



An integrated model of Neutrosophic TOPSIS with application in Multi-Criteria Decision-Making Problem

Rana Muhammad Zulqarnain¹, Xiao Long Xin^{1,2*}, Muhammad Saqlain¹, Florentin Smarandache³ and Muhammad Irfan Ahamad⁴

¹ School of Mathematics, Northwest University, Xian, 710127, China. E-mail: ranazulqarnain7777@gmail.com

² School of Science, Xi'an Polytechnic University, Xi'an 710048, China. E-mail: xlxin@nwu.edu.cn

¹ School of Mathematics, Northwest University, Xian, 710127, China. E-mail: msgondal0@gmail.com

³ Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA. E-mail: smarand@unm.edu

⁴ College of Urban and Environmental Sciences, Northwest University, Xian 710127, China. E-mail:

irfan@stumail.nwu.edu.cn

* Corresponding: E-mail: xlxin@nwu.edu.cn

Abstract: Multi-criteria decision making (MCDM) is the technique of selecting the best alternative from multiple alternatives and multiple conditions. The technique for order preference by similarity to an ideal solution (TOPSIS) is a crucial practical technique for ranking and selecting different options by using a distance measure. In this article, we protract the fuzzy TOPSIS technique to neutrosophic fuzzy TOPSIS and prove the accuracy of the method by explaining the MCDM problem with single-valued neutrosophic information and use the method for supplier selection in the production industry. We hope that this article will promote future scientific research on numerous existing issues based on multi-criteria decision making.

Keywords: Neutrosophic set, Single valued Neutrosophic set, TOPSIS, MCDM

1. Introduction

We faced a lot of complications in different areas of life which contain vagueness such as engineering, economics, modeling, and medical diagnoses, etc. However, a general question is raised that in mathematical modeling how we can express and use the uncertainty. A lot of researchers in the world proposed and recommended different approaches to solve those problems that contain uncertainty. In decision-making problems, multiple attribute decision making (MADM) is the most essential part which provides us to find the most appropriate and extraordinary alternative. However, choosing the appropriate alternative is very difficult because of vague information in some cases. To overcome such situations, Zadeh developed the notion of fuzzy sets (FSs) [1] to solve those problems which contain uncertainty and vagueness. Fuzzy sets are like sets whose components have membership (Mem) degrees. In the classical set theory, the Mem degree of the elements in the set is checked in binary form according to the bivalent condition of whether the elements completely belong to the set. In contrast, the fuzzy set theory allows modern ratings of the Mem of elements in the set. This is represented by the Mem function, and the effective unit interval of the Mem function is $[0, 1]$. The fuzzy set is the generalization of the classical set because the indicator function of the classic set is a special case of the Mem function of the fuzzy set if the latter only takes the value 0 or 1. In the fuzzy set theory, the classical bivalent set is usually called the crisp set. Fuzzy set theory can be used in a wide range of fields with incomplete or imprecise information.

It is observed that in some cases circumstances cannot be handled by fuzzy sets, to overcome such types of situations Turksen [2] gave the idea of interval-valued fuzzy sets (IVFSs). In some cases, we must deliberate membership unbiased as the non-membership values for the suitable representation of an object in uncertain and indeterminate conditions that could not be handled by FSs nor IVFSs. To overcome these difficulties Atanassov offered the concept of Intuitionistic fuzzy sets (IFSs) [3]. The theory which was presented by Atanassov only deals the insufficient data considering both the membership and non-membership values, but the intuitionistic fuzzy set theory cannot handle the incompatible and imprecise information. To deal with such incompatible and imprecise data Smarandache [4] extended the work of Atanassov IFSs and proposed a powerful tool comparative to FSs and IFSs to deal with indeterminate, incomplete, and inconsistent information's faced in real-life problems. Since the direct use of Neutrosophic sets (NSs) for TOPSIS is somewhat difficult. To apply the NSs, Wang et al. introduced a subclass of NSs known as single-valued Neutrosophic sets (SVNSs) in [5]. In [6] the author proposed a geometric interpretation by using NSs. Gulfam et al. [7] introduced a new distance formula for SVNSs and developed some new techniques under the Neutrosophic environment. The concept of a single-valued Neutrosophic soft expert set is proposed in [8] by combining the SVNSs and soft expert sets. To solve MCDM problems with single-valued Neutrosophic numbers (SVNNs) presented by Deli and Subas in [9], they constructed the concept of cut sets of SVNNs. On the base of the correlation of IFSs, the term correlation coefficient of SVNSs [10] introduced and proposed a decision-making method by using a weighted correlation coefficient or the weighted cosine similarity measure of SVNSs. In [11] the idea of simplified Neutrosophic sets introduced with some operational laws and aggregation operators such as real-life Neutrosophic weighted arithmetic average operator and weighted geometric average operator. They constructed an MCDM method based on proposed aggregation operators and cosine similarity measure for simplified neutrosophic sets. Sahin and Yiğider [12] extended the TOPSIS method to MCDM with a single-valued neutrosophic technique.

