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Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology

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Abstract. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [30, 31, 32] and Salama et al. in [4-29]. In Geographical information systems (GIS) there is a need to model spatial regions with indeterminate boundary and under indeterminacy. In this paper the structure of some classes of neutrosophic crisp nearly open sets are investigated and some applications are given. Finally we generalize the crisp topological and intuitioistic studies to the notion of neutrosophic crisp set. Possible applications to GIS topological rules are touched upon.

Keywords: Neutrosophic Crisp Set; Neutrosophic Crisp Topology; Neutrosophic Crisp Open Set; Neutrosophic Crisp Nearly Open Set; Neutrosophic GIS Topology.

1 Introduction

The fundamental concepts of neutrosophic set, introduced by Smarandache [30, 31, 32] and Salama et al. in [4-29]., provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 20, 21, 22, 23, 34] such as a neutrosophic set theory. In this paper the structure of some classes of neutrosophic crisp sets are investigated, and some applications are given. Finally we generalize the crisp topological and intuitioistic studies to the notion of neutrosophic crisp set.

2 2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [30, 31, 32] and Salama et al. [4-29]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^{-}0, 1^{+}\right[$ is non-standard unit interval. Salama et al. [9, 10, 13, 14, 16, 17] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following. Salama extended the concepts of topological space and intuitionistic topological space to the case of neutrosophic crisp sets.

Definition1. 2 [13]

A neutrosophic crisp topology (NCT for short) on a non-empty set X is a family Γ of neutrosophic crisp subsets in X satisfying the following axioms

i) $\phi_{N,} X_N \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma \quad \forall \ \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is called a neutrosophic crisp topological space ($NCTS$ for short) in X. The elements in Γ are called neutrosophic crisp open sets (NCOSs for short) in X. A neutrosophic crisp set F is closed if and only if its complement F^C is an open neutrosophic crisp set.

Let (X, Γ) be a NCTS (identified with its class of neutrosophic crisp open sets), and NCint and NCcl denote neutrosophic interior crisp set and neutrosophic crisp closure with respect to neutrosophic crisp topology

3 Nearly Neutrosophic Crisp Open Sets

Definition3. 1

Let (X,Γ) be a NCTS and $A=\langle A_1,A_2,A_3\rangle$ be a NCS in X, then A is called

Neutrosophic crisp $\alpha-$open set iff $A\subseteq NC\text{int}(NCcl(NC\text{int}(A)))$,

Neutrosophic crisp $\beta-$open set iff $A\subseteq NCcl(NC\text{int}(A))$.

Neutrosophic crisp semi-open set iff $A\subseteq NC\text{int}(NCcl(A))$.

We shall denote the class of all neutrosophic crisp $\alpha-$ open sets $NC\Gamma^{\alpha}$, the calls all neutrosophic crisp $\beta-$ open sets $NC\Gamma^{\beta}$, and the class of all neutrosophic crisp semi-open sets $NC\Gamma^{s}$.

Remark 3.1

A class consisting of exactly all a neutrosophic crisp $\alpha-$ structure (resp. NC $\beta-$ structure). Evidently $NC\Gamma\subseteq NC\Gamma^{\alpha}\subseteq NC\Gamma^{\beta}$.

We notice that every non- empty neutrosophic crisp $\beta-$open has NC $\alpha-$nonempty interior. If all neutrosophic crisp sets the following $\{B_i\}_{i\in I}$ are NC $\beta-$ open sets, then $\{\bigcup_{i\in I} B_i\}_{i\in I}\subset NCcl(NC\text{int}(B_i))\subset NCcl(NC\text{int}(B_i))$, that is A NC $\beta-$structure is a neutrosophic closed with respect to arbitrary neutrosophic crisp unions.

We shall now characterize $NC\Gamma^{\alpha}$ in terms $NC\Gamma^{\beta}$

Theorem 3.1

Let (X,Γ) be a NCTS. $NC\Gamma^{\alpha}$ Consists of exactly those neutrosophic crisp set A for which $A\cap B\in NC\Gamma^{\beta}$ for $B\in NC\Gamma^{\beta}$

Proof

Let $A\in NC\Gamma^{\alpha}$, $B\in NC\Gamma^{\beta}$, $p\in A\cap B$ and U be a neutrosophic crisp neighbourhood (for short NCnbd)of p. Clearly $U\cap NC\text{int}(NCcl(NC\text{int}(A)))$, too is a neutrosophic crisp open neighbourhood of p, so $V=(U\cap NC\text{int}(NCcl(NC\text{int}(A))))\cap NC\text{int}(B)$ is non-empty. Since $V\subset NCcl(NC\text{int}(A))$ this implies $(U\cap NC\text{int}(A)\cap NC\text{int}(B))=V\cap NC\text{int}(A)=\phi_N$. It follows that

