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Interval Neutrosophic Rough Set

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Abstract: This Paper combines interval- valued neutrouphic sets and rough sets. It studies roughness in interval- valued neutrosophic sets and some of its properties. Finally we propose a Hamming distance between lower and upper approximations of interval valued neutrosophic sets.

Keywords: interval valued neutrosophic sets, rough sets, interval valued neutrosophic sets.

1.Introduction

Neutrosophic set (NS for short), a part of neutrosophy introduced by Smarandache [1] as a new branch of philosophy, is a mathematical tool dealing with problems involving imprecise, indeterminacy and inconsistent knowledge. Contrary to fuzzy sets and intuitionistic fuzzy sets, a neutrosophic set consists of three basic membership functions independently of each other, which are truth, indeterminacy and falsity. This theory has been well developed in both theories and applications. After the pioneering work of Smarandache, In 2005, Wang [2] introduced the notion of interval neutrosophic sets (INS for short) which is another extension of neutrosophic sets. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval neutrosophic (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set, and Interval Neutrosophic Set (INS) provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information. The interval neutrosophic set generalize, the classical set ,fuzzy set [3] , the interval valued fuzzy set [4], intuitionistic fuzzy set [5] , interval valued intuitionstic fuzzy set [6] and so on. Many scholars have performed studies on neutrosophic sets , interval neutrosophic sets and their properties [7,8,9,10,11,12,13]. Interval neutrosophic sets have also been widely applied to many fields [14,15,16,17,18,19].

The rough set theory was introduced by Pawlak [20] in 1982, which is a technique for managing the uncertainty and imperfection, can analyze incomplete information effectively. Therefore, many models have been built upon different aspect, i.e, univers, relations, object, operators by many scholars [21,22,23,24,25,26] such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough, rough intuitionistic fuzzy set. intuitionistic fuzzy rough sets [27]. It has been successfully applied in many fields such as attribute reduction [28,29,30,31], feature selection [32,33,34], rule extraction [35,36,37,38] and so on. The rough sets theory approximates any subset of objects of the universe by two sets, called the lower and upper approximations. It focuses on the ambiguity caused by the limited discernibility of objects in the universe of discourse.

More recently, S.Broumi et al [39] combined neutrosophic sets with rough sets in a new hybrid mathematical structure called “rough neutrosophic sets” handling incomplete and indeterminate information . The concept of rough neutrosophic sets generalizes fuzzy rough sets and intuitionistic fuzzy rough sets. Based on the equivalence relation on the universe of discourse, A.Mukherjee et al [40] introduced lower and upper approximation of interval valued intuitionistic fuzzy set in Pawlak’s approximation space . Motivated by this ,we extend the interval intuitionistic fuzzy lower and upper approximations to the case of interval valued neutrosophic set. The concept of interval valued neutrosophic rough set is introduced by coupling both interval neutrosophic sets and rough sets.

The organization of this paper is as follow : In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3 , basic concept of rough approximation of an interval valued neutrosophic sets and their properties are presented. In section 4, Hamming distance between lower approximation and upper approximation of interval neutrosophic set is introduced, Finally, we concludes the paper.

2.Preliminaries

Throughout this paper, We now recall some basic notions of neutrosophic sets , interval valued neutrosophic sets , rough set theory and intuitionistic fuzzy rough sets. More can found in ref [1, 2,20,27].

Definition 1 [1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form A= {< x: $\boldsymbol{\mu}_{A(x)}$, $\boldsymbol{\nu}_{A(x)}$, $\boldsymbol{\omega}_{A(x)}$ >,x ∈ U}, where the functions $\boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\omega}$: U→]$^-$0,1$^+$[define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \le \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \le 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-$0,1$^+$[.so instead of]$^-$0,1$^+$[we need to take the interval [0,1] for technical applications, because]$^-$0,1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2 [2]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeteminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x) \in [0,1]$.

