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Interval-Valued Complex Neutrosophic Soft Set and its Applications in Decision-Making

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Abstract: In this paper, the notion of the interval valued complex neutrosophic soft set (IV-CNSS) is defined as a combination of interval valued complex neutrosophic set and soft set. Then, we introduce the IV-CNSS's operations such as union, intersection and complement. To explore the study further, we present some basic operational rules, and investigate their properties. A new algorithm is developed by transforming the interval complex neutrosophic soft values from complex space to the real space using a practical formula which gives a decision-making with a simple computational process without the need to carry out directed operations by complex numbers. This algorithm is then applied to evaluate two kinds of a certain product from a manufacturer and choose the most suitable one. IV-CNSS provides an interval-based membership structure to handle the uncertain data. This feature allows decision makers to record their hesitancy in assigning membership values which in turn best catch the obscurity and the complexity of such data.

Keywords: soft set; neutrosophic set; interval complex neutrosophic set; interval neutrosophic set; complex neutrosophic soft set.

1. Introduction

In 1999, the model of neu-trosophic set(NS) presented by Smarandache **[1]** as a popularization of fuzzy set **[3]**, intuitionistic fuzzy set **[6]**, interval-valued fuzzy sets[8]and interval-valued intuitionistic fuzzy sets **[13]** . it's also an important method and powerful tool to deal with incomplete, indeterminate, and inconsistent information in some real-life problem. The notion of neu-trosophic soft set(NSS) was grounded by Maji **[11]**, as a popularization of a soft set **[2]**, fuzzy soft set **[4]**, and intuitionist fuzzy soft set [6]. In some real life applications, such as decision-making processes, he also used this concept**[11].** This concept(NSS) deals with indeterminate data, while when the relationships are indefinite, the fuzzy soft set and the intuitionistic fuzzy soft set fail to work..Since the neu-trosophic set is difficult to use explicitly in real-life implementations, Maji,first of all proposed the idea of single-valued neu-trosophic soft set and supplied its theoretical practices and properties. Additionally, in numerous real-life problems, the degrees of membership, non membership, and ind-eterminacy of a proven statement may be suitably given by interval forms, instead of real numbers. Deli **[14],** deals with this case, proposed connotation of the interval neutrosophic soft set , which is described by the degrees of membership , falsehood membership and indeterminacy, that are values of which are intervals rather than true numbers. Mukherjee **[17]**, The numerous similarity measures of interval valued neutrosophic soft sets were presented and their application in problems with pattern recognition. Abdel-Basset et al **[19-26]**, suggested a solution to supply change problems, professional selection problems, time-cost tradeoffs, and leveling problems in construction using a neutrosophic environment. Recent studies have focused on designing systems using complex fuzzy sets**[30]** in (NSS) and (INSS)**[15,29,35,36,37,39,42**].To design and model real-life applications in a better way, the 'complex' feature is used to manage uncertainty and periodicity data at the same time. By adding to the concept of a complex fuzzy set, complex-valued non-membership grade **[30],** the definition of complex intuitionist fuzzy soft set was introduced by Kumar**[31].** A complex neutrosophic soft set was proposed by Broumi et all **[29],** which is an extension type of a complex fuzzy soft set and a complex intuitionistic soft .The complex neutrosophic soft set will deal with the redundant existence of insecurity, incompleteness, indeterminacy, inconsistency in periodic data. The advantage of complex neutrosophic soft sets over the neutrosophic soft sets is the fact that, in addition to the membership degree provided by the neutrosophic soft sets and represented in the complex neutrosophic soft sets by amplitude, The phase, which is an attribute degree characterizing the amplitude, is also given by the complex neutrosophic soft sets. Yet it is not easy to find a crisp (exact) neutrosophic soft membership degree in many real-life applications (as in the single-value neutrosophic soft set), because we deal with unclear and ambiguous details. So we must establish a new notion to solve this, which uses a neutrosophic soft membership degree interval. In this article, we first describe complex interval neutrosophical soft sets (IV-CNSSs) as a generalization the concept of the soft set, complex fuzzy soft set, interval valued complex fuzzy soft set [32,33,34], complex intuitionistic fuzzy soft set, interval complex valued intuitionistic fuzzy soft sets. We then add such definitions and operations of interval complex neutrosophic soft sets. Several properties of IV-CNSSs have been established that are related to activities. The goal of this paper is also to explore decision-making on the basis of interval-value complex neutrosophical soft sets. We develop an adaptable approach to decision-making based on interval-complex valued neutrosophic soft sets and include examples to demonstrate the established approach. the novelty of this work can be viewed:

- In this work, we have combined all of the following concepts Interval, Complex setting, Neutrosophic set, and Soft set .Thus we got a new model is interval valued complex neutrosophic soft set (IV-CNSS).

- We have used this hybrid model to solve one of the famous real-life problems, which is the decision-making problem.

The rest of this article is organized as follows. Section 2 recalls some basic concepts of neutrosophic set, soft set, complex fuzzy soft set, neutrosophic soft set, complex neutrosophic set, and their operations. Section 3 presents the formulation of the interval-valued complex neutrosophic set and some examples. Section 4 presenting Set-Theoretic Operations of Interval Valued Complex Neutrosophic Soft Set(IV-CNSs). Section 5 presenting Operational rules of operation Interval Valued Complex Neutrosophic Soft Sets (IV-CNSs). Section6 we introduce an application of our concept to a decision-making problem. Section 7 delineates conclusions and suggests further studies.

2. Preliminaries

Now, we present the basic meanings of the neutrosophic set in this section**[1,5],** soft set theory **[2],** and complex fuzzy soft set **[31,34],** signal valued complex neutrosophic soft set **[12]** that is helpful for subsequent discussions.

Let X be a space of points (objects) denoted as x with generic elements in X.

Definition 2.1. [5] A neutrosophical set A is an entity that has the structure $A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X\}$ where the functions T,I,F:[$^-0,^+1$], denote the membership functions of truth, indeterminacy, and falsehood, respectively, of the $x\in X$ element with regard to set. The condition must satisfy these membership functions $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. The $T_A(x), I_A(x)$and$F_A(x)$ functions are actual normal or non-standard subsets of the [$^-0,^+1$] interval.

Definition 2.2. **[2]** Let U be a discourse universe and A be a parameter set. Let a power set of U be *P(U)*. A soft set over U is called an ordered pair *(F,A)*, where F is a mapping given by$F : A \rightarrow P(U)$.

The parameterized family of subsets of the U set is a soft set. All $F(e)$, $e \in E$ set from this family can be interpreted as a set consisting of soft set *e*-elements (F, E) or as an *e*-approximate soft set member.

Definition 2.3. [34] Let U be an initial set and E be a set of parameters. Let $P(U)$ denote the power set of the complex fuzzy sets of U and let $A \subset E$. A pair (F, A) is called a *complex fuzzy soft set* over U, where F is a mapping given by $F : A \rightarrow P(U)$ such that

$$F(e_i) = \{(h_k, r_k(x). e^{iarg_k(x)}) | i \text{ is the number of parameter and } k\text{: is the number of sets}\}$$

Definition 2.4. [11] Let U be an initial universe,E be a set of parameters and let $NP(U)$ denotes the set of all neutrosophic sets. Then the pair(S, A) is termed to be the neutrosophic soft set over U , where S is a mapping given by $S: A \rightarrow NP(U)$.

Definition 2.5. [12] Let U be a universal set, E be a set of parameters under, $A \subseteq E$ and for all $x \in U$, Ψ_A is a complex neutrosophic set over U. Then a single-value complex neutrosophic soft set S_A over U is then defined as a mapping $S_A: E \rightarrow CN(U)$, Where the complex neutrosophic sets in U are denoted by *CN(U)*, and $\Psi_A(x) = \emptyset$ If x *no belong* A Here $\Psi_A(x)$ is referred to here as a complex neutrosophical approximate function ofS_A and the values of S_A are referred to as the x-elements of the CNSS for all $x \in U$. Here S_A then be represented in the following manner by a series of ordered pairs:

$$S_A = \{\langle x, \Psi_A(x) \rangle : x \in E, \Psi_A(x) \in CN(U)\}$$

Where $\Psi_A(x) = (\langle x, p_A(x). e^{i\omega_A(x)}, q_A(x). e^{i\varphi_A(x)}, r_A(\mathrm{x}). e^{i\gamma_A(x)} \rangle)$,

$p_A, q_A, r_A$are real-valued and lie in [0,1] and $\omega_A, \varphi_A, \gamma_A \in (0,2\pi]$. This is achieved in order to ensure that the CNSS model description refers to the original form of the complex fuzzy set on which the CNSS model is centered.

Definition 2.6. [12] Over the universe U, let S_A and S_B be two complex neutrosophic sets,we define the operations of complement, subset, union and intersection as follows.The complement of S_A, denoted by $S^c{}_{A,}$*is a CNSS* defined by $S^c{}_A = \{(x, \Psi^c{}_{A,}(x)) : x \in U\}$, where $\Psi^c{}_A(x)$ is the complex neutrosophic complement of $\Psi_A(x)$.

