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Modeling Customer Lifetime Value Under Uncertain Environment.

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Abstract: Customer lifetime value (CLV) is an essential measure to determine the level of profitability of a customer to a firm. Customer relationship management treats CLV as the most significant factor for measuring the level of purchases and, consequently, the profitability of a given customer. This motivates researchers to compete in developing models to maximize the value of CLV. Dynamic programming models in general—and the Q-learning model specifically—play a significant role in this area of research as a model-free algorithm. This maximizes the long-term future rewards of a certain agent, given their current state, set of possible actions, and the next state of that agent, assuming the customer represents the agent and CLV is their future reward. However, due to the stochastic nature of this problem, it is inaccurate to obtain a single crisp value for Q. In this paper, fuzzy logic and neutrosophic logic shall be utilized to search for the membership values of Q to capture the stochasticity and uncertainty of the problem. Both fuzzy Q-learning and neutrosophic Q-learning were implemented using two membership functions (i.e., trapezoidal, and triangular) to search for the optimal Q value that maximizes the customer's future rewards. The proposed algorithms were applied to two benchmark datasets: The Knowledge Discovery and Data Mining (KDD) cup 1998 direct mailing campaign dataset and the other from Kaggle, related to direct mailing campaigns. The proposed algorithms proved their effectiveness and superiority when comparing them to each other or the traditional deep Q-learning models.

Keywords: Customer Lifetime Value; Fuzzy Logic; Neutrosophic Logic; Q-Learning; Dynamic Programming; Uncertainty

1. Introduction

Customer lifetime value (CLV) is a crucial concept in customer relationship management (CRM). It is defined as the present value of all future profits that can be obtained from the customers over their lifetime of relationship with a specific firm (as presented in Figure 1). In short, direct marketing is about treating customers differently based on their level of profitability, and CLV is the most reliable indicator in direct marketing for measuring the profitability of customers [4, 20, 27]. CLV depends on many factors including customers' retention rate, acquisition rate, probability of churn, and Recency, Frequency, Monetary (RFM) values [10]. Many researchers competed in developing models that measure CLV [8]. Meanwhile, due to the effectiveness of CLV in determining the level of profitability of the customer, the researchers devoted more interest in the models that maximize the values of CLV, to help the firm in maximizing the long-term profitability of its customers and treat those customers accordingly [28].

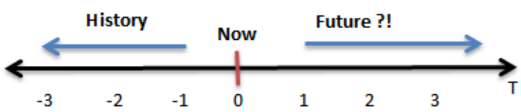

Figure 1. Historical and future periods for CLV

Q-learning model alongside deep learning models proved superiority in this area of research, by helping in maximizing CLV [31, 19]. Meanwhile, those models have a major drawback of overestimating the action values, and hence, recommending unrealistic actions. The rest of this section will be devoted to illustrating the main algorithms of the proposed models. Starting from Q-Learning, passing by Fuzzy logic and Neutrosophic logic; demonstrating the relationship between Fuzzy logic and Neutrosophic logic, and presenting their main ideas and applications. Q-Learning is an off-policy reinforcement learning algorithm that helps in approximating an optimal action to be taken, for the sake of maximizing the long-term reward given the current state. It is considered as an off-policy algorithm, as the Q function learns from actions outside the current policy (i.e., taking random actions) and this is why a policy is not needed. Traditionally, Q-learning was performed by constructing a Q-table that is a matrix of (states and actions) initialized to Zeros as demonstrated in Fig.2.

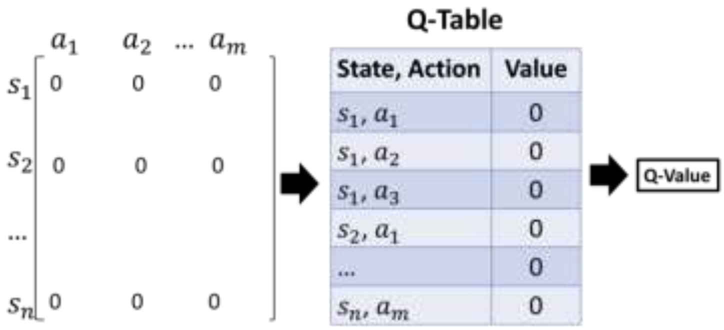

Figure 2. Q-table

In the Q table, states are on rows and actions are on columns and the goal is to select the action that gives the maximum Q value, and consequently the maximum long-term reward. Eq. (1) illustrates the relationship between Q value, current state, current action, next state, and immediate reward mathematically. Where Q(s_t,a_t) is the current Q value, r_t is the reward, Q(s_{t+1},a_t) is the expected reward from the action in the next state, ŋ is a learning rate, finally, γ is the discount factor.

$$Q(s_t, a_t) = Q(s_t, a_t) + ŋ[r_t + \gamma max_a Q(s_{t+1}, a) - Q(s_t, a_t)] \qquad (1)$$

Recently, the researchers integrated Q-learning with deep learning. In this integration, Q values are estimated using deep neural networks [31]. The latter empowers reinforcement learning especially in large and complex problems where finding an optimal solution is impossible, as DQN helps in finding the approximate solution for Q that helps in generalizing the results, as shown in Eq. (2); where $\mathbf{r_t}$ represents the immediate reward and Q(s_t, a_t) is the optimal Q value.

$$Q(s_t, a_t) = r_t + \gamma\ max_a Q^*(s_{t+1}, a) \qquad (2)$$

A fuzzy set is a special kind of sets whose elements have degrees of membership [33]. It differs from the classical set theory, as the latter assumes that the elements of a set have binary degrees of membership, and they either belong to this set or not [35]. This is why it is called the "Crisp" set. Meanwhile, in the Fuzzy set theory, the elements have a real-valued membership function, they belong to this set by a fractional value μ(x), where μ(x)∈[0,1]. Fig.3 demonstrates the difference between the membership function of crisp and fuzzy sets [38]. There are many types of fuzzy membership functions, including Triangular, Trapezoidal (i.e. will be discussed in detail in Section-2.2.1.), Sigmoid, and many others [17]. Fuzzy logic is used in many applications, including decision making, clustering, linguistics, and many more domains where the information is incomplete or imprecise. However, it is rarely used in marketing applications and tested on benchmark datasets alongside Q-learning. Although, it is expected to achieve superior results for generating optimal long term Q values, by relaxing the crisp Q value to different stochastic membership functions (i.e. triangular, trapezoidal, sigmoid, … etc.)

