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Extension of TOPSIS Method under Single-Valued Neutrosophic N-Soft Environment

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Abstract:

In this paper, we discuss aggregation operators for single-valued neutrosophic N-soft numbers. Further, we develop single-valued neutrosophic N-soft TOPSIS method based on single-valued neutrosophic N-soft aggregate operators in order to cumulate the decisions of all experts according to the worth of experts' opinion and parameters related to each alternative. For the final decision, we use revised closeness index depending upon the distance measures of alternatives from single-valued neutrosophic N-soft positive ideal solution and single-valued neutrosophic N-soft negative ideal solution. A numerical example is described to illustrate the importance of the proposed method. A comparison of single-valued neutrosophic N-soft TOPSIS method with single-valued neutrosophic TOPSIS method ensures the significance and trustworthiness of the proposed model.

Keywords: N-soft set, single-valued neutrosophic N-soft sets, TOPSIS method, MAGDM.

1 Introduction

In many field of life, the evaluation process is certainly switch from binary evaluation ($\{0, 1\}$) to non-binary evaluation ($\{0, 1, \ldots, N - 1\}$), that is, we are using the system of 5-stars, 4-stars or 3-stars instead of yes or no, in many disciplines of mathematical social sciences. Keeping in view the importance of ranking system, Fatima et al. [7] introduced N-soft sets and decision making methods to handle problems basis on non-binary evaluations. Apparently, N-soft set is an extension of soft set presented by Molodtsov [4], described all type of parametrization, while in N-soft sets grades are assigned to the parameters that actually representing the level of alternatives with respect to the attributes. Further, Akram et al. [28, 30] extended the concept of N-soft sets to fuzzy N-soft sets and intuitionistic fuzzy N-soft sets ($IFNS_fS$). The intuitionistic fuzzy N-soft set is describing the level of alternatives as well as the degree of membership and non-membership with their sum less than equal to zero. The Pythagorean fuzzy set (PFS) was firstly presented by Yager [39] in which squares sum of degree of membership and non-membership should not exceed one. Zhang [21] introduced the notion of Pythagorean fuzzy N-soft sets ($PFNS_fS$).

Human decision nature has indeterminacy within the judgments of yes or no that is actually prescribed the indecision for the related object. Since the PFSs and IFSs are not able to handle such part of decision nature independently, with limited range. Therefore, PFS and IFS will not be applicable. This is the origin of neutrosophic sets (NSs) presented by Smarandache [13] in 1999. Later on, Wang et al. [20] developed the concept of single-valued neutrosophic sets (SVNSs) to deal real life scientific problems having indeterminate information.Moreover, Singh [35, 36] presented theory of three-way and multi-granular based n-valued neutrosophic logics introduced by Smarandache [15] in 2014. On the other hand, Maji [34] and Jana et al. [2] combined the concept of soft sets with NSs and SVNSs, respectively. Many researchers work on TOPSIS method, like Chen [3], Chu and Kysely [41] and Alguliyev [38] extended the TOPSIS method in fuzzy environment for solving multi-attribute group decision making problems. Moreover, Gupta et al. [33] and Shen et al. [12] introduced the extended version of intuitionistic fuzzy TOPSIS method. Akram et al. [31, 29] developed a theoretical description for the Pythagorean fuzzy TOPSIS method. Similarly, and also motivated by SVNSs , Sahin and Yigider [40] used a single-valued neutrosophic-TOPSIS method to find the best supplier for production industry. Riaz et al. [32] being inspired by N-soft sets, presented a model of neutrosophic N-soft sets (NNS_fSs) with TOPSIS method that used relations and composition for evaluating the NNS_f positive ideal solution and negative ideal solution. They used similarity measures and choice function for solving MADM problem in medical diagnosis. In this paper, we discuss aggregation operators for single-valued neutrosophic N-soft numbers. Further, we develop single-valued neutrosophic N-soft TOPSIS method based on single-valued neutrosophic N-soft aggregate operators in order to cumulate the decisions of all experts according to the worth of experts' opinion and parameters related to each alternative. For the final decision, we use revised closeness index depending upon the distance measures of alternatives from single-valued neutrosophic N-soft positive ideal solution and single-valued neutrosophic N-soft negative ideal solution. A numerical example is described to illustrate the importance of the proposed method.

The rest of the paper is organized as follows: In Section 2, we represent the concept of $SVNNS_fS$ with related example. In Section 3, we define $SVNNS_fN$ with some properties and operations, like score function, accuracy function, comparison between two $SVNNS_fNs$, sum and product of $SVNNS_fNs$, inclusively. Section 4, describes intellectual basics for the $SVNNS_fS$-TOPSIS method for solving real life problems within an algorithm. Section 5, presenting a MAGDM problem, which is sorted out using $SVNNS_fS$-TOPSIS. In Section 6, we compare the proposed model with the SVN-TOPSIS method. In Section 7 we give conclusions about the paper and future directions for research.

Definition 1. *[13] Let Y be non-empty set. A neutrosophic set (NS) ρ over the universe of discourse Y is defined as:*

$$\rho = \langle y, \beta_\rho(y), \gamma_\rho(y), \delta_\rho(y) : y \in Y\rangle,$$

where, $\beta_\rho(y)$, $\gamma_\rho(y)$ and $\delta_\rho(y)$ are degree of satisfaction, degree of indeterminacy and degree of dissatisfaction, respectively, belongs to non-standard interval $]^-0, 1^+[$, for every $y \in Y$.

Definition 2. *[20] Let Y be non-empty set. A single-valued neutrosophic set $(SVNS)$ ρ over the universe of discourse Y is defined as:*

$$\rho = \langle y, \beta_\rho(y), \gamma_\rho(y), \delta_\rho(y) : y \in Y\rangle,$$

where, $\beta_\rho(y)$, $\gamma_\rho(y)$ and $\delta_\rho(y) \in [0, 1]$. For every $y \in Y$, $\beta_\rho(y)$, $\gamma_\rho(y)$ and $\delta_\rho(y)$, the degree of the satisfaction, degree of indeterminacy and degree of dissatisfaction, respectively, without any restriction on $\beta_\rho(y)$, $\gamma_\rho(y)$ and $\delta_\rho(y)$ or we can say that for all $y \in Y$,

$$0 \leq \beta_\rho(y) + \gamma_\rho(y) + \delta_\rho(y) \leq 3.$$

The triplet $(\beta_\rho(y), \gamma_\rho(y), \delta_\rho(y))$ is called single-valued neutrosophic number $(SVNN)$.

Definition 3. *[4] Let X be a non-empty set and $E \subseteq A$, A be a set of parameters. A pair $(\daleth, E)$ is called soft set S_fS over X denoted as:*

$$(\daleth, E) = \{\langle e_i, \daleth(e)\rangle : \forall e_i \in E\},$$

if $\daleth : E \to P(X)$, where $P(X)$ represents the family of all subsets of X.

Definition 4. *Let X be a non-empty set and $E \subseteq A$, A be a set of parameters. A pair (Υ, E) is called single-valued neutrosophic soft set $(SVNS_fS)$ over X, if $\Upsilon : E \to \mathcal{P}(X)$ is a mapping, which is denoted as:*

$$\Upsilon(e_i) = \{\langle x_j, (\beta_{ij}, \gamma_{ij}, \delta_{ij})\rangle : x_j \in X\},$$

where, $\mathcal{P}(X)$ represents the family of all SVNSs over X and $\beta_{ij}, \gamma_{ij}, \delta_{ij}$, which belongs to unit closed interval, are satisfying the condition

$$0 \leq \beta_{ij} + \gamma_{ij} + \delta_{ij} \leq 3, \forall x_j \in X.$$

Definition 5. *[7] Let X be a non-empty set and $E \subseteq A$, A be a set of parameters. Let $O = \{0, 1, 2, \ldots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple (H, E, N) is called N-soft set (NS_fS) over X if $H : E \to 2^{U\times G}$ is a mapping , with the property that for each $e_i \in E$ and $x_j \in X$ there exist a unique $(x_j, o_i^j) \in X \times O$ such that $(x_j, o_i^j) \in H(e_j), x_j \in X, o_i^j \in O$.*

2 Single-valued neutrosophic N-soft numbers

Definition 6. *Let X be a non-empty set and $E \subseteq A$, A be a set of parameters. Let $O = \{0, 1, 2, \ldots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. Let $H : E \to 2^{X\times O}$ be an NS_fS on X, and $T : E \to \mathcal{P}(SVNN)$, be a mapping, that $\mathcal{P}(SVNN)$ denotes the collection of single-valued neutrosophic numbers of X, then a triple (H_T, E, N) is called a single-valued neutrosophic N-soft set $(SVNNS_fS)$ on X, if $H_T : E \to (2^{X\times O} \times \mathcal{P}(SVNN))$ is a mapping, which is defined as:*

$$\begin{aligned} H_T(e_i) = \quad & \{\langle(H(e_i), T(e_i))\rangle : e_i \in E, H(e_i) \in 2^{W\times G}, T(e_i) \times \mathcal{P}(SVNN)\}, \\ = \quad & \{\langle((x_j, o_i^j), (\beta_{e_i}(x_j), \gamma_{e_i}(x_j), \delta_{e_i}(x_j)))\rangle\}, \\ = \quad & \{\langle((x_j, o_i^j), (\beta_{ij}, \gamma_{ij}, \delta_{ij}))\rangle\}, \end{aligned}$$

where, o_i^j denotes the level of attribute for the element x_j and $\beta_{ij}, \gamma_{ij}, \delta_{ij} \in [0, 1]$, satisfying the condition

$$0 \leq \beta_{ij} + \gamma_{ij} + \beta_{ij} \leq 3, \text{for all } x_j \text{ belongs to } X.$$

Example 1. *Mr. and Mrs. Bean decided to gift their child a bicycle on his* 17*th birthday because he needed a conveyance to go to college. For this purpose, they visited plenty of websites online, among these websites they found a website named as "Cycling weekly". This website provided ratings of bicycles according to the parameters filtered by Mr. and Mrs. Bean. For the selection of a best bicycle based on ratings, we will use* $SVNNS_fS$.
Let $X = \{x_1$ *= Merida Mission Road* 7000*-E,* x_2 *= Bianchi Infinity XE Ultegra Disc,* x_3 *= Strider* 12*,* x_4 *= Scott Iddict RC Pro,* x_5 *= Willier Cento* 10 *SL* $\}$ *be the set of five bicycles and the set of parameters be* $E = \{e_1$ *= Framework (stiffness and comfort frame),* e_2 *= weight,* e_3 *= Shape and quality,* e_4 *= Cost price* $\}$*. Following the ratings of bicycles according to the parameters, a* 6*-soft set is organized in Table 1, where*

Five checkmarks means 'Infinitely Good',

Four checkmarks means 'Extremely Good',

Three checkmarks means 'Good',

Two checkmarks means 'Bad',

One checkmarks means 'Extremely Bad',

Big dot means 'Infinitely Bad'

This level assessment by checkmarks can be represented by numbers as $O = \{0, 1, 2, 3, 4, 5\}$*, where*

0 *means* "•" ,

1 *means* "✓",

2 *means* "✓✓",

3 *means* "✓✓✓",

4 *means* "✓✓✓✓".