It is said that S_A is a CNS − subset of S_B and denoted by $S_A \subseteq S_B$ for all $x \in U$, $\Psi_A(x) \subseteq \Psi_B(x)$, that is conditions are satisfied:

$p_A(e) \leq p_B(e), q_A(e) \leq q_B(e), r_A(e) \leq r_B(e)$

$And\ \ \omega_A(e) \leq \omega_B(e), \varphi_A(e) \leq \varphi_B(e), \gamma_A(e) \leq \gamma_B(e)$

iii) The union(intersection) of $S_A$and S_B , denoted as $S_A \cup (\cap) S_B$ is defined as: $S_C = S_A \cup (\cap) S_B = \{(x, \Psi_A\ (x) \cup (\cap) \Psi_B\ (x)) : x \in U\}$

$$S_C(e) = \begin{Bmatrix} (x, \Psi_A\ (x)) \text{ if } \ e \in A - B \\ (x, \Psi_B\ (x)) \text{ if } \ e \in B - A \\ (x, \Psi_A\ (x) \cup (\cap) \Psi_B\ (x)) \text{ if } \ e \in A \cup (\cap) B \end{Bmatrix}$$

where $C = A \cup (\cap) B, x \in U, and$

$$\Psi_A\ (x) \cup (\cap) \Psi_B\ (x) = \begin{cases} p_A(x) \vee\ (\wedge) p_B(x).\, e^{j(\omega_A(x) \cup (\cap) \omega_B(x))}, \\ q_A(x) \wedge\ (\vee) q_B(x).\, e^{j(\varphi_A(x) \cup (\cap) \varphi_B(x))}, \\ r_A(x) \wedge\ (\vee) r_B(x).\, e^{j(\omega_A(x) \cup (\cap) \omega_B(x))}, \end{cases}$$

where $\vee$ and $\wedge$ respectively denote the maximum and minimal operators.

3. Interval Valued Complex Neutrosophic Soft Set

The interval complex neutrosophic soft set(IV-CNSS) model, which is a combination of the IV-CNS and softset models, is presented in this section. As seen below, the formal description of this model and some definitions related to this model are:

Defintion 3.1 : Let U be an initial universe, E be a set of parameters under consideration, $A \subset E$ and IV-CNS(U) denotes the set of IV-CNS-subset of U. Then a pair $(\bar{S}, A)$ is called an interval-valued complex neutrosophic soft set in short (IV-CNSS) over U , where $\bar{S}$ is a mapping given by $\bar{S}: A \to IVCNS(U)$ such that $\bar{S}_A(x) = \{\langle\ a, T_{\bar{S}_a}(x), I_{\bar{S}_a}(x), F_{\bar{S}_a}(x)\ \rangle : x \in U, a \in A\ \}$. The IV-CNSSs model is defined over a universe of discourse U by three membership function its truth membership function $T_{\bar{S}_a}$,an indeterminate membership function $I_{\bar{S}_a}$, and falsehood membership function $F_{\bar{S}_a}$ as follows:

$$T_{\bar{S}_a}: A \to ICNS(U), T_{\bar{S}_a}(x) = t_{\bar{S}_a}(x).\, e^{j\alpha\omega_{\bar{S}_a}(x)}$$

$$I_{\bar{S}_a}: A \to ICNS(U), I_{\bar{S}_a}(x) = i_{\bar{S}_a}(x).\, e^{j\beta\psi_{\bar{S}_a}(x)}$$

$F_{\bar{S}_a}: A \to ICNS(U), F_{\bar{S}_a}(x) = f_{\bar{S}_a}(x).e^{j\gamma\phi_{\bar{S}_a}(x)}$

In the above equations, the IV-CNS is a collection of complex neutrosophical interval sets,the function of the interval truth membership is $t_{\bar{S}_a}(x)$, $i_{\bar{S}_a}(x)$ is the interval indeterminate membership and $f_{\bar{S}_a}(x)$ is the interval falsehood membership function, while $e^{j\alpha\omega_{\bar{S}_a}(x)}$, $e^{j\beta\psi_{\bar{S}_a}(x)}$and $e^{j\gamma\phi_{\bar{S}_a}(x)}$ are the corresponding interval-valued phase terms, respectively, with $j = \sqrt{-1}$.The scaling factors α, β and γ lie within the interval $(0,2\pi]$. This study implies that the values $\alpha, \beta, \gamma = 2\pi$. An interval-complex neutrosophic soft set can be written in set theoretical form as:

$$(\bar{S}, A) = \{a, \langle \frac{T_{\bar{S}_a}(x) = t_{\bar{S}_a}(x).e^{j\alpha\omega_{\bar{S}_a}(x)}, I_{\bar{S}_a}(x) = i_{\bar{S}_a}(x).e^{j\beta\psi_{\bar{S}_a}(x)}, F_{\bar{S}_a}(x) = f_{\bar{S}_a}(x).e^{j\gamma\phi_{\bar{S}_a}(x)}}{x} \rangle : x \in U, a \in A\}$$

In the above set theoretic form, the amplitude interval-valued terms $t_{\bar{S}_a}(x), i_{\bar{S}_a}(x)$ and $f_{\bar{S}_a}(x)$ can be further split as $t_{\bar{S}_a}(x) = [t^L{}_{\bar{S}_a}(x), t^U{}_{\bar{S}_a}(x)], i_{\bar{S}_a}(x) = [i^L{}_{\bar{S}_a}(x), i^U{}_{\bar{S}_a}(x)]$ and $f_{\bar{S}_a}(x) = [f^L{}_{\bar{S}_a}(x), f^U{}_{\bar{S}_a}(x)]$,where $t^L{}_{\bar{S}_a}(x), i^L{}_{\bar{S}_a}(x), f^L{}_{\bar{S}_a}(x)$ represent the lower bound, while $t^U{}_{\bar{S}_a}(x), i^U{}_{\bar{S}_a}(x), f^U{}_{\bar{S}_a}(x)$ represent the upper bound in each interval, respectively. likewise, for the phases: $\omega_{\bar{S}_a}(x) = [\omega^L{}_{\bar{S}_a}(x), \omega^U{}_{\bar{S}_a}(x)], \psi_{\bar{S}_a}(x) = [\psi^L{}_{\bar{S}_a}(x), \psi^U{}_{\bar{S}_a}(x)], \phi_{\bar{S}_a}(x) = [\phi^L{}_{\bar{S}_a}(x), \phi^U{}_{\bar{S}_a}(x)]$.

Example3.2 Let U be a set of developing countries in the area of West Asia (WA), considered to be a set of criteria that characterize the economic indicators of a nation, and $A = \{e_1, e_2, e_3, e_4\} \subset E$,where these sets are as defined below:

$$U = \{u_1 = Iraq, u_2 = Kingdom\ of\ Saudi\ Arabia, u_3 = Jordan, u_4 = UAE\}$$

$$E = \{e_1 = inflation\ rate, e_2 = population\ growth, e_3 = GDP\ growth\ rate, e_4 = unemployment\ rate, e_5 = export\ volume\}.$$

The IV-CNSS $\bar{S}_A(e_1), \bar{S}_A(e_2), \bar{S}_A(e_3)and\bar{S}_A(e_4)$are defined as:

$$\bar{S}_A(e_1) =$$

$$\{(\frac{[0.4,0.6].e^{j2\pi[0.5,0.6]}, [0.1,0.7].e^{j2\pi[0.1,0.3]}, [0.3,0.5].e^{j2\pi[0.8,0.9]}}{u_1}), (\frac{[0.2,0.4].e^{j2\pi[0.3,0.6]}, [0.1,0.1].e^{j2\pi[0.7,0.9]}, [0.5,0.9].e^{j2\pi[0.2,0.5]}}{u_2}),$$

$$(\frac{[0.3,0.4].e^{j2\pi[0.7,0.8]}, [0.6,0.7].e^{j2\pi[0.6,0.7]}, [0.2,0.6].e^{j2\pi[0.6,0.8]}}{u_3}), (\frac{[0,0.9].e^{j2\pi[0.9,1]}, [0.2,0.3].e^{j2\pi[0.7,0.8]}, [0.3,0.5].e^{j2\pi[0.4,0.5]}}{u_4})\}$$

$$\bar{S}_A(e_2) =$$

$$\{(\frac{[0.2,0.5].e^{j2\pi[0.4,0.7]}, [0.5,0.7].e^{j2\pi[0.2,0.3]}, [0.6,0.8].e^{j2\pi[0.6,0.9]}}{u_1}), (\frac{[0.1,0.3].e^{j2\pi[0.1,0.3]}, [0.2,0.7].e^{j2\pi[0.6,0.8]}, [0.4,0.7].e^{j2\pi[0.1,0.5]}}{u_2}),$$

$$(\frac{[0.6,0.8].e^{j2\pi[0.8,0.9]}, [0.4,0.6].e^{j2\pi[0.4,0.7]}, [0.1,0.4].e^{j2\pi[0.7,0.8]}}{u_3}), (\frac{[0.3,0.8].e^{j2\pi[0.7,0.9]}, [0,0.1].e^{j2\pi[0.7,0.7]}, [0.2,0.4].e^{j2\pi[0.6,0.8]}}{u_4})\}$$

$$\bar{S}_A(e_3) =$$

$\{(\frac{[0.3,0.7].e^{j2\pi[0.5,0.6]},[0.1,0.7].e^{j2\pi[0.1,0.3]},[0.3,0.5].e^{j2\pi[0.8,0.9]}}{u_1}),(\frac{[0.4,0.4].e^{j2\pi[0.6,0.7]},[0.1,0.9].e^{j\pi[0.2,0.4]},[0.3,0.8].e^{j\pi[0.5,0.6]}}{u_2}),$

$(\frac{[0.37,0.64].e^{j\pi[0.47,0.50]},[0.36,0.57].e^{j\pi[0.64,0.7]},[0.28,0.66].e^{j\pi[0.16,0.20]}}{u_3}),(\frac{[0.15,0.52].e^{j\pi[0.1,0.2]},[0,0.5].e^{j\pi[0.6,0.7]},[0.3,0.3].e^{j\pi[0.6,0.7]}}{u_4})\}$

$\bar{S}_A(e_4) =$

$\{(\frac{[0.5,1].e^{j\pi[0.6,0.71]},[0.11,0.73].e^{j\pi[0.12,0.34]},[0.6,0.7].e^{j\pi[0.82,0.94]}}{u_1}),(\frac{[0.35,0.41].e^{j\pi[0.52,0.71]},[0.2,0.9].e^{j\pi[0.1,0.4]},[0.5,0.8].e^{j\pi[0.56,0.62]}}{u_2}),$

$(\frac{[0.21,0.63].e^{j\pi[0.41,0.55]},[0.31,0.52].e^{j\pi[0.11,0.67]},[0.17,0.49].e^{j\pi[0.26,0.40]}}{u_3}),(\frac{[0.22,0.42].e^{j\pi[0.2,0.35]},[0,3,0.65].e^{j\pi[0.36,0.88]},[0.4,0.6].e^{j\pi[0.74,0.91]}}{u_4})\}$

Then a selection of IV-CNSS of the form can be written by the interval valued complex neutrosophic soft sets $(\bar{S},A)$ such that $(\bar{S},A) = \{\bar{S}_A(e_1),\bar{S}_A(e_2),\bar{S}_A(e_3),\bar{S}_A(e_4)\}$.