For two IVNS, A= {<x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | x ∈ X } (2)

And B= {<x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | x ∈ X } the two relations are defined as follows:

(1)A ⊆ Bif and only if $\mu_A^L(x) \le \mu_B^L(x)$,$\mu_A^U(x) \le \mu_B^U(x)$, $\nu_A^L(x) \ge \nu_B^L(x)$,$\omega_A^U(x) \ge \omega_B^U(x)$, $\omega_A^L(x) \ge \omega_B^L(x)$,$\omega_A^U(x) \ge \omega_B^U(x)$

(2)A = B if and only if , $\mu_A(x) = \mu_B(x)$,$\nu_A(x) = \nu_B(x)$,$\omega_A(x) = \omega_B(x)$ for any x ∈ X

The complement of A_{IVNS} is denoted by A_{IVNS}^o and is defined by

A^o={ <x , $[\omega_A^L(x), \omega_A^U(x)]$> , $[1-\nu_A^U(x), 1-\nu_A^L(x)]$, $[\mu_A^L(x), \mu_A^U(x)]$ | x ∈ X }

A∩B ={ <x , [min($\mu_A^L(x)$,$\mu_B^L(x)$), min($\mu_A^U(x)$,$\mu_B^U(x)$)], [max($\nu_A^L(x)$,$\nu_B^L(x)$), max($\nu_A^U(x)$,$\nu_B^U(x)$], [max($\omega_A^L(x)$,$\omega_B^L(x)$), max($\omega_A^U(x)$,$\omega_B^U(x)$)] >: x ∈ X }

A∪B ={ <x , [max($\mu_A^L(x)$,$\mu_B^L(x)$), max($\mu_A^U(x)$,$\mu_B^U(x)$)], [min($\nu_A^L(x)$,$\nu_B^L(x)$), min($\nu_A^U(x)$,$\nu_B^U(x)$], [min($\omega_A^L(x)$,$\omega_B^L(x)$), min($\omega_A^U(x)$,$\omega_B^U(x)$)] >: x ∈ X }

0_N = {<x, [0, 0] ,[1 , 1], [1 ,1] >| x ∈ X}, denote the neutrosophic empty set ϕ

1_N = {<x, [0, 0] ,[0 , 0], [1 ,1] >| x ∈ X}, denote the neutrosophic universe set U

As an illustration, let us consider the following example.

Example 1. Assume that the universe of discourse U={x1, x2, x3}, where x1characterizes the capability, x2characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x1, x2 and x3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,

A = {< x1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 3 [20]

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R^*X and R_*X and are defined by

$R_* X = \{x \in U: [x]_R \subseteq X\}$,

$R^* X = \{ x \in U: [x]_R \cap X \neq \emptyset\}$.

Now if $R^*X = R_*X$, then X is called definable; otherwise X is called a rough set.

Definition 4 [27]

Let U be a universe and X , a rough set in U. An IF rough set A in U is characterized by a membership function μ_A $:U \to [0, 1]$ and non-membership function $\nu_A : U \to [0, 1]$ such that

$\mu_A(\underline{R}X) = 1$, $\nu_A(\underline{R}X) = 0$

Or $[\mu_A(x), \nu_A(x)] = [1, 0]$ if $x \in (\underline{R}X)$ and $\mu_A(U - \overline{R}X) = 0$, $\nu_A(U - \overline{R}X) = 1$

Or $[\mu_A(x), \nu_A(x)] = [0, 1]$ if $x \in U - \overline{R}X$,

$0 \leq \mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X) \leq 1$

Example 2: Example of IF Rough Sets

Let U= {Child, Pre-Teen, Teen, Youth, Teenager, Young-Adult, Adult, Senior, Elderly} be a universe.

Let the equivalence relation R be defined as follows:

R*= {[Child, Pre-Teen], [Teen, Youth, Teenager], [Young-Adult, Adult],[Senior, Elderly]}.

Let X = {Child, Pre-Teen, Youth, Young-Adult} be a subset of univers U.

We can define X in terms of its lower and upper approximations:

$\underline{R}X$ = {Child, Pre-Teen}, and $\overline{R}X$ = {Child, Pre-Teen, Teen, Youth, Teenager, Young-Adult, Adult}.

The membership and non-membership functions $\mu_A: U \to]1, 0[$ and $\nu_A: U \to]1, 0[$ on a set A are defined as follows:

μ_AChild) = 1, μ_A (Pre-Teen) = 1 and μ_A (Child) = 0, μ_A(Pre-Teen) = 0

μ_A (Young-Adult) = 0, μ_A (Adult) = 0, μ_A(Senior) = 0, μ_A (Elderly) = 0

3.Basic Concept of Rough Approximations of an Interval Valued Neutrosophic Set and their Properties.

In this section we define the notion of interval valued neutrosophic rough sets (in brief ivn- rough set) by combining both rough sets and interval neutrosophic sets. IVN- rough sets are the generalizations of interval valued intuitionistic fuzzy rough sets, that give more information about uncertain or boundary region.