5 *means* "✓✓✓✓✓".

Table 1: Evaluation data provided by the Website

X/E	e_1	e_2	e_3	e_4
x_1	✓	✓	✓✓✓	✓✓✓
x_2	✓✓	•	✓✓✓	✓✓✓✓
x_3	✓✓✓✓	✓✓✓	✓✓✓✓	✓✓✓✓✓
x_4	✓✓✓✓✓	✓✓✓✓	✓✓✓✓✓	✓✓✓
x_5	✓✓✓	✓✓	✓✓✓✓✓	✓✓✓✓

Table 2 can be adopted as natural convention of 5*-soft set model.*

Table 2: A 6-soft set

X/E	e_1	e_2	e_3	e_4
x_1	1	1	3	3
x_2	2	0	3	4
x_3	4	3	4	5
x_4	5	4	5	3
x_5	3	2	5	4

In coalition with the Definition 6, we describe for example $(x_3, o_2^3 = 3) \in H(e_2)$ *and* $(x_5, o_4^5 = 4) \in H(e_4)$*. This form of data is enough when it is extracted from real data, however, when there is ambiguity in the data and experts wants to describe the viewpoint of customers based on their satisfaction, hesitancy and dissatisfaction then we* $SVNNS_fSs$ *are appropriate which provide us information, how these grades are given* *to bicycles. The evaluation of bicycles follow this grading criteria;*

$$\begin{aligned}
when\ o_i^j = 0, &\quad -1.000 \le S_T < -0.787,\\
when\ o_i^j = 1, &\quad -0.787 \le S_T < -0.400,\\
when\ o_i^j = 2, &\quad -0.400 \le S_T < 0.000,\\
when\ o_i^j = 3, &\quad 0.000 \le S_T < 0.400,\\
when\ o_i^j = 4, &\quad 0.400 \le S_T < 0.787,\\
when\ o_i^j = 5, &\quad 0.787 \le S_T < 1.000.
\end{aligned}$$

According to above grading criteria, we can obtain Table 3.

Table 3: Grading criteria

o_i^j/T	Satisfaction degree	Indeterminacy degree	Dissatisfaction degree
grades	β_{ij}	γ_{ij}	δ_{ij}
$o_i^j = 0$	$[0.00, 0.15]$	$[0, 0.450)$	$[0.90, 1.00)$
$o_i^j = 1$	$[0.15, 0.30)$	$(0, 0.020)$	$(0.70, 0.90)$
$o_i^j = 2$	$[0.30, 0.50)$	$[0, 0.140)$	$(0.50, 0.70]$
$o_i^j = 3$	$[0.50, 0.70)$	$(0, 0.070]$	$[0.30, 0.50]$
$o_i^j = 4$	$(0.70, 0.90]$	$[0, 0.070)$	$[0.15, 0.30)$
$o_i^j = 5$	$(0.90, 1.00]$	$[0, 0.017)$	$[0.00, 0.15)$

Using Table 3 and Definition 6, a $SVN6S_fS$ that is also arranged in Table 4, is defined as:

$$\begin{aligned}
(\beta_{e_1}, \gamma_{e_1}, \delta_{e_1}) &= \{((x_1, 1), (0.160, 0.300, 0.870)), ((x_2, 2), (0.320, 0.015, 0.600)),\\
&\quad ((x_3, 4), (0.750, 0.012, 0.170)), ((x_4, 5), (0.950, 0.011, 0.120)),\\
&\quad ((x_5, 3), (0.550, 0.030, 0.420))\} \in SVNNS_fS,\\
(\beta_{e_2}, \gamma_{e_2}, \delta_{e_2}) &= \{((x_1, 1), (0.270, 0.017, 0.710)), ((x_2, 0), (0.120, 0.300, 0.950)),\\
&\quad ((x_3, 3), (0.560, 0.012, 0.380)), ((x_4, 4), (0.870, 0.025, 0.230)),\\
&\quad ((x_5, 2), (0.400, 0.120, 0.620))\} \in SVNNS_fS,\\
(\beta_{e_3}, \gamma_{e_3}, \delta_{e_3}) &= \{((x_1, 3), (0.520, 0.020, 0.350)), ((x_2, 3), (0.650, 0.010, 0.370)),\\
&\quad ((x_3, 4), (0.760, 0.033, 0.210)), ((x_4, 5), (0.970, 0.013, 0.040)),\\
&\quad ((x_5, 5), (0.920, 0.014, 0.14))\} \in SVNNS_fS,
\end{aligned}$$

$$\begin{aligned}
(\beta_{e_4}, \gamma_{e_4}, \delta_{e_4}) &= \{((x_1, 3), (0.550, 0.030, 0.360)), ((x_2, 4), (0.750, 0.032, 0.200)), &(1)\\
&\quad ((x_3, 5), (0.910, 0.016, 0.140)), ((x_4, 3), (0.660, 0.017, 0.360)), &(2)\\
&\quad ((x_5, 4), (0.780, 0.040, 0.290))\} \in SVNNS_fS. &(3)\\
& &(4)
\end{aligned}$$

Definition 7. *Let $H_T(e_i) = \{\langle(((x_j, o_i^j), (\beta_{ij}, \gamma_{ij}, \delta_{ij}))\rangle\}$ be a $SVNNS_fS$. Then the single-valued neutrosophic N-soft number $(SVNNS_fN)$ is defined as:*

$$\rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij})),$$

where β_{ij}, γ_{ij} and δ_{ij}, belong to unit interval, are the degree of membership, indeterminacy and non-membership, respectively.

Remark 8. *We see that:*

1. *For $N = 2$, $SVNNS_fS$ becomes single-valued neutrosophic soft set.*
2. *When $|E| = 1$, $SVNNS_fS$ becomes single-valued neutrosophic set.*

Table 4: A $SVN6S_fS$ $(H_T, E, 6)$

$(H_T, E, 6)$	e_1	e_2	e_3	e_4
x_1	$(1,(0.160,0.300,0.870))$	$(1,(0.270,0.017,0.710))$	$(3,(0.520,0.020,0.350))$	$(3,(0.550,0.030,0.360))$
x_2	$(2,(0.320,0.015,0.600))$	$(0,(0.120,0.300,0.950))$	$(3,(0.650,0.010,0.370))$	$(4,(0.750,0.032,0.200))$
x_3	$(4,(0.750,0.012,0.170))$	$(3,(0.560,0.012,0.380))$	$(4,(0.760,0.033,0.210))$	$(5,(0.910,0.016,0.140))$
x_4	$(5,(0.950,0.011,0.120))$	$(4,(0.870,0.025,0.230))$	$(5,(0.970,0.013,0.040))$	$(3,(0.660,0.017,0.360))$
x_5	$(3,(0.550,0.030,0.420))$	$(2,(0.400,0.120,0.620))$	$(5,(0.920,0.014,0.140))$	$(4,(0.780,0.040,0.290))$

Definition 9. *Consider a $SVNNS_fN$ $\rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$. The score function $Sc(\rho_{ij})$ is defined as:*

$$Sc(\rho_{ij}) = (\frac{o_i^j}{N-1}) + \beta_{ij} - \gamma_{ij} - \delta_{ij},$$

where $Sc(\rho) \in [-2, 2]$. The accuracy function $Ac(\rho_{ij})$ is defined as:

$$Ac(\rho_{ij}) = (\frac{o_i^j}{N-1}) + \beta_{ij} + \gamma_{ij} + \delta_{ij},$$

where $Ac(\rho) \in [0, 4]$, respectively.

Definition 10. *Let $\rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ and $\rho_{kj} = (o_k^j, (\beta_{kj}, \gamma_{kj}, \delta_{kj}))$, be two $SVNNS_fNs$.*

1. *If $Sc(\rho_{ij}) < Sc(\rho_{kj})$, then $\rho_{ij} < \rho_{kj}$,*
2. *If $Sc(\rho_{ij}) > Sc(\rho_{kj})$, then $\rho_{ij} > \rho_{kj}$,*
3. *If $Sc(\rho_{ij}) = Sc(\rho_{kj})$, then*

   (i) *$Ac(\rho_{ij}) < Ac(\rho_{kj})$, then $\rho_{ij} < \rho_{kj}$,*

   (ii) *$Ac(\rho_{ij}) > Ac(\rho_{kj})$, then $\rho_{ij} > \rho_{kj}$,*

   (iii) *$Ac(\rho_{ij}) = Ac(\rho_{kj})$, then $\rho_{ij} \sim \rho_{kj}$.*

Definition 11. *Let $\rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ and $\rho_{kj} = (o_k^j, (\beta_{kj}, \gamma_{kj}, \delta_{kj}))$ be two $SVNNS_fNs$ and $\zeta > 0$. The operations for $SVNNS_fNs$ can be defined as:*

$$\begin{aligned}
\rho_{ij} \cup \rho_{kj} &= \left(\max(o_i^j, o_k^j), (\max(\beta_{ij}, \beta_{kj}), \min(\gamma_{ij}, \gamma_{kj}), \min(\delta_{ij}, \delta_{kj}))\right), \\
\rho_{ij} \cap \rho_{kj} &= \left(\min(o_i^j, o_k^j), (\min(\beta_{ij}, \beta_{kj}), \max(\gamma_{ij}, \gamma_{kj}), \max(\delta_{ij}, \delta_{kj}))\right), \\
\zeta\rho_{ij} &= \left(o_i^j, 1-(1-\beta_{ij})^\zeta, \gamma_{ij}^\zeta, \delta_{ij}^\zeta\right), \\
\rho_{ij}^\zeta &= \left(o_i^j, \beta_{ij}^\zeta, 1-(1-\gamma_{ij})^\zeta, 1-(1-\delta_{ij})^\zeta\right), \\
\rho_{ij} \bigoplus \rho_{kj} &= \left(\max(o_i^j, o_k^j), \beta_{ij} + \beta_{kj} - \beta_{ij}\beta_{kj}, \gamma_{ij}\gamma_{kj}, \delta_{ij}\delta_{kj}\right), \\
\rho_{ij} \bigotimes \rho_{kj} &= \left(\min(o_i^j, o_k^j), \beta_{ij}\beta_{kj}, \gamma_{ij} + \gamma_{kj} - \gamma_{ij}\gamma_{kj}, \delta_{ij} + \delta_{kj} - \delta_{ij}\delta_{kj}\right).
\end{aligned}$$

Definition 12. *Let $\rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ and $\rho_{kj} = (o_k^j, (\beta_{kj}, \gamma_{kj}, \delta_{kj}))$ be any two $SVNNS_fNs$, then the following properties hold:*

1. $\rho_{ij} \bigoplus \rho_{kj} = \rho_{kj} \bigoplus \rho_{ij}$,
2. $\rho_{ij} \bigotimes \rho_{kj} = \rho_{kj} \bigotimes \rho_{ij}$,
3. $\zeta\rho_{ij} \bigoplus \zeta\rho_{kj} = \zeta(\rho_{kj} \bigoplus \rho_{ij}), \zeta > 0$,
4. $\zeta_1\rho_{ij} \bigoplus \zeta_1\rho_{ij} = (\zeta_1 + \zeta_2)\rho_{ij}, \zeta_1, \zeta_2 > 0$,
5. $\rho_{ij}^\zeta \bigotimes \rho_{kj}^\zeta = (\rho_{kj} \bigotimes \rho_{ij})^\zeta, \zeta > 0$,
6. $\rho_{ij}^{\zeta_1} \bigotimes \rho_{ij}^{\zeta_1} = \rho_{ij}^{(\zeta_1+\zeta_2)}. \zeta_1, \zeta_2 > 0.$