Definition 3.3 : Let $(\bar{S},A) and\ (\bar{S},B)$ be two IV-CNSSs over U. Then

$(\bar{S},A)$ is said to be a subset of $(\bar{S},B)$,denoted by $(\bar{S},A) \subset (\bar{S},B)$ iff $t^L{}_{\bar{s}_a}(x) \le t^L{}_{\bar{s}_b}(x), i^L{}_{\bar{s}_a}(x) \le i^L{}_{\bar{s}_b}(x), f^L{}_{\bar{s}_a}(x) \le f^L{}_{\bar{s}_b}(x)$ and $t^U{}_{\bar{s}_a}(x) \le t^U{}_{\bar{s}_b}(x), i^U{}_{\bar{s}_a}(x) \le t^U{}_{\bar{s}_b}(x), f^U{}_{\bar{s}_a}(x) \le f^U{}_{\bar{s}_b}(x)$ for the amplitude terms, and $\omega^L{}_{\bar{s}_a}(x) \le \omega^L{}_{\bar{s}_b}(x), \psi^L{}_{\bar{s}_a}(x) \le \psi^L{}_{\bar{s}_b}(x), \phi^L{}_{\bar{s}_a}(x) \le \phi^L{}_{\bar{s}_a}(x)$ and $\omega^U{}_{\bar{s}_a}(x) \le \omega^U{}_{\bar{s}_b}(x), \psi^U{}_{\bar{s}_a}(x) \le \psi^U{}_{\bar{s}_b}(x), \phi^U{}_{\bar{s}_a}(x) \le \phi^U{}_{\bar{s}_b}(x)$ for the phase terms for all $x \in U$.

$(\bar{S},A)$is said to be equal of $(\bar{S},B)$,denoted by $(\bar{S},A) = (\bar{S},B)$ iff $t^L{}_{\bar{s}_a}(x) = t^L{}_{\bar{s}_b}(x), i^L{}_{\bar{s}_a}(x) = i^L{}_{\bar{s}_b}(x), f^L{}_{\bar{s}_a}(x) = f^L{}_{\bar{s}_b}(x)$ and $t^U{}_{\bar{s}_a}(x) = t^U{}_{\bar{s}_b}(x), i^U{}_{\bar{s}_a}(x) = t^U{}_{\bar{s}_b}(x), f^U{}_{\bar{s}_a}(x) = f^U{}_{\bar{s}_b}(x)$ for the amplitude terms, and $\omega^L{}_{\bar{s}_a}(x) = \omega^L{}_{\bar{s}_b}(x), \psi^L{}_{\bar{s}_a}(x) = \psi^L{}_{\bar{s}_b}(x), \phi^L{}_{\bar{s}_a}(x) = \phi^L{}_{\bar{s}_b}(x)$ and $\omega^U{}_{\bar{s}_a}(x) = \omega^U{}_{\bar{s}_b}(x), \psi^U{}_{\bar{s}_a}(x) = \psi^U{}_{\bar{s}_b}(x), \phi^U{}_{\bar{s}_a}(x) = \phi^U{}_{\bar{s}_b}(x)$for the phase terms for all $x \in U$.

Definition 3.4: $(\bar{S},A)$ is said to be a null IV-CNSS, denoted by $(\bar{S},A)_\phi$if for all $x \in U, a \in A$,the amplitude and phase terms of the membership function are given by $t^L{}_{\bar{s}_a}(x) = t^U{}_{\bar{s}_a}(x), i^L{}_{\bar{s}_a}(x) = i^U{}_{\bar{s}_a}(x), f^L{}_{\bar{s}_a}(x) = f^U{}_{\bar{s}_a}(x) = 0\ and\ \omega^L{}_{\bar{s}_a}(x) = \omega^U{}_{\bar{s}_a}(x), \psi^L{}_{\bar{s}_a}(x) = \psi^U{}_{\bar{s}_a}(x), \phi^L{}_{\bar{s}_a}(x) = \phi^U{}_{\bar{s}_a}(x) = 0\pi$,respectively

Definition 3.5: $(\bar{S},A)$ is said to be an absolute IV-CNSS, denoted by $(\bar{S},A)_\delta$if for all $x \in U$, the amplitude and phase terms of the membership function are given by

$t^L{}_{\bar{s}_a}(x) = t^U{}_{\bar{s}_a}(x), i^L{}_{\bar{s}_a}(x) = i^U{}_{\bar{s}_a}(x), f^L{}_{\bar{s}_a}(x) = f^U{}_{\bar{s}_a}(x) = 1$ and $\omega^L{}_{\bar{s}_a}(x) = \omega^U{}_{\bar{s}_a}(x), \psi^L{}_{\bar{s}_a}(x) = \psi^U{}_{\bar{s}_a}(x), \phi^L{}_{\bar{s}_a}(x) = \phi^U{}_{\bar{s}_a}(x) = 2\pi$,respectively

The IV-CNSS effectively decreases to a crisp collection of the universe U in each of the cases defined in Definitions 3.4 and 3.5, i.e. $(\bar{S}, A)_{\Phi} = \left\{ (\frac{[0,0.].e^{j\pi[0.0]},[0.0].e^{j\pi[0,0]},[0,0].e^{j\pi[0,0]}}{x}) \right\} = \left\{ \left(\frac{0,0,0}{x} \right) \right\}$.

and $(\bar{S}, A)_{\delta} = \left\{ (\frac{[1,1].e^{j2\pi},[1.1].e^{j2\pi},[1,1].e^{j2\pi}}{x}) \right\} = \left\{ \left(\frac{1,1,1}{x} \right) \right\}$.

4. Set Theoretic Operations of Interval Valued Complex Neutrosophic Soft Set

The basic set of theoretical operations on IV-CNSS, namely the complement, union intersection, are described in this section.

Definition 4.1: Let $(\bar{S}, A) and\ (\bar{S}, B)$ be two IV-CNSSs over U. The union of $(\bar{S}, A)$and $(\bar{S}, B)$ is an IV-CNSS $(\bar{S}, C)$ where $= A \cup B$, $a \in A, b \in B$ and $x \in U$**.** to define the union we consider three cases :

Case 1: if $c \in A - B$.then

$$T_{\bar{S}_a}(x) = \left[inf t_{\bar{S}_a}(x), sup t_{\bar{S}_a}(x)\right]. e^{j\alpha\omega_{\bar{S}_a}(x)}$$

$$I_{\bar{S}_a}(x) = \left[inf i_{\bar{S}_a}(x), sup i_{\bar{S}_a}(x)\right]. e^{j\alpha\psi_{\bar{S}_a}(x)}$$

$$F_{\bar{S}_a}(x) = \left[inf f_{\bar{S}_a}(x), sup f_{\bar{S}_a}(x)\right]. e^{j\alpha\phi_{\bar{S}_a}(x)}$$

Case 2: if $c \in B - A$.then

$$T_{\bar{S}_b}(x) = \left[inf t_{\bar{S}_b}(x), sup t_{\bar{S}_b}(x)\right]. e^{j\alpha\omega_{\bar{S}_b}(x)}$$

$$I_{\bar{S}_b}(x) = \left[inf i_{b}(x), sup i_{\bar{S}_b}(x)\right]. e^{j\alpha\psi_{\bar{S}_b}(x)}$$

$$F_{\bar{S}_b}(x) = \left[inf f_{\bar{S}_b}(x), sup f_{\bar{S}_b}(x)\right]. e^{j\alpha\phi_{\bar{S}_b}(x)}$$

Case 3: if $c \in A \cap B$.then

$$T_{\bar{S}_C}(x) = \left[inf t_{\bar{S}_C}(x), sup t_{\bar{S}_C}(x)\right]. e^{j\alpha\omega_{\bar{S}_C}(x)}$$

$$I_{\bar{S}_C}(x) = \left[inf i_{\bar{S}_C}(x), sup i_{\bar{S}_C}(x)\right]. e^{j\alpha\psi_{\bar{S}_C}(x)}$$

$$F_{\bar{S}_C}(x) = \left[inf f_{\bar{S}_C}(x), sup f_{\bar{S}_C}(x)\right]. e^{j\alpha\phi_{\bar{S}_C}(x)}$$

Where

$$inf t_{\bar{S}_C}(x) = \vee\left(inf t_{\bar{S}_A}(x), inf t_{\bar{S}_B}(x)\right), sup t_{\bar{S}_C}(x) = \vee\left(sup t_{\bar{S}_A}(x), sup t_{\bar{S}_B}(x)\right);$$

$$inf i_{\bar{S}_C}(x) = \wedge\left(inf i_{\bar{S}_A}(x), inf i_{\bar{S}_B}(x)\right), sup i_{\bar{S}_C}(x) = \wedge\left(sup i_{\bar{S}_A}(x), sup i_{\bar{S}_B}(x)\right);$$

$$inf f_{\bar{S}_C}(x) = \wedge\left(inf f_{\bar{S}_A}(x), inf f_{\bar{S}_B}(x)\right), sup i_{\bar{S}_C}(x) = \wedge\left(sup f_{\bar{S}_A}(x), sup f_{\bar{S}_B}(x)\right);$$

The union of the phase terms are the same as defined for the union of the amplitude terms . The symbols ∨,∧ represent respectively max and min operators.