Definition 5 : Let (U,R) be a pawlak approximation space ,for an interval valued neutrosophic set

$A = \{<x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)]> \mid x \in X\}$ be interval neutrosophic set. The lower approximation $\underline{A}_R$ and $\overline{A}_R$ upper approximations of A in the pawlak approwimation space (U, R) are defined as:

$\underline{A}_R = \{<x, [\wedge_{y \in [x]_R}\{\mu_A^L(y)\}, \wedge_{y \in [x]_R}\{\mu_A^U(y)\}], [\vee_{y \in [x]_R}\{\nu_A^L(y)\}, \vee_{y \in [x]_R}\{\nu_A^U(y)\}], [\vee_{y \in [x]_R}\{\omega_A^L(y)\}, \vee_{y \in [x]_R}\{\omega_A^U(y)\}]>: x \in U\}$.

$\overline{A}_R = \{<x, [\vee_{y \in [x]_R}\{\mu_A^L(y)\}, \vee_{y \in [x]_R}\{\mu_A^U(y)\}], [\wedge_{y \in [x]_R}\{\nu_A^L(y)\}, \wedge_{y \in [x]_R}\{\nu_A^U(y)], [\wedge_{y \in [x]_R}\{\omega_A^L(y)\}, \wedge_{y \in [x]_R}\{\omega_A^U(y)\}]: x \in U\}$.

Where " ∧ " means " min" and " ∨ " means " max", R denote an equivalence relation for interval valued neutrosophic set A.

Here $[x]_R$ is the equivalence class of the element x. It is easy to see that

$[\wedge_{y \in [x]_R}\{\mu_A^L(y)\}, \wedge_{y \in [x]_R}\{\mu_A^U(y)\}] \subset [0,1]$

$[\vee_{y \in [x]_R}\{\nu_A^L(y)\}, \vee_{y \in [x]_R}\{\nu_A^U(y)\}] \subset [0,1]$

$[\vee_{y \in [x]_R}\{\omega_A^L(y)\}, \vee_{y \in [x]_R}\{\omega_A^U(y)\}] \subset [0,1]$

And

$\mathbf{0} \leq \wedge_{y \in [x]_R}\{\mu_A^U(y)\} + \vee_{y \in [x]_R}\{\nu_A^U(y)\} + \vee_{y \in [x]_R}\{\omega_A^U(y)\} \leq \mathbf{3}$

Then, $\underline{A}_R$ is an interval neutrosophic set

Similarly , we have

$[\vee_{y\in[x]_R}\{\mu^L_A(y)\},\ \vee_{y\in[x]_R}\{\mu^U_A(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R}\{\nu^L_A(y)\},\ \wedge_{y\in[x]_R}\{\nu^U_A(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R}\{\omega^L_A(y)\},\ \wedge_{y\in[x]_R}\{\omega^U_A(y)\}] \subset [0,1]$

And

$\mathbf{0} \le \vee_{y\in[x]_R}\{\mu^U_A(y)\} + \wedge_{y\in[x]_R}\{\nu^U_A(y)\} + \wedge_{y\in[x]_R}\{\omega^U_A(y)\} \le \mathbf{3}$

Then, $\overline{A}_R$ is an interval neutrosophic set

If $\underline{A}_R = \overline{A}_R$,then A is a definable set, otherwise A is an interval valued neutrosophic rough set, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval valued neutrosophic set with respect to approximation space (U, R), respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$.

In the following , we employ an example to illustrate the above concepts

Example:

Theorem 1. Let A, B be interval neutrosophic sets and $\underline{A}$ and $\overline{A}$ the lower and upper approximation of interval –valued neutrosophic set A with respect to approximation space (U, R) ,respectively. $\underline{B}$ and $\overline{B}$ the lower and upper approximation of interval –valued neutrosophic set B with respect to approximation space (U,R) ,respectively.Then we have

i. $\underline{A} \subseteq A \subseteq \overline{A}$
ii. $\overline{A\cup B} = \overline{A}\cup\overline{B}$, $\underline{A\cap B} = \underline{A}\cap\underline{B}$
iii. $\underline{A}\cup\underline{B} = \underline{A\cup B}$, $\overline{A\cap B} = \overline{A}\cap\overline{B}$
iv. $\underline{(\overline{A})} = \overline{(\overline{A})} = \overline{A}$, $\underline{(\underline{A})} = \overline{(\underline{A})} = \underline{A}$
v. $\underline{U} = U$; $\overline{\phi} = \phi$
vi. If A ⊆ B ,then $\underline{A} \subseteq \underline{B}$ and $\overline{A} \subseteq \overline{B}$
vii. $\underline{A^c} = (\overline{A})^c$, $\overline{A^c} = (\underline{A})^c$

