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On Measures of Similarity for Neutrosophic Sets with Applications in Classification and Evaluation Processes

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Abstract: In the last decades, different researchers have considerably incorporated the notion of neutrosophic sets, their properties and different measures for managing the uncertainty, impreciseness and vagueness in the information. It may be noted that neutrosophic set is a popular defined procedures for solving the classification problem and evaluation problem of decision-making. Numerical examples for the classification problem and the decision-making problem have also been presented and compared the obtained results with the well established existing approaches.

Keywords: Neutrosophic set, Similarity measure, Classification problem, Decision-making

1. Introduction

In the fields of expert system, information & belief system, the concept of belongingness of fuzzy set (FS) [5] does not remain the single key-term to be taken care for the evident but also the non-belongingness grade to be taken into consideration. The intuitionistic fuzzy sets (IFSs) [6] take belongingness/non-belongingness both into account to manage the incomplete/imprecise information other than the indeterminate information of a belief system (if any). The technical literature of FSs and IFSs have been utilized in many real-world applications in the field of decision-making, pattern recognition problems, financial economics etc.

The concept of a neutrosophic set (NS) was first given by Smarandache [7] as an additional generalization for mathematically model the uncertainty/impreciseness, incompleteness/ inconsistency found in the problems. As in the words of Smarandache - "*Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra*"[7]. It may certainly be noted that the notion of neutrosophic set can be taken as a formalized general structure of the crisp set, FS, IFS etc. Single valued neutrosophic set (SVNS) is a particular case of neutrosophic sets explained by Wang et al. [4]. In the available research, different extensions of SVNSs are found having a composed structure of soft neutrosophic set, rough neutrosophic, hesitant NS etc. Many researchers have enhanced the literature of FS and IFS by studying various information measures of similarity, entropy, divergence etc. as having different applications in various fields. It is to mention that the indeterminacy degree of IFS is dependent on the membership & non-membership grade. In this way, a decision maker is bounded and restricted for quantifying the sense of impreciseness. The theory of neutrosophic set certainly have the capability to deal with such restrictions and proved to be effective in information-based applications.

The generalization of fuzzy set to neutrosophic set may be well understood by the geometric presentation in Figure 1 showing the better coverage of the imprecise information.

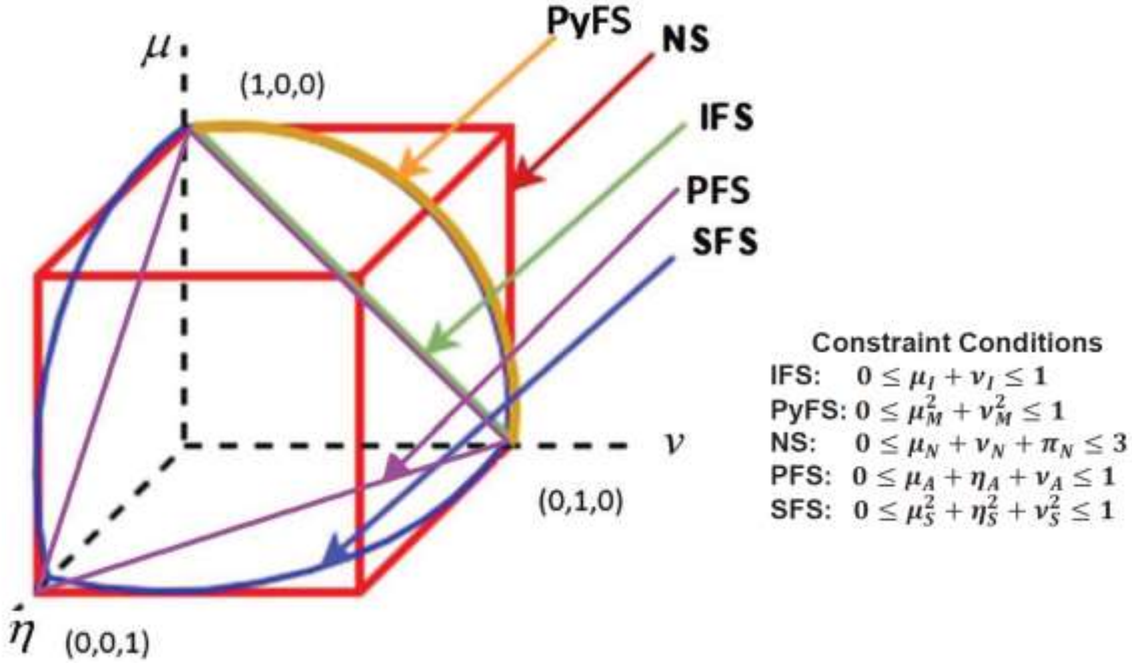

Figure 1: Extension of Fuzzy Set to Neutrosophic set

A brief literature survey on measures of neutrosophic sets is given below:

"Different kinds of similarity/distance measures of NSs have been well studied by Broumi and Smarandache [8]. Utilizing the distance measure between two SVNSs, Majumdar and Samanta [9] defined some important measures of similarity along with their characteristics. Ye [28] presented the three different similarity measures between SVNSs as an extension of the Jaccard, Dice, and cosine similarity measures in vector space and utilized then to solve the MCDM problem under simplified neutrosophic information. Mondal and Pramanik [29] proposed a new trigonometric measure called tangent similarity measure as an improvement of cosine similarity and used this to solve the applications problem of "selection of educational stream" and "medical diagnosis". Ye [10] has given different similarity measures for the interval neutrosophic sets based on distance measures with application in decision processes [11]. Next, Ye et al. [12] [13 and Wu et al. [15] discussed the problem of diagnosis based on the similarity measures for SVNSs.."