Example 4.2. Let $U=\{u_1,u_2\}$ be a universe of discourse, $(\bar{S},A) and\ (\bar{S},B)$ be two interval complex neutrosophic soft sets such that $A=\{e_1,e_2,e_3\}$, $B=\{e_2,e_3,e_4\}$ defined on U as follows:

$$(\bar{S},A)=$$

$$\{e_1,(\frac{[0.1,0.8].e^{j2\pi[0.22,0.48]},[0.4,0.8].e^{j2\pi[0.55,0.62]},[0.37,0.46].e^{j2\pi[0.57,0.83]}}{u_1}),(\frac{[0.26,0.39].e^{j2\pi[0.53,0.86]},[0.65,0.86].e^{j2\pi[0.43,0.61]},[0.53,0.9].e^{j2\pi[0.2,0.5]}}{u_2}),$$

$$e_2,(\frac{[0.4,0.6].e^{j2\pi[0.5,0.6]},[0.1,0.7].e^{j2\pi[0.1,0.3]},[0.3,0.5].e^{j2\pi[0.8,0.9]}}{u_1}),(\frac{[0.2,0.4].e^{j2\pi[0.3,0.6]},[0.1,0.1].e^{j2\pi[0.7,0.9]},[0.5,0.9].e^{j2\pi[0.2,0.5]}}{u_2})),$$

$$e_3,(\frac{[0.2,0.5].e^{j2\pi[0.4,0.7]},[0.5,0.7].e^{j2\pi[0.2,0.3]},[0.6,0.8].e^{2j\pi[0.6,0.9]}}{u_1}),(\frac{[0.1,0.3].e^{j2\pi[0.1,0.3]},[0.2,0.7].e^{2j\pi[0.6,0.8]},[0.4,0.7].e^{j2\pi[0.1,0.5]}}{u_2})\}$$

$$(\bar{S},B)=$$

$$\{e_2,(\frac{[0.3,0.7].e^{j2\pi[0.7,0.8]},[0.4,0.9].e^{j2\pi[0.3,0.5]},[0.6,0.8].e^{j2\pi[0.5,0.6]}}{u_1}),(\frac{[0.4,0.4].e^{j2\pi[0.6,0.7]},[0.1,0.9].e^{j2\pi[0.2,0.4]},[0.3,0.8].e^{j2\pi[0.5,0.6]}}{u_2})$$

$$,e_3,(\frac{[0.3,0.7].e^{j2\pi[0.4,0.5]},[0.3,0.5].e^{j2\pi[0.6,0.7]},[0.2,0.6].e^{j2\pi[0.16,0.3]}}{u_1}),(\frac{[0.21,0.63].e^{j2\pi[0.41,0.55]},[0.31,0.52].e^{j2\pi[0.11,0.67]},[0.3,0.73].e^{j2\pi[0.2,0.58]}}{u_2})$$

$$e_4,(\frac{[0.2,0.6].e^{j2\pi[0.6,0.7]},[0.3,0.8].e^{j2\pi[0.2,0.4]},[0.5,0.7].e^{j2\pi[0.4,0.5]}}{u_1}),(\frac{[0.5,0.6].e^{j2\pi[0.7,0.8]},[0.2,0.8].e^{j2\pi[0.3,0.5]},[0.4,0.9].e^{j2\pi[0.6,0.7]}}{u_2})\}$$

Then the union between two IV-CNSS defined as:

$$(\bar{S},A)\cup(\bar{S},B)=(\bar{S},C)=$$

$$\{e_1,(\frac{[0.1,0.8].e^{j2\pi[0.22,0.48]},[0.4,0.8].e^{j2\pi[0.55,0.62]},[0.37,0.46].e^{j2\pi[0.57,0.83]}}{u_1}),(\frac{[0.26,0.39].e^{j2\pi[0.53,0.86]},[0.65,0.86].e^{j2\pi[0.43,0.61]},[0.53,0.9].e^{j2\pi[0.2,0.5]}}{u_2})),$$

$$\{(e_2,(\frac{[0.4,0.7].e^{j2\pi[0.7,0.8]},[0.1,0.7].e^{j2\pi[0.1,0.3]},[0.3,0.5].e^{j2\pi[0.5,0.6]}}{u_1}),(\frac{[0.4,0.4].e^{j2\pi[0.6,0.7]},[0.1,0.1].e^{j2\pi[0.2,0.4]},[0.4,0.91].e^{j2\pi[0.58,0.69]}}{u_2})),$$

$$e_3,(\frac{[0.3,0.7].e^{j2\pi[0.4,0.7]},[0.3,0.7].e^{j2\pi[0.6,0.7]},[0.2,0.6].e^{j2\pi[0.16,0.3]}}{u_1}),(\frac{[0.21,0.63].e^{j2\pi[0.41,0.55]},[0.2,0.52].e^{j2\pi[0.11,0.67]},[0.3,0.8].e^{j2\pi[0.1,0.5]}}{u_2}),$$

$$e_4,(\frac{[0.2,0.6].e^{j2\pi[0.6,0.7]},[0.3,0.8].e^{j2\pi[0.2,0.4]},[0.5,0.7].e^{j2\pi[0.4,0.5]}}{u_1}),(\frac{[0.5,0.6].e^{j2\pi[0.7,0.8]},[0.2,0.8].e^{j2\pi[0.3,0.5]},[0.4,0.9].e^{j2\pi[0.6,0.7]}}{u_2})\}$$

$$where\ C=A\cup B.$$

Definition 4.3. Let $(\bar{S},A)$and $(\bar{S},B)$ be two IV-CNSs over U. The intersection of $(\bar{S},A)$and $(\bar{S},B)$ is an IV-CNSS $(\bar{S},C)$ where $C=A\cap B, a\in A, b\in B$ and $x\in U$. to define the intersection we consider three cases:

Case 1: if $c\in A-B$.then

$$T_{\bar{S}_a}(x)=\left[inf t_{\bar{S}_a}(x), sup t_{\bar{S}_a}(x)\right].e^{j\alpha\omega_{\bar{S}a}(x)}$$

$$I_{\bar{S}_a}(x)=\left[inf i_{\bar{S}_a}(x), sup i_{\bar{S}_a}(x)\right].e^{j\alpha\psi_{\bar{S}a}(x)}$$

$$F_{\bar{S}_a}(x) = \left[inff_{\bar{S}_a}(x), supf_{\bar{S}_a}(x)\right].e^{j\alpha\phi_{\bar{S}_a}(x)}$$

Case 2: if $c \in B - A$.then

$$T_{\bar{S}_b}(x) = \left[inft_{\bar{S}_b}(x), supt_{\bar{S}_b}(x)\right].e^{j\alpha\omega_{\bar{S}_b}(x)}$$

$$I_{\bar{S}_b}(x) = \left[infi_b(x), supi_{\bar{S}_b}(x)\right].e^{j\alpha\psi_{\bar{S}_b}(x)}$$

$$F_{\bar{S}_b}(x) = \left[inff_{\bar{S}_b}(x), supf_{\bar{S}_b}(x)\right].e^{j\alpha\phi_{\bar{S}_b}(x)}$$

Case 3: if $c \in A \cap B$.then

$$T_{\bar{S}_C}(x) = \left[inft_{\bar{S}_C}(x), supt_{\bar{S}_C}(x)\right].e^{j\alpha\omega_{\bar{S}_C}(x)}$$

$$I_{\bar{S}_C}(x) = \left[infi_{\bar{S}_C}(x), supi_{\bar{S}_C}(x)\right].e^{j\alpha\psi_{\bar{S}_C}(x)}$$

$$F_{\bar{S}_C}(x) = \left[inff_{\bar{S}_C}(x), supf_{\bar{S}_C}(x)\right].e^{j\alpha\phi_{\bar{S}_C}(x)}$$

Where

$$inft_{\bar{S}_C}(x) = \wedge\left(inft_{\bar{S}_A}(x), inft_{\bar{S}_B}(x)\right), supt_{\bar{S}_C}(x) = \wedge\left(supt_{\bar{S}_A}(x), supt_{\bar{S}_B}(x)\right);$$

$$infi_{\bar{S}_C}(x) = \vee\left(infi_{\bar{S}_A}(x), infi_{\bar{S}_B}(x)\right), supi_{\bar{S}_C}(x) = \vee\left(supi_{\bar{S}_A}(x), supi_{\bar{S}_B}(x)\right);$$

$$inff_{\bar{S}_C}(x) = \vee\left(inff_{\bar{S}_A}(x), inff_{\bar{S}_B}(x)\right), supi_{\bar{S}_C}(x) = \vee\left(supf_{\bar{S}_A}(x), supf_{\bar{S}_B}(x)\right);$$

The intersection of the phase terms are the same as defined for the intersection of the amplitude terms. The symbols $\vee, \wedge$ represent respectively max and min operators.