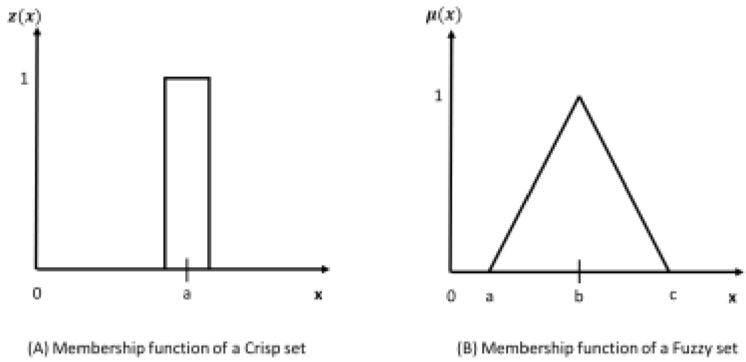

Figure 3. Membership function of crisp and fuzzy sets

Finally, the term Neutrosophic means neutral thought knowledge. It is a combination of two terms (Neuter) and (Sophia), wherein Latin Neuter means "Neutral" and Sophia means "Wisdom". In general, Neutrosophic set and logic are generalizations of classical fuzzy and intuitionistic fuzzy [40], while neutrosophic Probability and Statistics are generalizations of classical and imprecise probability and statistics [3]. Neutrosophic Logic (NL) is a framework for the unification of many existing logics, such as fuzzy logic, paraconsistent logic, intuitionistic logic, etc. [34, 37]. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic space, where each dimension of that space represents respectively the truth (T), the indeterminacy (I), and the falsehood (F) of the statement under consideration; where T, I, and F are standard or non-standard real subsets from $]0, 1[$ with not necessarily any connection between them [2]. Many examples can be represented only by neutrosophic logic and neither by fuzzy, nor intuitionistic fuzzy. One of those examples is "Voting" [36]. In general, the neutrosophic set depends on three membership functions (T, I, and F). These functions are independent, and their sum does not add up to 1. Meanwhile, it should add up to 3 [39]. Neutrosophic logic is considered a bigger umbrella of Fuzzy logic. Also, it has many applications however it has not been used so far alongside Q-learning. Although, by combining it with Q-learning, more realistic, and flexible long-term values for Q are expected to be obtained.

Due to the significance of CLV and the effectiveness of Q-learning, fuzzy logic, and neutrosophic logic algorithms; many researchers compete in developing models to utilize these algorithms separately in the marketing context. Meanwhile, each of their implementations has a certain drawback. For instance, neutrosophic logic is not applied yet in a real-life marketing context to maximize CLV [11]. Also, fuzzy logic is not utilized to maximize CLV, but for many other purposes, including clustering the customer base according to their level of profitability to the firm or also measuring it with RFM values instead of CLV [28, 6]. Finally, Q-learning has been combined with different machine learning and deep learning algorithms for that purpose. For instance, some researchers utilized deep learning to predict the optimal value of Q that maximized the long-term profitability of the customers within the firm [31, 19]. Meanwhile, these algorithms overestimated the action values of Q, hence, generated unrealistic actions [14].

Consequently, this paper proposes two models, Fuzzy Q-learning (FQL), and Neutrosophic Q-learning (NQL). The former combines fuzzy logic with Q-learning, to search for an optimal Q value that maximizes long term future rewards. Also, neutrosophic logic is utilized for the same purpose. Both models are implemented using two types of membership functions (triangular, and trapezoidal). Each of them is applied to two benchmark datasets. Also, both of those models are expected to overcome the limitations of the traditional models that overestimate the action values and hence, generate unrealistic actions. The proposed models expected to overcome this by capturing the stochastic nature of the problem through recommending fuzzy membership values for Q instead of crisp ones, in case of fuzzy logic, and replacing these crisp Q values with the neutrosophic three-values membership (T, I, F) in case of neutrosophic logic. The rest of the paper is organized as follows; Section-2 lists the work that is related to this point of research. Section-3 presents the proposed models; while the datasets and the experimental results are presented in Section-4. Section-5 lists the managerial implications of the proposed algorithms, while Section-6 mentions the limitations of this research and future research directions. Finally, Section-7 concludes the proposed work.

2. Background and Related Work

The following two subsections present the related work of utilizing Fuzzy sets, and Neutrosophic sets in the field of machine learning and decision making to empower marketing decisions. The main focus of those sections is the illustration of triangular and trapezoidal membership functions, while the last subsection illustrates the power of Q-learning as a dynamic programming approach in the area of maximizing customer lifetime value.