Proof. 1. $\rho_{ij} \bigoplus \rho_{kj}$

$$\begin{aligned} &= \left(\max(o_i^j, o_k^j), \beta_{ij} + \beta_{kj} - \beta_{ij}\beta_{kj}, \gamma_{ij}\gamma_{kj}, \delta_{ij}\delta_{kj}\right), \\ &= \left(\max(o_k^j, o_i^j), \beta_{kj} + \beta_{ij} - \beta_{kj}\beta_{ij}, \gamma_{kj}\gamma_{ij}, \delta_{kj}\delta_{ij}\right), \\ &= \rho_{kj} \bigoplus \rho_{ij}. \end{aligned}$$

2. $\rho_{ij} \bigotimes \rho_{kj}$

$$\begin{aligned} &= \left(\min(o_i^j, o_k^j), \beta_{ij}\beta_{ij}, \gamma_{ij} + \gamma_{kj} - \gamma_{ij}\gamma_{kj}, \delta_{ij} + \delta_{kj} - \delta_{ij}\delta_{kj}\right) \\ &= \left(\min(o_k^j, o_i^j), \beta_{kj}\beta_{ij}, \gamma_{kj} + \gamma_{ij} - \gamma_{kj}\gamma_{ij}, \delta_{kj} + \delta_{ij} - \delta_{kj}\delta_{ij}\right) \\ &= \rho_{kj} \bigotimes \rho_{ij}. \end{aligned}$$

3. $\zeta\rho_{ij} \bigoplus \zeta\rho_{kj}$

$$\begin{aligned} &= \left(o_i^j, [1-(1-\beta_{ij})^\zeta], \gamma_{ij}^\zeta, \delta_{ij}^\zeta) \bigoplus (o_k^j, [1-(1-\beta_{kj})^\zeta], \gamma_{kj}^\zeta, \delta_{kj}^\zeta\right) \\ &= \left(\max(o_i^j, o_k^j), [1-(1-\beta_{ij})^\zeta] + [1-(1-\beta_{kj})^\zeta] - [1-(1-\beta_{ij})^\zeta][1-(1-\beta_{kj})^\zeta], \gamma_{ij}^\zeta\gamma_{kj}^\zeta, \delta_{ij}^\zeta\delta_{kj}^\zeta\right) \\ &= \left(\max(o_i^j, o_k^j), [1-(1-\beta_{ij}+\beta_{kj}-\beta_{ij}\beta_{kj})^\zeta], (\gamma_{ij}\gamma_{kj})^\zeta, (\delta_{ij}\delta_{kj})^\zeta\right) \\ &= \zeta(\max(o_i^j, o_k^j), \beta_{ij} + \beta_{kj} - \beta_{ij}\beta_{kj}, \gamma_{ij}\gamma_{kj}, \delta_{ij}\delta_{kj}) \\ &= \zeta\left(\rho_{ij} \bigoplus \rho_{kj}\right). \end{aligned}$$

4. $\zeta_1\rho_{ij} \bigoplus \zeta_1\rho_{ij}$

$$\begin{aligned} &= \left(o_i^j, 1-(1-\beta_{ij})^{\zeta_1}, \gamma_{ij}^{\zeta_1}, \delta_{ij}^{\zeta_1}) \bigoplus (o_i^j, 1-(1-\beta_{ij})^{\zeta_2}, \gamma_{ij}^{\zeta_2}, \delta_{ij}^{\zeta_2}\right) \\ &= \left(\max(o_i^j, o_i^j), [1-(1-\beta_{ij})^{\zeta_1}] + [1-(1-\beta_{ij})^{\zeta_2}] - [1-(1-\beta_{ij})^{\zeta_1}][1-(1-\beta_{ij})^{\zeta_2}], \gamma_{ij}^{\zeta_1}\gamma_{ij}^{\zeta_2}, \delta_{ij}^{\zeta_1}\delta_{ij}^{\zeta_2}\right) \\ &= \left(o_i^j, 1-(1-\beta_{ij})^{\zeta_1+\zeta_2}, \gamma_{ij}^{\zeta_1+\zeta_2}, \delta_{ij}^{\zeta_1+\zeta_2}\right) \\ &= (\zeta_1+\zeta_2)\rho_{ij}. \end{aligned}$$

5. $\rho_{ij}^\zeta \bigotimes \rho_{kj}^\zeta$

$$\begin{aligned} &= \left(o_i^j, \beta_{ij}^\zeta, [1-(1-\gamma_{ij})^\zeta], [1-(1-\delta_{ij})^\zeta]) \bigotimes (o_k^j, \beta_{kj}^\zeta, [1-(1-\gamma_{kj})^\zeta], [1-(1-\delta_{kj})^\zeta]\right) \\ &= \Big(\min(o_i^j, o_k^j), \beta_{ij}^\zeta\beta_{kj}^\zeta, [1-(1-\gamma_{ij})^\zeta] + [1-(1-\gamma_{kj})^\zeta] - [1-(1-\gamma_{ij})^\zeta][1-(1-\gamma_{kj})^\zeta] \\ &\quad , [1-(1-\delta_{ij})^\zeta] + [1-(1-\delta_{kj})^\zeta] - [1-(1-\delta_{ij})^\zeta][1-(1-\delta_{kj})^\zeta]\Big). \\ &= \left(\min(o_i^j, o_k^j), (\beta_{ij}\beta_{kj})^\zeta, [1-(1-\gamma_{ij}+\gamma_{kj}-\gamma_{ij}\gamma_{kj})^\zeta], [1-(1-\gamma_{ij}+\gamma_{kj}-\gamma_{ij}\gamma_{kj})^\zeta]\right) \\ &= (\rho_{kj} \bigotimes \rho_{ij})^\zeta. \end{aligned}$$

6. $\rho_{ij}^{\zeta_1} \bigotimes \rho_{ij}^{\zeta_1}$

$$
\begin{aligned}
&= \Big(o_i^j, \beta_{ij}^{\zeta_1}, [1-(1-\gamma_{ij})^{\zeta_1}], [1-(1-\delta_{ij})^{\zeta_1}]\Big) \bigotimes \Big(o_k^j, \beta_{ij}^{\zeta_2}, [1-(1-\gamma_{ij})^{\zeta_2}], [1-(1-\delta_{ij})^{\zeta_2}]\Big) \\
&= \Big(\min(o_i^j, o_i^j), \beta_{ij}^{\zeta_1}\beta_{ij}^{\zeta_2}, [1-(1-\gamma_{ij})^{\zeta_1}] + [1-(1-\gamma_{ij})^{\zeta_2}] - [1-(1-\gamma_{ij})^{\zeta_1}][1-(1-\gamma_{ij})^{\zeta_2}] \\
&\quad , [1-(1-\delta_{ij})^{\zeta_1}] + [1-(1-\delta_{ij})^{\zeta_2}] - [1-(1-\delta_{ij})^{\zeta_1}][1-(1-\delta_{ij})^{\zeta_2}]\Big) \\
&= \Big(o_i^j, \beta_{ij}^{(\zeta_1+\zeta_2)}, [1-(1-\gamma_{ij})^{(\zeta_1+\zeta_2)}], [1-(1-\delta_{ij})^{(\zeta_1+\zeta_2)}]\Big) \\
&= \rho_{ij}^{(\zeta_1+\zeta_2}.
\end{aligned}
$$

□

Definition 13. *Let $\rho_{ij} = \rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ $(i = 1, 2, \ldots, l)$ be a collection of $SVNNS_fNs$ and θ_i be the weight vectors (WV) of ρ_{ij} with $\theta_i > 0$ and $\sum_{i=1}^{l} \theta_i = 1$. The single-valued neutrosophic N-soft weighted average operator $(SVNNS_fWA)$ is a mapping $SVNNS_fWA : \mathcal{B}^l \to \mathcal{B}$, where $\mathcal{B}$ is the set of $SVNNS_fNs$, defined as follows:*

$$
\begin{aligned}
SVNNS_fWA(\rho_{1j}, \rho_{2j}, \ldots, \rho_{lj}) &= \bigoplus_{i=1}^{l} (\theta_i \rho_{ij}) && (5)\\
&= \left(\max_{i=1}^{l}(o_i^j), 1 - \Pi_{i=1}^{l}(1-\beta_{ij})^{\theta_i}, \Pi_{i=1}^{l}(\gamma_{ij})^{\theta_i}, \Pi_{i=1}^{l}(\delta_{ij})^{\theta_i}\right). && (6)
\end{aligned}
$$

Definition 14. *Let $\rho_{ij} = \rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ $(i = 1, 2, \ldots, l)$ be a collection of $SVNNS_fNs$ and θ_i be the weight vectors (WV) of ρ_{ij} with $\theta_i > 0$ and $\sum_{i=1}^{l} \theta_i = 1$. The single-valued neutrosophic N-soft ordered weighted average operator $(SVNNS_fOWA)$ is a mapping $SVNNS_fOWA :$ $\mathcal{B}^l \to \mathcal{B}$, where $\mathcal{B}$ is the set of $SVNNS_fNs$, defined as follows:*

$$
\begin{aligned}
SVNNS_fOWA(\rho_{1j}, \rho_{2j}, \ldots, \rho_{lj}) &= \left(\theta_1\rho_{\phi(1j)} \bigoplus \theta_2\rho_{\phi(2j)} \bigoplus \ldots \bigoplus \theta_l\rho_{\phi(lj)}\right) \\
&= \left(\max_{i=1}^{l}(o_i^j), 1 - \Pi_{i=1}^{l}(1-\beta_{\phi(1j)})^{\theta_i}, \Pi_{i=1}^{l}(\gamma_{\phi(1j)})^{\theta_i}, \Pi_{i=1}^{l}(\delta_{\phi(1j)})^{\theta_i}\right),
\end{aligned}
$$

where, $(\phi(1j), \phi(2j), \ldots, \phi(lj))$ is a permutation of $(1j, 2j, \ldots, lj)$ such that $\rho_{\phi(ij)} \geq \rho_{\phi(kj)}$, for all $i < k$, $(i, k = 1, 2, \ldots, l)$ and $(j = 1, 2, \ldots, m)$.