Hwang and Yoon [13] established TOPSIS to solve the general difficulties of DM. The TOPSIS method can effectively maintain the minimum distance from the ideal solution, thereby helping to select the finest choice. After the TOPSIS technique came out, some investigators utilized the TOPSIS technique for DM and protracted the TOPSIS technique to several other hybrid structures of FS. The most important determinant of current scientific research is to present an integrated model for neutrosophic TOPSIS to solve the MCDM problem. Chen & Hwang [14] extended the idea of the TOPSIS method and proposed a new TOPSIS model. The author uses the newly proposed decision-making method to solve uncertain data [15]. Zulqarnain et al. [16] utilized the TOPSIS method for the prediction of diabetic patients in medical diagnosis. They also utilized the TOPSIS extensions of different hybrid structures of FS [17–19] and used them for decision making. Pramanik et al. [21] established the TOPSIS to resolve the multi-attribute decision-making problem under a single-valued neutrosophic soft set expert scenario. Zulqarnain et al. [21] presented the generalized neutrosophic TOPSIS to solve the MCDM problem. Zulqarnain et al. [22] utilized fuzzy TOPSIS to solve the MCDM problem. Maji [23] proposed the concept of neutrosophic soft sets (NSSs) with some properties and operations. The authors studied NSSs and gave some new definitions on NSSs [24], they also gave the idea of neutrosophic soft matrices with some operations and proposed a decision-making method. Many researchers developed the decision-making models by using the NSSs reported in the literature [25–27]. Elhassouny and Smarandache [28] extended the work on a simplified TOPSIS method and by using single-valued Neutrosophic information they proposed Neutrosophic simplified TOPSIS method. The concept of single-valued neutrosophic cross-entropy measure introduced by Jun [29], he also constructed an MCDM method and claimed that this proposed method is more appropriate than previous methods for decision making.

Saha and Broumi [31] studied the interval-valued neutrosophic sets (IVNSs) and developed some new set-theoretic operations on IVNSs with their properties. The idea of an Interval-valued generalized single valued neutrosophic trapezoidal number (IVGSVTrN) was presented by Deli [32] with some operations and discussed their properties based on neutrosophic numbers. Hashim et al

[33], studied the vague set and interval neutrosophic set and established a new theory known as interval neutrosophic vague set (INVS), they also presented some operations for INVS with their properties and derived the properties by using numerical examples. Abdel basset et al. [34] applied TODIM and TOPSIS methods based on the best-worst method to increase the accuracy of evaluation under uncertainty according to the NSs. They also used the Plithogenic set theory to resolve the indeterminate information and evaluate the economic performance of manufacturing industries, they used the AHP method to find the weight vector of the financial ratios to achieve this goal after that they used the VIKOR and TOPSIS methods to utilize the companies ranking [35, 36]. Nabeeh et al. [37] utilized the integrating neutrosophic analytical hierarchy process (AHP) with the TOPSIS for personal selection. Nabeeh et al. [38] developed the AHP neutrosophic by merging the AHP and NS. Abdel-Basset et al. [39] merged the AHP, MCDM approach, and NS to handle the indefinite and irregularity in decision making. Abdel-Basset et al. [40] constructed the TOPSIS technique for type-2 neutrosophic numbers and utilized the presented approach for supplier selection. Abdel-Basset et al. [41] utilized the neutrosophic TOPSIS for the selection of medical instruments and many. Saqlain *et al.* applied TOPSIS for the prediction of sports, and in MCDM problems [42-44].

The FS and IFS theories do not provide any information about the indeterminacy part of the object. Because the above work is considered to examine the environment of linear inequality between the degree of membership (MD) and the degree of non-membership (NMD) of the considered attributes. However, all existing studies only deal with the scenario by using MD and NMD of attributes. If any decision-maker considers the truthiness, falsity, and indeterminacy of any attribute of the alternatives, then clearly, we can see that it cannot be handled by the above-mentioned FS and IFS theories. To overcome the above limitations, Smarandache [4] proposed the NS to solve uncertain objects by considering the truthiness, falsity, and indeterminacy. In the following article, we explain some positive impacts of this research. The concentration of this study is to evaluate the best supplier for the production industry. This research is a very suitable illustration of Neutrosophic TOPSIS. A group of decision-makers chooses the best supplier for the production industry. The Neutrosophic TOPSIS method increases alternative performances based on the best and worst solutions. Classical TOPSIS uses clear techniques for language assessment, but due to the imprecision and ambiguity of language assessment, we propose neutrosophic TOPSIS. In this paper, we discuss the NSs and SVNNSs with some operations. We presented the generalization of TOPSIS for the SVNNSs and use the proposed method for supplier selection.

In Section 2, some basic definitions have been added, which will help us to design the structure of the current article. In section 3, we develop an integrated model to solve the MCDM problem under single-valued neutrosophic information. We also established the graphical and mathematical structure of the proposed TOPSIS approach. To ensure the validity of the developed methodology we presented a numerical illustration for supplier selection in the production industry in section 4.

2. Preliminaries

In this section, we remind some basic definitions such as NSs and SVNNSs with some operations that will be used in the following sequel.

Neutrosophic Set (NS) [30]: Let X be a space of points and x be an arbitrary element of X . A neutrosophic set A in X is defined by a Truth-membership function $T_A(x)$, an Indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ i.e.; $T_A(x), I_A(x), F_A(x): X \rightarrow]0^-, 1^+[$, and $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Single Valued Neutrosophic Sets [5]: Let E be a universe. An SVNNS over E is an NS over E , but truthiness, indeterminacy, and falsity membership functions are defined

$T_A(x): X \rightarrow [0, 1]$, $I_A(x): X \rightarrow [0, 1]$, $F_A(x): X \rightarrow [0, 1]$, and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Multiplication of SVN S [11]: Let $A = \{\alpha_1, \alpha_2, \alpha_3\}$ and $B = \{\beta_1, \beta_2, \beta_3\}$ are two SVN numbers, then their multiplication is defined as follows $A \otimes B = (\alpha_1\beta_1, \alpha_2 + \beta_2 - \alpha_2\beta_2, \alpha_3 + \beta_3 - \alpha_3\beta_3)$.

3. Neutrosophic TOPSIS [11]

3. 1. Algorithm for Neutrosophic TOPSIS using SVN S

To explain the procedure of Neutrosophic TOPSIS using SVN S the following steps are followed. Let $A = \{A_1, A_2, A_3, \dots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2, C_3, \dots, C_n\}$ be a set of evaluation criteria and DM be a set of "l" decision-makers as follows $DM = \{DM_1, DM_2, DM_3, \dots, DM_l\}$. In the form of linguistic variables, the importance of the evaluation criteria, DMs, and alternative ratings are given in Table 1.