2.1. Fuzzy Models

Fuzzy sets and Fuzzy logic are attractive area of research for many researchers, to be utilized in the area of CLV. A proposal for customer segmentation using fuzzy c-means clustering and customer ranking using an optimized version of fuzzy AHP has been done [28]. Their proposed model was applied to a large IT company in Iran and proved its effectiveness in grouping the customer base into nine segments. One of their limitations was that they applied their proposed model to only a single industry. Hence, its results were not generalized. Other researchers proposed six fuzzy key performance indicators to measure customer retention and loyalty. They concluded with the effectiveness of these indicators in determining the retention and loyalty of the customers. On top of their limitations was that their study had a limited number of respondents and from a particular management level, for a certain segment from a particular company [32]; while other researchers applied a fuzzy linguistic model that related customer segmentation with campaign activities for more interpretability to the results. Their work was well presented with many implementation details. Meanwhile, it was applied to a single company without generalization, besides the fact that the segmentation based on RFM usually causes a lake of precision [6]. This is a bit different from the work in [5], which added the "Length" dimension to RFM values in their LRFM model, and hence, considered customer loyalty. They calculated the length as (the number of days from the first to the last visit date in a given period). They also performed clustering analysis using LRFM. The main drawback of their work was that most of the illustrative charts are not clear. Others proposed a modeling framework algorithm that estimated the class conditional density functions of Bayesian decision theory for the discrete values, using frequency probability, this stemmed from a set of statistically independent simulations. Meanwhile, for the continuous variables, they assigned a fuzzy logic. Their model outperformed the traditional risk scorecards. Their idea was well presented, however, its applicability was not ensured, as they did not mention any experimental results although it was mentioned that this was a practical approach [24]. Another contribution was an interval type-2 fuzzy model for the quality of web service. Their proposed model showed a greater capability and outperformance over the traditional fuzzy sets in managing the uncertainty of the problem [13]. Their well-structured work would be much more valuable and effective if it was applied to many other industries. Table 1 summarizes the work of applying fuzzy logic in the field of customer lifetime value.

2.2. Neutrosophic Models

In this section, the neutrosophic Q-learning model is presented. Two types of membership functions for the NQL model are illustrated (Trapezoidal and Triangular). The goal is to utilize the neutrosophic model to learn the optimal Q value that maximizes long term rewards. The stochastic nature of the problem is captured by assuming three values for Q (i.e. T, I, and F) instead of a single value, each of which follows the Trapezoidal or Triangular membership function illustrated in the upcoming sub-sections.

2.2.1. Trapezoidal Neutrosophic Q-Learning

In light of neutrosophic logic's definition mentioned in Section-1, which depends upon 3 core values (T, I, and F); this section illustrates how to calculate these values, and how to calculate the model performance measurements [17].

Let H be a universal set, hence, a single-valued neutrosophic set B in H is calculated in Eq. (3)

$$B = \{h, < T_B(h), I_B(h), F_B(h) > | h \in H\}, \quad (3)$$

Where truth membership function (T_B (h)), indeterminacy membership function (I_B (h)), and falsity membership function (F_B (h)) satisfy the following conditions:

$$T_S(z) = \begin{cases} \frac{(z-k)t_S}{(l-k)}, & k \le z < l \\ t_S, & l \le z \le m \\ \frac{(n-z)t_S}{(n-m)}, & m < z \le n \\ 0, & otherwise \end{cases} \quad (4)$$

$$I_S(z) = \begin{cases} \frac{(l-z)+(z-k')i_S}{(l-k')}, & k' \le z < l \\ i_S, & l \le z \le m \\ \frac{z-m+(n'-z)i_S}{(n'-m)}, & m < z \le n' \\ 1, & otherwise \end{cases} \quad (5)$$

$$F_S(z) = \begin{cases} \frac{(l-z)+(z-k'')f_S}{(l-k'')}, & k'' \le z < l \\ f_S, & l \le z \le m \\ \frac{z-m+(n\prime'-z)f_S}{(n''-m)}, & m < z \le n'' \\ 1, & otherwise \end{cases} \quad (6)$$

Where S is a trapezoidal neutrosophic number, $\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n} \in \boldsymbol{R}$. Then $\boldsymbol{S} = ([\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}];\ \boldsymbol{t_s}, \boldsymbol{i_s}, \boldsymbol{f_s})$ is called trapezoidal neutrosophic number (TrNN); and it has one of three possibilities (Positive TrNN, negative TrNN, or normalized TrNN). $\boldsymbol{m}$ is called positive TrNN, if $\boldsymbol{0 \le k \le m \le n}$. While, if $\boldsymbol{k \le l \le m \le n \le 0}$, then $\boldsymbol{S}$ is called negative TrNN. If $\boldsymbol{0 \le k \le l \le m \le n \le 1}$ and $\boldsymbol{T_s}, \boldsymbol{I_s}, \boldsymbol{F_s} \in [\boldsymbol{0}, \boldsymbol{1}]$, then $\boldsymbol{X}$ is called normalized TrNN. The membership function is demonstrated in Fig.4.

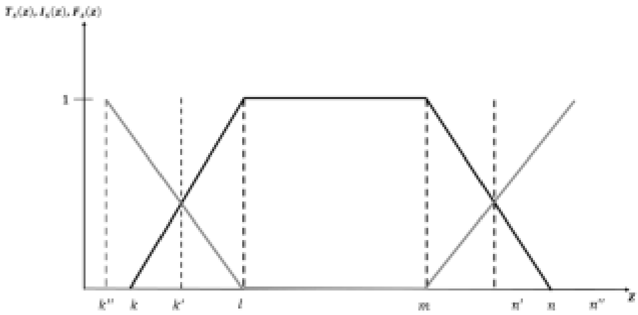

Figure 4. TrNN membership function for truth, indeterminacy, and falsity functions