Example 4.4. As in Example 4.2, let $(\bar{S}, A)$and $(\bar{S}, B)$ be two two interval value complex neutrosophic soft sets . Then, $(\bar{S}, C)$ is given by the intersection of two interval value complex neutrosophic soft sets:

$$(\bar{S}, A) \cap (\bar{S}, B) = (\bar{S}, C) =$$

$$\{e_1, (\frac{[0.1,0.8].e^{j2\pi[0.22,0.48]}, [0.4,0.8].e^{j2\pi[0.55,0.62]}, [0.37,0.46].e^{j2\pi[0.57,0.83]}}{u_1}), (\frac{[0.26,0.39].e^{j2\pi[0.53,0.86]}, [0.65,0.86].e^{j2\pi[0.43,0.61]}, [0.53,0.9].e^{j2\pi[0.2,0.5]}}{u_2}),$$

$$e_2, (\frac{[0.3,0.6].e^{j2\pi[0.5,0.6]}, [0.4,0.9].e^{j2\pi[0.3,0.5]}, [0.6,0.8].e^{j2\pi[0.8,0.9]}}{u_1}), (\frac{[0.2,0.4].e^{j2\pi[0.3,0.6]}, [0.1,0.9].e^{j2\pi[0.7,0.9]}, [0.5,0.9].e^{j2\pi[0.5,0.6]}}{u_2}),$$

$$e_3, (\frac{[0.2,0.5].e^{j2\pi[0.4,0.7]}, [0.5,0.7].e^{j2\pi[0.8,0.7]}, [0.2,0.8].e^{j2\pi[0.6,0.9]}}{u_1}), (\frac{[0.1,0.3].e^{j2\pi[0.1,0.3]}, [0.31,0.7].e^{j2\pi[0.6,0.8]}, [0.4,0.73].e^{j2\pi[0.2,0.58]}}{u_2}),$$

$$e_4, (\frac{[0.2,0.6].e^{j2\pi[0.6,0.7]}, [0.3,0.8].e^{j2\pi[0.2,0.4]}, [0.5,0.7].e^{j2\pi[0.4,0.5]}}{u_1}), (\frac{[0.5,0.6].e^{j2\pi[0.7,0.8]}, [0.2,0.8].e^{j2\pi[0.3,0.5]}, [0.4,0.9].e^{j2\pi[0.6,0.7]}}{u_2})\}$$

$$Where\ C = A \cap B$$

Definition4.5. Let $(\bar{S}, A)$ be IV-CNSs over U. The complement of $(\bar{S}, A)$, denoted by $(\bar{S}, A)^{c}$ is as defined below:

$(\bar{S},A)^{c} = (\bar{S}^{c},A) = \{a,\langle \frac{T_{\bar{S}_{A}c}(x)=t_{\bar{S}_{A}c}(x).e^{j2\pi\omega_{\bar{S}_{A}c}(x)}, I_{\bar{S}_{A}c}(x)=i_{\bar{S}_{A}c}(x).e^{j2\pi\psi_{\bar{S}_{A}c}(x)}, F_{\bar{S}_{A}c}(x)=f_{\bar{S}_{A}c}(x).e^{j2\pi\phi_{\bar{S}_{A}c}(x)}}{x}\rangle : x \in U, a \in A\}$.

Where $t_{\bar{S}_{A^c}}(x) = f_{\bar{S}_A}(x)$ and $\omega_{\bar{S}_{A^c}}(x) = 2\pi - \omega_{\bar{S}_A}(x)$. Similarly $i_{\bar{S}_{A^c}}(x) = \left(inf\,i_{\bar{S}_{A^c}}(x), sup\,i_{\bar{S}_{A^c}}(x)\right)$where $inf\,i_{\bar{S}_{A^c}}(x) = 1 - sup\,i_{\bar{S}_A}(x)$ and $sup\,i_{\bar{S}_{A^c}}(x) = 1 - inf\,i_{\bar{S}_A}(x)$,with phase term $\psi_{\bar{S}_{A^c}}(x) = 2\pi - \psi_{\bar{S}_A}(x)$ Also, $f_{\bar{S}_{A^c}}(x) = t_{\bar{S}_A}(x)$,while the phase term $\phi_{\bar{S}_{A^c}}(x) = 2\pi - \phi_{\bar{S}_A}(x)$.

Proposition 4.6. Let $(\bar{S},A)$ is a *IV-CNSs* over *U*, then, $((\bar{S},A)^{C})^{C} = (\bar{S},A)$.

Proof.From Definition 4.5, we have

$(\bar{S},A)^{c} = (\bar{S}^{c},A) = \{a,\langle T_{\bar{S}_{A}c}(x), I_{\bar{S}_{A}c}(x), F_{\bar{S}_{A}c}(x)\rangle : x \in U, a \in A\}$.

$=\{a,\langle t_{\bar{S}_{A^c}}(x).\,e^{j2\pi\omega_{\bar{S}_{A^c}}(x)}, i_{\bar{S}_{A^c}}(x).\,e^{j2\pi\psi_{\bar{S}_{A^c}}(x)}, f_{\bar{S}_{A^c}}(x).\,e^{j2\pi\phi_{\bar{S}_{A^c}}(x)}\rangle : x \in U, a \in A\}$.

$=\{a,\langle f_{\bar{S}_A}(x).\,e^{j2\pi(2\pi-\omega_{\bar{S}_A}(x))}, (inf\,i_{\bar{S}_{A^c}}(x), sup\,i_{\bar{S}_{A^c}}(x)).\,e^{j2\pi(2\pi-\psi_{\bar{S}_A}(x))}, t_{\bar{S}_A}(x).\,e^{j2\pi(2\pi-\phi_{\bar{S}_A}(x))}\rangle : x \in U, a \in A\}$.

$=\{a,\langle f_{\bar{S}_A}(x).\,e^{j2\pi(2\pi-\omega_{\bar{S}_A}(x))}, (1 - sup\,i_{\bar{S}_A}(x), 1 - inf\,i_{\bar{S}_A}(x)).\,e^{j2\pi(2\pi-\psi_{\bar{S}_A}(x))}, f_{\bar{S}_{A^c}}(x).\,e^{j2\pi(2\pi-\phi_{\bar{S}_A}(x))}\rangle : x \in U, a \in A\}$.

Thus

$((\bar{S},A)^{c})^{c}$

$=\{a,\langle f_{\bar{S}_{A^c}}(x).\,e^{j2\pi(2\pi-(2\pi-\omega_{\bar{S}_{A^c}}(x)))}\,(1 - sup\,i_{\bar{S}_{A^c}}(x), 1 - inf\,i_{\bar{S}_{A^c}}).\,e^{j2\pi(2\pi-\psi_{\bar{S}_{A^c}}(x))}, t_{\bar{S}_{A^c}}(x).\,e^{j2\pi(2\pi-(2\pi-\phi_{\bar{S}_{A^c}}(x)))}\rangle : x \in U, a \in A\}$.

$=\{a,\langle f_{\bar{S}_{A^c}}(x).\,e^{j2\pi(2\pi-(2\pi-(2\pi-\omega_{\bar{S}_A}(x)))}\,(1 - (1 - inf\,i_{\bar{S}_A}(x)), 1 - (1 - sup\,i_{\bar{S}_A}(x)).\,e^{j2\pi(2\pi-(2\pi-\psi_{\bar{S}_A}(x))}, t_{\bar{S}_{A^c}}(x).\,e^{j2\pi(2\pi-(2\pi-\phi_{\bar{S}_A}(x)}\rangle : x \in U, a \in A\}$.

$=\{a,\langle t_{\bar{S}_A}(x).\,e^{j2\pi\omega_{\bar{S}_A}(x)}, i_{\bar{S}_A}(x).\,e^{j2\pi\psi_{\bar{S}_A}(x)}, f_{\bar{S}_A}(x).\,e^{j2\pi\phi_A(x)}\rangle : x \in U, a \in A\}$.

$=(\bar{S},A)$.

Example 4.7. Consider Example 1. The complement of $(\bar{S},A)$ is given by $(\bar{S},A)^{c} = \{\bar{S}_A{}^{c}(e_1), \bar{S}_A{}^{c}(e_2), \bar{S}_A{}^{c}(e_3), \bar{S}_A{}^{c}(e_4)\}$, we just give the complement to $\bar{S}_A{}^{c}(e_1)$below for the sake of brevity

$$\bar{S}_A{}^{c}(e_1) =$$

$$\{(\frac{[0.3,0.5].\,e^{j2\pi[0.5,0.6]}, [0.3,0.9].\,e^{j2\pi[0.1,0.3]}, [0.4,0.6].\,e^{j2\pi[0.8,0.9]}}{u_1}), (\frac{[0.5,0.9].\,e^{j2\pi[0.3,0.6]}, [0.9,0.9].\,e^{j2\pi[0.7,0.9]}, [0.2,0.4].\,e^{j2\pi[0.2,0.5]}}{u_2}),$$

$$(\frac{[0.2,0.6].\,e^{j2\pi[0.7,0.8]}, [0.3,0.4].\,e^{j2\pi[0.6,0.7]}, [0.3,0.4].\,e^{j2\pi[0.6,0.8]}}{u_3}), (\frac{[0.3,0.5].\,e^{j2\pi[0.9,1]}, [0.7,0.8].\,e^{j2\pi[0.7,0.8]}, [0,0.9].\,e^{j2\pi[0.4,0.5]}}{u_4})\}$$