Step 1: Computation of weights of the DMs

Let the SVN number for rating the k^{th} DM is denoted by

$$D_k = (T_k^{dm}, I_k^{dm}, F_k^{dm})$$

The weight of the k^{th} DM can be found by the following formula

$$\lambda_k = \frac{1 - \left[\frac{1}{3} \left\{ (1 - T_k^{dm}(x))^2 + (I_k^{dm}(x))^2 + (F_k^{dm}(x))^2 \right\} \right]^{0.5}}{\sum_{k=1}^l \left(1 - \left[\frac{1}{3} \left\{ (1 - T_k^{dm}(x))^2 + (I_k^{dm}(x))^2 + (F_k^{dm}(x))^2 \right\} \right]^{0.5} \right)} ; \text{ where } \lambda_k \geq 0 \text{ and } \sum_{k=1}^l \lambda_k = 1 \tag{1}$$

Step 2: Computation of the Aggregated Neutrosophic Decision Matrix (ANDM)

The ANDM is given as follows

$$D = \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix} = [r_{ij}]_{m \times n} \tag{2}$$

where r_{ij} can be defined as

$$r_{ij} = (T_{ij}, I_{ij}, F_{ij}) = (T_{A_i}(x_j), I_{A_i}(x_j), F_{A_i}(x_j)), \text{ where } i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n$$

Therefore, ANDM written as follows

$$D = \begin{bmatrix} (T_{A_1}(x_1), I_{A_1}(x_1), F_{A_1}(x_1)) & (T_{A_1}(x_2), I_{A_1}(x_2), F_{A_1}(x_2)) & \dots & (T_{A_1}(x_n), I_{A_1}(x_n), F_{A_1}(x_n)) \\ (T_{A_2}(x_1), I_{A_2}(x_1), F_{A_2}(x_1)) & (T_{A_2}(x_2), I_{A_2}(x_2), F_{A_2}(x_2)) & \dots & (T_{A_2}(x_n), I_{A_2}(x_n), F_{A_2}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{A_m}(x_1), I_{A_m}(x_1), F_{A_m}(x_1)) & (T_{A_m}(x_2), I_{A_m}(x_2), F_{A_m}(x_2)) & \dots & (T_{A_m}(x_n), I_{A_m}(x_n), F_{A_m}(x_n)) \end{bmatrix}$$

rating for the i^{th} alternative w.r.t. the j^{th} criterion by the k^{th} DM

$$r_{ij}^{(k)} = (T_{ij}^{(k)}, I_{ij}^{(k)}, F_{ij}^{(k)})$$

For DM weights and alternative ratings r_{ij} can be calculated by using a single-valued neutrosophic weighted averaging operator (SVNWAO)

$$r_{ij} = [1 - \prod_{k=1}^l (1 - T_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^l (I_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^l (F_{ij}^{(k)})^{\lambda_k}] \tag{3}$$

Step 3: Computation of the weights for the criteria

Let an SVN N allocated to the criterion by X_j the k^{th} DM is denoted as

$$w_j^{(k)} = (T_j^{(k)}, I_j^{(k)}, F_j^{(k)})$$

SVNWAO to compute the weights of the criteria is given as follows

$$w_j = [1 - \prod_{k=1}^l (1 - T_j^{(k)})^{\lambda_k}, \prod_{k=1}^l (I_j^{(k)})^{\lambda_k}, \prod_{k=1}^l (F_j^{(k)})^{\lambda_k}] \tag{4}$$

The aggregated weight for the criterion X_j is represented as

$$w_j = (T_j, I_j, F_j) \quad j = 1, 2, 3, \dots, n$$

$$W = [w_1, w_2, w_3, \dots, w_n]^{Transpose}$$

Step 4: Computation of Aggregated Weighted Neutrosophic Decision Matrix (AWNDM)

The AWNDM is calculated as follows

$$R' = \begin{bmatrix} r'_{11} & r'_{12} & \dots & r'_{1n} \\ r'_{21} & r'_{22} & \dots & r'_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r'_{m1} & r'_{m2} & \dots & r'_{mn} \end{bmatrix} = [r'_{ij}]_{m \times n} \tag{5}$$

where $r'_{ij} = (T_{A_i.W}(x_j), I_{A_i.W}(x_j), F_{A_i.W}(x_j))$ where $i = 1, 2, 3, \dots, m$; $j = 1, 2, 3, \dots, n$.

Therefore, R' can be written as

$$R' = \begin{bmatrix} (T_{A_1.W}(x_1), I_{A_1.W}(x_1), F_{A_1.W}(x_1)) & (T_{A_1.W}(x_2), I_{A_1.W}(x_2), F_{A_1.W}(x_2)) & \dots & (T_{A_1.W}(x_n), I_{A_1.W}(x_n), F_{A_1.W}(x_n)) \\ (T_{A_2.W}(x_1), I_{A_2.W}(x_1), F_{A_2.W}(x_1)) & (T_{A_2.W}(x_2), I_{A_2.W}(x_2), F_{A_2.W}(x_2)) & \dots & (T_{A_2.W}(x_n), I_{A_2.W}(x_n), F_{A_2.W}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{A_m.W}(x_1), I_{A_m.W}(x_1), F_{A_m.W}(x_1)) & (T_{A_m.W}(x_2), I_{A_m.W}(x_2), F_{A_m.W}(x_2)) & \dots & (T_{A_m.W}(x_n), I_{A_m.W}(x_n), F_{A_m.W}(x_n)) \end{bmatrix}$$

To find $T_{A_i.W}(x_j)$, $I_{A_i.W}(x_j)$ and $F_{A_i.W}(x_j)$ we used

$$R \otimes W = \{ \langle x, T_{A_i.W}(x) \rangle, \langle x, I_{A_i.W}(x) \rangle, \langle x, F_{A_i.W}(x) \rangle \mid x \in X \} \tag{6}$$