2.2.1.Triangular Neutrosophic Q-Learning

Assume E is a universe, the triangular neutrosophic number $\boldsymbol{\bar{a}}$ for every z ∈E as ((a, b, c); $\boldsymbol{w_{\bar{a}}}$, $\boldsymbol{u_{\bar{a}}}$, $\boldsymbol{y_{\bar{a}}}$) and the truth, indeterminacy, and falsity membership functions are defined in Eq. (7, 8, and 9) respectively, and demonstrated in Fig.4 . The vector $\boldsymbol{\bar{a}}$ takes one of two forms, if a ≥ 0 and a<b<c, then $\boldsymbol{\bar{a}}$ is called a positive triangular neutrosophic number, while, if a ≤ 0 and a>b>c then $\boldsymbol{\bar{a}}$ is called a negative triangular neutrosophic number [1].

$$T_{\bar{a}}(z) = \begin{cases} \frac{(z-a)w_{\bar{a}}}{(b-a)}, & a \le z < b \\ w_{\bar{a}}, & z = b \\ \frac{(c-z)w_{\bar{a}}}{(c-b)}, & b < z \le c \\ 0, & otherwise \end{cases} \quad (7)$$

$$I_{\bar{a}}(z) = \begin{cases} \frac{(b-z+(z-a)u_{\bar{a}})}{(b-a)}, & a \le z < b \\ u_{\bar{a}}, & z = b \\ \frac{z-b+(c-z)u_{\bar{a}}}{(c-b)}, & b < z \le c \\ 1, & otherwise \end{cases} \quad (8)$$

$$F_{\bar{a}}(z) = \begin{cases} \frac{(b-z+(z-a)y_{\bar{a}})}{(b-a)}, & a \leq z < b \\ y_{\bar{a}}, & z = b \\ \frac{z-b+(c-z)y_{\bar{a}}}{(c-b)}, & b < z \leq c \\ 1, & otherwise \end{cases} \quad (9)$$

There are three main performance measurements to evaluate the output of the trapezoidal neutrosophic set [9]. These are score function $\boldsymbol{Sc}$, accuracy function Ac, and certainty function E. Assuming a neutrosophic function (g), these measures can be stated as demonstrated in Eqs. (10, 11, and 12).

$$Sc(g) = \frac{2+T_g-I_g-F_g}{3}, \quad (10)$$

$$Ac(g) = \left(T_g - F_g\right), \quad (11)$$

$$E(g) = T_g, \quad (12)$$

Based on the score function mentioned in Eq. (10) that was also stated in the work in [22], classified the score of single-valued neutrosophic sets to three major zones (Highly Acceptable Zone, Tolerable Acceptable Zone, and Unacceptable Zone). The three zones and their corresponding intervals are demonstrated in Fig.5. Ranking these scores in descending order helps in selecting the most effective and significant attributes in the decision marking problem at hand. While, the accuracy values range from [-1, 1].

Figure 5. Score zones of single-valued neutrosophic set.

The score and accuracy in Eqs. (10, and 11) are very essential in determining the ranking of the set of alternatives at hand. The score and accuracy of each neutrosophic numbers (x, y) are compared, as mentioned in Eqs. (13, 14, and 15)

$$If \ \mathrm{Sc(x)} > \mathrm{Sc(y)} \ Then \ x > y \quad (13)$$

$$If \ \mathrm{Sc(x)} = \mathrm{Sc(y)} \ \& \ Ac(x) > Ac(y) \ Then \ x > y \quad (14)$$

$$If \ \mathrm{Sc(x)} = \mathrm{Sc(y)} \ \& \ Ac(x) < Ac(y) \ Then \ x < y \quad (15)$$

One of the most significant steps in neutrosophic logic is the concept of de-neutrosophication [9]. In this step, the three neutrosophic values (T, I, F) are converted to a single crisp value using Eq. (16)

$$\Psi = 1 - \sqrt{\frac{(1-T_x)^2 + I_x{}^2 + F_x{}^2}{3}} \quad (16)$$

Researchers utilized neutrosophic logic in many domains including Physics [29], speech recognition [26], supply chain [12], or in decision-making process that is much more relevant to the work in [25]. Other researchers proposed an approach to the binary classification problem using ensemble neural networks based on interval neutrosophic set and bagging technique. They built two neural networks to predict the degree of truth and falsity membership values and estimated the degree of indeterminacy. Their proposed algorithm was tested on three benchmark datasets on UCI and proved its superiority over the single pair of neural networks. Their work was well presented, meanwhile, only applied to a medical application, and it is recommended to be applied to other areas to test its robustness [17]. While others utilized neural networks to a bit similar mechanism but for a multi-class classification task. They could provide an assessment for the uncertain predicted values by utilizing two neural networks also to predict the true and false membership values. The indeterminacy value was estimated as well. Their proposed algorithm was tested on different benchmark datasets from UCI and expected to be applied on a real-life "oil and gas" dataset. Meanwhile, their work was not compared to other work to prove its superiority [18]. On the other hand, there was another contribution of single-valued neutrosophic set logic in data mining tasks including neutrosophy decision trees, neutrosophy prototypes, and neutrosophy clustering. They proposed a novel way to calculate the score of the alternatives at the multi-criteria decision-making problem. Meanwhile, their proposed model was not applied to a real-life dataset to test its robustness and effectiveness [22].

Finally, one of the well-structured and well-organized literature survey papers in neutrosophic was written by other researchers who related machine learning tasks to neutrosophic logic and mentioned the contribution of the research in each research direction. For instance, how neutrosophic set dramatically enhanced the traditional clustering techniques and prediction models. They concluded their paper with the fact that relating neutrosophic to Q-learning and deep learning is an untouched research direction, and this is one of the motivations of this paper [11]. The major contribution of neutrosophic set logic in machine learning is listed in Table 1. Meanwhile, none of the work in the literature tackled the problem of maximizing CLV using neutrosophic logic or neutrosophic Q-learning, and this is one of the main contributions of this research.