Proposition 4.8. Let $(\bar{S},A)$, $(\bar{S},B)$ and $(\bar{S},C)$ be three interval complex neutrosophic soft sets over *U*. *Then:*

i. $(\bar{S}, A) \cup (\bar{S}, B) = (\bar{S}, B) \cup\ (\bar{S}, A)$ ($Commutative$ law)

ii. $(\bar{S}, A) \cap (\bar{S}, B) = (\bar{S}, B) \cap\ (\bar{S}, A)$ ($Commutative$ law)

iii. $(\bar{S}, A) \cup \big((\bar{S}, B) \cup (\bar{S}, C)\big) = \big((\bar{S}, A) \cup (\bar{S}, B)\big) \cup (\bar{S}, C)$ ($Associative$ law)

iv. $(\bar{S}, A) \cap \big((\bar{S}, B) \cap (\bar{S}, C)\big) = \big((\bar{S}, A) \cap (\bar{S}, B)\big) \cap (\bar{S}, C)$ ($Associative$ law)

v. $(\bar{S}, A) \cup ((\bar{S}, B) \cap\ (\bar{S}, C))= \big((\bar{S}, A) \cup (\bar{S}, B)) \cap ((\bar{S}, A)\big) \cup (\bar{S}, C))$ ($Distribution$ law)

vi. $(\bar{S}, A) \cap ((\bar{S}, B) \cup\ (\bar{S}, C))= \big((\bar{S}, A) \cap (\bar{S}, B)) \cup ((\bar{S}, A)\big) \cap (\bar{S}, C))$ ($Distribution$ law)

vii. $(\bar{S}, A) \cup ((\bar{S}, A) \cap (\bar{S}, B)) = (\bar{S}, A)$

viii. $(\bar{S}, A) \cap ((\bar{S}, A) \cup (\bar{S}, B)) = (\bar{S}, A)$

ix. $(\bar{S}, A) \cup (\bar{S}, B)^c = (\bar{S}, A)^c \cap (\bar{S}, B)^c$ (De $Morgan's$ law)

x. $((\bar{S}, A) \cap (\bar{S}, B))^c = ((\bar{S}, A)^c \cup (\bar{S}, B))^c$ (De $Morgan's$ law)

Proof: All of these assertions can be directly proven.

Theorem 4.9. Let $(\bar{S}, A)$ and $(\bar{S}, B)$be two interval complex neutrosophic soft set,Then The smallest one containing both $(\bar{S}, A)$ and $(\bar{S}, B)$is $(\bar{S}, A) \cup (\bar{S}, B)$.

Proof: Directly

Theorem 4.10. Let $(\bar{S}, A)$ and $(\bar{S}, B)$be two interval complex neutrosophic soft set,then the largest one present in both$(\bar{S}, A)$ and $(\bar{S}, B)$is $(\bar{S}, A) \cap (\bar{S}, B)$.

Proof: Directly

Theorem 4.11. Let $(\bar{S}, A)$ and $(\bar{S}, B)$. be two interval complex neutrosophic soft sets on U. Then, $(\bar{S}, A) \leq (\bar{S}, B)$iff $(\bar{S}, B)^c \leq (\bar{S}, A)^c$

Proof: Directly

Theorem 4.12. Let $\bar{P}$ be the power set of all interval complex neutrosophic soft set. Then, $(\bar{P},\cup,\cap)$ forms a distributive lattice.

Proof: Directly

5. Operational rules of operation Interval Valued Complex Neutrosophic Soft Sets

Let$(\bar{S},A)=\{a,\left(\left[T^{L}{}_{\bar{s}_A},T^{U}{}_{\bar{s}_A}\right],\left[I^{L}{}_{\bar{s}_A},I^{U}{}_{\bar{s}_A}\right],\left[F^{L}{}_{\bar{s}_A},F^{U}{}_{\bar{s}_A}\right]\right),a\in A\}$.and$(\bar{S},B)=\{b,\left(\left[T^{L}{}_{\bar{s}_B},T^{U}{}_{\bar{s}_B}\right],\left[I^{L}{}_{\bar{s}_B},I^{U}{}_{\bar{s}_B}\right],\left[F^{L}{}_{\bar{s}_B},F^{U}{}_{\bar{s}_B}\right]\right),b\in B\}$. be two interval valued complex neutrosophic soft sets over U which are defined by

$\left[T^{L}{}_{\bar{s}_A},T^{U}{}_{\bar{s}_A}\right]=\left[t^{L}{}_{\bar{s}_A}(x),t^{U}{}_{\bar{s}_A}(x)\right].e^{j\alpha\left[\omega^{L}{}_{\bar{s}_A}(x),\omega^{U}{}_{\bar{s}_A}(x)\right].}$, $\left[I^{L}{}_{\bar{s}_A},I^{U}{}_{\bar{s}_A}\right]=$

$\left[i^{L}{}_{\bar{s}_A}(x),i^{U}{}_{\bar{s}_A}(x)\right].e^{j\beta\left[\psi^{L}{}_{\bar{s}_A}(x),\psi^{U}{}_{\bar{s}_A}(x)\right].}$, $\left[F^{L}{}_{\bar{s}_A},F^{U}{}_{\bar{s}_A}\right]=$

$\left[f^{L}{}_{\bar{s}_A}(x),f^{U}{}_{\bar{s}_A}(x)\right].e^{j\gamma\left[\Phi^{L}{}_{\bar{s}_A}(x),\Phi^{U}{}_{\bar{s}_A}(x)\right].}$and

$\left[T^{L}{}_{\bar{s}_B},T^{U}{}_{\bar{s}_B}\right]=\left[t^{L}{}_{\bar{s}_B}(x),t^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\omega^{L}{}_{\bar{s}_B}(x),\omega^{U}{}_{\bar{s}_B}(x)\right].}$, $\left[I^{L}{}_{\bar{s}_B},I^{U}{}_{\bar{s}_B}\right]=$

$\left[i^{L}{}_{\bar{s}_B}(x),i^{U}{}_{\bar{s}_B}(x)\right].e^{j\beta\left[\psi^{L}{}_{\bar{s}_B}(x),\psi^{U}{}_{\bar{s}_B}(x)\right].}$, $\left[F^{L}{}_{\bar{s}_B},F^{U}{}_{\bar{s}_B}\right]=$

$\left[f^{L}{}_{\bar{s}_B}(x),f^{U}{}_{\bar{s}_B}(x)\right].e^{j\gamma\left[\Phi^{L}{}_{\bar{s}_B}(x),\Phi^{U}{}_{\bar{s}_B}(x)\right].}$,

respectively. Then,some operational rules of IV-CNSSs as follows, are described:

(i) The product of $(\bar{S},A)$ and $(\bar{S},B)$. denoted as $(\bar{S},A)\times(\bar{S},B)$ is:
{⟨(a,b), $T_{\bar{S}_{A\times B}}(x)$, $I_{\bar{S}_{A\times B}}(x)$, $F_{\bar{S}_{A\times B}}(x)$⟩:(a,b)∈ $A\times B$}, where

$$T_{\bar{S}_{A\times B}}(x)=\left[t^{L}{}_{\bar{s}_A}(x).t^{L}{}_{\bar{s}_B}(x),t^{U}{}_{\bar{s}_A}(x).t^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\omega^{L}{}_{\bar{s}_{A\times B}}(x),\omega^{U}{}_{\bar{s}_{A\times B}}(x)\right].},$$

$$I_{\bar{S}_{A\times B}}(x)=\left[i^{L}{}_{\bar{s}_A}(x)+i^{L}{}_{\bar{s}_B}(x)-i^{L}{}_{\bar{s}_A}(x).i^{L}{}_{\bar{s}_B}(x),i^{U}{}_{\bar{s}_A}(x)+i^{U}{}_{\bar{s}_B}(x)-i^{U}{}_{\bar{s}_A}(x).i^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\psi^{L}{}_{\bar{s}_{A\times B}}(x),\psi^{U}{}_{\bar{s}_{A\times B}}(x)\right].},$$

$$F_{\bar{S}_{A\times B}}(x)=\left[f^{L}{}_{\bar{s}_A}(x)+f^{L}{}_{\bar{s}_B}(x)-f^{L}{}_{\bar{s}_A}(x).f^{L}{}_{\bar{s}_B}(x),f^{U}{}_{\bar{s}_A}(x)+f^{U}{}_{\bar{s}_B}(x)-f^{U}{}_{\bar{s}_A}(x).f^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\Phi^{L}{}_{\bar{s}_{A\times B}}(x),\Phi^{U}{}_{\bar{s}_{A\times B}}(x)\right].},$$

The product of phase terms is defined below:

$\omega^{L}{}_{\bar{s}_{A\times B}}(x)=\omega^{L}{}_{\bar{s}_A}(x)\omega^{L}{}_{\bar{s}_B}(x)$, $\omega^{U}{}_{\bar{s}_{A\times B}}(x)=\omega^{U}{}_{\bar{s}_A}(x)\omega^{U}{}_{\bar{s}_B}(x)$

$\psi^{L}{}_{\bar{s}_{A\times B}}(x)=\psi^{L}{}_{\bar{s}_A}(x)\psi^{L}{}_{\bar{s}_B}(x)$, $\psi^{U}{}_{\bar{s}_{A\times B}}(x)=\psi^{U}{}_{\bar{s}_A}(x)\psi^{U}{}_{\bar{s}_B}(x)$

$\Phi^{L}{}_{\bar{s}_{A\times B}}(x)=\Phi^{L}{}_{\bar{s}_A}(x)\Phi^{L}{}_{\bar{s}_B}(x)$, $\Phi^{U}{}_{\bar{s}_{A\times B}}(x)=\Phi^{U}{}_{\bar{s}_A}(x)\Phi^{U}{}_{\bar{s}_B}(x)$.

