to seek the determined fault pattern by utilizing the proposed model.

In steam turbine fault diagnosis, assume that we take a fault testing sample, which is represented by the following information in the form of single-valued neutrosophic sets.

$$\begin{aligned} A = \{&\langle y_1, \langle 0, 0, 1\rangle\rangle, \langle y_2, \langle 0, 0, 1\rangle\rangle,\\ &\langle y_3, \langle 0.1, 0, 0.9\rangle\rangle, \langle y_4, \langle 0.9, 0, 0.1\rangle\rangle,\\ &\langle y_5, \langle 0, 0, 1\rangle\rangle, \langle y_6, \langle 0, 0, 1\rangle\rangle, \langle y_7, \langle 0, 0, 1\rangle\rangle,\\ &\langle y_8, \langle 0, 0, 1\rangle\rangle, \langle y_9, \langle 0, 0, 1\rangle\rangle\}. \end{aligned}$$

According to Definitions 3.2 and 3.3, we calculate the lower and upper approximations of optimistic and pessimistic SVN multigranulation rough sets over two universes of A in terms of (U, V, R_i), respectively.

$$\begin{aligned} \underline{\sum_{i=1}^{3} R_i}^{O}(A) = \{&\langle x_1, \langle 0, 0, 0.85\rangle\rangle,\\ &\langle x_2, \langle 0.69, 0, 0.28\rangle\rangle, \langle x_3, \langle 0.38, 0, 0.4\rangle\rangle,\\ &\langle x_4, \langle 0.18, 0, 0.77\rangle\rangle, \langle x_5, \langle 0.79, 0, 0.18\rangle\rangle,\\ &\langle x_6, \langle 0.55, 0, 0.37\rangle\rangle, \langle x_7, \langle 0.88, 0, 0.1\rangle\rangle,\\ &\langle x_8, \langle 0.68, 0, 0.27\rangle\rangle, \langle x_9, \langle 0.07, 0, 0.85\rangle\rangle,\\ &\langle x_{10}, \langle 0.17, 0, 0.77\rangle\rangle\}, \end{aligned}$$

$$\begin{aligned} \overline{\sum_{i=1}^{3} R_i}^{O}(A) = \{&\langle x_1, \langle 0, 0.02, 1\rangle\rangle,\\ &\langle x_2, \langle 0.55, 0.04, 0.3\rangle\rangle, \langle x_3, \langle 0, 0.07, 1\rangle\rangle,\\ &\langle x_4, \langle 0.08, 0.04, 0.89\rangle\rangle, \langle x_5, \langle 0.08, 0.02, 0.88\rangle\rangle, \end{aligned}$$

Table 1

Knowledge of system fault given by engineer 1

R_1	y_1	y_2	y_3	y_4	y_5
x_1	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0.85, 0.15, 0⟩
x_2	⟨0, 0, 1⟩	⟨0.28, 0.03, 0.69⟩	⟨0.09, 0.03, 0.88⟩	⟨0.55, 0.15, 0.3⟩	⟨0, 0, 1⟩
x_3	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0.3, 0.28, 0.42⟩
x_4	⟨0.09, 0.02, 0.89⟩	⟨0.78, 0.04, 0.18⟩	⟨0, 0, 1⟩	⟨0.08, 0.03, 0.89⟩	⟨0, 0, 1⟩
x_5	⟨0.09, 0.03, 0.88⟩	⟨0.09, 0.02, 0.89⟩	⟨0.08, 0.04, 0.88⟩	⟨0.09, 0.03, 0.88⟩	⟨0.18, 0.03, 0.79⟩
x_6	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0.18, 0.04, 0.78⟩
x_7	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0.08, 0.04, 0.88⟩	⟨0.86, 0.07, 0.07⟩	⟨0, 0, 1⟩
x_8	⟨0, 0, 1⟩	⟨0.27, 0.05, 0.68⟩	⟨0.08, 0.04, 0.88⟩	⟨0.54, 0.08, 0.38⟩	⟨0, 0, 1⟩
x_9	⟨0.85, 0.08, 0.07⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩
x_{10}	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩

R_1	y_6	y_7	y_8	y_9
x_1	⟨0.04, 0.02, 0.94⟩	⟨0.04, 0.03, 0.93⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩
x_2	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0.08, 0.05, 0.87⟩
x_3	⟨0.4, 0.22, 0.38⟩	⟨0.08, 0.05, 0.87⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩
x_4	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩
x_5	⟨0.08, 0.05, 0.87⟩	⟨0.08, 0.05, 0.87⟩	⟨0.08, 0.04, 0.88⟩	⟨0.08, 0.04, 0.88⟩
x_6	⟨0.12, 0.05, 0.83⟩	⟨0.37, 0.08, 0.55⟩	⟨0, 0, 1⟩	⟨0.22, 0.06, 0.72⟩
x_7	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩
x_8	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩
x_9	⟨0, 0, 1⟩	⟨0, 0, 1⟩	⟨0.08, 0.04, 0.88⟩	⟨0, 0, 1⟩
x_{10}	⟨0.77, 0.06, 0.17⟩	⟨0.19, 0.04, 0.77⟩	⟨0, 0, 1⟩	⟨0, 0, 1⟩