2.2. *Q-Learning Models*

Q-learning is a very reliable and robust model-free algorithm, that has been applied in many research areas; either standalone or alongside other optimization algorithms (i.e. Artificial Neural Network (ANN) or Deep Learning) to enhance its performance and experimental results, through generalizing its results. Other researchers tried to utilize Q-learning to solve the model-free optimal tracking control problem. They approximated the Q function using ANN. Their method showed superiority over traditional exploration methods. Although they tested the effectiveness of their proposed algorithm on a set of simulation studies, it would have been much more informative, if they tested it on real-life datasets [21]. Others combined policy gradient with an off-policy Q-learning. Hence, they could estimate the Q values from the action preferences of the policy. Their model showed outperforming results when it was tested on a set of numerical examples and Atari Games. However, it was not tested in real-life industrial applications [23].

Other researchers explored how DQN could be used to predict CLV in video games. To test their model, they compared the performance of DQN to parametric models (i.e. Pareto/NBD) and it outperformed it [7]. Similar to the research of this manuscript are two publications [31, 19]. The former proposed a framework that utilized deep Q networks to accomplish two major contributions [31]. First, introducing a modified version of RFM value that can be used to define the state space of the donors; meanwhile, FRM values are ambiguous, and using a deterministic nature problem setup is inappropriate. Second, they tried to determine the optimal marketing action in both discrete and continuous action spaces. They applied their proposed algorithm to the KDD cup 1998 mailing dataset. The researchers in [19] built on their work. They worked on the same dataset and with the same algorithm but had a set of differences. The latter utilized deep learning mainly to learn the representation of the states in a partially observable environment. Furthermore, they proposed a hybrid approach that combined supervised learning to learn the hidden states and reinforcement learning to select the optimal action [19]. Yet, their proposed algorithms had the main limitation of overestimating the action values and consequently, resulted in unrealistic actions; and this is the major drawback of combining deep learning with reinforcement learning [14]; and the main motivation of the work of this paper. The major contributions of applying reinforcement learning in CLV are summarized in Table 1.

Table 1 Major contribution of reinforcement learning in CLV

Publication's Title	Proposed Algorithm	Reference
Model-free optimal tracking control via critic-only Q-learning	Reinforcement Learning	[21]
Combining policy gradient and Q-learning	Reinforcement Learning	[23]
Autonomous CRM control via CLV approximation with deep reinforcement learning in discrete and continuous action space	Reinforcement Learning	[31]
Recurrent reinforcement learning: a hybrid approach	Reinforcement Learning	[19]
Customer lifetime value in video games using deep learning and parametric models	Reinforcement Learning	[7]
Machine learning in Neutrosophic Environment: A Survey	Neutrosophic Logic	[11]
Role of neutrosophic logic in data mining	Neutrosophic Logic	[22]
Ensemble neural networks using interval neutrosophic sets and bagging	Neutrosophic Logic	[17]
Multiclass classification using neural networks and interval neutrosophic sets	Fuzzy Logic	[18]
Customer lifetime value determination based on RFM model	Fuzzy Logic	[28]
Fuzzy indicators for customer retention	Fuzzy Logic	[32]
A Fuzzy Linguistic RFM Model Applied to Campaign Management	Fuzzy Logic	[6]
New Approach for Customer Clustering by Integrating the LRFM Model and Fuzzy Inference System	Fuzzy Logic	[5]
Consumer credit limit assignment using Bayesian decision theory and Fuzzy Logic–a practical approach	Fuzzy Logic	[24]
An interval type-2 fuzzy model of compliance monitoring for quality of web service	Fuzzy Logic	[13]

Each of the algorithms applied by other researchers has a set of advantages and disadvantages. Table 2 lists some of them. Combining those algorithms in the proposed algorithm avoids their disadvantages and tries to make the best out of their advantages.

Table 2: Advantages and Limitations of the traditional techniques

Algorithm	Advantages	Limitations
Reinforcement Learning	1. Able to solve very complex problems. 2. Can correct the errors that occurred during the training process. 3. In the absence of a training dataset, it can learn from its experience. 4. In outperforms humans in many tasks [30] 5. Achieves the ideal behavior of the model while mainlining the balance between exploration and exploitation	1. It needs a lot of data. 2. Needs a lot of computations. 3. It assumes the world is Markovian, which is not always the case. 4. To obtain the best of it, one can combine it with other algorithms (i.e., Deep learning)
Fuzzy Logic	1. Can be used to solve complex problems 2. The structure of it is easy and understandable 3. It can offer accurate and acceptable reasoning 4. Deal with uncertainty	1. Setting “exact” fuzzy rules and membership functions are difficult tasks 2. Its results are not always accurate 3. Expensive validation and verification 4. Doesn’t support real-time response
Neutrosophic Logic	1. An effective way to handle antinomies or uncertainties 2. Indeterminacy plays an essential role in NL, while it’s ignored in other methods 3. Based on the above two advantages, NL has more ability to assess cause-effect relationships 4. Perfectly handle the situations that contain incomplete information	1. Although NL proved its effectiveness in many cases, it might have some limitations in its applicability in a few real- life case studies

2.3. *Customer Lifetime Value*

Data mining played a significant role in measuring CLV. Meanwhile, traditional data mining techniques mainly tried to segment the customers according to their CLV [16], classified them accordingly, or even identified the potential of risky customers [40]. Yet, those contributions indirectly supported the business decision. Thus, this research aims to close the decision-making process loop, by utilizing reinforcement learning techniques (i.e. Q-learning) alongside stochastic programming methods (Fuzzy logic and Neutrosophic logic) to provide actions that directly contribute to maximizing the CLV of the customers. The closest contributions to the work of this paper are [31, 19]. Each of them utilized Q-learning for the same purpose. They trained machine learning [31] or deep learning [19] algorithms to learn the Q value. The proposed models of this paper integrate Q-learning with either fuzzy logic or neutrosophic logic instead of deep learning models. This is expected to generalize the Q values, generate realistic actions, and overcome the overestimation issue caused by learning the Q values using deep learning algorithms. The proposed models are illustrated in more detail in Section-3.