Definition 15. *Let $\rho_{ij} = \rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ $(i = 1, 2, \ldots, l)$ be a collection of $SVNNS_fNs$ and θ_i be the weight vectors (WV) of ρ_{ij} with $\theta_i > 0$ and $\sum_{i=1}^{l} \theta_i = 1$. The single-valued neutrosophic N-soft weighted geometric operator $(SVNNS_fWG)$ is a mapping $SVNNS_fWG : \mathcal{B}^l \to \mathcal{B}$, where $\mathcal{B}$ is the set of $SVNNS_fNs$, defined as follows:*

$$
\begin{aligned}
SVNNS_fWG(\rho_{1j}, \rho_{2j}, \ldots, \rho_{lj}) &= \bigotimes_{i=1}^{l} (\rho_{ij})^{\theta_i} && (7)\\
&= \left(\min_{i=1}^{l}(o_i^j), \Pi_{i=1}^{l}(\beta_{ij})^{\theta_i}, 1 - \Pi_{i=1}^{l}(1-\gamma_{ij})^{\theta_i}, 1 - \Pi_{i=1}^{l}(1-\delta_{ij})^{\theta_i}\right). && (8)
\end{aligned}
$$

Definition 16. *Let $\rho_{ij} = \rho_{ij} = (o_i^j, (\beta_{ij}, \gamma_{ij}, \delta_{ij}))$ $(i = 1, 2, \ldots, l)$ be a collection of $SVNNS_fNs$ and θ_i be the weight vectors (WV) of ρ_{ij} with $\theta_i > 0$ and $\sum_{i=1}^{l} \theta_i = 1$. The single-valued neutrosophic N-soft ordered weighted geometric operator $(SVNNS_fOWG)$ is a mapping $SVNNS_fOWG : \mathcal{B}^l \to \mathcal{B}$, where $\mathcal{B}$ is the set of $SVNNS_fNs$, defined as follows:*

$$SVNNS_fOWG(\rho_{1j},\rho_{2j},\ldots,\rho_{lj}) = \left(\rho_{\phi(1j)^{\theta_1}} \bigotimes \rho_{\phi(2j)^{\theta_2}} \bigotimes \cdots \bigotimes \rho_{\phi(lj)^{\theta_l}}\right)$$
$$= \left(\min_{i=1}^{l}(o_i^j), \Pi_{i=1}^{l}(\beta_{\phi(1j)})^{\theta_i}, 1-\Pi_{i=1}^{l}(1-\gamma_{\phi(1j)})^{\theta_i}, 1-\Pi_{i=1}^{l}(1-\delta_{\phi(1j)})^{\theta_i}\right),$$

where, $(\phi(1j),\phi(2j),\ldots,\phi(lj))$ *is a permutation of* $(1j,2j,\ldots,lj)$ *such that* $\rho_{\phi(ij)} \geq \rho_{\phi(kj)}$, *for all* $i<k$, $(i,k=1,2,\ldots,l)$ *and* $(j=1,2,\ldots,m)$.

3 Single-valued neutrosophic N-soft TOPSIS method

In this section, we extend TOPSIS method to the environment of $SVNNS_fSs$ that will be used to find out an alternative that is nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS) as the feasible solution of MAGDM problem. Let $E=\{E_1,E_2,E_3,\ldots,E_m\}$ denote the set of attributes decided by the experts $\widetilde{D}_1,\widetilde{D}_2,\widetilde{D}_3,\ldots,\widetilde{D}_p$, for the alternatives $X=\{X_1,X_2,X_3,\ldots,X_q\}$, according to the MAGDM problems. The experts decisions weighted through the weight vector $\theta=(\theta_1,\theta_2,\theta_3,\ldots,\theta_p)^T$ with $\sum_{r=1}^{p}\theta=1$, where $\theta_r\in[0,1]$. The step by step procedure for $SVNNS_f$-TOPSIS method is as follows:

3.1 Formulation of decision matrices of each experts

Each expert assigns ranking, corresponding to each linguistic term, to the alternatives after thoroughly observing the attributes and MAGDM problem. The ranking provided by the experts is actually denoting NS_fS related to each expert. According to the proficiencies of the MAGDM problem, grading criteria defined by the experts according to which $SVNNS_fN$ is assigned to NS_fS, that is associated with each expert. Further, a single-valued neutrosophic N-soft decision matrix $(SVNNS_fDM)$ $G^{(r)}=(G_{ij}^{(r)})_{j\times i}$, is assembled by rth expert $\widetilde{D}_r$. So p $SVNNS_fDMs$, $G^{(1)},G^{(2)},\ldots,G^{(p)}$, are formed as follows:

$$G^{(r)}=\begin{pmatrix} (o_1^{1(r)},\beta_{11}^{(r)},\gamma_{11}^{(r)},\delta_{11}^{(r)}) & (o_2^{1(r)},\beta_{12}^{(r)},\gamma_{12}^{(r)},\delta_{12}^{(r)}) & \ldots & (o_m^{1\ (r)},\beta_{1m}^{(r)},\gamma_{1m}^{(r)},\delta_{1m}^{(r)}) \\ (o_1^{2(r)},\beta_{21}^{(r)},\gamma_{21}^{(r)},\delta_{21}^{(r)}) & (o_2^{2(r)},\beta_{22}^{(r)},\gamma_{22}^{(r)},\delta_{22}^{(r)}) & \ldots & (o_m^{2\ (r)},\beta_{2m}^{(r)},\gamma_{2m}^{(r)},\delta_{2m}^{(r)}) \\ \vdots & \vdots & \ddots & \vdots \\ (o_1^{q(r)},\beta_{q1}^{(r)},\gamma_{q1}^{(r)},\delta_{q1}^{(r)}) & (o_2^{q}(r),\beta_{q2}^{(r)},\gamma_{q2}^{(r)},\delta_{q2}^{(r)}) & \ldots & (o_m^{q\ (r)},\beta_{qm}^{(r)},\gamma_{qm}^{(r)},\delta_{qm}^{(r)}) \end{pmatrix},$$

where, $G_{ij}^{(r)}=((o_i^j)^{(r)},\beta_{ij}^{(r)},\gamma_{ij}^{(r)},\delta_{ij}^{(r)})$, $j=\{1,2,3,\ldots,q\}$, $i=\{1,2,3,\ldots,m\}$ and $r=\{1,2,3,\ldots,p\}$.

3.2 Formulation of aggregated single-valued neutrosophic N-soft decision matrix

The $SVNNS_fWA$ operator or $SVNNS_fWG$ operator, given in Equations 5 and 7, are used to summarize the $SVNNS_fDMs$ related to each expert, known as aggregated single-valued neutrosophic N-soft decision matrix $(ASVNNS_fDM)$, is calculated as follows:

$$\mathbb{G}=SVNNS_fWA(G_{ij}^{(1)},G_{ij}^{(2)},\ldots,G_{ij}^{(r)});$$

or

$$\mathbb{G}=SVNNS_fWG(G_{ij}^{(1)},G_{ij}^{(2)},\ldots,G_{ij}^{(r)});$$

The $ASVNN_fSDM$ denoted as:

$$\mathbb{G}=\begin{pmatrix} (o_1^1,\beta_{11},\gamma_{11},\delta_{11}) & (o_2^1,\beta_{12},\gamma_{12},\delta_{12}) & \ldots & (o_m^1,\beta_{1m},\gamma_{1m},\delta_{1m}) \\ (o_1^2,\beta_{21},\gamma_{21},\delta_{21}) & (o_2^2,\beta_{22},\gamma_{22},\delta_{22}) & \ldots & (o_m^2,\beta_{2m},\gamma_{2m},\delta_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (o_1^q,\beta_{q1},\gamma_{q1},\delta_{q1}) & (o_2^q,\beta_{q2},\gamma_{q2},\delta_{q2}) & \ldots & (o_m^q,\beta_{qm},\gamma_{qm},\delta_{qm}) \end{pmatrix}.$$

3.3 Calculation for weight vector of attributes

The value and importance of the attributes variate according to the MAGDM problem. The experts assigned rank to each attribute as weightage, keeping in view the expertise of the alternatives in the MAGDM problem. Using grading criteria, $SVNNS_fN$ assigned to each rank, i.e., $\mu_i^{(r)}=(o_i^{(r)},\beta_i^{(r)},\gamma_i^{(r)},\delta_i^{(r)})$ be the weight of ith attribute given by the rth expert in the decision maker panel. The weight vector $\mu=(\mu_1,\mu_2,\ldots,\mu_m)^T=(o_i,\beta_i,\gamma_i,\delta_i)$ is accumulated, by using the $SVNNS_fWA$ operator or $SVNNS_fWG$ operator given in Equations 5 and 7, as follows:

$$\mu_i=SVNNS_fWA(\mu_1^{(r)},\mu_2^{(r)},\ldots,\mu_m^{(r)});$$

or

$$\mu_i = SVNNS_f WG(\mu_1^{(r)}, \mu_2^{(r)}, \ldots, \mu_m^{(r)}).$$

3.4 Formulation of aggregated weighted single-valued neutrosophic N-soft decision matrix

The $ASVNN_fSDM$ and the weightage μ_i corresponding to each attribute E_i are used to calculate the aggregated weighted single-valued neutrosophic N-soft decision matrix
$(AWSVNNS_fDM)$ as follows:

$$\begin{aligned} \widetilde{\mathbb{G}} &= \mathbb{G} \bigotimes \mu_i \\ &= (\min((o_i^j), o_i), \beta_{ij}\beta_i, \gamma_{ij} + \gamma_i - \gamma_{ij}\gamma_i, \delta_{ij} + \gamma_i - \gamma_{ij}\gamma_i) \\ &= (\widetilde{o}_i^j, \widetilde{\beta}_{ij}, \widetilde{\gamma}_{ij}, \widetilde{\delta}_{ij}). \end{aligned}$$

So that the $AWSVNNS_fDM$ is:

$$\widetilde{\mathbb{G}} = \begin{pmatrix} (\widetilde{o}_1^1, \widetilde{\beta}_{11}, \widetilde{\gamma}_{11}, \widetilde{\delta}_{11}) & (\widetilde{o}_2^1, \widetilde{\beta}_{12}, \widetilde{\gamma}_{12}, \widetilde{\delta}_{12}) & \ldots & (\widetilde{o}_m^1, \widetilde{\beta}_{1m}, \widetilde{\gamma}_{1m}, \widetilde{\delta}_{1m}) \\ (\widetilde{o}_1^2, \widetilde{\beta}_{21}, \widetilde{\gamma}_{21}, \widetilde{\delta}_{21}) & (\widetilde{o}_2^2, \widetilde{\beta}_{22}, \widetilde{\gamma}_{22}, \widetilde{\delta}_{22}) & \ldots & (\widetilde{o}_m^2, \widetilde{\beta}_{2m}, \widetilde{\gamma}_{2m}, \widetilde{\delta}_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\widetilde{o}_1^q, \widetilde{\beta}_{q1}, \widetilde{\gamma}_{q1}, \widetilde{\delta}_{q1}) & (\widetilde{o}_2^q, \widetilde{\beta}_{q2}, \widetilde{\gamma}_{q2}, \widetilde{\delta}_{q2}) & \ldots & (\widetilde{o}_m^q, \widetilde{\beta}_{qm}, \widetilde{\gamma}_{qm}, \widetilde{\delta}_{qm}) \end{pmatrix}.$$

3.5 Formulation of single-valued neutrosophic N-soft ideal solution

The score value and the accuracy value are used to evaluate the single-valued neutrosophic positive ideal solution $SVNN_fS$-PIS and single-valued neutrosophic N-soft negative ideal solution $SVNNS_f$-NIS on the basis of cost-type attributes and benefit-type attributes. Let $\mathfrak{A}_\mathfrak{c}$ and $\mathfrak{A}_\mathfrak{b}$ be the collection of cost-type attributes and benefit-type attributes, respectively, that are chosen according to the nature of the MAGDM problem. Now, relative to the attribute E_i the $SVNNS_f$-PIS can be calculated as follows:

$$\overline{\mathbb{G}}_i = \begin{cases} \max_{j=1}^{q} \widetilde{\mathcal{G}}_{ij}, & \text{if } E_i \in \mathfrak{A}_\mathfrak{b}, \\ \min_{j=1}^{q} \widetilde{\mathcal{G}}_{ij}, & \text{if } E_i \in \mathfrak{A}_\mathfrak{c}, \end{cases} \tag{9}$$

and the $SVNNS_f$-NIS is computed as:

$$\underline{\mathbb{G}}_i = \begin{cases} \max_{j=1}^{q} \widetilde{\mathcal{G}}_{ij}, & \text{if } E_i \in \mathfrak{A}_\mathfrak{c}, \\ \min_{j=1}^{q} \widetilde{\mathcal{G}}_{ij}, & \text{if } E_i \in \mathfrak{A}_\mathfrak{b}. \end{cases} \tag{10}$$

The $SVNNS_f$-PIS and $SVNNS_f$-NIS are denoted as: $\overline{\mathbb{G}}_i = (\overline{o}_i, \overline{\beta}_i, \overline{\gamma}_i, \overline{\delta}_i)$, and $\underline{\mathbb{G}}_i = (\underline{o}_i, \underline{\beta}_i, \underline{\gamma}_i, \underline{\delta}_i)$, respectively.