Table 2
The knowledge of system fault given by engineer 2

R_2	y_1	y_2	y_3	y_4	y_5
x_1	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.86, 0.14, 0\rangle$
x_2	$\langle 0, 0, 1\rangle$	$\langle 0.29, 0.02, 0.69\rangle$	$\langle 0.08, 0.05, 0.87\rangle$	$\langle 0.58, 0.12, 0.3\rangle$	$\langle 0, 0, 1\rangle$
x_3	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.32, 0.28, 0.4\rangle$
x_4	$\langle 0.08, 0.03, 0.89\rangle$	$\langle 0.77, 0.06, 0.17\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.09, 0.06, 0.85\rangle$	$\langle 0, 0, 1\rangle$
x_5	$\langle 0.08, 0.04, 0.88\rangle$	$\langle 0.09, 0.03, 0.88\rangle$	$\langle 0.09, 0.06, 0.85\rangle$	$\langle 0.08, 0.03, 0.89\rangle$	$\langle 0.19, 0.02, 0.79\rangle$
x_6	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.2, 0.03, 0.77\rangle$
x_7	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.09, 0.05, 0.86\rangle$	$\langle 0.88, 0.06, 0.06\rangle$	$\langle 0, 0, 1\rangle$
x_8	$\langle 0, 0, 1\rangle$	$\langle 0.29, 0.04, 0.67\rangle$	$\langle 0.1, 0.03, 0.87\rangle$	$\langle 0.55, 0.09, 0.36\rangle$	$\langle 0, 0, 1\rangle$
x_9	$\langle 0.86, 0.07, 0.07\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_{10}	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$

R_2	y_6	y_7	y_8	y_9
x_1	$\langle 0.06, 0.01, 0.93\rangle$	$\langle 0.05, 0.02, 0.93\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_2	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.1, 0.04, 0.86\rangle$
x_3	$\langle 0.42, 0.2, 0.38\rangle$	$\langle 0.09, 0.06, 0.85\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_4	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_5	$\langle 0.06, 0.07, 0.87\rangle$	$\langle 0.08, 0.04, 0.88\rangle$	$\langle 0.08, 0.05, 0.87\rangle$	$\langle 0.08, 0.06, 0.86\rangle$
x_6	$\langle 0.14, 0.04, 0.82\rangle$	$\langle 0.39, 0.06, 0.55\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.21, 0.07, 0.72\rangle$
x_7	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_8	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_9	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.09, 0.03, 0.88\rangle$	$\langle 0, 0, 1\rangle$
x_{10}	$\langle 0.8, 0.04, 0.16\rangle$	$\langle 0.2, 0.03, 0.77\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$

Table 3
The knowledge of system fault given by engineer 3

R_3	y_1	y_2	y_3	y_4	y_5
x_1	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.88, 0.12, 0\rangle$
x_2	$\langle 0, 0, 1\rangle$	$\langle 0.28, 0.04, 0.68\rangle$	$\langle 0.06, 0.07, 0.87\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0, 0, 1\rangle$
x_3	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.31, 0.3, 0.39\rangle$
x_4	$\langle 0.08, 0.04, 0.88\rangle$	$\langle 0.79, 0.06, 0.15\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.1, 0.04, 0.86\rangle$	$\langle 0, 0, 1\rangle$
x_5	$\langle 0.08, 0.03, 0.89\rangle$	$\langle 0.09, 0.04, 0.87\rangle$	$\langle 0.08, 0.05, 0.87\rangle$	$\langle 0.07, 0.03, 0.9\rangle$	$\langle 0.2, 0.01, 0.79\rangle$
x_6	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.19, 0.05, 0.76\rangle$
x_7	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.08, 0.05, 0.87\rangle$	$\langle 0.87, 0.07, 0.06\rangle$	$\langle 0, 0, 1\rangle$
x_8	$\langle 0, 0, 1\rangle$	$\langle 0.28, 0.06, 0.66\rangle$	$\langle 0.09, 0.04, 0.87\rangle$	$\langle 0.56, 0.08, 0.36\rangle$	$\langle 0, 0, 1\rangle$
x_9	$\langle 0.87, 0.07, 0.06\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_{10}	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$