3. Proposed Models

This section presents two proposed models; one of them is a novel one, that combines neutrosophic logic with Q-learning. The other model combines fuzzy logic with Q-learning. The latter is not considered as a novel model, yet its implementation in a marketing context on two benchmark datasets including the Paralyzed Veterans of America (PVA) dataset, and Kaggle direct marketing dataset^1 is its source of novelty. Each of these two models is applied using two membership functions (i.e. Triangular and Trapezoidal). Both of the proposed models are applied to two datasets as will be illustrated in the following subsections.

3.1. Fuzzy Q-Learning

In Fuzzy Q-Learning (FQL) the goal and/or the constraints are fuzzy, however, the system under control is not necessarily being fuzzy [15]. FQL(s_t, a_t) estimates the value of taking action a at state s at a certain time t. The value of the state s is defined as the optimal state-action pair, as demonstrated in Eq. (17). Hence, FQL is a combination of the immediate rewards plus the discounted value of the next state $s_{t+1}$and the constraints on selecting the action $a_t$in state s_t, as illustrated in Eq. (18), while Eq. (19) demonstrates the update rule of FQL. Algorithm-1 lists the main steps of the FQL algorithm, assuming γ is the discount factor, and ŋ is the learning rate.

$$V(s) = Max_a FQL(s_t, a_t) \tag{17}$$

$$FQL(s_t, a_t) = E[(r_t + \gamma V(s_{t+1})) \wedge \mu_c(s_t, a)] \tag{18}$$

$$\Delta FQL(s_t, a_t) \leftarrow ŋ[(r + \gamma V(s_{t+1})) \wedge \mu_c(s_t, a_t) - FQL(s_t, a_t)] \tag{19}$$

Algorithm 1: FQL

Step-1: Input γ and ŋ where $\gamma \in [0, 1]$ and ŋ $\in [0, 1]$

Step-2: Initialize FQL values

$$FQL(s_t, a_t) \leftarrow 0$$

Step-3: Until FQL values converge do

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3.1. $s_t \leftarrow$ current state

3.2. Select action (a) with the highest FQL (*if multiple exist, select one of them randomly*)

3.3. Apply action (a) and observe the new state (s_{t+1}) and a reward (r_t)

3.4. Update Eqs. (21, 22)

$$FQLnew = ŋ[(r_t + \gamma V(s_{t+1})) \wedge \mu_c(s_t, a_t) - FQL(s_t, a_t)] \tag{20}$$

$$FQL(s_t, a_t) \leftarrow FQL(s_t, a_t) + FQLnew \tag{21}$$

}

3.2. Neutrosophic Q-Learning

The idea of the proposed neutrosophic Q-learning (NQL) algorithm is to utilize the three values (T, I, F) of neutrosophic to learn the optimal long term reward of Q. Hence, in short, the process of the proposed algorithm starts with replacing the single value of Q with the three neutrosophic components (T, I, F), calculating the score of each value, then applying de-neutrosophication to convert the results back to a single value to be able to inject it in Eq. (2).

The de-neutrosophication is applied using many techniques, the most popular is either to rank the alternatives based on their score ranges mentioned in Fig.5 or apply Eq. (17). The main goal of utilizing NQL is to capture the stochasticity of the problem in the neutrosophic three values; such that at each state Q is represented by three independent values (true, indeterminate, and false) instead of a single crisp value. This is expected to learn the optimal value of Q without overestimating its action values and consequently, generate reliable proposed actions. The main steps of NQL are listed in Algorithm 2.

Algorithm 2: NQL

Step-1: Input γ and ŋ where $\gamma \in [0,1]$ and ŋ $\in [0,1]$

Step-2: Initialize NQL values

$$Q \leftarrow 0$$

$$NQL(s_t, a_t) \leftarrow 0$$

Step-3: Until NQL values converge do

{

3.1. $s_t \leftarrow current\ state$

3.2. $Calculate\ three\ values\ (T, I, F)$

$If\ Q\ follows\ the\ Trapezoidal\ membership\ function$

$apply\ Eq. (4, 5, 6)$

$If\ Q\ follows\ the\ Traigular\ membership\ function$

$apply\ Eq. (7, 8, 9)$

3.3. Calculate the score function using Eq. (11)

Determine the three zones (Highly acceptable, Tolerance acceptable, or Unacceptable), based on the score range of values (mentioned in Fig.5)

Apply de-neurosophication (whether by ranking the attributes or applying Eq. (17))

3.6. $Update\ Q\ value$

$$QCurrState = r_t + \gamma max_a Q(s_{t+1}, a) - NQL(s_t, a_t) \quad (22)$$

$$NQL(s_t, a_t) = NQL(s_t, a_t) + ŋQ\_nextState \quad (23)$$

3.7. Select action (a) with the highest NQL (if multiple exists, select one of them randomly)

3.8. Calculate Q value

$$NQL(s_t, a_t) = r_t + \gamma\ max_a NQL^*(s_{t+1}, a) \quad (24)$$

}

4. Experimental Results

This section presents the results of the experiments that have been done. Two proposed algorithms (FQL and NQL) are applied to two benchmark datasets. First, KDD cup 1998 direct mailing campaign dataset [19], and the second one is a direct marketing dataset from Kaggle^1. Each algorithm is applied using two different membership functions (i.e. Trapezoidal and Triangular) on different train-test data split types, to test the effectiveness of each algorithm on different data sizes.