"Also, a new multi-attribute decision making method has been developed based on the proposed information measures with a numerical example of city pollution evaluation. Thao and Smarandache [16] proposed new divergence measure for neutrosophic set with some properties and utilized to solve the medical diagnosis problem and the classification problem. Recently, the notion of NSs theory and its various generalizations have been explored in various field of research by different researchers. Abdel-Basset et al. [17] developed a new model to handle the hospital medical care evaluation system based on plithogenic sets and also studied intelligent medical decision support model [18] based on soft computing and internet of things. In addition to this, a hybrid plithogenic approach [19] by utilizing the quality function in the supply chain management has also been developed. Further, a new systematic framework for providing aid and support to the cancer patients by using neutrosophic sets has been successfully suggested by Abdel-Basset et al. [20]. Based on neutrosophic sets, some

new decision-making models have also been successfully presented for project selection [21] and heart disease diagnosis [22] with advantages and defined limitations. In subsequent research, Abdel-Basset et al. [23] have proposed a modified forecasting model based on neutrosophic time series analysis and a new model for linear fractional programming based on triangular neutrosophic numbers [24]. Also, Yang et al. [25] have studied some new similarity and entropy measures of the interval neutrosophic sets on the basis of new axiomatic definition along with its application in MCDM problem."

Recently, Abdel-Basset et al. [30] proposed an integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries. Additionally, a novel decision-making model has been provided for sustainable supply chain finance under uncertainty environment [31]. Also, a novel framework to evaluate innovation value proposition for smart product–service systems has been well developed by Abdel-Basset et al. [32]. Guleria et al. [26] proposed a parametric divergence measure and along with it presented some methodologies for solving the classification problem and MCDM problem in neutrosophic set up. Guleria and Bajaj [27] provided a new technique for the dimensionality reduction of informational data under the neutrosophic soft matrices environment and utilized it to get the solution for the decision-making problem. Looking at the recent literature discussions accomplished above, it is being stated that the information measures of NSs deal with the concerns in connection with uncertainty/vagueness.

Nabeeh et al. [36] proposed the technique of N-MCDMF which integrates the theory of neutrosophic sets using different methods of MCDM for evaluating the GCP in the direction of environment. Further, Nabeeh et al. [37] [38] also enhanced the process of the management of the resources and clearly explained the internet of things connection in case of smart village by using the method of neutrosophic AHP and TOPSIS which helps the decision makers to solve the problem of reaching the goal of the companies respectively. Additionally, various examples have been presented which make the readers understand the utility of the used methods more accurately. The problem faced by the IoT industries was further explained by Basset et al. [39] and presented the solution to the traditional process using the non-traditional method in contrast with the methods of AHP and theory of neutrosophic.

In this paper, we have incorporated the exponential function for framing the new similarity measures for the neutrosophic sets along with their weighted form and utilized them for the solving a standard classification problem of pattern recognition and the decision-making problem. Various other researchers have also discussed various types of similarity measures

The structure of the presented manuscript is as follows:

Some fundamental definitions, standard operations and existing similarity measures of the neutrosophic sets are presented in Section 2. In section 3, we have proposed some new exponential similarity measures with proof of their validity and also presented several counter-intuitive cases to show the efficacy of the exponential measures. In order to show the applicability of the exponential similarity measures, we have presented the two illustrative examples - one related to the classification problem (pattern recognition) and other related to the evaluation problem of decision-making in Section 4. In addition, some important comparative remarks have been enumerated. Finally, we have concluded the paper in Section 5.

2. Preliminaries

First, we present soe basic preliminaries and fundamenta definitons in connection with neutrosophic set, similarity measures and its properties which are available in literature.

Definition 1 *[5]* "An intuitionistic fuzzy set (IFS) I in U (universe of discourse) is given by

$$I = \{< u, \mu_1(u), \nu_1(u) > | u \in U\};$$

Where $\mu_1: U \rightarrow [0,1]$ and $\nu_1: U \rightarrow [0,1]$ degree of membership and non-membership respectively and for every $u \in U$ satisfies the condition

$$0 \leq \mu_1(u) + \nu_1(u) \leq 1;$$

And the degree of indeterminacy for any IFS I and $u \in U$ is given by $\pi_1 = 1 - \mu_1(u) - \nu_1(u)$".