The components of the product given as

$$T_{A_i.W}(x) = T_{A_i}(x) \cdot T_j$$

$$I_{A_i.W}(x) = I_{A_i}(x) + I_j(x) - I_{A_i}(x) \times I_j(x)$$

$$F_{A_i.W}(x) = F_{A_i}(x) + F_j(x) - F_{A_i}(x) \times F_j(x)$$

Step 5: Computation of Single Valued Neutrosophic Positive Ideal Solution (SVN-PIS) and Single Valued Neutrosophic Positive Ideal Solution (SVN-NIS)

Let J_1 be the benefit criteria and J_2 be the cost criteria. A^* be an SVN-PIS and A' be an SVN-NIS as follows

$$A^* = (T_{A^*.W}(x_j), I_{A^*.W}(x_j), F_{A^*.W}(x_j)) \text{ and}$$

$$A' = (T_{A'.W}(x_j), I_{A'.W}(x_j), F_{A'.W}(x_j))$$

The components of SVN-PIS and SVN-NIS are following

$$T_{A^*.W}(x_j) = \left(\left(\max_i T_{A_i.W}(x_j) \mid j \in j_1 \right), \left(\min_i T_{A_i.W}(x_j) \mid j \in j_2 \right) \right)$$

$$I_{A^*.W}(x_j) = \left(\left(\min_i I_{A_i.W}(x_j) \mid j \in j_1 \right), \left(\max_i I_{A_i.W}(x_j) \mid j \in j_2 \right) \right)$$

$$F_{A^*.W}(x_j) = \left(\left(\min_i F_{A_i.W}(x_j) \mid j \in j_1 \right), \left(\max_i F_{A_i.W}(x_j) \mid j \in j_2 \right) \right)$$

$$T_{A'.W}(x_j) = \left(\left(\min_i T_{A_i.W}(x_j) \mid j \in j_1 \right), \left(\max_i T_{A_i.W}(x_j) \mid j \in j_2 \right) \right)$$

$$I_{A'.W}(x_j) = \left(\left(\max_i I_{A_i.W}(x_j) \mid j \in j_1 \right), \left(\min_i I_{A_i.W}(x_j) \mid j \in j_2 \right) \right)$$

$$F_{A'.W}(x_j) = \left(\left(\max_i F_{A_i.W}(x_j) \mid j \in j_1 \right), \left(\min_i F_{A_i.W}(x_j) \mid j \in j_2 \right) \right)$$

Step 6: Computation of Separation Measures

For the separation measures d^* and d' , Normalized Euclidean Distance is used as given as

$$d_i^* = \left(\frac{1}{3n} \sum_{j=1}^n \left[\left(T_{A_i.W}(x_j) - T_{A^*.W}(x_j) \right)^2 + \left(I_{A_i.W}(x_j) - I_{A^*.W}(x_j) \right)^2 + \left(F_{A_i.W}(x_j) - F_{A^*.W}(x_j) \right)^2 \right] \right)^{0.5} \quad (7)$$

$$d_i' = \left(\frac{1}{3n} \sum_{j=1}^n \left[\left(T_{A_i.W}(x_j) - T_{A'.W}(x_j) \right)^2 + \left(I_{A_i.W}(x_j) - I_{A'.W}(x_j) \right)^2 + \left(F_{A_i.W}(x_j) - F_{A'.W}(x_j) \right)^2 \right] \right)^{0.5} \quad (8)$$

Step 7: Computation of Relative Closeness Coefficient (RCC)

The RCC of an alternative A_i w.r.t. the SVN-PIS A^* is computed as

$$RCC_i = \frac{d_i'}{d_i' + d_i^*} \quad \text{where } 0 \leq RCC_i \leq 1 \quad (9)$$

Step 8: Ranking alternatives

After computation of RCC_i for each alternative A_i , the rank of the alternatives presented in descending orders of RCC_i .

The flow chart of the presented technique can be seen in Figure 1.