4.1. KDD Dataset

The proposed models are applied to the KDD cup 1998 direct mailing campaign dataset [19]. It has been collected by Paralyzed Veterans of America or PVA for short. It is a non-profit organization that has programs and services for United States veterans with spinal cord injuries or diseases. Hence, the training data of this dataset contains a record for every donor who received a PVA donation mailing campaign and didn't make a donation in the last 12 months. It has been collected for 23 distinct periods for a total number of donors of 95,412. It describes whether and how each of them donated as well as their donation amount. It consists of 477 independent variables and two types of dependent variables represent the donation flag and amount. The proposed model is applied to the only subset of these variables to construct the Q-learning tuple (current state, action, next state, and reward). The current state of each donor is assumed to be a five-dimensional vector describes (how recently the donor donated last $(\boldsymbol{r_0})$, how frequently he donates $(\boldsymbol{f_0})$, their average donation amount $(\boldsymbol{m_0})$, how many times PVA sends him an email in the last six months $(\boldsymbol{ir_0})$, and how many times PVA has sent her emails $(\boldsymbol{if_0})$). The next state is also assumed to be a 5-dimensional tuple as $(\boldsymbol{r_1}, \boldsymbol{f_1}, \boldsymbol{m_1}, \boldsymbol{ir_1}, \boldsymbol{if_1})$, the transaction from a current state to the next state was through taking an action (a). A direct mailing campaign is a well-known task in CRM, where the goal is to decide which mailing type to send to the customer to maximize their long-term profitability (i.e. donation amount). Consequently, the KDD dataset consists of 12 mailing types to choose from (i.e. sending a thank you mail, blank cards, Christmas cards with labels, etc.). The rewards represent the donation amount of each donor, these are range from ($0 to $1000) in the training data. The proposed models of FQL and NQL are implemented using the Python programming language. Table 5 and Table 6 summarize the values of average rewards ($) of the proposed models using different membership functions, different split types of train-test data, and different action selection policies; where the real policy represents the action selection policy stated in the dataset while, uniformly random policy is a selection of the actions based on uniform distribution. This is what was exactly done by other researchers to be able to compare the results of FQL and NQL with the results of their proposed algorithm [19]. Finally, the average reward reported in Table 3 and Table 4 is the average of all Q values at every iteration out of 10 iterations.

Table 3. Avg. Reward ($) of FQL Using Different Train-Test Data Split Type and Different Membership Functions

	Real Policy (R)		Uniformly Random Policy (U)	
Train-Test Data Split	Trapezoidal Function	Triangular Function	Trapezoidal Function	Triangular Function
10 Fold CV	9.98	9.19	9.42	9.48
50-50	9.56	9.43	9.36	9.26
70-30	9.45	8.90	9.55	9.48
80-20	9.54	8.90	9.54	9.49

The results of Table 3 and Table 4 are compared to the results of the researchers in [19] as they utilized DQN on the same dataset and for the same purpose of this study, that is searching for the Q value that maximizes the long-term reward of every donor. During their study, they performed 3 action selection criteria that is whether being uniformly random policy (U), a probability matching policy (M), or a Real Policy (R). Only the results of the former and latter criteria are reported in Table 5, as it matches the same action selection criterion of the proposed models.

Table 4. Avg. Reward ($) of NQL Using Different Train-Test Data Split Type and Different Membership Functions

	Real Policy (R)		Uniformly Random Policy (U)	
Train-Test Data Split	Trapezoidal Function	Triangular Function	Trapezoidal Function	Triangular Function
10 Fold CV	9.45	9.80	9.73	9.27
50-50	9.81	9.25	9.67	9.61
70-30	9.40	9.47	9.52	9.29
80-20	9.48	9.45	9.54	9.25

Table 5. Average rewards of Deep Reinforcement Learning models

Reinforcement Learning Models	Avg. Rewards ($) – "Uniform Policy"	Avg. Rewards ($) – "Real Policy"
DQN	9.44	7.03
RL_RNN	9.65	7.62
RL_LSTM	9.60	7.27
SL-RNN + RL_DQN	9.86	7.80
SL_LSTM + RL_DQN	9.81	7.91

The experimental results reported in Table 3, Table 4, and Table 5 proves the superiority of both Fuzzy Q-Learning and Neutrosophic Q-learning in generating higher average reward values than in the deep reinforcement learning, in the case of real policy action selection and mainly using trapezoidal membership function. Meanwhile, this is not usually the case, in the case of the uniform policy. Table 5 lists the average reward values of deep reinforcement learning under different dataset sizes. While, Table 6, demonstrates the results of FQL under different dataset sizes and different action selection policies vs maximum avg. rewards generated from a deep reinforcement learning method. Comparing the results of Table 3 to the results of Table 4, it is obvious that FQL using 10-fold CV type, generated a slightly higher average reward value than SL-RNN + RL_DQN on a dataset of size 500K; however, this is not the case if Table 4 results are compared with Table 6, and the same finding can be by comparing deep reinforcement learning results in Table 6 to NQL results in Table 7.