Definition 2 [7] "Let U be a fixed class points (objects) with a generic element u in U. A neutrosophic set P in U is specified by a truth-membership function $T_P(u)$, an indeterminacy-membership function $I_P(u)$ and falsity-membership function $F_P(u)$, where $T_P(u)$, $I_P(u)$ and $F_P(u)$ are real standard or nonstandard subsets of the interval $(^{-}0\,, 1^{+})$ such that $T_P(u): U \rightarrow (^{-}0\,, 1^{+}), I_P(u): U \rightarrow (^{-}0\,, 1^{+}), F_P(u): U \rightarrow (^{-}0\,, 1^{+})$ and the sum of these function viz. $T_P(u) + I_P(u) + F_P(u)$ satisfies the requirement

$$^{-}0 \leq supT_P(u) + supI_P(u) + supF_P(u) \leq 3^{+}."$$

We denote the neutrosophic set $I = \{(u, T_P(u),\ I_P(u),\ F_P(u) | u \in U\}$.

"In case of neutrosophic set, indeterminacy gets quantified in an explicit way, while truth-membership, indeterminacy-membership and falsity-membership are independent terms. Such framework is found to be very useful in the applications of information fusion where the data are logged from different sources. For scientific and engineering applications, Wang et al. [4] defined a single valued neutrosophic set (SVNS) as an instance of a neutrosophic set as follows:"

Definition 3 [4] "Let U be a fixed class of points (objects) with a generic element u in U. A single valued neutrosophic set P in U is characterzied by a truth-membership function $T_P(u)$, an interminacy-membership function $I_P(u)$ and a falsity-membership function $F_P(u)$. Foe each point $u \in U$, where $I_P(u), T_P(u), F_P(u) \in [0,1]$. A single valued neutrosophic set P can be denoted by

$$P = \{< I_P(u)\,, T_P(u), F_P(u) | u \in U\}."$$

It may be noted that $I_P(u) + T_P(u) + F_P(u) \in [0,3]$.

We denote *SVNS(U)* as the collection of all the *SVNSs* on U. For any two single valued neutrosophic sets $P, Q \epsilon SVNS(U)$ (Refer [4]):

- **Union of P and Q:**

$$P \cup Q = \{\left(u, T_{P\cup Q}(u), I_{P\cup Q}(u), F_{P\cup Q}(u)\right| u \in U\};$$

where $T_{P\cup Q}(u) = max\{T_P(u), T_Q(u)\}$, $I_{P\cup Q}(u) = min\{I_P(u), I_Q(u)\}$ and

$F_{P\cup Q}(u) = min\{F_P(u), F_Q(u)\}$; for all $u \in U$

- **Intersection of P and Q:**

$$P \cap Q = \{(u, T_{P\cup Q}(u), I_{P\cup Q}(u), F_{P\cup Q}(u) | u \in U\};$$

where $T_{P\cap Q}(u) = min\{T_P(u), T_Q(u)\}$, $I_{P\cap Q}(u) = max\{I_P(u), I_Q(u)\}$ and $F_{P\cap Q}(u) = max\{F_P(u), F_Q(u)\}$; for all $u \in U$

- **Containment:**

$P \subseteq Q$ if and only if $T_P(u) \leq T_Q(u)$, $I_P(u) \geq I_Q(u), F_P(u) \geq F_Q(u)$, for all $u \in U$.

- **Complement:** The complement of P, denoted by $\overline{P}$, characterized by

$T_P(u) = 1 - T_P(u), T_P(u) = 1 - T_P(u)$, $T_P(u) = 1 - T_P(u),$; for all $u \in U$.

Definition 4 "A function $S: SVNS(U) \times SVNS(U) \Rightarrow [0,1]$ is called a similarity measure for single value neutrosophic sets, if the following conditions are satisfied:

For any $P, Q, O \epsilon SVNS(U)$,

I. $0 \leq S(P, Q) \leq 1$

II. $S(P, Q) = 1$ if and only if$P = Q$;

III. $S(P, Q) = S(Q, P)$;

IV. $P \subseteq Q \subseteq O$, then $S(P, O) \leq S(P, Q); S(P, O) \leq S(Q, O)$."

<u>**Existing Similarity Measures**</u>

In the literature, different similarity measures have been proposed by various researchers. For the sake of understanding, some of them are being presented below.

Let $P = \{T_P(u_i), I_P(u_i), F_P(u_i) | u_i \in U\}$ & $Q = \{T_Q(u_i), I_Q(u_i), F_Q(u_i) | u_i \in U, i = 1,2, \ldots n\}$ be the two-single valued neutrosophic sets. Then the existing similarity measures between P and Q are as follows:

- **Jaccard's Similarity Measure** [28]

$$S_j(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{T_P(u_i)T_Q(u_i) + I_P(u_i)I_Q(u_i) + F_P(u_i)F_Q(u_i)}{T_P^2(u_i) + I_P^2(u_i) + F_P^2(u_i) + T_Q^2(u_i) + I_Q^2(u_i) + F_Q^2(u_i) - \left(T_P(u_i)T_Q(u_i) + I_P(u_i)I_Q(u_i) + F_P(u_i)F_Q(u_i)\right)} \quad (2.1)$$

- **Dice Similarity Measure** [28]

$$S_D(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_P(u_i)T_Q(u_i) + I_P(u_i)I_Q(u_i) + F_P(u_i)F_Q(u_i))}{T_P^2(u_i) + I_P^2(u_i) + F_P^2(u_i) + T_Q^2(u_i) + I_Q^2(u_i) + F_Q^2(u_i)} \quad (2.2)$$