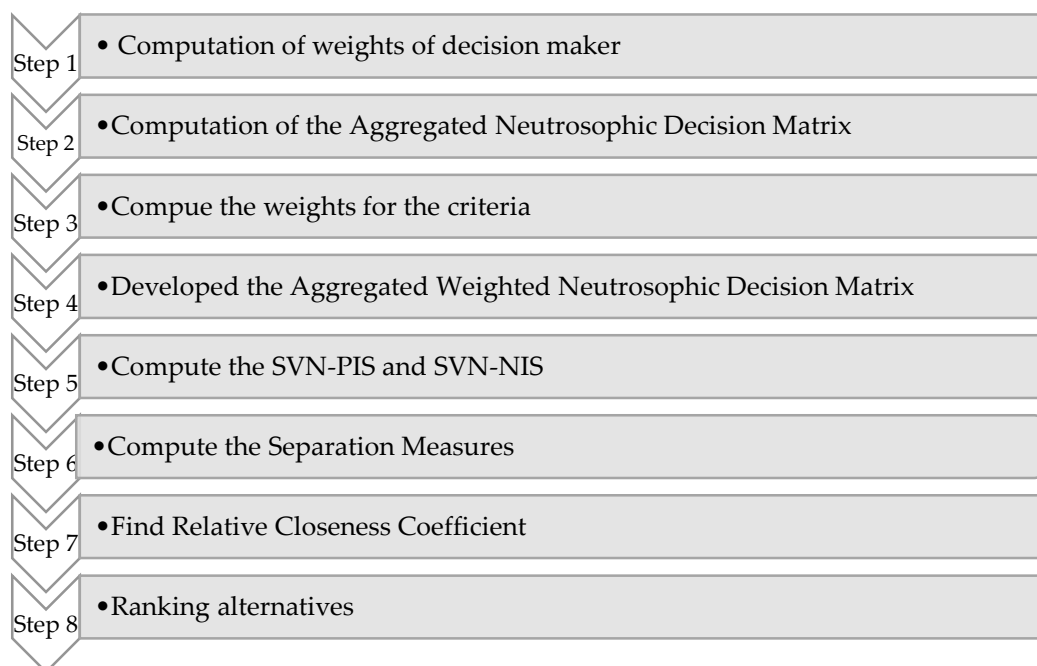


Figure 1: Flow chart of the presented approach

4. Application of Neutrosophic TOPSIS in decision making

A production industry wants to hire a supplier, for the selection of supplier managing director of the industry decides the criteria for supplier selection. The industry hires a team of decision-makers for the selection of the best supplier. Consider $A = \{A_i: i = 1, 2, 3, 4, 5\}$ be a set of supplier and $DM = \{DM_1, DM_2, DM_3, DM_4\}$ be a team of decision-makers ($l = 4$). The evaluation criteria ($n = 5$) for the selection of supplier given as follows,

$$C = \begin{cases} \text{Benefit Criteria} \\ \text{Cost Criteria} \end{cases} \quad j_1 = \begin{cases} X_1: \text{Delivery} \\ X_2: \text{Quality} \\ X_3: \text{Flexibility} \\ X_4: \text{Service} \end{cases} \quad j_2 = \{X_5: \text{Price}\}$$

Calculations of the problem using the proposed SVN-TOPSIS for the importance of criteria and DMs SVN rating scale is given in the following Table

Table 1. Linguistic variables LV's for rating the importance of criteria and decision-makers

| LVs | SVNNs |
|-----|-----------------|
| VI | (.90, .10, .10) |
| I | (.75, .25, .20) |
| M | (.50, .50, .50) |
| UI | (.35, .75, .80) |
| VUI | (.10, .90, .90) |

Where VI, I, M, UI, VUI stand for very important, important, medium, unimportant, very unimportant respectively. The alternative ratings are given in the following table

Table 2. Alternative Ratings for Linguistic Variables

| LVs | SVNNs |
|-----|-----------------|
| EG | (1.0, 0.0,0.0) |
| VVG | (.90, .10, .10) |
| VG | (.80, .15, .20) |
| G | (.70, .25, .30) |
| MG | (.60, .35, .40) |
| M | (.50, .50, .50) |
| MB | (.40, .65, .60) |
| B | (.30, .75, .70) |
| VB | (.20, .85, .80) |
| VVB | (.10, .90, .90) |
| EB | (0.0,1.0,1.0) |

Where EG, VVG, VG, G, MG, M, MB, B, VB, VVB, EB are representing extremely good, very very good, very good, good, medium good, medium, medium bad, bad, very bad, very very bad, extremely bad respectively.

Step 1: Determine the weights of the DMs

By using Equation 1, weights for the DMs are calculated as follows:

$$\lambda_k = \frac{1 - \left[\frac{1}{3} \left\{ (1 - T_k^{dm}(x))^2 + (I_k^{dm}(x))^2 + (F_k^{dm}(x))^2 \right\} \right]^{0.5}}{\sum_{k=1}^l \left(1 - \left[\frac{1}{3} \left\{ (1 - T_k^{dm}(x))^2 + (I_k^{dm}(x))^2 + (F_k^{dm}(x))^2 \right\} \right]^{0.5} \right)} ; \lambda_k \geq 0 \text{ and } \sum_{k=1}^l \lambda_k = 1$$

$$\lambda_1 = \frac{1 - \left[\frac{1}{3} \left\{ (1 - T_1^{dm}(x))^2 + (I_1^{dm}(x))^2 + (F_1^{dm}(x))^2 \right\} \right]^{0.5}}{\sum_{k=1}^l \left(1 - \left[\frac{1}{3} \left\{ (1 - T_k^{dm}(x))^2 + (I_k^{dm}(x))^2 + (F_k^{dm}(x))^2 \right\} \right]^{0.5} \right)}$$

$$\lambda_1 = \frac{1 - \left[\frac{1}{3} \left\{ (1 - T_1^{dm}(x))^2 + (I_1^{dm}(x))^2 + (F_1^{dm}(x))^2 \right\} \right]^{0.5}}{1 - \left[\frac{1}{3} \left\{ (1 - T_1^{dm}(x))^2 + (I_1^{dm}(x))^2 + (F_1^{dm}(x))^2 \right\} \right]^{0.5} + 1 - \left[\frac{1}{3} \left\{ (1 - T_2^{dm}(x))^2 + (I_2^{dm}(x))^2 + (F_2^{dm}(x))^2 \right\} \right]^{0.5} + 1 - \left[\frac{1}{3} \left\{ (1 - T_3^{dm}(x))^2 + (I_3^{dm}(x))^2 + (F_3^{dm}(x))^2 \right\} \right]^{0.5} + 1 - \left[\frac{1}{3} \left\{ (1 - T_4^{dm}(x))^2 + (I_4^{dm}(x))^2 + (F_4^{dm}(x))^2 \right\} \right]^{0.5}}$$

$$\lambda_1 = \frac{1 - \left[\frac{1}{3} \left\{ (1 - 0.9)^2 + (0.10)^2 + (0.10)^2 \right\} \right]^{0.5}}{1 - \left[\frac{1}{3} \left\{ (1 - 0.9)^2 + (0.10)^2 + (0.10)^2 \right\} \right]^{0.5} + 1 - \left[\frac{1}{3} \left\{ (1 - 0.75)^2 + (0.25)^2 + (0.20)^2 \right\} \right]^{0.5} + 1 - \left[\frac{1}{3} \left\{ (1 - 0.50)^2 + (0.50)^2 + (0.50)^2 \right\} \right]^{0.5} + 1 - \left[\frac{1}{3} \left\{ (1 - 0.35)^2 + (0.75)^2 + (0.80)^2 \right\} \right]^{0.5}}$$