3.6 Evaluation of normalized Euclidean distance

To find out best solution, we have to identify the nearest and farthest alternative from the $SVNNS_f$-PIS and $SVNNS_f$-NIS, respectively. For this purpose, we computed normalized Euclidean distance of $SVNNS_f$-PIS and $SVNNS_f$-NIS from each alternative, simultaneously, as follows:

$$d(\overline{\mathbb{G}}_i, X_j) = \Big(\frac{1}{4i}\sum_{i=1}^{m}\Big[((\frac{\overline{o}_i}{N-1}) - (\frac{\widetilde{o}_i^j}{N-1}))^2 + (\overline{\beta}_i - \widetilde{\beta}_{ij})^2 + (\overline{\gamma}_i - \widetilde{\gamma}_{ij})^2 + (\overline{\delta}_i - \widetilde{\delta}_{ij})^2\Big]\Big).$$

The normalized Euclidean distance between the $SVNNS_f$-NIS and any of the alternative X_j, can be evaluated as follows:

$$d(\underline{\mathbb{G}}_i, X_j) = \Big(\frac{1}{4i}\sum_{i=1}^{m}\Big[((\frac{\underline{o}_i}{N-1}) - (\frac{\widetilde{o}_i^j}{N-1}))^2 + (\underline{\beta}_i - \widetilde{\beta}_{ij})^2 + (\underline{\gamma}_i - \widetilde{\gamma}_{ij})^2 + (\underline{\delta}_i - \widetilde{\delta}_{ij})^2\Big]\Big).$$

3.7 Computation of revised closeness index

We have to use some ranking index to compare the alternatives as we have alternatives having maximum distance from $SVNNS_f$-PIS as well as the minimum distance from $SVNNS_f$-NIS. Therefore, the revised closeness index modified by Gundogdu and Kahraman [11] for the selection of optimal solution is as follows:

$$\psi(X_j) = \frac{d(\overline{\mathbb{G}}_i, X_j)}{\min\limits_j d(\overline{\mathbb{G}}_i, X_j)} - \frac{d(\underline{\mathbb{G}}_i, X_j)}{\max\limits_j d(\underline{\mathbb{G}}_i, X_j)}, \tag{11}$$

where, $i = 1, 2, \ldots, m$.
Clearly, the closed index in Equation 14, generates zero or negative outputs, therefore we prefer this modified relation given in Equation 11 for $SVNNS_f$-TOPSIS method as it gives zero or positive results.

3.8 Order of alternatives

The alternatives are arranged in ascending order with respect to the revised closeness index and the alternative with lowest value is considered as the most suitable solution of the MAGDM problem.
The algorithm and the flowchart of the proposed $SVNNS_f$-TOPSIS method is given in Algorithm 1. For solving a MAGDM problem, the Algorithm 1 is given as:

Algorithm 1: Steps to deal MAGDM problem by $SVNN_f$-TOPSIS method

1. **Input**:
   X : Set of alternatives,
   E : Set of attributes,
   θ : WV for experts $\widetilde{D}_r$,
   $NS_fS : (H, E, N)$ with $O = \{0, 1, 2, 3, \ldots, N-1\}$, $N \in \{1, 2, 3, \ldots\}$,
2. Construct the $SVNNS_fDM$ $G^{(r)}$, corresponding to each ordered grade for the element X_j.
3. Evaluate the $ASVNNS_fDM$ using equation

$$\mathbb{G}_{ij} = \left(\max_{r=1}^{p} (o_i^j)^{(r)}, 1 - \prod_{r=1}^{p} (1 - (\beta_{ij}^{(r)}))^{\theta_r}, \prod_{r=1}^{p} (\gamma_{ij}^{(r)})^{\theta_r}, \prod_{r=1}^{p} (\delta_{ij}^{(r)})^{\theta_r} \right).$$

4. Calculating the weight vector $\mu = (\mu_1, \mu_2, \ldots, \mu_m)^T$ for attributes as follows:

$$\mu_i = \left(\max_{r=1}^{p} (o_i^{(r)}), 1 - \prod_{r=1}^{p} (1 - (\beta_i^{(r)}))^{\theta_r}, \prod_{r=1}^{p} (\gamma_i^{(r)})^{\theta_r}, \prod_{r=1}^{p} (\delta_i^{(r)})^{\theta_r} \right).$$

5. Compute the $AWSVNNS_fDM$ using $ASVNNS_fDM$ and the weight vector of attributes μ_i, as follows:

$$\widetilde{\mathbb{G}} = (\min((o_i^j), o_i), \beta_{ij}\beta_i, \gamma_{ij} + \gamma_i - \gamma_{ij}\gamma_i, \delta_{ij} + \delta_i - \delta_{ij}\delta_i).$$

6. Identify the $SVNNS_f$ PIS and $SVNNS_f$ NIS, using Equations (9) and (10).
7. Compute the normalized Euclidean distance of $CSVNNS_f$ PIS and $CSVNNS_f$ NIS from each alternative, respectively.
8. Calculate the revised closeness index.
9. Rank the alternatives in ascending order with respect to the revised closeness index.

Output: The alternative with least revised closeness index will be the decision.

4 Application

In this section, we solve a multi-attribute group decision making (MADM) problem using $SVNNS_f - TOPSIS$ method for the selection of branch manager in Quiqup company(courier company), UAE.

4.1 Selection for the post of branch manager in Quiqup company, UAE

The courier companies are serving as a bridge between the sellers and the customers that enhance the M-Commerce which is a shopping online through smartphone. M-Commerce has enabled us to have a lot of free time that we can sell or buy anything, anytime within a seconds and through courier companies. In UAE, the online shopping arena has been making tremendous growth in the past 10-years. For this purpose there are so many companies in UAE, one of them is Quiqup company in which courier drivers are specifically appointed for placing orders at the right place where the branch manager has to look after the overall records of the couriers. For the post of branch manager, three decision makers shortlisted five courier drivers for further evaluations. The experts $\widetilde{D}_1$, $\widetilde{D}_2 and \widetilde{D}_3$ analyzed courier drivers, named as $\{X_1 = Bahzad, X_2 = Naqash, X_3 = Zakwan, X_4 = Soreach, X_5 = Waqas\}$, on the basis of the following parameters $\{E_1 =$ Experience, $X_2 =$ Education, $X_3 =$ courier services , $X_4 =$ Fines and Expenditures, $X_5 =$ Behaviour $\}$. The weight vector for the experts is $\theta = (0.4, 0.3, 0.3)^T$ according to this MAGDM problem.

Step 1: According to these attributes each expert model 6-soft set in Table 5, where,

Five stars means 'Infinitely Good',

Four stars means 'Extremely Good',

Three stars means 'Good',

Two stars means 'Bad',

One stars means 'Extremely Bad',

Big dot means 'Infinitely Bad'

Table 3 represents the grading criteria, used for assigning the $SVNNS_fN$ corresponding to each rank by the expert $\widetilde{D}_1$, $\widetilde{D}_2$ and $\widetilde{D}_3$ arranged in Tables 6, 7 and 8, respectively.

Table 5: Experts' opinion related to parameters

$Parameters$	$Alternatives$	$\widetilde{D}_1$	$\widetilde{D}_2$,	$\widetilde{D}_3$
E_1	X_1	$** = 2$	$*** = 3$	$* = 1$
	X_2	$* = 1$	$** = 2$	$** = 2$
	X_3	$***** = 5$	$***** = 5$	$**** = 4$
	X_4	$** = 2$	$*** = 3$	$*** = 3$
	X_5	$\bullet = 0$	$* = 1$	$** = 2$
E_2	X_1	$**** = 4$	$***** = 5$	$*** = 3$
	X_2	$* = 1$	$\bullet = 0$	$* = 1$
	X_3	$**** = 4$	$***** = 5$	$***** = 5$
	X_4	$*** = 3$	$* = 1$	$\bullet = 0$
	X_5	$** = 2$	$* = 1$	$** = 2$
E_3	X_1	$*** = 3$	$**** = 4$	$***** = 5$
	X_2	$**** = 4$	$*** = 3$	$**** = 4$
	X_3	$***** = 5$	$***** = 5$	$**** = 4$
	X_4	$* = 1$	$*** = 3$	$*** = 3$
	X_5	$*** = 3$	$**** = 4$	$*** = 3$
E_4	X_1	$**** = 4$	$*** = 3$	$***** = 5$
	X_2	$**** = 4$	$***** = 5$	$***** = 5$
	X_3	$*** = 3$	$** = 2$	$**** = 4$
	X_4	$**** = 4$	$*** = 3$	$**** = 4$
	X_5	$***** = 5$	$**** = 4$	$**** = 4$
E_5	X_1	$**** = 4$	$**** = 4$	$**** = 4$
	X_2	$** = 2$	$** = 2$	$** = 2$
	X_3	$***** = 5$	$***** = 5$	$***** = 5$
	X_4	$*** = 3$	$*** = 3$	$*** = 3$
	X_5	$* = 1$	$* = 1$	$* = 1$

Table 6: $SVNNS_fDM$ of expert $\widetilde{D}_1$

$(H_J^{(1)}, E, 6)$	E_1	E_2	E_3	E_4	E_5
X_1	$(2, (0.410, 0.125, 0.610))$	$(4, (0.710, 0.030, 0.250))$	$(3, (0.690, 0.068, 0.480))$	$(4, (0.720, 0.040, 0.260))$	$(4, (0.730, 0.050, 0.270))$
X_2	$(1, (0.290, 0.018, 0.810))$	$(1, (0.280, 0.017, 0.790))$	$(4, (0.740, 0.060, 0.220))$	$(4, (0.750, 0.550, 0.170))$	$(2, (0.460, 0.132, 0.160))$
X_3	$(5, (0.980, 0.010, 0.020))$	$(4, (0.870, 0.012, 0.160))$	$(5, (0.970, 0.015, 0.016))$	$(3, (0.680, 0.0350, 0.410))$	$(5, (0.990, 0.010, 0.014))$
X_4	$(2, (0.430, 0.129, 0.630))$	$(3, (0.660, 0.036, 0.430))$	$(1, (0.270, 0.016, 0.780))$	$(4, (0.760, 0.057, 0.180))$	$(3, (0.670, 0.034, 0.420))$
X_5	$(0, (0.500, 0.300, 0.800))$	$(2, (0.420, 0.127, 0.620))$	$(3, (0.650, 0.037, 0.440))$	$(5, (0.910, 0.016, 0.140))$	$(1, (0.260, 0.015, 0.770))$

Step 2: The $ASVNNS_fDM$ formulated by aggregation formula defined in Algorithm 1(3). The accumulated opinions of all experts is shown in Table 9.