Proof: we prove only i,ii,iii, the others are trivial

(i)

Let $A = \{<x , [\mu^L_A(x), \mu^U_A(x)], [\nu^L_A(x), \nu^U_A(x)], [\omega^L_A(x), \omega^U_A(x)] > | x \in X\}$ be interval neutrosophic set

From definition of $\underline{A}_R$ and $\overline{A}_R$, we have

Which implies that

$\mu^L_{\underline{A}}(x) \le \mu^L_A(x) \le \mu^L_{\overline{A}}(x)$; $\mu^U_{\underline{A}}(x) \le \mu^U_A(x) \le \mu^U_{\overline{A}}(x)$ for all $x \in X$

$\nu^L_{\underline{A}}(x) \ge \nu^L_A(x) \ge \nu^L_{\overline{A}}(x)$; $\nu^U_{\underline{A}}(x) \ge \nu^U_A(x) \ge \nu^U_{\overline{A}}(x)$ for all $x \in X$

$\omega^L_{\underline{A}}(x) \ge \omega^L_A(x) \ge \omega^L_{\overline{A}}(x)$; $\omega^U_{\underline{A}}(x) \ge \omega^U_A(x) \ge \omega^U_{\overline{A}}(x)$ for all $x \in X$

$([\mu^L_{\underline{A}}, \mu^U_{\underline{A}}], [\nu^L_{\underline{A}}, \nu^U_{\underline{A}}], [\omega^L_{\underline{A}}, \omega^U_{\underline{A}}]) \subseteq ([\mu^L_A, \mu^U_A], [\nu^L_A, \nu^U_A], [\omega^L_A, \omega^U_A]) \subseteq ([\mu^L_{\overline{A}}, \mu^U_{\overline{A}}], [\nu^L_{\overline{A}}, \nu^U_{\overline{A}}], [\omega^L_{\overline{A}}, \omega^U_{\overline{A}}])$ .Hence $\underline{A}_R \subseteq A \subseteq \overline{A}_R$

(ii) Let $A = \{<x , [\mu^L_A(x), \mu^U_A(x)], [\nu^L_A(x), \nu^U_A(x)], [\omega^L_A(x), \omega^U_A(x)] > | x \in X\}$ and

$B = \{<x, [\mu^L_B(x), \mu^U_B(x)], [\nu^L_B(x), \nu^U_B(x)], [\omega^L_B(x), \omega^U_B(x)] > | x \in X\}$ are two intervalvalued neutrosophic set and

$\overline{A\cup B} = \{<x , [\mu^L_{\overline{A\cup B}}(x), \mu^U_{\overline{A\cup B}}(x)] , [\nu^L_{\overline{A\cup B}}(x), \nu^U_{\overline{A\cup B}}(x)] , [\omega^L_{\overline{A\cup B}}(x), \omega^U_{\overline{A\cup B}}(x)] > | x \in X\}$

$\overline{A}\cup\overline{B} = \{x, [\max(\mu^L_{\overline{A}}(x), \mu^L_{\overline{B}}(x)), \max(\mu^U_{\overline{A}}(x), \mu^U_{\overline{B}}(x))], [\min(\nu^L_{\overline{A}}(x), \nu^L_{\overline{B}}(x)), \min(\nu^U_{\overline{A}}(x), \nu^U_{\overline{B}}(x))], [\min(\omega^L_{\overline{A}}(x), \omega^L_{\overline{B}}(x)), \min(\omega^U_{\overline{A}}(x), \omega^U_{\overline{B}}(x))]$

for all $x \in X$

$\mu^L_{\overline{A\cup B}}(x) = \vee\{\mu^L_{A\cup B}(y) \mid y \in [x]_R\}$

$= \vee\{\mu^L_A(y) \vee \mu^L_B(y) \mid y \in [x]_R\}$

$= (\vee \mu^L_A(y) \mid y \in [x]_R) \vee (\vee \mu^L_A(y) \mid y \in [x]_R)$

$= (\mu^L_{\overline{A}} \vee \mu^L_{\overline{B}})(x)$

$\mu^U_{\overline{A\cup B}}(x) = \vee\{\mu^u_{A\cup B}(y) \mid y \in [x]_R\}$

$= \vee\{\mu^U_A(y) \vee \mu^U_B(y) \mid y \in [x]_R\}$

$= (\vee\ \mu_A^u(y) \mid y \in [x]_R) \vee (\vee\ \mu_A^U(y) \mid y \in [x]_R)$

$=(\mu_{\overline{A}}^U \vee \mu_{\overline{B}}^U)(x)$

$\nu_{\overline{A\cup B}}^L(x)=\wedge\{\nu_{A\cup B}^L(y) \mid y \in [x]_R\}$

$= \wedge \{\nu_A^L(y) \wedge \nu_B^L(y) \mid y \in [x]_R\}$

$= (\wedge\ \nu_A^L(y) \mid y \in [x]_R) \wedge (\wedge\ \nu_B^L(y) \mid y \in [x]_R)$