$$\lambda_1 = \frac{0.9}{0.9+0.76548+0.5+0.26402}$$

$$\lambda_1 = \frac{0.9}{2.42950} = 0.37045$$

$$\lambda_1 = 0.37045$$

Similarly, we get the weights for the other decision-makers as follows

$$\lambda_2 = \frac{0.76548}{2.42950} = 0.31508$$

$$\lambda_2 = 0.31508$$

$$\lambda_3 = \frac{0.5}{2.42950} = 0.20580$$

$$\lambda_3 = 0.20580$$

$$\lambda_4 = \frac{0.26402}{2.42950} = 0.10867$$

$$\lambda_4 = 0.10867$$

The weights for DMs are given in the following Table

Table 3. Weights of Decision Makers

| Criteria | Alternatives | Decision Makers | | | |
|----------------|----------------|--|--|--|--|
| | | DM ₁ | DM ₂ | DM ₃ | DM ₄ |
| X ₁ | A ₁ | VG (0.80,0.15,0.20) $r_{11}^{(1)} = (T_{11}^{(1)}, I_{11}^{(1)}, F_{11}^{(1)})$ | MG (0.60,0.35,0.40) $r_{11}^{(2)} = (T_{11}^{(2)}, I_{11}^{(2)}, F_{11}^{(2)})$ | VG (0.80,0.15,0.20) $r_{11}^{(3)} = (T_{11}^{(3)}, I_{11}^{(3)}, F_{11}^{(3)})$ | G (0.70,0.25,0.30) $r_{11}^{(4)} = (T_{11}^{(4)}, I_{11}^{(4)}, F_{11}^{(4)})$ |
| | A ₂ | G (0.70,0.25,0.30) $r_{21}^{(1)} = (T_{21}^{(1)}, I_{21}^{(1)}, F_{21}^{(1)})$ | VG (0.80,0.15,0.20) $r_{21}^{(2)} = (T_{21}^{(2)}, I_{21}^{(2)}, F_{21}^{(2)})$ | MG (0.60,0.35,0.40) $r_{21}^{(3)} = (T_{21}^{(3)}, I_{21}^{(3)}, F_{21}^{(3)})$ | MG (0.60,0.35,0.40) $r_{21}^{(4)} = (T_{21}^{(4)}, I_{21}^{(4)}, F_{21}^{(4)})$ |
| | A ₃ | M (0.50,0.50,0.50) $r_{31}^{(1)} = (T_{31}^{(1)}, I_{31}^{(1)}, F_{31}^{(1)})$ | G (0.70,0.25,0.30) $r_{31}^{(2)} = (T_{31}^{(2)}, I_{31}^{(2)}, F_{31}^{(2)})$ | MG (0.60,0.35,0.40) $r_{31}^{(3)} = (T_{31}^{(3)}, I_{31}^{(3)}, F_{31}^{(3)})$ | M (0.50,0.50,0.50) $r_{31}^{(4)} = (T_{31}^{(4)}, I_{31}^{(4)}, F_{31}^{(4)})$ |
| | A ₄ | G (0.70,0.25,0.30) $r_{41}^{(1)} = (T_{41}^{(1)}, I_{41}^{(1)}, F_{41}^{(1)})$ | MG (0.60,0.35,0.40) $r_{41}^{(2)} = (T_{41}^{(2)}, I_{41}^{(2)}, F_{41}^{(2)})$ | G (0.70,0.25,0.30) $r_{41}^{(3)} = (T_{41}^{(3)}, I_{41}^{(3)}, F_{41}^{(3)})$ | MG (0.60,0.35,0.40) $r_{41}^{(4)} = (T_{41}^{(4)}, I_{41}^{(4)}, F_{41}^{(4)})$ |
| | A ₅ | MG (0.60,0.35,0.40) $r_{51}^{(1)} = (T_{51}^{(1)}, I_{51}^{(1)}, F_{51}^{(1)})$ | G (0.70,0.25,0.30) $r_{51}^{(2)} = (T_{51}^{(2)}, I_{51}^{(2)}, F_{51}^{(2)})$ | VG (0.80,0.15,0.20) $r_{51}^{(3)} = (T_{51}^{(3)}, I_{51}^{(3)}, F_{51}^{(3)})$ | VG (0.80,0.15,0.20) $r_{51}^{(4)} = (T_{51}^{(4)}, I_{51}^{(4)}, F_{51}^{(4)})$ |
| X ₂ | A ₁ | G (0.70,0.25,0.30) $r_{12}^{(1)} = (T_{12}^{(1)}, I_{12}^{(1)}, F_{12}^{(1)})$ | G (0.70,0.25,0.30) $r_{12}^{(2)} = (T_{12}^{(2)}, I_{12}^{(2)}, F_{12}^{(2)})$ | MG (0.60,0.35,0.40) $r_{12}^{(3)} = (T_{12}^{(3)}, I_{12}^{(3)}, F_{12}^{(3)})$ | G (0.70,0.25,0.30) $r_{12}^{(4)} = (T_{12}^{(4)}, I_{12}^{(4)}, F_{12}^{(4)})$ |
| | A ₂ | VG (0.80,0.15,0.20) $r_{22}^{(1)} = (T_{22}^{(1)}, I_{22}^{(1)}, F_{22}^{(1)})$ | MG (0.60,0.35,0.40) $r_{22}^{(2)} = (T_{22}^{(2)}, I_{22}^{(2)}, F_{22}^{(2)})$ | M (0.50,0.50,0.50) $r_{22}^{(3)} = (T_{22}^{(3)}, I_{22}^{(3)}, F_{22}^{(3)})$ | MG (0.60,0.35,0.40) $r_{22}^{(4)} = (T_{22}^{(4)}, I_{22}^{(4)}, F_{22}^{(4)})$ |
| | A ₃ | M (0.50,0.50,0.50) $r_{32}^{(1)} = (T_{32}^{(1)}, I_{32}^{(1)}, F_{32}^{(1)})$ | VG (0.80,0.15,0.20) $r_{32}^{(2)} = (T_{32}^{(2)}, I_{32}^{(2)}, F_{32}^{(2)})$ | G (0.70,0.25,0.30) $r_{32}^{(3)} = (T_{32}^{(3)}, I_{32}^{(3)}, F_{32}^{(3)})$ | G (0.70,0.25,0.30) $r_{32}^{(4)} = (T_{32}^{(4)}, I_{32}^{(4)}, F_{32}^{(4)})$ |
| | A ₄ | MG (0.60,0.35,0.40) $r_{42}^{(1)} = (T_{42}^{(1)}, I_{42}^{(1)}, F_{42}^{(1)})$ | M (0.50,0.50,0.50) $r_{42}^{(2)} = (T_{42}^{(2)}, I_{42}^{(2)}, F_{42}^{(2)})$ | VG (0.80,0.15,0.20) $r_{42}^{(3)} = (T_{42}^{(3)}, I_{42}^{(3)}, F_{42}^{(3)})$ | M (0.50,0.50,0.50) $r_{42}^{(4)} = (T_{42}^{(4)}, I_{42}^{(4)}, F_{42}^{(4)})$ |
| | A ₅ | G (0.70,0.25,0.30) $r_{52}^{(1)} = (T_{52}^{(1)}, I_{52}^{(1)}, F_{52}^{(1)})$ | G (0.70,0.25,0.30) $r_{52}^{(2)} = (T_{52}^{(2)}, I_{52}^{(2)}, F_{52}^{(2)})$ | MG (0.60,0.35,0.40) $r_{52}^{(3)} = (T_{52}^{(3)}, I_{52}^{(3)}, F_{52}^{(3)})$ | VG (0.80,0.15,0.20) $r_{52}^{(4)} = (T_{52}^{(4)}, I_{52}^{(4)}, F_{52}^{(4)})$ |
| X ₃ | A ₁ | MG (0.60,0.35,0.40) | MG (0.60,0.35,0.40) | M (0.50,0.50,0.50) | M (0.50,0.50,0.50) |