Step 3: According to the MAGDM problem, experts assigned ratings to parameters to explain their significance related to each alternatives. Further, the ratings are replaced by $SVNNS_fNs$, shown in Table 10, and the weight vector μ cumulated using Algorithm 1(step 4) is

Table 7: $SVNNS_fDM$ of expert $\widetilde{D}_2$

$(H_J^{(2)}, E, 6)$	E_1	E_2	E_3	E_4	E_5
X_1	$(3,(0.640,0.038,0.450))$	$(5,(0.950,0.015,0.130))$	$(4,(0.780,0.058,0.190))$	$(3,(0.630,0.039,0.460))$	$(4,(0.790,0.059,0.210))$
X_2	$(2,(0.440,0.130,0.640))$	$(0,(0.510,0.310,0.810))$	$(3,(0.620,0.040,0.470))$	$(5,(0.920,0.016,0.140))$	$(2,(0.470,0.133,0.670))$
X_3	$(5,(0.980,0.011,0.009))$	$(5,(0.995,0.008,0.007))$	$(5,(0.975,0.007,0.006))$	$(2,(0.450,0.131,0.650))$	$(5,(0.960,0.004,0.040))$
X_4	$(3,(0.610,0.041,0.480))$	$(1,(0.250,0.014,0.760))$	$(3,(0.620,0.042,0.490))$	$(3,(0.630,0.043,0.350))$	$(3,(0.640,0.044,0.360))$
X_5	$(1,(0.240,0.013,0.750))$	$(1,(0.230,0.012,0.740))$	$(4,(0.810,0.061,0.220))$	$(4,(0.820,0.062,0.230))$	$(1,(0.220,0.011,0.730))$

Table 8: $SVNNS_fDM$ of expert $\widetilde{D}_3$

$(H_J^{(3)}, E, 6)$	E_1	E_2	E_3	E_4	E_5
X_1	$(1,(0.210,0.010,0.720))$	$(3,(0.510,0.045,0.370))$	$(5,(0.915,0.013,0.120))$	$(5,(0.925,0.014,0.100))$	$(4,(0.830,0.064,0.250))$
X_2	$(2,(0.490,0.135,0.550))$	$(1,(0.200,0.009,0.710))$	$(4,(0.820,0.063,0.240))$	$(5,(0.930,0.010,0.110))$	$(2,(0.480,0.134,0.680))$
X_3	$(4,(0.710,0.015,0.165))$	$(5,(0.970,0.005,0.006))$	$(4,(0.840,0.065,0.260))$	$(4,(0.850,0.066,0.270))$	$(5,(0.983,0.005,0.050))$
X_4	$(3,(0.520,0.046,0.380))$	$(0,(0.520,0.320,0.820))$	$(3,(0.530,0.047,0.390))$	$(4,(0.860,0.067,0.280))$	$(3,(0.540,0.048,0.290))$
X_5	$(2,(0.350,0.136,0.560))$	$(2,(0.360,0.137,0.570))$	$(3,(0.550,0.049,0.330))$	$(4,(0.870,0.068,0.290))$	$(1,(0.190,0.008,0.700))$

Table 9: Aggregated single-valued neutrosophic N-soft decision matrix

$\mathbb{G}$	E_1	E_2	E_3	E_4	E_5
X_1	$(3,(0.466,0.043,0.572))$	$(5,(0.821,0.026,0.219))$	$(5,(0.801,0.042,0.245))$	$(5,(0.778,0.030,0.250))$	$(4,(0.780,0.056,0.242))$
X_2	$(2,(0.398,0.060,0.677))$	$(1,(0.354,0.040,0.776))$	$(4,(0.729,0.052,0.293))$	$(5,(0.878,0.058,0.142))$	$(2,(0.468,0.132,0.668))$
X_3	$(5,(0.964,0.011,0.026))$	$(5,(0.971,0.008,0.024))$	$(5,(0.957,0.016,0.022))$	$(4,(0.680,0.065,0.434))$	$(5,(0.981,0.006,0.028))$
X_4	$(3,(0.522,0.066,0.504))$	$(3,(0.511,0.044,0.616))$	$(3,(0.480,0.029,0.557))$	$(4,(0.756,0.054,0.254))$	$(3,(0.630,0.040,0.362))$
X_5	$(2,(0.379,0.082,0.715))$	$(2,(0.344,0.056,0.646))$	$(4,(0.699,0.047,0.321))$	$(5,(0.874,0.036,0.200))$	$(1,(0.229,0.011,0.738))$

given as follows:

$$\mu = \begin{pmatrix} (5,(0.932,0.027,0.204)) \\ (3,(0.815,0.037,0.541)) \\ (4,(0.914,0.026,0.266)) \\ (4,(0.525,0.047,0.499)) \\ (5,(0.657,0.035,0.278)) \end{pmatrix}.$$

Table 10: Ratings of experts about parameters

	E_1	E_2	E_3	E_4	E_5
$\widetilde{D}_1$	$(4,(0.820,0.040,0.250))$	$(3,(0.600,0.020,0.40))$	$(4,(0.800,0.025,0.200))$	$(1,(0.200,0.040,0.850))$	$(2,(0.350,0.100,0.600))$
$\widetilde{D}_2$	$(5,(0.920,0.010,0.550))$	$(2,(0.370,0.090,0.550))$	$(3,(0.660,0.030,0.410))$	$(4,(0.760,0.030,0.220))$	$(4,(0.750,0.020,0.210))$
$\widetilde{D}_3$	$(3,(0.680,0.061,0.041))$	$(1,(0.270,0.030,0.554))$	$(4,(0.770,0.025,0.230))$	$(2,(0.360,0.120,0.670))$	$(5,(0.950,0.015,0.127))$

Step 4: We used $\mathbb{G}$ and weight vector μ of parameters for availing the $AWSVNNS_fDM$ summarized in Table 11.

Table 11: Aggregated weighted single-valued neutrosophic N-soft decision matrix

$\mathbb{G}$	E_1	E_2	E_3	E_4	E_5
X_1	$(3,(0.430,0.068,0.659))$	$(3,(0.699,0.062,0.641))$	$(4,(0.732,0.066,0.446))$	$(4,(0.408,0.076,0.624))$	$(4,(0.512,0.089,0.452))$
X_2	$(2,(0.367,0.085,0.742))$	$(1,(0.288,0.075,0.897))$	$(4,(0.666,0.076,0.481))$	$(4,(0.460,0.102,0.570))$	$(2,(0.307,0.162,0.760))$
X_3	$(5,(0.890,0.038,0.224))$	$(3,(0.791,0.044,0.552))$	$(4,(0.874,0.042,0.282))$	$(4,(0.460,0.108,0.716))$	$(5,(0.644,0.040,0.298))$
X_4	$(3,(0.481,0.091,0.605))$	$(3,(0.416,0.079,0.824))$	$(3,(0.438,0.054,0.674))$	$(4,(0.396,0.098,0.626))$	$(3,(0.414,0.074,0.539))$
X_5	$(2,(0.350,0.106,0.773))$	$(2,(0.280,0.090,0.838))$	$(4,(0.638,0.072,0.502))$	$(4,(0.458,0.081,0.599))$	$(1,(0.150,0.046,0.810))$

Step 5: The parameters experiences, customer services, education and behaviour are benefit-type parameters while the fines and expenditures is cost-type parameter .Keeping in view the nature of parameters and applying Equation 9 and 10 $SVNNS_f$-PIS and $SVNNS_f$-NIS are evaluated, arranged in Table 12.

Table 12: $SVNNS_f$-PIS and $CSVNNS_f$-NIS

Attribute	$CSVNNS_f$-PIS	$SVNNS_f$-NIS
z_1	$(5,(0.890,0.038,0.224))$	$(2,(0.350,0.106,0.773))$
z_2	$(3,(0.791,0.044,0.552))$	$(1,(0.288,0.075,0.897))$
z_3	$(4,(0.874,0.042,0.282))$	$(3,(0.438,0.054,0.674))$
z_4	$(4,(0.460,0.108,0.716))$	$(4,(0.460,0.102,0.570))$
z_5	$(5,(0.644,0.040,0.298))$	$(1,(0.150,0.046,0.810))$

Step 6: The normalized Euclidean distance, from each alternative to $SVNNS_f$-PIS and $SVNNS_f$-NIS, is given in Table 13.

Table 13: Normalized Euclidean distance from ideal solution

Alternative	$d(\overline{\mathbb{G}}_k, X_j)$	$d(\underline{\mathbb{G}}_k, X_j)$
X_1	0.0361	0.0640
X_2	0.1122	0.0390
X_3	0.0005	0.1540
X_4	0.0679	0.0302
X_5	0.1327	0.0070

Step 7: The revised closeness index of each alternative is calculated by utilizing Equation 11, given in Table 14.

Table 14: Revised closeness index of each alternative

Alternative	$\psi(X_j)$
X_1	6.8044
X_2	22.186
X_3	0
X_4	13.3838
X_5	26.4945

Step 8: Since X_3 has minimum revised closeness index, therefore Zakwan is the most suitable courier driver for branch manager post. The ranking of alternatives is shown in Table 15.

Table 15: Ranking according to the revised closeness index

Alternative	X_1	X_2	X_3	X_4	X_5
Ranking	2	4	1	3	5

5 Comparison

In this section, we solve the MAGDM problem "selection for the post of branch manager in Quiqup company, UAE" using single-valued neutrosophic TOPSIS method, proposed by Sahi and Yigider. [40], to demonstrate the significance of the proposed model. The solution by single-valued neutrosophic TOPSIS method is as follows:

Step 1 The linguistic term corresponding to each rank asses by the experts, are same as given in Table 5. To apply the SVN-TOPSIS method the grading part is excluded from the $SVNNS_fN$ and SVNNs are assigning by each expert $\widetilde{D}_1$, $\widetilde{D}_2$ and $\widetilde{D}_3$, are arranged in Tables 16,17 and 18, respectively, according to the grading criteria define in Table 3.