$=(\nu_{\overline{A}}^L \wedge \nu_{\overline{B}}^L)(x)$

$\nu_{\overline{A\cup B}}^U(x)=\wedge\{\nu_{A\cup B}^U(y) \mid y \in [x]_R\}$

$= \wedge \{\nu_A^U(y) \wedge \nu_B^U(y) \mid y \in [x]_R\}$

$= (\wedge\ \nu_A^U(y) \mid y \in [x]_R) \wedge (\wedge\ \nu_B^U(y) \mid y \in [x]_R)$

$=(\nu_{\overline{A}}^U(y) \wedge \nu_{\overline{B}}^U(y))(x)$

$\omega_{\overline{A\cup B}}^L(x)=\wedge\{\omega_{A\cup B}^L(y) \mid y \in [x]_R\}$

$= \wedge \{\omega_A^L(y) \wedge \omega_B^L(y) \mid y \in [x]_R\}$

$= (\wedge\ \omega_A^L(y) \mid y \in [x]_R) \wedge (\wedge\ \omega_B^L(y) \mid y \in [x]_R)$

$=(\omega_{\overline{A}}^L \wedge \omega_{\overline{B}}^L)(x)$

$\omega_{\overline{A\cup B}}^U(x)=\wedge\{\omega_{A\cup B}^U(y) \mid y \in [x]_R\}$

$= \wedge \{\omega_A^U(y) \wedge \nu_B^U(y) \mid y \in [x]_R\}$

$= (\wedge\ \omega_A^U(y) \mid y \in [x]_R) \wedge (\wedge\ \omega_B^U(y) \mid y \in [x]_R)$

$=(\omega_{\overline{A}}^U \wedge \omega_{\overline{B}}^U)(x)$

Hence, $\overline{A \cup B} = \overline{A} \cup \overline{B}$

Also for $\underline{A \cap B} = \underline{A} \cap \underline{B}$ for all x ∈ A

$\mu_{\underline{A\cap B}}^L(x) = \wedge\{\mu_{A\cap B}^L(y) \mid y \in [x]_R\}$

$= \wedge \{\mu_A^L(y) \wedge \mu_B^L(y) \mid y \in [x]_R\}$

$= \wedge\ (\mu_A^L(y) \mid y \in [x]_R) \wedge (\vee\ \mu_B^L(y) \mid y \in [x]_R)$

$=\mu_{\underline{A}}^L(x) \wedge \mu_{\underline{B}}^L(x)$

$=(\mu_{\underline{A}}^L \wedge \mu_{\underline{B}}^L)(x)$

Also

$\mu_{\underline{A\cap B}}^U(x) = \wedge\{\mu_{A\cap B}^U(y) \mid y \in [x]_R\}$

$= \wedge \{\mu_A^U(y) \wedge \mu_B^U(y) \mid y \in [x]_R\}$

$= \wedge\ (\mu_A^U(y) \mid y \in [x]_R) \wedge (\vee\ \mu_B^U(y) \mid y \in [x]_R)$