| | | | | | |
|----------------|----------------|---|---|---|---|
| | | $r_{13}^{(1)} = (T_{13}^{(1)}, I_{13}^{(1)}, F_{13}^{(1)})$ | $r_{13}^{(2)} = (T_{13}^{(2)}, I_{13}^{(2)}, F_{13}^{(2)})$ | $r_{13}^{(3)} = (T_{13}^{(3)}, I_{13}^{(3)}, F_{13}^{(3)})$ | $r_{13}^{(4)} = (T_{13}^{(4)}, I_{13}^{(4)}, F_{13}^{(4)})$ |
| | A ₂ | VG (0.80,0.15,0.20) | G (0.70,0.25,0.30) | VG (0.80,0.15,0.20) | VG (0.80,0.15,0.20) |
| | | $r_{23}^{(1)} = (T_{23}^{(1)}, I_{23}^{(1)}, F_{23}^{(1)})$ | $r_{23}^{(2)} = (T_{23}^{(2)}, I_{23}^{(2)}, F_{23}^{(2)})$ | $r_{23}^{(3)} = (T_{23}^{(3)}, I_{23}^{(3)}, F_{23}^{(3)})$ | $r_{23}^{(4)} = (T_{23}^{(4)}, I_{23}^{(4)}, F_{23}^{(4)})$ |
| | A ₃ | M (0.50,0.50,0.50) | G (0.70,0.25,0.30) | MG (0.60,0.35,0.40) | MG (0.60,0.35,0.40) |
| | | $r_{33}^{(1)} = (T_{33}^{(1)}, I_{33}^{(1)}, F_{33}^{(1)})$ | $r_{33}^{(2)} = (T_{33}^{(2)}, I_{33}^{(2)}, F_{33}^{(2)})$ | $r_{33}^{(3)} = (T_{33}^{(3)}, I_{33}^{(3)}, F_{33}^{(3)})$ | $r_{33}^{(4)} = (T_{33}^{(4)}, I_{33}^{(4)}, F_{33}^{(4)})$ |
| | A ₄ | G (0.70,0.25,0.30) | MG (0.60,0.35,0.40) | G (0.70,0.25,0.30) | MG (0.60,0.35,0.40) |
| | | $r_{43}^{(1)} = (T_{43}^{(1)}, I_{43}^{(1)}, F_{43}^{(1)})$ | $r_{43}^{(2)} = (T_{43}^{(2)}, I_{43}^{(2)}, F_{43}^{(2)})$ | $r_{43}^{(3)} = (T_{43}^{(3)}, I_{43}^{(3)}, F_{43}^{(3)})$ | $r_{43}^{(4)} = (T_{43}^{(4)}, I_{43}^{(4)}, F_{43}^{(4)})$ |
| | A ₅ | MG (0.60,0.35,0.40) | G (0.70,0.25,0.30) | VG (0.80,0.15,0.20) | G (0.70,0.25,0.30) |
| X ₄ | A ₁ | $r_{53}^{(1)} = (T_{53}^{(1)}, I_{53}^{(1)}, F_{53}^{(1)})$ | $r_{53}^{(2)} = (T_{53}^{(2)}, I_{53}^{(2)}, F_{53}^{(2)})$ | $r_{53}^{(3)} = (T_{53}^{(3)}, I_{53}^{(3)}, F_{53}^{(3)})$ | $r_{53}^{(4)} = (T_{53}^{(4)}, I_{53}^{(4)}, F_{53}^{(4)})$ |
| | | G (0.70,0.25,0.30) | M (0.50,0.50,0.50) | MG (0.60,0.35,0.40) | M (0.50,0.50,0.50) |
| | | $r_{14}^{(1)} = (T_{14}^{(1)}, I_{14}^{(1)}, F_{14}^{(1)})$ | $r_{14}^{(2)} = (T_{14}^{(2)}, I_{14}^{(2)}, F_{14}^{(2)})$ | $r_{14}^{(3)} = (T_{14}^{(3)}, I_{14}^{(3)}, F_{14}^{(3)})$ | $r_{14}^{(4)} = (T_{14}^{(4)}, I_{14}^{(4)}, F_{14}^{(4)})$ |
| | A ₂ | VG (0.80,0.15,0.20) | VG (0.80,0.15,0.20) | M (0.50,0.50,0.50) | G (0.70,0.25,0.30) |
| | | $r_{24}^{(1)} = (T_{24}^{(1)}, I_{24}^{(1)}, F_{24}^{(1)})$ | $r_{24}^{(2)} = (T_{24}^{(2)}, I_{24}^{(2)}, F_{24}^{(2)})$ | $r_{24}^{(3)} = (T_{24}^{(3)}, I_{24}^{(3)}, F_{24}^{(3)})$ | $r_{24}^{(4)} = (T_{24}^{(4)}, I_{24}^{(4)}, F_{24}^{(4)})$ |