- **Cosine Similarity Measure** [28]

$$S_C(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{T_P(u_i)T_Q(u_i) + I_P(u_i)I_Q(u_i) + F_P(u_i)F_Q(u_i)}{\sqrt{T_P^2(u_i) + I_P^2(u_i) + F_P^2(u_i)}\sqrt{T_Q^2(u_i) + I_Q^2(u_i) + F_Q^2(u_i)}} \quad (2.3)$$

- **Tangent Similarity Measure** [29]

$S_T(P,Q) = 1 - \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{12}\left(\left|T_P(u_i) - T_Q(u_i)\right| + \left|I_P(u_i) - I_Q(u_i)\right| + \right.\right.$

$\left.\left.\left|F_P(u_i) - F_Q(u_i)\right|\right)\right).$ (2.4)

- The similarity measure of SVNSs between P and Q is defined as follows [9]:

$$S_1 = \frac{\sum_{i=1}^{n}\left(min\left(T_P(u_i),T_Q(u_i)\right) + min\left(I_P(u_i),I_Q(u_i)\right) + min\left(F_P(u_i),F_Q(u_i)\right)\right)}{\sum_{i=1}^{n}\left(max\left(T_P(u_i),T_Q(u_i)\right) + max\left(I_P(u_i),I_Q(u_i)\right) + max\left(F_P(u_i),F_Q(u_i)\right)\right)}$$

(2.5)

- Similarity Measures Based on Theoretic Approach.[40]

$$S_{2T}(P,Q) = \frac{1}{n}\left(\sum_{1}^{N}\left[\frac{min\left(T_P(u_i),T_Q(u_i)\right)}{max\left(T_P(u_i),T_Q(u_i)\right)}\right]\right)$$

$$S_{2F}(P,Q) = \frac{1}{n}\left(\sum_{1}^{N}\left[\frac{min\left(I_P(u_i),I_Q(u_i)\right)}{max\left(I_P(u_i),I_Q(u_i)\right)}\right]\right)$$

$$S_{2F}(P,Q) = \frac{1}{n}\left(\sum_{1}^{N}\left[\frac{min\left(F_P(u_i),F_Q(u_i)\right)}{max\left(F_P(u_i),F_Q(u_i)\right)}\right]\right)$$

and $S_1 = \left(S_{2T}(P,Q), S_{2I}(P,Q), S_{2F}(P,Q)\right).$ (2.6)

3. Similarity Measure of Neutrosophic Sets

In this section, we mainly introduced some new similarity measures for the single valued neutrosophic sets based on the exponential function. Let U be the universe of discourse.

Definition 5 Consider $P = \{(T_P(u_i), I_P(u_i), F_P(u_i))|u_i \epsilon U\}$ and $Q = \{(T_Q(u_i), I_Q(u_i), F_Q(u_i))|u_i \epsilon U, i = 1,2,\dots,n\}$ be two valued neutrosophic sets, then the similarity measure $SM_1(P,Q)$ between P and Q is defined as:

$SM_1(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left(SM_i^T(u_i) \times SM_i^I(u_i) \times SM_i^F(u_i)\right);$ (3.1)

where $SM_i^T(u_i) = e^{-|T_P(u_i)-T_Q(u_i)|}$; $SM_i^I(u_i) = e^{-|I_P(u_i)-I_Q(u_i)|}$ & $SM_i^F(u_i) = e^{-|F_P(u_i)-F_Q(u_i)|}$.

Definition 6 Consider $P = \{(T_P(u_i), I_P(u_i), F_P(u_i))|u_i \epsilon U\}$ and $Q = \{(T_Q(u_i), I_Q(u_i), F_Q(u_i))|u_i \epsilon U, i = 1,2,\dots,n\}$ be two valued neutrosophic sets, then the weighted similarity measure $SM_1^w(P,Q)$ between P and Q is defined as:

$SM_1^w(P,Q) = \sum_{i=1}^{n} w_i \times \left(SM_i^T(u_i) \times SM_i^I(u_i) \times SM_i^F(u_i)\right);$ (3.2)

where $SM_i^T(u_i) = e^{-|T_P(u_i)-T_Q(u_i)|}$; $SM_i^I(u_i) = e^{-|I_P(u_i)-I_Q(u_i)|}$ & $SM_i^F(u_i) = e^{-|F_P(u_i)-F_Q(u_i)|}$.

Definition 7 Suppose $P = \{(T_P(u_i), I_P(u_i), F_P(u_i))|u_i \epsilon U\}$ and

$Q = \{(T_Q(u_i), I_Q(u_i), F_Q(u_i)) | u_i \epsilon U, i = 1,2, \dots, n\}$ be two valued neutrosophic sets, then the similarity measure $SM_2(P,Q)$ between P and Q is defined as:

$$SM_2(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{SM_i^T(u_i)+SM_i^I(u_i)+SM_i^F(u_i)}{3}\right); \tag{3.3}$$

where $SM_i^T(u_i) = e^{-|T_P(u_i)-T_Q(u_i)|}$; $SM_i^I(u_i) = e^{-|I_P(u_i)-I_Q(u_i)|}$ & $SM_i^F(u_i) = e^{-|F_P(u_i)-F_Q(u_i)|}$.