R_3	y_6	y_7	y_8	y_9
x_1	$\langle 0.06, 0.02, 0.92\rangle$	$\langle 0.05, 0.03, 0.92\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_2	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.08, 0.06, 0.86\rangle$
x_3	$\langle 0.43, 0.19, 0.38\rangle$	$\langle 0.1, 0.07, 0.83\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_4	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_5	$\langle 0.07, 0.06, 0.87\rangle$	$\langle 0.09, 0.04, 0.87\rangle$	$\langle 0.09, 0.05, 0.86\rangle$	$\langle 0.09, 0.07, 0.84\rangle$
x_6	$\langle 0.13, 0.05, 0.82\rangle$	$\langle 0.4, 0.06, 0.54\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.22, 0.08, 0.7\rangle$
x_7	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_8	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$
x_9	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0.07, 0.04, 0.89\rangle$	$\langle 0, 0, 1\rangle$
x_{10}	$\langle 0.81, 0.04, 0.15\rangle$	$\langle 0.21, 0.04, 0.75\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$

$$\langle x_6, \langle 0, 0.05, 1\rangle\rangle, \langle x_7, \langle 0.86, 0.05, 0.1\rangle\rangle,$$
$$\langle x_8, \langle 0.54, 0.04, 0.38\rangle\rangle, \langle x_9, \langle 0, 0.04, 1\rangle\rangle,$$
$$\langle x_{10}, \langle 0, 0.04, 1\rangle\rangle\},$$

$$\underline{\sum_{i=1}^{3} R_i}^{P}(A) = \{\langle x_1, \langle 0, 0, 0.88\rangle\rangle,$$
$$\langle x_2, \langle 0.68, 0, 0.29\rangle\rangle, \langle x_3, \langle 0.38, 0, 0.43\rangle\rangle,$$
$$\langle x_4, \langle 0.15, 0, 0.79\rangle\rangle, \langle x_5, \langle 0.79, 0, 0.2\rangle\rangle,$$
$$\langle x_6, \langle 0.54, 0, 0.4\rangle\rangle, \langle x_7, \langle 0.86, 0, 0.1\rangle\rangle,$$
$$\langle x_8, \langle 0.66, 0, 0.29\rangle\rangle, \langle x_9, \langle 0.06, 0, 0.87\rangle\rangle,$$
$$\langle x_{10}, \langle 0.15, 0, 0.81\rangle\rangle\},$$

$$\overline{\sum_{i=1}^{3} R_i}^{P}(A) = \{\langle x_1, \langle 0, 0.01, 1\rangle\rangle,$$

$\langle x_2, \langle 0.6, 0.02, 0.3\rangle\rangle$, $\langle x_3, \langle 0, 0.05, 1\rangle\rangle$,
$\langle x_4, \langle 0.1, 0.02, 0.85\rangle\rangle$, $\langle x_5, \langle 0.09, 0.01, 0.85\rangle\rangle$,
$\langle x_6, \langle 0, 0.03, 1\rangle\rangle$, $\langle x_7, \langle 0.88, 0.04, 0.1\rangle\rangle$,
$\langle x_8, \langle 0.56, 0.03, 0.36\rangle\rangle$, $\langle x_9, \langle 0, 0.03, 1\rangle\rangle$,
$\langle x_{10}, \langle 0, 0.03, 1\rangle\rangle\}$.

Based on above results, we further calculate the sets: $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ and $\frac{1}{2}(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)) \oplus \frac{1}{2}(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A))$. Then, suppose that $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ and $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ are two alternatives, the weighted decision criterion is the reference point. In prospect theory, Kahneman and Tversky [1] experimentally determined the values of parameters: $\alpha = \beta = 0.88$, $\lambda = 2.25$, $\gamma = 0, 61$, $\delta = 0.69$, which are consistent with empirical data. And we suppose the probability of each fault pattern occurs is equal, i.e., $p_i = 0.1$. According to the prospect value function, we can obtain the prospect value of the optimistic decision criterion is $V = 0.0267$, while the prospect value of the pessimistic decision criterion is $V = 0.0435$. Thus, the pessimistic decision criterion can be considered as the best alternative. Finally, in the pessimistic decision criterion, the ranking result of all fault patterns is: $x_7 > x_2 > x_8 > x_5 > x_6 > x_3 > x_4 > x_{10} > x_9 > x_1$. That is, the vibration faults of the steam turbine generator unit are originated from damage of antithrust bearing (x_7) at first, next pneumatic force couple (x_2), and then surge (x_8), and so on. Compared with other existing works, the proposed decision-making approach can reflect the risk attitude of decision-makers well, that overcomes the drawback of expected utility theory. And SVN multigranulation rough sets over two universes take advantages of both single-valued neutrosophic sets and multigranulation rough sets theories, decision-makers can better depict the uncertain preferences and aggregate several experts' opinions, which is more in consistent with practical decisions-making. Thus, the proposed method provides a framework for fault diagnosis under risk and uncertainty.

6. Conclusions

Aiming at combining the theories of multigranulation rough sets over two universes with single-valued neutrosophic sets, we put forward SVN multigranulation rough sets over two universes in this paper. In the framework of the proposed model, we have investigated basic definitions of SVN multigranulation rough sets over two universes. Then, we have developed a general approach to decision-making within the background of steam turbine fault diagnosis by utilizing the proposed model. At last, to verify the effectiveness of the decision-making approach, a case study has been studied. As future work, we consider the attribute reduction approaches, uncertainty measures and topological structures of the proposed rough set model. It is also meaningful to further investigate the proposed rough set model under interval neutrosophic environment and apply it to data mining and knowledge discovery.

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