Step 2 Using the weight vector of experts $\theta = (0.4, 0.3, 0.3)^T$ and single-valued neutrosophic weighted average $(SVNWA)$ operator [40], we can calculate the aggregated single-valued neutrosophic decision matrix $(ASVNDM)$, whose entries are evaluated by the formula defined as follows:

$$\mathbb{G}_{ij} = \left(1 - \prod_{r=1}^{p}(1-(\beta_{ij}^{(r)}))^{\theta_r}, \prod_{r=1}^{p}(\gamma_{ij}^{(r)})^{\theta_r}, \prod_{r=1}^{p}(\delta_{ij}^{(r)})^{\theta_r}\right).$$

Table 16: $SVNDM$ of expert $\widetilde{D}_1$

	E_1	E_2	E_3	E_4	E_5
X_1	$(0.410, 0.125, 0.610)$	$(0.710, 0.030, 0.250)$	$(0.690, 0.068, 0.480)$	$(0.720, 0.040, 0.260)$	$(0.730, 0.050, 0.270)$
X_2	$(0.290, 0.018, 0.810)$	$(0.280, 0.017, 0.790)$	$(0.740, 0.060, 0.220)$	$(0.750, 0.550, 0.170)$	$(0.460, 0.132, 0.160)$
X_3	$(0.980, 0.010, 0.020)$	$(0.870, 0.012, 0.160)$	$(0.970, 0.015, 0.016)$	$(0.680, 0.035, 0.410)$	$(0.990, 0.010, 0.014)$
X_4	$(0.430, 0.129, 0.630)$	$(0.660, 0.036, 0.430)$	$(0.270, 0.016, 0.780)$	$(0.760, 0.057, 0.180)$	$(0.670, 0.034, 0.420)$
X_5	$(0.500, 0.300, 0.800)$	$(0.420, 0.127, 0.620)$	$(0.650, 0.037, 0.440)$	$(0.910, 0.016, 0.140)$	$(0.260, 0.015, 0.770)$

Table 17: $SVNDM$ of expert $\widetilde{D}_2$

	E_1	E_2	E_3	E_4	E_5
X_1	$(0.640, 0.038, 0.450)$	$(0.950, 0.015, 0.130)$	$(0.780, 0.058, 0.190)$	$(0.630, 0.039, 0.460)$	$(0.790, 0.059, 0.210)$
X_2	$(0.440, 0.130, 0.640)$	$(0.510, 0.310, 0.810)$	$(0.620, 0.040, 0.470)$	$(0.920, 0.016, 0.140)$	$(0.470, 0.133, 0.670)$
X_3	$(0.980, 0.011, 0.009)$	$(0.995, 0.008, 0.007)$	$(0.975, 0.007, 0.006)$	$(0.450, 0.131, 0.650)$	$(0.960, 0.004, 0.040)$
X_4	$(0.610, 0.041, 0.480)$	$(0.250, 0.014, 0.760)$	$(0.620, 0.042, 0.490)$	$(0.630, 0.043, 0.350)$	$(0.640, 0.044, 0.360)$
X_5	$(0.240, 0.013, 0.750)$	$(0.230, 0.012, 0.740)$	$(0.810, 0.061, 0.220)$	$(0.820, 0.062, 0.230)$	$(0.220, 0.011, 0.730)$

Table 18: $SVNDM$ of expert $\widetilde{D}_3$

	E_1	E_2	E_3	E_4	E_5
X_1	$(0.210, 0.010, 0.720)$	$(0.510, 0.045, 0.370)$	$(0.915, 0.013, 0.120)$	$(0.925, 0.014, 0.100)$	$(0.830, 0.064, 0.250)$
X_2	$(0.490, 0.135, 0.550)$	$(0.200, 0.009, 0.710)$	$(0.820, 0.063, 0.240)$	$(0.930, 0.010, 0.110)$	$(0.480, 0.134, 0.680)$
X_3	$(0.710, 0.015, 0.165)$	$(0.970, 0.005, 0.006)$	$(0.840, 0.065, 0.260)$	$(0.850, 0.066, 0.270)$	$(0.983, 0.005, 0.050)$
X_4	$(0.520, 0.046, 0.380)$	$(0.520, 0.320, 0.820)$	$(0.530, 0.047, 0.390)$	$(0.860, 0.067, 0.280)$	$(0.540, 0.048, 0.290)$
X_5	$(0.350, 0.136, 0.560)$	$(0.360, 0.137, 0.570)$	$(0.550, 0.049, 0.330)$	$(0.870, 0.068, 0.290)$	$(0.190, 0.008, 0.700)$

The $ASVNDM$ is arranged in Table 19.

Table 19: Aggregated single-valued neutrosophic decision matrix

$\mathbb{G}$	E_1	E_2	E_3	E_4	E_5
X_1	$(0.466, 0.043, 0.572)$	$(0.821, 0.026, 0.219)$	$(0.801, 0.042, 0.245)$	$(0.778, 0.030, 0.250)$	$(0.780, 0.056, 0.242)$
X_2	$(0.398, 0.060, 0.677)$	$(0.354, 0.040, 0.776)$	$(0.729, 0.052, 0.293)$	$(0.878, 0.058, 0.142)$	$(0.468, 0.132, 0.668)$
X_3	$(0.964, 0.011, 0.026)$	$(0.971, 0.008, 0.024)$	$(0.957, 0.016, 0.022)$	$(0.680, 0.065, 0.434))$	$(0.981, 0.006, 0.028)$
X_4	$(0.522, 0.066, 0.504)$	$(0.511, 0.044, 0.616)$	$(0.480, 0.029, 0.557)$	$(0.756, 0.054, 0.254)$	$(0.630, 0.040, 0.362)$
X_5	$(0.379, 0.082, 0.715)$	$(0.344, 0.056, 0.646)$	$(0.699, 0.047, 0.321)$	$(0.874, 0.036, 0.200)$	$(0.229, 0.011, 0.738)$

Step 3 The experts opinion about the importance of attributes are given in Table 20. The experts opinion are combined using $(SVNWA)$ operator [40], to formulate the weight vector μ for the attributes, defined as follows:

$$\mathbb{G}_{ij} = \left(1 - \prod_{r=1}^{p}(1-(\beta_{ij}^{(r)}))^{\theta_r}, \prod_{r=1}^{p}(\gamma_{ij}^{(r)})^{\theta_r}, \prod_{r=1}^{p}(\delta_{ij}^{(r)})^{\theta_r}\right).$$

Thus we have,

$$\mu = \begin{pmatrix} (0.932, 0.027, 0.204) \\ (0.815, 0.037, 0.541) \\ (0.914, 0.026, 0.266) \\ (0.525, 0.047, 0.499) \\ (0.657, 0.035, 0.278) \end{pmatrix}.$$

Step 4 The aggregated weighted single-valued neutrosophic decision matrix (AWSVNDM) arranged in Table 21, where the entries of AWSVNDM

Table 20: Ratings of experts about parameters in single-valued neutrosophic environment

	E_1	E_2	E_3	E_4	E_5
$\widetilde{D}_1$	$(0.820, 0.040, 0.250)$	$(0.600, 0.020, 0.400)$	$(0.800, 0.025, 0.200)$	$(0.200, 0.040, 0.850)$	$(0.350, 0.100, 0.600)$
$\widetilde{D}_2$	$(0.920, 0.010, 0.550)$	$(0.370, 0.090, 0.550)$	$(0.660, 0.030, 0.410)$	$(0.760, 0.030, 0.220)$	$(0.750, 0.020, 0.210)$
$\widetilde{D}_3$	$(0.680, 0.061, 0.041)$	$(0.270, 0.030, 0.554)$	$(0.770, 0.025, 0.230)$	$(0.360, 0.120, 0.670)$	$(0.950, 0.015, 0.127)$

are calculated using the formula:

$$\widetilde{\mathbb{G}} \quad = \quad (\beta_{ij}\beta_i, \gamma_{ij} + \gamma_i - \gamma_{ij}\gamma_i, \delta_{ij} + \delta_i - \delta_{ij}\delta_i).$$

Table 21: Aggregated weighted single-valued neutrosophic decision matrix

$\mathbb{G}$	E_1	E_2	E_3	E_4	E_5
X_1	$(0.430, 0.068, 0.659)$	$(0.699, 0.062, 0.641)$	$(0.732, 0.066, 0.446)$	$(0.408, 0.076, 0.624)$	$(0.512, 0.089, 0.452)$
X_2	$(0.367, 0.085, 0.742)$	$(0.288, 0.075, 0.897)$	$(0.666, 0.076, 0.481)$	$(0.460, 0.102, 0.570)$	$(0.307, 0.162, 0.760)$
X_3	$(0.890, 0.038, 0.224)$	$(0.791, 0.044, 0.552)$	$(0.874, 0.042, 0.282)$	$(0.460, 0.108, 0.716)$	$(0.644, 0.040, 0.298)$
X_4	$(0.481, 0.091, 0.605)$	$(0.416, 0.079, 0.824)$	$(0.438, 0.054, 0.674)$	$(0.396, 0.098, 0.626)$	$(0.414, 0.074, 0.539)$
X_5	$(0.350, 0.106, 0.773)$	$(0.280, 0.090, 0.838)$	$(0.638, 0.072, 0.502)$	$(0.458, 0.081, 0.599)$	$(0.150, 0.046, 0.810)$

Step 5 To evaluate the single-valued neutrosophic positive ideal solution (SVN-PIS) and negative ideal solution (SVN-NIS) are to be calculated by the formula:

$$\overline{\mathbb{G}}_i = \begin{cases} (\max_j \widetilde{\beta}_{ij}, \min_j \widetilde{\gamma}_{ij}, \min_j \widetilde{\delta}_{ij}), & \text{if } E_i \in \mathfrak{A}_{\mathfrak{b}}, \\ (\min_j \widetilde{\beta}_{ij}, \max_j \widetilde{\gamma}_{ij}, \max_j \widetilde{\delta}_{ij}), & \text{if } E_i \in \mathfrak{A}_{\mathfrak{c}}, \end{cases}$$

and

$$\underline{\mathbb{G}}_i = \begin{cases} (\max_j \widetilde{\beta}_{ij}, \min_j \widetilde{\gamma}_{ij}, \min_j \widetilde{\delta}_{ij}), & \text{if } E_i \in \mathfrak{A}_{\mathfrak{c}}, \\ (\min_j \widetilde{\beta}_{ij}, \max_j \widetilde{\gamma}_{ij}, \max_j \widetilde{\delta}_{ij}), & \text{if } E_i \in \mathfrak{A}_{\mathfrak{b}}, \end{cases}$$

So that, the SVN-PIS and SVN-NIS found given in Table 22.