Table 6. Average rewards of FQL vs Deep Reinforcement Learning models under different data sizes

	Fuzzy Q-Learning		**Deep Reinforcement Learning Algorithms**
Data Size	Trapezoidal Function	Triangular Function	
50K	9.22	9.38	9.74
100K	9.28	9.14	9.69
200K	9.26	9.34	9.78

Table 7. Average rewards of NQL vs Deep Reinforcement Learning models under different data sizes

	Neutrosophic Q-Learning		**Deep Reinforcement Learning Algorithms**
Data Size	Trapezoidal Function	Triangular Function	
50K	9.50	9.18	9.74
100K	9.26	9.52	9.69
200K	9.63	9.33	9.78

4.2. Kaggle Dataset

This is one of Kaggle's direct mailing campaign datasets. It includes data from one of the direct marketers, who sells his products only via a direct email. The marketer sends catalogs with product characteristics to customers who then order directly from the catalogs. He has developed customer records to learn what makes some customers spend more than others. This dataset includes data for 1000 customers each of which has a set of variables (represent their state in the developed Q learning model) including their age, gender, whether he owns a home or not, their marital status, their location, salary, number of children he has, history of their previous purchases, number of catalogs sent to him, and their purchased amounts ($). The purchasing decision of each customer takes them to the next state that is also represented by these components, while the rewards are the monetary value of the purchases. Both FQL and NQL have been applied to the dataset to optimizing the Q value using either fuzzy logic or neutrosophic logic with their different membership functions (i.e. triangular, and trapezoidal). Fig.6 demonstrates the average rewards of the FQL algorithm generated by each of its membership functions in different cross-validation types. It is obvious that none of the membership functions strictly dominates the other, meanwhile, we can trust the trapezoidal membership function as it generates a higher average reward in (70-30 and 80-20) train-test split types where more training data is provided.

On another hand, NQL is used for the same purpose of optimizing Q value using the same membership functions of triangular, and trapezoidal. Meanwhile, the outperformance of trapezoidal is obvious in NQL and all train-test data split types, as demonstrated in Fig.7.

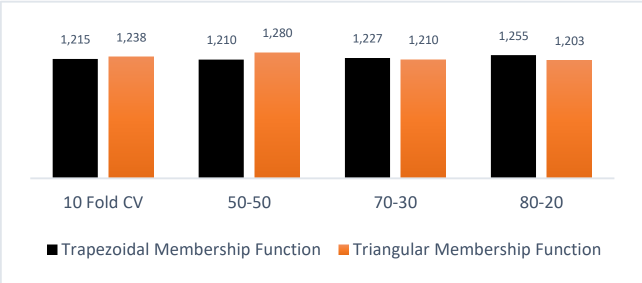

Figure 6 Avg. reward ($) for FQL

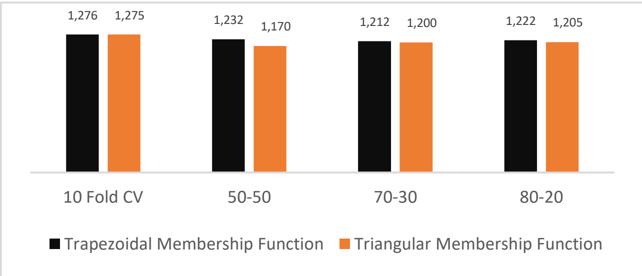

Figure 7 Avg. reward ($) for NQL

5. Managerial Implications

This section presents the managerial implications of the proposed models and how each of them can help in the decision-making process. The proposed models have a set of advantages that boost their flexibility and applicability; including the fact that both depend only on a few parameters, and match the stochastic nature that exists in most real-life situations. Finally, the ease of the implantation of both models, and the possibility of their generalization, promote their applicability in many business situations. Consequently, the proposed models are expected to be of interest to both managers and researchers. The former can apply them on real-life datasets to maximize CLV. While, researchers might apply the proposed models on other datasets to test their robustness, modify them to fill any observed gap or limitation.

6. Limitations and Future Research

This research can be an atom for many future research directions. Meanwhile, it has a set of limitations, including being applied to benchmark datasets, not on real datasets. Moreover, it was not applied to many benchmark datasets to test its robustness and reliability, meanwhile, this is because most of the contributions of the literature review have been done on either hypothetical data or rarely on benchmark datasets, or even built only on a theoretical model. Hence, it was difficult to find many datasets to test the proposed models. Also, it utilized the basic versions of fuzzy logic, neutrosophic logic, and Q-learning models, without contributing to them. Meanwhile, for future research, an advanced version of each algorithm might be applied, for instance, a weighted version of neutrosophic numbers might be utilized [9]. Furthermore, the parameters of Fuzzy and Neutrosophic logic can be optimized using one of the optimization algorithms (i.e. artificial neural network). Another direction is to combine deep reinforcement learning with neutrosophic Q-learning, to avoid the main drawback of overestimating the action values generated from one of the most popular deep reinforcement learning algorithms (i.e. deep Q-learning algorithm). The last but not least research direction is to apply the proposed models on other datasets or applications to test their robustness and reliability.

7. Conclusion

Customer lifetime value (CLV) plays a significant role in determining the value of a customer's profitability within a firm. This motivated the researchers to compete in developing models that maximize CLV. A bunch of those researchers utilized the Q-Learning model for this purpose. They combined Q-learning with deep learning to be able to select the action that would maximize the long term profitability of the customers. In this paper, two models were proposed (Fuzzy Q-Learning and Neutrosophic Q-Learning). The former combined Fuzzy logic with Q-learning while the latter combined Neutrosophic logic with Q-learning. Both models were utilized to select a stochastic value for Q that would maximize the long-term reward, instead of having a single crisp value that may overestimate the action values and make them unrealistic. Two membership values were utilized in each model (i.e. Trapezoidal and Triangular). The proposed models were applied to two different datasets. KDD cup 1998 direct mailing campaign dataset was the first one. While Kaggle direct marketing campaign dataset was the second. The proposed models were applied to both using different data split types and were compared to deep reinforcement learning models in the case of the KDD dataset. The proposed algorithms showed superiority, whether under different action selection criteria or different dataset sizes. The results of FQL and NQL were compared to each other in the case of the Kaggle dataset as it was not utilized in any of the previous research. Trapezoidal membership function generated higher average reward values in most of the cross-validation types, especially, in the case of NQL.

Conflict of Interest/ Competing interests
The authors declare that there is no conflict of interest in the research.

Availability of data and material
This manuscript depends on two benchmark datasets; both are available online and already cited within the paper.

Code availability
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