Definition 8 Consider $P = \{(T_P(u_i), I_P(u_i), F_P(u_i)) | u_i \epsilon U\}$ and $Q = \{(T_Q(u_i), I_Q(u_i), F_Q(u_i)) | u_i \epsilon U, i = 1,2, \dots, n\}$ be two valued neutrosophic sets, then the weighted similarity measure $SM_2^w(P,Q)$ between P and Q is defined as:

$$SM_2^w(P,Q) = \sum_{i=1}^{n} w_i \times \left(\frac{SM_i^T(u_i)+SM_i^I(u_i)+SM_i^F(u_i)}{3}\right); \tag{3.4}$$

where $SM_i^T(u_i) = e^{-|T_P(u_i)-T_Q(u_i)|}$; $SM_i^I(u_i) = e^{-|I_P(u_i)-I_Q(u_i)|}$ & $SM_i^F(u_i) = e^{-|F_P(u_i)-F_Q(u_i)|}$

Theorem 1 The measure proposed in Definition 5 is a valid similarity measure.

Proof: For this, we need to show that the similarity measure $SM_1(P,Q)$ between two neutrosophic sets P and Q holds the conditions as defined in Definition 4.

(i) We know that $T_P(u_i), T_Q(u_i) \leq 1$, which implies $|T_P(u_i) - T_Q(u_i)| \leq 1$. This can also be written as $-1 \leq |T_P(u_i) - T_Q(u_i)| \leq 0$.

Hence,

$$0 \leq e^{-|T_P(u_i)-T_Q(u_i)|} \leq 1 \Rightarrow 0 \leq SM_i^T(u_i) \leq 1.$$

Also $0 \leq SM_i^I(u_i), SM_i^F(u_i) \leq 1$. Therefore, from equation (3.1) we conclude that $0 \leq SM_1(P,Q) \leq 1$.

(ii) We know that $SM_i^T(u_i) = 1$, $SM_i^I(u_i) = 1$ and $SM_i^F(u_i) = 1$ if and only if $P = Q$, so we have $SM_1(P,Q) = 1 \Leftrightarrow P = Q$.

(iii) As $SM_i^T(u_i), SM_i^I(u_i), SM_i^F(u_i)$ are symmetric for neutrosophic sets. Hence, we observe that $SM_1(P,Q) = SM_1(Q,P)$.

(iv) If $P \subseteq Q \subseteq O$, then for $u_i \in U$ we have

$$0 \leq T_P(u_i) \leq T_Q(u_i) \leq T_O(u_i) \leq 1;$$
$$0 \geq I_P(u_i) \geq I_Q(u_i) \geq I_O(u_i) \geq 1;$$

and

$$0 \leq F_P(u_i) \leq F_Q(u_i) \leq F_O(u_i) \leq 1.$$

It means that

$$-|T_P(u_i) - T_Q(u_i)| \leq \min\{|T_P(u_i) - T_Q(u_i)|, |T_Q(u_i) - T_O(u_i)|\};$$
$$-|I_P(u_i) - I_Q(u_i)| \leq \min\{|I_P(u_i) - I_Q(u_i)|, |I_Q(u_i) - I_O(u_i)|\};$$

and

$$-\left|F_P(u_i)-F_Q(u_i)\right| \le \min\{\left|F_P(u_i)-F_Q(u_i)\right|, \left|F_Q(u_i)-F_O(u_i)\right|\};$$

This implies that

$$SM_i^T(P,Q) \le \min\{SM_i^T(P,Q), SM_i^T(Q,O)\};$$

$$SM_i^I(P,Q) \le \min\{SM_i^I(P,Q), SM_i^I(Q,O)\};$$

and

$$SM_i^F(P,Q) \le \min\{SM_i^F(P,Q), SM_i^F(Q,O)\}.$$

Thus, based on this, equation (3.1) becomes $SM_1(P,Q) \le SM_1(P,Q)$ and $SM_1(P,Q) \le SM_1(Q,O)$. Hence, the proposed measure in the Definition 5 is the valid similarity measure over two neutrosophic sets.

Theorem 2 The measure proposed in the Definition 6 is a valid similarity measure.

Proof: For this, we need to show the similarity measure $SM_1(P,Q)$ between two neutrosophic sets P and Q holds the conditions defined in Definition 4.

(i) We know that $T_P(u_i), T_Q(u_i) \le 1$, which implies $\left|T_P(u_i)-T_Q(u_i)\right| \le 1$. This can also be written as

$$-1 \le \left|T_P(u_i)-T_Q(u_i)\right| \le 0.$$

Hence, $0 \le e^{-\left|T_P(u_i)-T_Q(u_i)\right|} \le 1 \Rightarrow 0 \le SM_i^T(u_i) \le 1.$

Also, $0 \le SM_i^I(u_i), SM_i^F(u_i) \le 1.$

Therefore, from equation (3.1) we conclude that

$$0 \le SM_1^w(P,Q) \le \sum_{i=1}^{n} w_i = 1.$$

(ii) We know that $SM_i^T(u_i)=1$, $SM_i^I(u_i)=1$ and $SM_i^F(u_i)=1$ if only if $P=Q$ because, $\sum_{i=1}^{n} w_i = 1$, so we have , $SM_1^w(P,Q)=1 \Leftrightarrow P=Q$.