The addition of $(\bar{S},A)$ and $(\bar{S},B)$, denoted as $(\bar{S},A)+(\bar{S},B)$,is defined as :

$$T_{\bar{S}_{A+B}}(x)=\left[t^{L}{}_{\bar{s}_A}(x)+t^{L}{}_{\bar{s}_B}(x)-t^{L}{}_{\bar{s}_A}(x).t^{L}{}_{\bar{s}_B}(x),t^{U}{}_{\bar{s}_A}(x)+t^{U}{}_{\bar{s}_B}(x)-t^{U}{}_{\bar{s}_A}(x).t^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\omega^{L}{}_{\bar{s}_{A+B}}(x),\omega^{U}{}_{\bar{s}_{A+B}}(x)\right].}$$

$$I_{\bar{S}_{A+B}}(x)=\left[i^{L}{}_{\bar{s}_A}(x).i^{L}{}_{\bar{s}_B}(x),i^{U}{}_{\bar{s}_A}(x).i^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\psi^{L}{}_{\bar{s}_{A+B}}(x),\psi^{U}{}_{\bar{s}_{A+B}}(x)\right].},$$

$$F_{\bar{S}_{A+B}}(x)=\left[f^{L}{}_{\bar{s}_A}(x).f^{L}{}_{\bar{s}_B}(x),f^{U}{}_{\bar{s}_A}(x).f^{U}{}_{\bar{s}_B}(x)\right].e^{j\alpha\left[\Phi^{L}{}_{\bar{s}_{A+B}}(x),\Phi^{U}{}_{\bar{s}_{A+B}}(x)\right].},$$

below is the addition of phase terms is defined :

${\omega^L}_{\bar{s}_{A+B}}(x)= {\omega^L}_{\bar{s}_A}(x)+{\omega^L}_{\bar{s}_B}(x)$, ${\omega^U}_{\bar{s}_{A+B}}(x)= {\omega^U}_{\bar{s}_A}(x)+{\omega^U}_{\bar{s}_B}(x)$

${\psi^L}_{\bar{s}_{A+B}}(x)= {\psi^L}_{\bar{s}_A}(x) + {\psi^L}_{\bar{s}_B}(x)$, ${\psi^U}_{\bar{s}_{A+B}}(x)= {\psi^U}_{\bar{s}_A}(x) + {\psi^U}_{\bar{s}_B}(x)$

${\Phi^L}_{\bar{s}_{A+B}}(x)= {\Phi^L}_{\bar{s}_A}(x) + {\Phi^L}_{\bar{s}_B}(x)$, ${\Phi^U}_{\bar{s}_{A+B}}(x)= {\Phi^U}_{\bar{s}_A}(x) + {\Phi^U}_{\bar{s}_B}(x)$.

(iii) The scalar multiplication of $(\bar{S}, A)$ is an interval valued complex neutrosophic soft set denoted as $(\bar{S}, C)$ $=k(\bar{S}, A)$ and defined as:

{⟨a, $T_{\bar{S}_c}(x)$, $I_{\bar{S}_c}(x)$, $F_{\bar{S}_c}(x)$⟩: a∈ A}, where

$$T_{\bar{S}_C}(x) = \left[1-(1-{t^L}_{\bar{S}_C}(x))^k, 1-(1-{t^U}_{\bar{S}_A}(x))^k\right].\, e^{j2\pi\left[{\omega^L}_{\bar{s}_C}(x)\,,{\omega^U}_{\bar{s}_C}(x)\right]},$$

$$I_{\bar{S}_C}(x) = \left[({i^L}_{\bar{S}_A}(x))^k, ({i^U}_{\bar{S}_A}(x))^k\right].\ e^{j2\pi\left[{\psi^L}_{\bar{s}_C}(x)\,,{\psi^U}_{\bar{s}_C}(x)\right]},$$

$$F_{\bar{S}_C}(x) = \left[({f^L}_{\bar{S}_A}(x))^k, ({f^U}_{\bar{S}_A}(x))^k\right].\ e^{j2\pi\left[{\Phi^L}_{\bar{s}_C}(x)\,,{\Phi^U}_{\bar{s}_C}(x)\right]},$$

below is the scalar of phase terms defined :

${\omega^L}_{\bar{s}_C}(x)= {\omega^L}_{\bar{s}_A}(x).k$, ${\omega^U}_{\bar{s}_C}(x)= {\omega^U}_{\bar{s}_A}(x).k$,

${\psi^L}_{\bar{s}_C}(x)= {\psi^L}_{\bar{s}_A}(x).k$, ${\psi^U}_{\bar{s}_C}(x)= {\psi^U}_{\bar{s}_A}(x).k$,

${\Phi^L}_{\bar{s}_C}(x)= {\Phi^L}_{\bar{s}_A}(x).k$, ${\Phi^U}_{\bar{s}_C}(x)= {\Phi^U}_{\bar{s}_A}(x).k$,

6. Interval Valued Complex Neutrosophic Soft Set Approach to Problem-Making Decisions

In this here section, by considering the following case, we introduce an application of IV-CNSSs to a decision-making problem.

Example 6.1. Assume that two kinds of a single commodity from a source must be compared by a merchant company and pick the most appropriate one. Assume that the current phase an expert view in two phases on these two categories of products: once before using the products and once again after reviewing a trial of one of the two types of products. Assume that the universe of the two alternatives consists of $U = \{u_1, u_2\}$ (the two product types) and $E = \{e_1, e_2, e_3\}$ is the set of attributes, where e_1 symbolize "easy to use", e_2 symbolize " functional " and e_3 symbolize "durable". The expert is now asked to decide on the most suitable choice based on the goals and constraints of setting up the IV-CNSS.

$$(\bar{S}, A) =$$

$$\left\{\left(e_1, \left(\left\{\left(\frac{[0.4,0.6].\, e^{j2\pi[0.5,0.6]}, [0.1,0.7].\, e^{j2\pi[0,8,0.9]}, [0.3,0,5].\, e^{j2\pi[0.8,0.9]}}{u_1}\right), \left(\frac{[0.2,0.4].\, e^{j2\pi[0.3,0.6]}, [0.1,0.1].\, e^{j2\pi[0.7,0.9]}, [0.5,0.9].\, e^{j2\pi[0.2,0.5]}}{u_2}\right)\right\}\right)\right),\right.$$

$$\left(e_2, \left(\left\{\left(\frac{[0.2,0.5].\, e^{j2\pi[0.4,0.7]}, [0.5,0.7].\, e^{j2\pi[0.5,0.6]}, [0.2,0.8].\, e^{j2\pi[0.6,0.9]}}{u_1}\right), \left(\frac{[0.1,0.3].\, e^{j2\pi[0.1,0.3]}, [0.31,0.7].\, e^{j2\pi[0.6,0.8]}, [0.4,0.61].\, e^{j2\pi[0.2,0.58]}}{u_2}\right)\right\}\right)\right),$$

$$\left.\left(e_3, \left(\left\{\left(\frac{[0.2,0.7].\, e^{j2\pi[0.7,0.8]}, [0.4,0.9].\, e^{j2\pi[0.3,0.5]}, [0.6,0.8].\, e^{j2\pi[0.5,0.6]}}{u_1}\right), \left(\frac{[0.15,0.52].\, e^{j2\pi[0.1,0.3]}, [0,0.5].\, e^{j2\pi[0.6,0.8]}, [0.3,0.3].\, e^{j2\pi[0.6,0.7]}}{u_2}\right)\right\}\right)\right)\right\}$$

The opinions of the experts in stage one reflect the amplitude of true membership, falsehood membership and indeterminate membership (before the products are used) in the IV-CNSS$(\bar{S}, A)$ above, while the terms of membership, non-membership and indeterminacy process reflect the opinions of the experts in the second phase (in accordance with having tried a selection of both of the product's two types). Thus, the amplitude term of the truth membership of the first phase and the phase term of the membership of the second phase form a complex-valued function of the truth membership of the IV-CNSS$(\bar{S}, A)$. Similarly, a complex-valued falsity membership function is generated by the non-membership amplitude term in phase one and the falsehood membership phase term in the second phase. In addition, the amplitude term of undecidedness in the first phase and the phase term of indeterminacy in the second phase form the complex-valued indeterminate membership function. Our problem now is to choose the most suitable product type for the merchant company. We use IV-CNSS $(\bar{S}, A)$ along with the proposed algorithm to disband this decision-making problem. Using a functional formula that allows fast computational decision-making without the need to perform guided operations on complex numbers, this algorithm converts interval complex neutrosophic soft values (I-CNSVs) to interval neutrosophic soft values (INSVs). This algorithm. After that we convert interval neutrosophic soft values (INSVs) to single neutrosophic soft values (SNSVs) by taking the arithmetic average of $T_{\bar{S}_A}(x), I_{\bar{S}_A}(x)$ and $F_{\bar{S}_A}(x)$ respectively. In this formula, we assign a weight to the amplitude terms (a weight to the expert's opinion previous to using the product) by multiplying the weight vector to each amplitude term. Similarly, by multiplying the weight vector for each phase term, we assign a weight to the phase terms (a weight to the opinion of the expert after using the product). Then, to obtain the IV-CNSVs that together reflect the views of the experts on both phases, We combine the values and phase terms of the weighted amplitude terms. After conducting these simple arithmetic for all membership functions of the IVCNSS $(\bar{S}, A)$, we then lead to the final decision using the single neutrosophic soft method.