$A\cap B\subset NCcl(Nc\text{int}(A)\cap NC\text{int}(B))=$

$NCcl(NC\text{int}(A\cap B))$ i.e. $A\cap B\in NC\Gamma^{\beta}$. Conversely, let $A\cap B\in NC\Gamma^{\beta}$ for all $B\in NC\Gamma^{\beta}$ then in particular $A\in NC\Gamma^{\beta}$. Assume that $p\in A\cap(NC\text{int}(NCcl(A)\cap NC\text{int}(A)))^{c}$. Then $p\in NCcl(B)$, where $(NCcl(NC\text{int}(A)))^{c}$. Clearly $\{p\}\cup B\in NC\Gamma^{\beta}$, and consequently $A\cap\{\{p\}\cup B\}\in NC\Gamma^{\beta}$. But $A\cap\{\{p\}\cup B\}=\{p\}$. Hence $\{p\}$ is a neutrosophic crisp open. As $p\in(NCcl(NC\text{int}(A))$ this implies $p\in NC\text{int}(NCcl(NC\text{int}(A)))$, contrary to assumption. Thus $p\in A$ implies $p\in NCcl(NC\text{int}(A))$ and $A\in NC\Gamma^{\alpha}$. This completes the proof. Thus we have found that $NC\Gamma^{\alpha}$ is complete determined by $NC\Gamma^{\beta}$ i.e. all neutrosophic crisp topologies with the same NC β-structure also determined the same NC α-structure, explicitly given Theorem 3.1.

We shall that conversely all neutrosophic crisp topologies with the same NC α-structure, so that $NC\Gamma^{\beta}$ is completely determined by $NC\Gamma^{\alpha}$.

Theorem 3.2

Every neutrosophic crisp NC α-structure is a neutrosophic crisp topology.

Proof

$NC\Gamma^{\beta}$ Contains the neutrosophic crisp empty set and is closed with respect to arbitrary unions. A standard result gives the class of those neutrosophic crisp sets A for which $A\cap B\in NC\Gamma^{\beta}$ for all $B\in NC\Gamma^{\beta}$ constitutes a neutrosophic crisp topology, hence the theorem. Hence forth we shall also use the term NC α-topology for NC α-structure two neutrosophic crisp topologies deterring the same NC α-structure shall be called NC α-equivalent , and the equivalence classes shall be called NC α-classes.

We may now characterize $NC\Gamma^{\beta}$ in terms of $NC\Gamma^{\alpha}$ in the following way.

Proposition 3.1

Let (X,Γ) be a NCTS. Then $NC\Gamma^{\beta}=NC\Gamma^{\alpha\beta}$ and hence NC α-equivalent topologies determine the same NC β-structure.

Proof

Let NC $\alpha-cl$ and NC $\alpha-\text{int}$ denote neutrosophic closure and Neutrosophic crisp interior with respect to

$NC\Gamma^{\alpha}$. If $p \in B \in NC\Gamma^{\beta}$ and $p \in B \in NC\Gamma^{\alpha}$, then $(NC\,\text{int}(NCcl(NC\,\text{int}(A))) \cap NC\,\text{int}(B)) \neq \phi_N$. Since $NC\,\text{int}(NCcl(NC\,\text{int}(A)))$ is a crisp neutrosophic neighbourhood of point p. So certainly $NC\,\text{int}(B)$ meets $NCcl(NC\,\text{int}(A))$ and therefore (bing neutrosophic open) meets $NC\,\text{int}(A)$, proving $A \cap NC\,\text{int}(B) \neq \phi_N$ this means $B \subset NC\alpha cl(NC\,\text{int}(B))$ i.e. $B \in NC\Gamma^{\alpha\beta}$ on the other hand let $A \in NC\Gamma^{\alpha\beta}$, $p \in A$ and $p \in V \in NC\Gamma$. As $V \in NC\Gamma^{\alpha}$ and $p \in NCcl(NC\,\text{int}(A))$ we have $V \cap NC\,\text{int}(A) \neq \phi_N$ and there exist a nutrosophic crisp set $W \in \Gamma$ such that $W \subset V \cap NC\alpha\,\text{int}(A) \subset A$.
In other words $V \cap NC\,\text{int}(A) \neq \phi_N$ and $p \in NCcl(NC\,\text{int}(A))$. Thus we have verified $NC\Gamma^{\alpha\beta} \subset NC\Gamma^{\alpha}$, and the proof is complete combining Theorem 1 and Proposition 1. We get $NC\Gamma^{\alpha\alpha} = NC\Gamma^{\alpha}$.

Corollary 3.2

A neutrosophic crisp topology $NC\Gamma$ a $NC\alpha-$topology iff $NC\Gamma = NC\Gamma^{\alpha}$. Thus an $NC\alpha-$ topology belongs to the $NC\alpha-$class if all its determining a Neutrosophic crisp topologies, and is the finest topology of finest neutrosophic topology of this class. Evidently $NC\Gamma^{\beta}$ is a neutrosophic crisp topology iff $NC\Gamma^{\alpha} = NC\Gamma^{\beta}$. In this case $NC\Gamma^{\beta\beta} = NC\Gamma^{\alpha\beta} = NC\Gamma^{\beta}$.