$=\mu_{\underline{A}}^U(x) \wedge \mu_{\underline{B}}^U(x)$

$=(\mu_{\underline{A}}^U \wedge \mu_{\underline{B}}^U)(x)$

$\nu_{\underline{A\cap B}}^L(x) = \vee\{\nu_{A\cap B}^L(y) \mid y \in [x]_R\}$

$= \vee \{\nu_A^L(y) \vee \nu_B^L(y) \mid y \in [x]_R\}$

$= \vee\ (\nu_A^L(y) \mid y \in [x]_R) \vee (\vee\ \nu_B^L(y) \mid y \in [x]_R)$

$=\nu_{\underline{A}}^L(x) \vee \nu_{\underline{B}}^L(x)$

$=(\nu_{\underline{A}}^L \vee \nu_{\underline{B}}^L)(x)$

$\nu_{\underline{A\cap B}}^U(x) = \vee\{\nu_{A\cap B}^U(y) \mid y \in [x]_R\}$

$= \vee \{\nu_A^U(y) \vee \nu_B^U(y) \mid y \in [x]_R\}$

$= \vee\ (\nu_A^U(y) \mid y \in [x]_R) \vee (\vee\ \nu_B^U(y) \mid y \in [x]_R)$

$=\nu_{\underline{A}}^U(x) \vee \nu_{\underline{B}}^U(x)$

$=(\nu_{\underline{A}}^U \vee \nu_{\underline{B}}^U)(x)$

$\omega_{\underline{A\cap B}}^L(x) = \vee\{\omega_{A\cap B}^L(y) \mid y \in [x]_R\}$

$= \vee \{\omega_A^L(y) \vee \omega_B^L(y) \mid y \in [x]_R\}$

$= \vee\ (\omega_A^L(y) \mid y \in [x]_R) \vee (\vee\ \omega_B^L(y) \mid y \in [x]_R)$

$=\omega_{\underline{A}}^L(x) \vee \nu\omega_{\underline{B}}^L(x)$

$=(\omega_{\underline{A}}^L \vee \omega_{\underline{B}}^L)(x)$

$\omega_{\underline{A\cap B}}^U(x) = \vee\{\omega_{A\cap B}^U(y) \mid y \in [x]_R\}$

$= \vee \{\omega_A^U(y) \vee \omega_B^U(y) \mid y \in [x]_R\}$

$= \vee\ (\omega_A^U(y) \mid y \in [x]_R) \vee (\vee\ \omega_B^U(y) \mid y \in [x]_R)$

$=\omega_{\underline{A}}^U(x) \vee \omega_{\underline{B}}^U(x)$

$=(\omega^{\mathrm{U}}_{\underline{A}} \vee \omega^{\mathrm{U}}_{\underline{B}})(\mathrm{x})$

(iii)

$\mu^{\mathrm{U}}_{\overline{A \cap B}}(\mathrm{x}) = \vee\{ \mu^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \vee \{\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \wedge \mu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$=(\vee (\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)) \wedge (\vee (\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R))$

$= \mu^{\mathrm{U}}_{\overline{A}}(\mathrm{x}) \vee \mu^{\mathrm{U}}_{\overline{B}}(\mathrm{x})$

$=(\mu^{\mathrm{U}}_{\overline{A}} \vee \mu^{\mathrm{U}}_{\overline{B}})(\mathrm{x})$

$\nu^{\mathrm{U}}_{\overline{A \cap B}}(\mathrm{x}) = \wedge\{ \nu^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \wedge \{\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \wedge \nu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$=(\wedge (\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)) \vee (\wedge (\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R))$

$= \nu^{\mathrm{U}}_{\overline{A}}(\mathrm{x}) \vee \nu^{\mathrm{U}}_{\overline{B}}(\mathrm{x})$

$=(\nu^{\mathrm{U}}_{\overline{A}} \vee \nu^{\mathrm{U}}_{\overline{B}})(\mathrm{x})$

$\omega^{\mathrm{U}}_{\overline{A \cap B}}(\mathrm{x}) = \wedge\{ \omega^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \wedge \{\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \wedge \omega\nu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$=(\wedge (\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)) \vee (\wedge (\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R))$

$= \omega^{\mathrm{U}}_{\overline{A}}(\mathrm{x}) \vee \omega^{\mathrm{U}}_{\overline{B}}(\mathrm{x})$

$=(\omega^{\mathrm{U}}_{\overline{A}} \vee \omega^{\mathrm{U}}_{\overline{B}})(\mathrm{x})$

Hence follow that $\overline{A \cap B} = \overline{A} \cap \overline{B}$ .we get $\underline{A} \cup \underline{B} = \underline{A \cup B}$ by following the same procedure as above.

Definition 6:

Let (U,R) be a pawlak approximation space ,and A and B two interval valued neutrosophic sets over U.

If $\underline{A} = \underline{B}$,then A and B are called interval valued neutrosophic lower rough equal.

If $\overline{A} = \overline{B}$, then A and B are called interval valued neutrosophic upper rough equal.

If $\underline{A} = \underline{B}$, $\overline{A} = \overline{B}$, then A and B are called interval valued neutrosophic rough equal.

Theorem 2 .