Table 22: SVN-PIS and SVN-NIS

Parameters	SVN-PIS	SVN-NIS
E_1	$(0.802, 0.038, 0.028)$	$(0.340, 0.106, 0.703)$
E_2	$(0.452, 0.052, 0.554)$	$(0.166, 0.184, 0.848)$
E_3	$(0.720, 0.034, 0.260)$	$(0.370, 0.042, 0.591)$
E_4	$(0.458, 0.089, 0.634)$	$(0.458, 0.250, 0.541)$
E_5	$(0.426, 0.032, 0.758)$	$(0.786, 0.046, 0.271)$

Step 6 The Euclidean distance of each alternative from SVN-PIS and SVN-NIS, evaluated by Equations 12 and 13, respectively, is given in Table 23.

$$d_E(\overline{\mathbb{G}}_i, X_j) = \sqrt{\left(\frac{1}{3}\sum_{i=1}^{m}\left[(\overline{\beta}_i - \widetilde{\beta}_{ij})^2 + (\overline{\gamma}_i - \widetilde{\gamma}_{ij})^2 + (\overline{\delta}_i - \widetilde{\delta}_{ij})^2\right]\right)}. \tag{12}$$

and

$$d_E(\underline{\mathbb{G}}_i, X_j) \quad = \quad \sqrt{\left(\frac{1}{3}\sum_{i=1}^{m}\left[(\underline{\beta}_i - \widetilde{\beta}_{ij})^2 + (\underline{\gamma}_i - \widetilde{\gamma}_{ij})^2 + (\underline{\delta}_i - \widetilde{\delta}_{ij})^2\right]\right)}. \tag{13}$$

Table 23: single-valued neutrosophic Euclidean distance

Alternative	$d(\overline{\mathbb{G}}_k, X_j)$	$d(\underline{\mathbb{G}}_k, X_j)$
X_1	0.4159	0.4830
X_2	0.6753	0.2120
X_3	0.0369	0.7092
X_4	0.5759	0.2898
X_5	0.7239	0.1084

Step 7: The revised closeness index of each alternative, evaluated by Equations 14, is tabulated in Table 24 and the ratings are tabulated in Table 25 in descending order, according to which X_3 is the best choice for the post of branch manager in Quiqup company, UAE.

$$\psi(X_j) = \frac{d(\underline{\mathbb{G}}_i, X_j)}{d(\overline{\mathbb{G}}_i, X_j) + d(\underline{\mathbb{G}}_i, X_j)} \tag{14}$$

where, $i = 1, 2, \ldots, m$.

Table 24: Revised closeness index of each alternative

Alternative	$\psi(X_j)$
X_1	0.5373
X_2	0.23900
X_3	0.9505
X_4	0.3347
X_5	0.1302

Table 25: Ranking in single-valued neutrosophic environment

Alternative	X_1	X_2	X_3	X_4	X_5
Ranking	2	4	1	3	5

5.1 Discussion

1. We conclude that the comparison of the proposed $SVNNS_f$-TOPSIS method with the existing technique SVN-TOPSIS method results the same courier driver for the post of branch manager as well as the order of ranking of the remaining alternatives remain same, given in Table 26.
   The accuracy and reliability of the outcomes in comparison proves the superiority of the the proposed method from the SVN methods.

Table 26: Comparison

Method	Ranking	Best candidate
SVN-TOPSIS [40]	$X_3 > X_1 > X_4 > X_2 > X_5$	X_3
$SVNNS_f$-TOPSIS (Proposed)	$X_3 > X_1 > X_4 > X_2 > X_5$	X_3

2. The $SVNNS_f$-TOPSIS method has ability to handle MAGDM problems under the framework of $IFNS_fS$ and $PFNS_fS$ but these models have no capacity to deal the hesitancy opinion of human nature independently.
3. The existing models, specifically the generalized model SVNSs are impotent to handle modern problems described by parameterized rating systems but our model has potential to grip such type of modern problems.
4. By substituting $N = 2$, we switch from $SVNNS_f$ environment to $SVNS_f$ environment so that the $SVNNS_f$-TOPSIS method could be applied to the $SVNS_f$ environment in a satisfactory manners.

6 Conclusion

In this paper, we have mainly contributed in TOPSIS method precisely for group decision making under the most generalized environment of $SVNNS_fSs$. The $SVNNS_f$-TOPSIS method is an advanced technique to evaluate the optimal alternative nearest to $SVNNS_f$-PIS and farthest from $SVNNS_f$-NIS. For the extension of TOPSIS method, we have presented the aggregate operators to assess the $SVNNS_f$ aggregated and weighted aggregated decision matrix that are further used to spot the $SVNNS_f$-PIS and $SVNNS_f$-NIS heeding the benefit and cost type parameters. We have defined normalized Euclidian distance for $SVNNS_fNs$ so that we can evaluate the revised closeness index regarding to each alternative. We have illustrated practical examples of the MAGDM problem that is the selection of the branch manager post in Quiqup company, UAE, to intimate the application of the proposed method and have performed the comparison with SVN-TOPSIS technique that signify the legitimacy of the proposed method. For future direction, we can apply the presented method to solve many other MAGDM problems like for designer selection or management system. We can develop theory for the following techniques under the $SVNNS_f$-framework: (1) AHP method (2) VIKOR method.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] B.C. Cuong and V. Kreinovich, *Picture fuzzy sets-a new concept for computational intelligence problems*, In 2013 Third World Congress on Information and Communication Technologies (WICT 2013) (2013), 1-6.

[2] C. Jana and M. Pal, *A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making*, Symmetry, **11**(2019), 110.

[3] C.T. Chen, *Extension of TOPSIS for group decision-making under fuzzy environment*, Fuzzy Sets and Systems, **114**(1)(2000), 1-9.

[4] D.A. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, **37**(4-5)(1999), 19-31.

[5] D. Ramot, R. Milo, M. Friedman and A. Kandel, *Complex fuzzy sets*, IEEE Transactions on Fuzzy Systems, **10**(2)(2002), 171-186.

[6] F. Chiclana, F. Herrera and E. Herrera-Viedma, *The ordered weighted geometric operator, Properties and application*, In: Proc of 8th International Conference on Information Processing and Management of Uncertainty in Knowledgebased Systems, Madrid, (2000), 985-991.

[7] F. Fatima, D. Rosadi, R.B.F. Hakim and J.C.R. Alcantud, *N-soft sets and their decision-making algorithms*, Soft Computing, **22**(12)(2018), 3829-3842.

[8] F. Feng, *Soft rough sets applied to multi criteria group decision making*, Ann Fuzzy Math Inform, **2**(1)(2011), 69-80.

[9] F. Feng, X. Liu, V. Fotea, Y. B. Jun, *Soft sets and soft rough sets*, Information Sciences, 181(6)(2011), 1125–1137.

[10] F. Feng, M. Akram, B. Davvaz and V. Fotea, *Attribute analysis of information systems based on elementary soft implications*, Knowledge-Based Systems, **70**(2014), 281-292.

[11] F.K. Gundogdu and C. Kahraman, *Spherical fuzzy sets and spherical fuzzy TOPSIS method*, Journal of Intelligent and Fuzzy system, **36**(1)(2019), 337-352.

[12] F. Shen, X. Ma, Z. Li, Xu and D. Cai, *An extended intuitionistic fuzzy TOPSIS method based on new distance measure with an application to credit risk evaluation*, Information Sciences, **428**(2018), 105-119.

[13] F. Smarandache, *Neutrosophy: Neutrosophic probabillity, set and logic* , American Research Press, (1998), Rehoboth, USA.

[14] F. Smarandache, *Neutrosophic set - a generalization of the intuitionistic fuzzy sets*, International Journal of Pure and Applied Mathematics, **24**(3)(2005), 287-297.

[15] F. Smarandache, *n-valued neutrosophic logic and its application to physics*, Progress in Physics, **4**(2014), 143-146.

[16] G. Beliakov, A. Pradera and T. Calvo, *Aggregation functions*, A Guide For Practitioners. Springer Heidelberg, Germany, 2007.

[17] G.T. Zhang, S, Dillon, K.Y. Cai, J. Ma and J. Lu, *Operation properties and δ-equalities of complex fuzzy sets*, International Journal of Approximate Reasoning, **50**(2009), 1227-1249.

[18] G. Wei, *Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making*, Applied Soft Computing, **10**(2)(2010), 423-431.

[19] H. Garg, *Some Picture fuzzy aggregation operators and thier applications to muti-criteria decision making*, Arabian Journal for Science and Engineering, **42**(12)(2017), 5275-5290.

[20] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sundarraman, *Single valued neutrosophic set* , Multispace and Multistructure, **4**(2011), 410-413.

[21] H. Zhang, D. Jia-Hua and C. Yan, *Multi-attribute group decision-making methods based on Pythagorean fuzzy N-soft sets*, IEEE Access, **8**(2020), 62298-62309.

[22] J.C.R. Alcantud and A. Laruelle, *Dis and approval voting: a characterization*, Social Choice and Welfare, **43**(1)(2014), 1-10.

[23] J.C.R. Alcantud *A novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set*, Information Fusion, **29**(2016), 142-148.

[24] J. Figueira, S. Greco and M. Ehrgott, *Multiple criteria decision analysis*, Springer New York, (2016).

[25] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20**(1)(1986), 87-96.

[26] L.A. Zadeh, *Fuzzy sets*, Information and Control, **8**(3)(1965), 338-356.

[27] M. Akram, A. Adeel, and J. C. R. Alcantud, *Fuzzy N-soft sets: A novel model with applications*, Journal of Intelligent and Fuzzy Systems, **35**(4)(2018), 4757-4771.

[28] M. Akram, A. Adeel and J.C.R. Alcantud, *Group decision-making methods based on hesitant N- soft sets*, Expert System with Applications, **115**(2019), 95-105.

[29] M. Akram, F.Ilyas and H. Garg, *Multi-criteria group decision making based on ELECTRE I method in Pythagorean fuzzy information*, Soft Computing, **24**(2020), 3425-3453.

[30] M. Akram, G. Ali, and J. C. R. Alcantud, *New decision-making hybrid model: Intuitionistic fuzzy N-soft rough sets*, Soft Computing, **23**(20)(2019), 9853-9868.

[31] M. Akram, W.A. Dudek and F.Ilyas, *Group decision making based on Pythagorean fuzzy TOPSIS method*, International Journal of Intelligent Systems, **34**(2019), 1455-1475.

[32] M. Riaz, K. Naeem, I. Zareef and D. Afzal, *Neutrosophic N-soft set with TOPSIS method for multiple attribute decision making*, Neutrosophic sets and systems, **32** (2020), 1-24

[33] P. Gupta, M.K. Mehlawat and N. Grover, *A generalized TOSIS method for intuitionistic fuzzy multiple attribute group decision making considering different scenearios of attributes weight information*, International Journal of Fuzzy Systems, **21**(2019), 369-387.

[34] P. K. Maji, *Neutrosophic soft set*, Annal of Fuzzy Mathematics and Informatics, **5**(1)(2013), 157-168.

[35] P. K. Singh, *Three-way n-valued neutrosophic concept lattice at different granulation*, International Journal of Machine Learning and Cybernetics, **9**(11)(2018), 1839-1855.

[36] P. K. Singh, *Multi-granular-based n-valued neutrosophic context analysis*, Granular Computing, **5**(2020), 287-301.

[37] Q. Song, A. Kandel and M. Schneider, *Parametrised fuzzy operator in fuzzy decision making*, International Journal of Intelligent Systems, **18**(9)(2003), 971-987.

[38] R. Alguliyev, R. Aliguliyev and F. Yusifov, *Modified fuzzy TOPSIS + TFNs ranking model for candidate selection using the qualifying criteria*, Soft Computing, **24**(1)(2020), 681-695.

[39] R.R. Yager, *Pythagorean membership grades in multicriteria decision making*, IEEE Transaction on Fuzzy Systems, **22**(4)(2013), 958-965.

[40] R. Sahin and M. Yigider, *A multi-criteria neutrosophic group decision making metod based TOPSIS for supplier selection*, Applied Mathematics and Information Sciences, **10**(5)(2016), 1843-1852.

[41] T. Chu and M. Kysely, *Ranking objectives of advertisement on Facebook by a fuzzy TOPSIS method*, Electronic Commerce Research (2020), https://doi.org/10.1007/s10660-019-09394-z.

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