Corollary 3.3

$NC\beta-$Structure B is a neutrosophic crisp topology, then $\mathrm{B} = \mathrm{B}^{\alpha} = \mathrm{B}^{\beta}$.

We proceed to give some results an the neutrosophic structure of neutrosophic crisp $NC\alpha-$topology

Proposition 3.4

The $NC\alpha-$open with respect to a given neutrosophic crisp topology are exactly those sets which may be written as a difference between a neutrosophic crisp open set and neutrosophic crisp nowhere dense set

If $A \in NC\Gamma^{\alpha}$ we have $A = (NC\,\text{int}(NCcl(NC\,\text{int}(A)) \cap$ $(NC\,\text{int}(NCcl(NC\,\text{int}(A)) \cap A^{C})^{C}$, where $(NC\,\text{int}(NCcl(NC\,\text{int}(A)) \cap A^{C})$ clearly is neutrosophic crisp nowhere dense set , we easily see that $B \subset NCcl(NC\,\text{int}(A))$ and consequently $A \subset B \subset NC\,\text{int}(NCcl(NC\,\text{int}(A))$ so the proof is complete.

Corollary 3.4

A neutrosophic crisp topology is a $NC\alpha-$topology iff all neutrosophic crisp nowhere dense sets are neutrosophic crisp closed.

For a neutrosophic crisp $NC\alpha-$topology may be characterized as neutrosophic crisp topology where the difference between neutrosophic crisp open and neutrosophic crisp nowhere dense set is again a neutrosophic crisp open, and this evidently is equivalent to the condition stated.

Proposition 3.5

Neutrosophic crisp topologies which are $NC\alpha-$ equivalent determine the same class of neutrosophic crisp nowhere dense sets.

Definition 3.2

We recall a neutrosophic crisp topology a neutrosophic crisp extremely disconnected if the neutrosophic crisp closure of every neutrosophic crisp open set is a neutrosophic crisp open .

Proposition 3.6

If $NC\alpha-$Structure B is a neutrosophic crisp topology, all a neutrosophic crisp topologies Γ for which $\Gamma^{\beta} = \mathrm{B}$ are neutrosophic crisp extremely disconnected.

In particular: Either all or none of the neutrosophic crisp topologies of a $NC\alpha-$class are extremely disconnected.

Proof

Let $\Gamma^{\beta} = \mathrm{B}$ and suppose there is a $A \in \Gamma$ such that $NCcl(A) \notin \Gamma$ Let $p \in NCcl(A) \cap (NC\,\text{int}(NCcl(A))^{C}$ with $\mathrm{B} = \{p\} \cup NC\,\text{int}(NCcl(A))$, $M = (NC\,\text{int}(NCcl(A)))^{C}$ we have $\{p\} \subset M = (NC\,\text{int}(NCcl(A)))^{C} = NCcl(NC\,\text{int}(M)$, $\{p\} \subset NCcl(A) = NCcl(NC\,\text{int}(NCcl(A))$ $\subset NCcl(NC\,\text{int}(B))$. Hence both B and M are in Γ^{β}. The intersection $\mathrm{B} \cap M = \{p\}$ is not neutrosophic crisp open since $p \in NCcl(A) \cap M^{C}$, hence not $NC\beta-$ open so. $\Gamma^{\beta} = \mathrm{B}$ is not a neutrosophic crisp topology. Now suppose B is not a topology, and $\Gamma^{\beta} = \mathrm{B}$ There is a $\mathrm{B} \in \Gamma^{\beta}$ such that $\mathrm{B} \notin \Gamma^{\alpha}$. Assume that $NCcl(NC\,\text{int}(\mathrm{B}) \in \Gamma$. Then $\mathrm{B} \subset NCcl(NC\,\text{int}(\mathrm{B}) = NC\,\text{int}(NCcl(NC\,\text{int}(\mathrm{B}))$ .i.e. $\mathrm{B} \in \Gamma^{\alpha}$, contrary to assumption. Thus we have produced a neutrosophic crisp open set whose neutrosophic crisp closure is not neutronsophic crisp open, which completes the proof.

Corollary 3.5

A neutrosophic crisp topology Γ is a neutrosophic crisp extremely disconnected if and only if Γ^{β} is a neutrosophic crisp topology.

4 Conclusion and future work

Neutrosophic set is well equipped to deal with missing data. By employing NSs in spatial data models, we can express a hesitation concerning the object of interest. This article has gone a step forward in developing methods that can be used to define neutrosophic spatial regions and their relationships. The main contributions of the paper can be described as the following: Possible applications have been listed after the definition of NS. Links to other models have been shown. We are defining some new operators to describe objects, describing a simple neutrosophic region. This paper has demonstrated that spatial object may profitably be addressed in terms of neutrosophic set. Implementation of the named applications is necessary as a proof of concept.

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