(iii) As $SM_i^T(u_i)$, $SM_i^I(u_i)$, $SM_i^F(u_i)$ are symmetric for neutrosophic sets. Hence, we observe that $SM_1^w(P,Q)=SM_1(Q,P)$.

(iv) For $P \subseteq Q \subseteq O$ and $u_i \in U$, we have

$$SM_i^T(P,Q) \le \min\{SM_i^T(P,Q), SM_i^T(Q,O)\};$$

$$SM_i^I(P,Q) \le \min\{SM_i^I(P,Q), SM_i^I(Q,O)\};$$

and

$$SM_i^F(P,Q) \le \min\{SM_i^F(P,Q), SM_i^F(Q,O)\}.$$

Thus, based on this, equation (3.2) becomes $SM_1^w(P,Q) \le SM_1^w(P,Q)$ and $SM_1^w(P,Q) \le SM_1^w(Q,O)$.

Hence, the proposed measure in the Definition 6 is the valid similarity measure over two neutrosophic sets.

3.1 Comparison with Existing Similarity Measures

In order to show the effectiveness, performance and advantages of the proposed similarity measures, we present the following comparative analysis with existing measures presented in Equation (2.1), Equation (2.2), Equation (2.3) and Equation (2.4).

Thus, to carry out the comparison of the proposed similarity measures with the existing ones in the literature, we consider five different cases consisting of two neutrosophic sets as follows:

$$\text{Case 1: A} = \{0.2,0.3,0.4\} \ \& \ \text{B} = \{0.2, 0.3, 0.4\}$$

$$\text{Case 2: A} = \{0.3,0.2,0.4\} \ \& \ \text{B} = \{0.4, 0.2, 0.3\}$$

$$\text{Case 3: A} = \{1,0.0,0.0\} \ \& \ \text{B} = \{0.0, 1, 1\}$$

$$\text{Case 4: A} = \{1,0.0,0.0\} \ \& \ \text{B} = \{0.0, 0.0, 0.0\}$$

$$\text{Case 5: A} = \{0.4,0.2,0.6\} \ \& \ \text{B} = \{0.2, 0.1, 0.3\}$$

Based on the computational analysis, the values obtained by the proposed similarity measures and existing similarity measures for each case have been tabulated in the Table 1.

Table 1: Comparison of Proposed Similarity Measure with Existing One's

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
SM_1	1	0.8187	0.0497	0.3678	0.5488
SM_2	1	0.978	0.3678	0.7892	0.8214
S_J[28]	1	0.93	0.0	0.0	0.666
S_D[28]	1	0.965	0.0	0.0	0.8
S_C[28]	1	0.965	0.0	Null	1
S_T[28]	1	-2.10	0.954	0.984	0.259

In view of the computed values obtained by the different measures, we can conclude that the proposed similarity measures are quite effective and give distinguished result whereas the existing ones are not able to perform good in some cases (indicated by the bold values).

Remark: "Null" represents the case when the degree of similarity can not be computed due to the problem "division by zero".

4. Applications of Neutrosophic Similarity Measures

4.1 Classification Problem

Consider a standard classification problem where we have m different classes (say) $C_1, C_2, C_3, \dots, C_m$ of known patterns over the universe of discourse $U = \{u_1, u_2, u_3, \dots, u_n\}$. Suppose we choose one sample (say) $P_1, P_2, P_3, \dots, P_m$ from each class and have an unknown sample Q where the information in each known and unknown pattern is featured under the neutrosophic environment. Thus, our main objective is to classify the unknown sample into one of the known classes.

In order to solve this classification problem, we calculate the similarity measure of unknown sample Q with each known pattern $P_i (i = 1, 2, 3, \dots, m)$ and then allocate the unknown sample to one of the classes which has highest similarity index among all.

Example 1: Let us consider three existing patterns P_1, P_2 and P_3 being described by the neutrosophic sets in $U = \{u_1, u_2, u_3\}$ as following:

$$P_1 = \{(u_1, 0.5, 0.4, 0.2), (u_2, 0.4, 0.3, 0.4), (u_3, 0.4, 0.5, 0.1)\};$$

$$P_2 = \{(u_1, 0.6, 0.5, 0.1), (u_2, 0.5, 0.1, 0.3), (u_3, 0.5, 0.5, 0.1)\};$$

$$P_3 = \{(u_1, 0.4, 0.4, 0.2), (u_2, 0.4, 0.5, 0.2), (u_3, 0.3, 0.3, 0.4)\};$$

Let us take an unknown pattern Q given by

$$Q = \{(u_1, 0.4, 0.4, 0.2), (u_2, 0.5, 0.6, 0.1), (u_3, 0.3, 0.4, 0.4)\}.$$

Now, the main task to be accomplished in the problem is to find the class to which Q belongs.

We present the computational procedure of solving the classification problem under consideration with the help of following Figure 2.