Let (U,R) be a pawlak approximation space ,and A and B two interval valued neutrosophic sets over U. then

1. $\underline{A} = \underline{B} \Leftrightarrow \underline{A \cap B} = \underline{A}$, $\underline{A \cap B} = \underline{B}$
2. $\overline{A} = \overline{B} \Leftrightarrow \overline{A \cup B} = \overline{A}$, $\overline{A \cup B} = \overline{B}$
3. If $\overline{A} = \overline{A'}$ and $\overline{B} = \overline{B'}$,then $\overline{A \cup B} = \overline{A' \cup B'}$
4. If $\underline{A} = \underline{A'}$ and $\underline{B} = \underline{B'}$,Then
5. If $A \subseteq B$ and $\underline{B} = \underline{\phi}$,then $\underline{A} = \underline{\phi}$
6. If $A \subseteq B$ and $\underline{B} = \underline{U}$,then $\underline{A} = \underline{U}$
7. If $\underline{A} = \underline{\phi}$ or $\underline{B} = \underline{\phi}$ or then $\underline{A \cap B} = \underline{\phi}$
8. If $\overline{A} = \overline{U}$ or $\overline{B} = \overline{U}$,then $\overline{A \cup B} = \overline{U}$
9. $\overline{A} = \overline{U} \Leftrightarrow$ A = U
10. $\overline{A} = \overline{\phi} \Leftrightarrow A = \phi$

Proof: the proof is trial

4.Hamming distance between Lower Approximation and Upper Approximation of IVNS

In this section , we will compute the Hamming distance between lower and upper approximations of interval neutrosophic sets based on Hamming distance introduced by **Ye [41]** of interval neutrosophic sets.

Based on Hamming distance between two interval neutrosophic set A and B as follow:

$$\mathrm{d(A,B)}=\frac{1}{6}\sum_{i=1}^{n}\left[\left|\mu^{\mathrm{L}}_{\mathrm{A}}(\mathrm{x_i}) - \mu^{\mathrm{L}}_{\mathrm{B}}(\mathrm{x_i})\right| + \left|\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{x_i}) - \mu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{x_i})\right| + \left|\nu^{\mathrm{L}}_{\mathrm{A}}(\mathrm{x_i}) - \nu^{\mathrm{L}}_{\mathrm{B}}(\mathrm{x_i})\right| + \left|\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{x_i}) - \nu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{x_i})\right| + \left|\omega^{\mathrm{L}}_{\mathrm{A}}(\mathrm{x_i}) - \omega^{\mathrm{L}}_{\mathrm{B}}(\mathrm{x_i})\right| + \left|\omega^{\mathrm{L}}_{\mathrm{A}}(\mathrm{x_i}) - \nu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{x_i})\right|\right]$$

we can obtain the standard hamming distance of $\underline{A}$ and $\overline{A}$ from

$$d_H(\underline{A} ,\overline{A}) = \frac{1}{6}\sum_{i=1}^{n}\left[\left|\mu^{\mathrm{L}}_{\underline{A}}(\mathrm{x_j}) - \mu^{\mathrm{L}}_{\overline{A}}(\mathrm{x_j})\right| + \left|\mu^{\mathrm{U}}_{\underline{A}}(\mathrm{x_j}) - \mu^{\mathrm{U}}_{\overline{A}}(\mathrm{x_j})\right| + \left|\nu^{\mathrm{L}}_{\underline{A}}(\mathrm{x_j}) - \nu^{\mathrm{L}}_{\overline{A}}(\mathrm{x_j})\right| + \left|\nu^{\mathrm{U}}_{\underline{A}}(\mathrm{x_j}) - \nu^{\mathrm{U}}_{\overline{A}}(\mathrm{x_j})\right| + \left|\omega^{\mathrm{L}}_{\underline{A}}(\mathrm{x_j}) - \omega^{\mathrm{L}}_{\overline{A}}(\mathrm{x_j})\right| + \left|\omega^{\mathrm{U}}_{\underline{A}}(\mathrm{x_j}) - \omega^{\mathrm{U}}_{\overline{A}}(\mathrm{x_j})\right|\right]$$

Where

$\underline{A}_R$=**{<x, [**$\wedge_{y\in[x]_R}\{\mu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\mu_A^U(y)\}$], [$\vee_{y\in[x]_R}\{\nu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\nu_A^U(y)\}$], [$\vee_{y\in[x]_R}\{\omega_A^L(y)\}$, $\vee_{y\in[x]_R}\{\omega_A^U(y)\}$]>:x ∈ U}.

$\overline{A}_R$=**{<x, [**$\vee_{y\in[x]_R}\{\mu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\mu_A^U(y)\}$], **[**$\wedge_{y\in[x]_R}\{\nu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\nu_A^U(y)\}$], **[**$\wedge_{y\in[x]_R}\{\omega_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\omega_A^U(y)\}$]:x ∈ U}.