Algorithm:

Step 1.Input the IV-CNSS $(\bar{S}, A)$,

Step 2.Convert IV-CNSS $(\bar{\mathrm{S}}, \mathrm{A})$,to IVNSS (S,A) By gaining the values of the weighted aggregation of $T_{\bar{S}_A}(x), I_{\bar{S}_A}(x)$ and $F_{\bar{S}_A}(x)$, $\forall a \in A$ and $\forall x \in U$ as the following Formulas:

$$T_{\bar{S}_A}(x) = \left[w_1 t^L{}_{\bar{S}_A}(x) + w_2 \left(\frac{1}{2\pi}\right) \alpha\omega^L{}_{\bar{S}_A}(x)\, , w_1 t^U{}_{\bar{S}_A}(x) + w_2 \left(\frac{1}{2\pi}\right) \alpha\omega^U{}_{\bar{S}_A}(x)\right],$$

$$I_{\bar{S}_A}(x) = \left[w_1 i^L{}_{\bar{S}_A}(x) + w_2 \left(\frac{1}{2\pi}\right) \beta\psi^L{}_{\bar{S}_A}(x)\, , w_1 i^U{}_{\bar{S}_A}(x) + w_2 \left(\frac{1}{2\pi}\right) \beta\psi^U{}_{\bar{S}_A}(x)\right],$$

$$F_{\bar{S}_A}(x) = \left[w_1 f^L{}_{\bar{S}_A}(x) + w_2 \left(\frac{1}{2\pi}\right) \gamma\Phi^L{}_{\bar{S}_A}(x)\, , w_1 f^U{}_{\bar{S}_A}(x) + w_2 \left(\frac{1}{2\pi}\right) \gamma\Phi^U{}_{\bar{S}_A}(x)\right],$$

where $t^L{}_{\bar{S}_A}(x)$, $t^U{}_{\bar{S}_A}(x), i^L{}_{\bar{S}_A}(x), i^U{}_{\bar{S}_A}(x)$ and $f^L{}_{\bar{S}_A}(x), f^U{}_{\bar{S}_A}(x)$are the amplitude terms and $\omega^L{}_{\bar{S}_A}(x)$, $\omega^U{}_{\bar{S}_A}(x), \psi^L{}_{\bar{S}_A}(x), \psi^U{}_{\bar{S}_A}(x)$ and $\Phi^L{}_{\bar{S}_A}(x), \Phi^U{}_{\bar{S}_A}(x)$ are the phase terms in the the Interval Complex Neutrosophic Soft Set $(\bar{S}, A)$, respectively. $T_{\bar{S}_A}(x), I_{\bar{S}_A}(x)$ and $F_{\bar{S}_A}(x)$ are truth membership function ,anindeterminate membership function,and falsehood membership function in IV-NSS (S, A), respectively and w_1, w_2 the weights for the terms of the amplitude and phase terms, respectively, where $w_1, w_2 \in [0,1]$ and $w_1 + w_2 = 1$.

Step 3. Convert IVNSS (S,A) to SVNSS by taking the arithmetic average of $T_{\bar{S}_A}(x), I_{\bar{S}_A}(x)$ and $F_{\bar{S}_A}(x)$ respectively.

Step 4. Compute the comparison matrix of the SVNSS. Comparison matrix of SVNSS [40], is a matrix whose rows are labelled by the object names $u_1, u_2, \ldots, u_n$ and the columns are labelled by the parameters $e_1, e_2, \ldots, e_m$ The entries c_{ij} are calculated by $c_{ij} = \mathrm{a_{ij}} + (\mathrm{b_{ij}} - \mathrm{c_{ij}})$, where 'a' is the integer calculated as `how many times $T_{e_i}(u_j)$ exceeds or equal to$T_{e_i}(u_k)$, for $u_j \neq u_k, \forall\, u_j \in U$, 'b' is the integer calculated as `how many times $I_{e_i}(u_j)$ exceeds or equal to$I_{e_i}(u_k)$, for $u_J \neq u_k, \forall\, u_J \in U$, and 'c' is the integer calculated as `how many times $F_{e_i}(u_j)$ exceeds or equal to $I_{e_i}(u_k)$ ', for $u_i \neq u_k, \forall\, u_j \in U$.

Step 5.Calculate the scorec_i of u_i, $\forall i$. The score of an object u_i of c_i its calculated as $c_i = \sum_j c_{ij}$.

Step 6. The decision is to select u_i if $c_k = max_{u_i \in U} c_i$

Step 7. if we has more than one decision then any one of $u_i$could be the preferable choice.

Now, to change the form of the IV-CNSS $(\bar{\mathrm{S}}, \mathrm{A})$,to IV-NSS (S,A) we assume that the weight vectors are $w_1 = 0.6$ *and* $w_2 = 0.4$. To illustrate this step, we calculate $T_{e_1}(u_1)$, $I_{e_1}(u_1)$ and $F_{e_1}(u_1)$, as shown below:

$$T_{e_1}(u_1) = \left[w_1 t^L{}_{\bar{s}_{e_1}}(u_1) + w_2 \left(\frac{1}{2\pi}\right) \alpha\omega^L{}_{\bar{s}_{e_1}}(u_1)\,, w_1 t^U{}_{\bar{s}_{e_1}}(u_1) + w_2 \left(\frac{1}{2\pi}\right) \alpha\omega^U{}_{\bar{s}_{e_1}}(u_1)\right]$$

$$=\left[0.6(0.4) + 0.4 \left(\tfrac{1}{2\pi}\right)(2\pi)(0.5), 0.6(0.6) + 0.4 \left(\tfrac{1}{2\pi}\right)(2\pi)(0.6)\right]$$

$$=[0.44, 0.6]$$

$$I_{e_1}(u_1) = \left[w_1 i^L{}_{\bar{s}_{e_1}}(u_1) + w_2 \left(\frac{1}{2\pi}\right) \beta\, \psi^L{}_{\bar{s}_{e_1}}(u_1)\,, w_1 i^U{}_{\bar{s}_{e_1}}(u_1) + w_2 \left(\frac{1}{2\pi}\right) \beta\, \psi^U{}_{\bar{s}_{e_1}}(u_1)\right]$$

$$=\left[0.6(0.1) + 0.4 \left(\tfrac{1}{2\pi}\right)(2\pi)(0.8), 0.6(0.7) + 0.4 \left(\tfrac{1}{2\pi}\right)(2\pi)(0.9)\right]$$

$$=[0.38, 0.78].$$

$$F_{e_1}(u_1) = \left[w_1 f^L{}_{\bar{s}_{e_1}}(u_1) + w_2 \left(\frac{1}{2\pi}\right) \gamma\Phi^L{}_{\bar{s}_{e_1}}(u_1)\,, w_1 f^U{}_{\bar{s}_{e_1}}(u_1) + w_2 \left(\frac{1}{2\pi}\right) \gamma\Phi^U{}_{\bar{s}_{e_1}}(u_1)\right]$$

$$= \left[0.6(0.3) + 0.4 \left(\tfrac{1}{2\pi}\right)(2\pi)(0.8), 0.6(0.5) + 0.4 \left(\tfrac{1}{2\pi}\right)(2\pi)(0.9)\right]$$

$$= [0.5, 0.66]$$

Then the IV-NSS

$$\left(\left[T^L{}_{\bar{s}_{e_1}}(u_1), T^U{}_{\bar{s}_{e_1}}(u_1)\right], \left[I^L{}_{\bar{s}_{e_1}}(u_1), I^U{}_{\bar{s}_{e_1}}(u_1)\right], \left[F^L{}_{\bar{s}_{e_1}}(u_1), F^U{}_{\bar{s}_{e_1}}(u_1)\right]\right)$$

$$= ([0.44, 0.6], [0.38, 0.78], [0.5, 0.66]).$$

we measure the other IV-NSSS in the same way. $\forall\, e_i \in A$ and $\forall\, u_j \in U$. as shown.Table 1.

Table1.values of IV-NSS

U	u_1	u_2
e_1	([0.44,0.6], [0.38, 0.78], [0.5,0.66])	([0.24,0.48], [0.34, 0.42], [0.38,0.74])
e_2	([0.28,0.58], [0.5,0.66], [0.36,0.84])	([0.10,0.30] ,[0.43,0.74] ,[0.32,0.59])
e_3	([0.4,0.74] ,[0.36,0.74] ,[0.68,0.72])	([0.13,0.43] ,[0.24,0.62] ,[0.42,0.46])

Now we convert the IV-NSVS to SVNSV by taking the arithmetic average of $T_{\bar{S}_A}(x), I_{\bar{S}_A}(x)$ and $F_{\bar{S}_A}(x)$ respectively as shown Table2

Table2 . values of SVNSS

U	u_1	u_2
e_1	(0.52,0.58,0.58)	(0.36,0.38 ,0.56)
e_2	(0.43,0.58,0.6)	(0.2,0.50,0.46)
e_3	(0.57,0.55,0.7)	(0.28 ,0.43 ,0.44)

Table 3. comparison matrix of the SVNSS

U	u_1	u_2
e_1	3	0
e_2	1	-1
e_3	0	2

Table 4: compute the score c_i

U	u_1	u_2
e_1	3	0
e_2	1	-1
e_3	0	2
Score(c_i)	4	1

Decision: The best option is to select u_1. Since $c_1 = max_{u_i \in U} c_i = u_1$. The expert advice therefore selects the form u_1 of this product as u_1 desirable alternative.

7. Conclusion

We established the concept of IV-CNSS by combining the two concepts of interval complex neutrosophic sets with soft sets. The basic operations on IV-CNSS, namely complement, subset, union, intersection operations, were defined. Subse-quently, the basic properties of these operations such as De Morgan's laws and other relevant laws pertain-ing to the concept of IV-CNSS were proven. Finally, a new algorithm is introduced and applied to the IV-CNSS model to solve a hypothetical decision-making problem, and its superiority and feasibility are further verified by comparison with other existing methods. This new extension will provide a significant addition to existing theories for handling indeterminacy, where time plays a vital role in the decision process, and spurs more developments of further research and pertinent applications. For further research, we intend to take into account unknown weight information to develop some real applications of IV-CNSS in other areas, where the phase term may represent other variables such as distance, speed, and temperature.

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