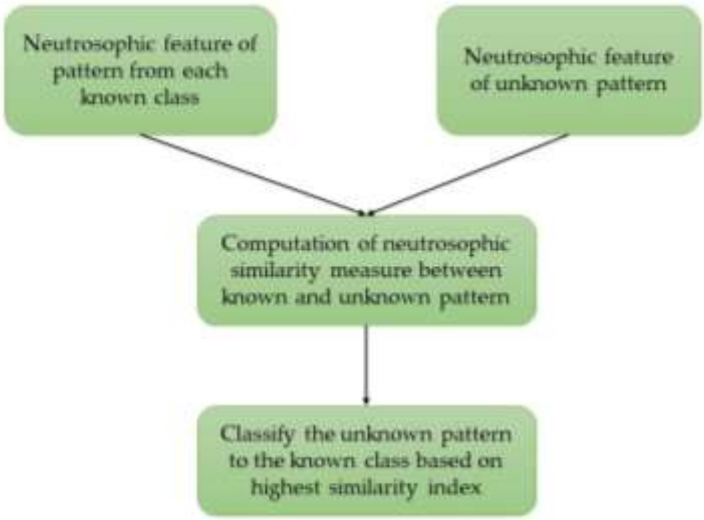

Figure 2: Computational Procedure for Classification Problem

With the help of proposed similarity measures given by equations (3.1), (3.2), (3.3) & (3.4), and choosing the arbitrary weight vector $\boldsymbol{w} = (\mathbf{0.3}, \mathbf{0.4}, \mathbf{0.3})$ (may be selected on the decision maker's choice) of the elements of U, we compute the desired values and tabulate them in Table 2.

Table 2: Computed Values of Similarity Measures

	$(\boldsymbol{P_1}, \mathbf{Q})$	$(\boldsymbol{P_2}, \mathbf{Q})$	$(\boldsymbol{P_3}, \mathbf{Q})$

$\boldsymbol{SM_1}$	0.6725	0.5611	0.5322
$\boldsymbol{SM_1^w}$	0.6659	0.5656	0.5530
$\boldsymbol{SM_2}$	0.880	0.8226	0.804
$\boldsymbol{SM_2^w}$	0.876	0.824	0.814

Based on the obtained values in Table 2, we conclude that the unknown pattern Q belongs to the class $\boldsymbol{P_1}$. The results obtained by utilizing the proposed similarity measures are certainly found to be consistent with the results obtained in [2]. The values obtained are also more prominent and decisive in nature.

4.2 Evaluation Process in Decision Making

In view of the general format of a decision-making problem, we consider a set of available alternatives (say) $\{\boldsymbol{Z_1}, \boldsymbol{Z_2}, \dots, \boldsymbol{Z_m}\}$ and the set of criteria (say) $\boldsymbol{O_1}, \boldsymbol{O_2}, \dots, \boldsymbol{O_n}$. The main goal of the problem is to select the optimal and the best alternatives out of the m available alternatives with respect to n criteria.

The procedure for ranking of the alternatives is based on transforming the neutrosophic decision matrix and computing the similarity index between the alternatives and the ideal solution which has been clearly represented with the help of the following block diagram given in Figure 3:

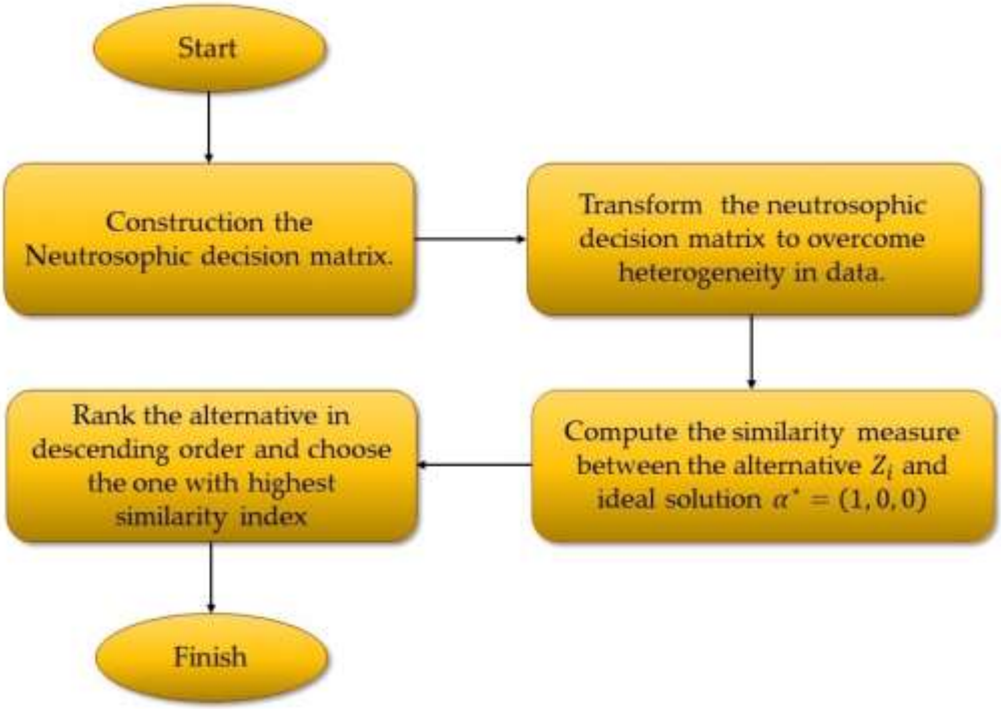

Figure 3: Ranking Procedure for Decision Making with Similarity Measures

Example 2: Consider there is a financial private limited firm whose objective is to invest a significant amount of money in the best possible sector. Suppose there are four possible investment sectors selected on the basis of an initial survey, say,