$\mu_{\underline{A}}^L(x_j) = \wedge_{y\in[x]_R}\{\mu_A^L(y)\}$; $\mu_{\underline{A}}^U(x_j) = \wedge_{y\in[x]_R}\{\mu_A^U(y)\}$

$\nu_{\underline{A}}^L(x_j) = \vee_{y\in[x]_R}\{\nu_A^L(y)\}$; $\nu_{\underline{A}}^U(x_j) = \vee_{y\in[x]_R}\{\nu_A^U(y)\}$

$\omega_{\underline{A}}^L(x_j) = \vee_{y\in[x]_R}\{\omega_A^L(y)\}$; $\omega_{\underline{A}}^U(x_j) = \vee_{y\in[x]_R}\{\omega_A^U(y)\}$

$\mu_{\overline{A}}^L(x_j) = \vee_{y\in[x]_R}\{\mu_A^L(y)\}$; $\mu_{\overline{A}}^U(x_j) = \vee_{y\in[x]_R}\{\mu_A^U(y)\}$

$\mu_{\overline{A}}^L(x_j) = \wedge_{y\in[x]_R}\{\nu_A^L(y)\}$; $\mu_{\overline{A}}^U(x_j) = \wedge_{y\in[x]_R}\{\nu_A^U(y)\}$

$\omega_{\overline{A}}^L(x_j) = \wedge_{y\in[x]_R}\{\omega_A^L(y)\}$; $\omega_{\overline{A}}^U(x_j) = \wedge_{y\in[x]_R}\{\omega_A^U(y)\}$

Theorem 3. Let (U, R) be approximation space, A be an interval valued neutrosophic set over U . Then

(1) If d ($\underline{A}$, $\overline{A}$) = 0, then A is a definable set.

(2) If 0 < d($\underline{A}$, $\overline{A}$) < 1, then A is an interval-valued neutrosophic rough set.

Theorem 4. Let (U, R) be a Pawlak approximation space, and A and B two interval-valued neutrosophic sets over U . Then

1. d ($\underline{A}$, $\overline{A}$) ≥ d ($\underline{A}$, A) and d ($\underline{A}$, $\overline{A}$) ≥ d (A , $\overline{A}$);
2. d ($\overline{A \cup B}$, $\overline{A} \cup \overline{B}$) = 0, d ($\underline{A \cap B}$, $\underline{A} \cap \underline{B}$) = 0.
3. d ($\underline{A} \cup \underline{B}$, A ∪ B) ≥ d($\underline{A} \cup \underline{B}$, $\underline{A \cup B}$)
   and d($\underline{A} \cup \underline{B}$, A ∪ B) ≥ d($\underline{A \cup B}$, A ∪ B) ;
   and d(A ∩ B, $\overline{A} \cap \overline{B}$) ≥ d(A ∩ B, $\overline{A \cap B}$)
   and d(A ∩ B, $\overline{A} \cap \overline{B}$) ≥ $d(\overline{A \cap B}, \overline{A} \cap \overline{B})$
4. d($\overline{(\overline{A})}$, $\underline{(\overline{A})}$)= 0 , d($\overline{(\overline{A})}$, $\overline{A}$) = 0 , d($\underline{(\overline{A})}$, $\overline{A}$)= 0;
   d($\underline{(\underline{A})}$, $\overline{(\underline{A})}$) = 0 , d($\overline{(\underline{A})}$, , $\overline{A}$) = 0 , d($\underline{(\overline{A})}$, $\underline{A}$) = 0,
5. d ($\underline{U}$, U) =0 , d($\overline{\phi}$, ϕ) = 0
6. if A B ,then d($\overline{A}$,B) ≥ d($\underline{A}$, $\underline{B}$) and d($\underline{A}$, B) ≥ d($\underline{B}$,B)
   d(A , $\overline{B}$) ≥d(A, $\overline{A}$) and d(A, $\overline{B}$)= ≥d($\overline{A}$, $\overline{B}$)
7. d($\underline{A^c}$,$(\overline{A})^c$)= 0, d($\overline{A^c}$,$(\underline{A})^c$) = 0

5-Conclusion

In this paper we have defined the notion of interval valued neutrosophic rough sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and interval valued neutrosophic set theory. Further, we have introduced the Hamming distance between two interval neutrosophic rough sets. We hope that our results can also be extended to other algebraic system.

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