- $\boldsymbol{Z_1}$: Automobile Sector,
- $\boldsymbol{Z_2}$: Food & Beverages Service Sector,
- $\boldsymbol{Z_3}$: Information Technology Sector,
- $\boldsymbol{Z_4}$: Ammunition Production Sector.

The investment company must take a decision according to the following three important criteria:

- $\boldsymbol{O_1}$: Risk Factor,
- $\boldsymbol{O_2}$: Growth Prospects,
- $\boldsymbol{O_3}$: Ecological Impact.

Suppose that the management and the decision-makers assign suitable weights to each criteria based on their experience and risk bearing capability given by $w = (0.35, 0.25, 0.4)$. The necessary information has been taken from the experts/decision makers for the sake of evaluation of the alternatives $Z_i's$ with respect to each criterion $O_j's$.
The opinion values of each alternative with respect to each criteria have been expressed as a neutrosophic information, and the following neutrosophic decision matrix has been provided:

$$\mathcal{R} = \begin{array}{c} \\ \mathcal{Z}_1 \\ \mathcal{Z}_2 \\ \mathcal{Z}_3 \\ \mathcal{Z}_4 \end{array} \begin{array}{c} \begin{array}{ccc} O_1 & O_2 & O_3 \end{array} \\ \begin{bmatrix} (0.4, 0.2, 0.3) & (0.4, 0.2, 0.3) & (0.2, 0.2, 0.5) \\ (0.6, 0.1, 0.2) & (0.6, 0.1, 0.2) & (0.5, 0.2, 0.2) \\ (0.3, 0.2, 0.3) & (0.5, 0.2, 0.3) & (0.5, 0.3, 0.2) \\ (0.7, 0.0, 0.1) & (0.6, 0.1, 0.2) & (0.4, 0.3, 0.2) \end{bmatrix} \end{array}$$

The ideal solution in such decision-making problems can be as $\boldsymbol{\alpha}^* = (\mathbf{1}, \mathbf{0}, \mathbf{0})$. However, it may be noted that the ideal solution generally does not exist in practice but a closer value is accepted. Our decision can be obtained by calculating the values proposed similarity measures between each alternative $\boldsymbol{Z_i}(\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4})$ and the ideal solution $\boldsymbol{\alpha}^*$. In view of the procedure presented in Figure 3, these values have been computed and tabulated in the Table 3.

Table 3: Computed values of Similarity measure

	$\boldsymbol{SM_1}$	$\boldsymbol{SM_1^w}$	$\boldsymbol{SM_2}$	$\boldsymbol{SM_2^w}$
$(\boldsymbol{Z_1}, \boldsymbol{\alpha}^*)$	0.2962	0.2889	0.6768	0.6716
$(\boldsymbol{Z_2}, \boldsymbol{\alpha}^*)$	0.4665	0.4605	0.7813	0.7779
$(\boldsymbol{Z_3}, \boldsymbol{\alpha}^*)$	0.3456	0.3445	0.7098	0.7092
$(\boldsymbol{Z_4}, \boldsymbol{\alpha}^*)$	0.6703	0.4919	0.7942	0.7892

On the basis of the computed values, the ranking order of the four alternatives in the above problem is

$$Z_4 > Z_2 > Z_3 > Z_1$$

Thus, we have that the alternative Z_4 is the best choice among all the alternatives. The results obtained by utilizing the proposed similarity measures are consistent with the results obtained by Ye [3] and Wang et al. [1].

5. Conclusions & Scope for Future Work

We have successfully introduced some new measures of similarity for the neutrosophic sets in terms of the exponential functions of the truth membership, indeterminacy-membership and falsity-membership. The efficiency of the proposed measure has been validated by presenting few counter-intuitive cases which show that the existing measures fail under some certain cases, while the proposed measures classify them more accurately and precisely. Furthermore, to illustrate the applicability of the proposed similarity measures, an example of classification problem and an example of decision-making problem under neutrosophic environment have been successfully solved. Finally, we conclude that the proposed types of exponential similarity measures are better than the existing measures. The proposed measures produce a reasonable and distinguishable results which is the main outcome and advantage in contrast with other existing methods. Also, it may clearly be observed that the proposed measures are very simple and have the minimum computational burden as compared with other existing methods. The psoposed exponenital similarity measure for the the neutrosophic sets can be extended for single and multi-valued neutrosophic hypersoft set also along with the relvant application which will certainly give an added advantage in the literature. The proposed strategy utilizing the exponential similarity measure can further be applied in various other decision-making problems.

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