| | A ₃ | MG (0.60,0.35,0.40) | MG (0.60,0.35,0.40) | MG (0.60,0.35,0.40) | MG (0.60,0.35,0.40) |
| | | $r_{34}^{(1)} = (T_{34}^{(1)}, I_{34}^{(1)}, F_{34}^{(1)})$ | $r_{34}^{(2)} = (T_{34}^{(2)}, I_{34}^{(2)}, F_{34}^{(2)})$ | $r_{34}^{(3)} = (T_{34}^{(3)}, I_{34}^{(3)}, F_{34}^{(3)})$ | $r_{34}^{(4)} = (T_{34}^{(4)}, I_{34}^{(4)}, F_{34}^{(4)})$ |
| | A ₄ | M (0.50,0.50,0.50) | MB (0.40,0.65,0.60) | MG (0.60,0.35,0.40) | VG (0.80,0.15,0.20) |
| | | $r_{44}^{(1)} = (T_{44}^{(1)}, I_{44}^{(1)}, F_{44}^{(1)})$ | $r_{44}^{(2)} = (T_{44}^{(2)}, I_{44}^{(2)}, F_{44}^{(2)})$ | $r_{44}^{(3)} = (T_{44}^{(3)}, I_{44}^{(3)}, F_{44}^{(3)})$ | $r_{44}^{(4)} = (T_{44}^{(4)}, I_{44}^{(4)}, F_{44}^{(4)})$ |
| | A ₅ | MG (0.60,0.35,0.40) | G (0.70,0.25,0.30) | VG (0.80,0.15,0.20) | G (0.70,0.25,0.30) |
| X ₅ | A ₁ | $r_{54}^{(1)} = (T_{54}^{(1)}, I_{54}^{(1)}, F_{54}^{(1)})$ | $r_{54}^{(2)} = (T_{54}^{(2)}, I_{54}^{(2)}, F_{54}^{(2)})$ | $r_{54}^{(3)} = (T_{54}^{(3)}, I_{54}^{(3)}, F_{54}^{(3)})$ | $r_{54}^{(4)} = (T_{54}^{(4)}, I_{54}^{(4)}, F_{54}^{(4)})$ |
| | | M (0.50,0.50,0.50) | MG (0.60,0.35,0.40) | VG (0.80,0.15,0.20) | M (0.50,0.50,0.50) |
| | | $r_{15}^{(1)} = (T_{15}^{(1)}, I_{15}^{(1)}, F_{15}^{(1)})$ | $r_{15}^{(2)} = (T_{15}^{(2)}, I_{15}^{(2)}, F_{15}^{(2)})$ | $r_{15}^{(3)} = (T_{15}^{(3)}, I_{15}^{(3)}, F_{15}^{(3)})$ | $r_{15}^{(4)} = (T_{15}^{(4)}, I_{15}^{(4)}, F_{15}^{(4)})$ |
| | A ₂ | VG (0.80,0.15,0.20) | M (0.50,0.50,0.50) | G (0.70,0.25,0.30) | G (0.70,0.25,0.30) |
| | | $r_{25}^{(1)} = (T_{25}^{(1)}, I_{25}^{(1)}, F_{25}^{(1)})$ | $r_{25}^{(2)} = (T_{25}^{(2)}, I_{25}^{(2)}, F_{25}^{(2)})$ | $r_{25}^{(3)} = (T_{25}^{(3)}, I_{25}^{(3)}, F_{25}^{(3)})$ | $r_{25}^{(4)} = (T_{25}^{(4)}, I_{25}^{(4)}, F_{25}^{(4)})$ |
| | A ₃ | G (0.70,0.25,0.30) | G (0.70,0.25,0.30) | M (0.50,0.50,0.50) | MG (0.60,0.35,0.40) |
| | | $r_{35}^{(1)} = (T_{35}^{(1)}, I_{35}^{(1)}, F_{35}^{(1)})$ | $r_{35}^{(2)} = (T_{35}^{(2)}, I_{35}^{(2)}, F_{35}^{(2)})$ | $r_{35}^{(3)} = (T_{35}^{(3)}, I_{35}^{(3)}, F_{35}^{(3)})$ | $r_{35}^{(4)} = (T_{35}^{(4)}, I_{35}^{(4)}, F_{35}^{(4)})$ |
| | A ₄ | M (0.50,0.50,0.50) | M (0.50,0.50,0.50) | MG (0.60,0.35,0.40) | G (0.70,0.25,0.30) |
| | | $r_{45}^{(1)} = (T_{45}^{(1)}, I_{45}^{(1)}, F_{45}^{(1)})$ | $r_{45}^{(2)} = (T_{45}^{(2)}, I_{45}^{(2)}, F_{45}^{(2)})$ | $r_{45}^{(3)} = (T_{45}^{(3)}, I_{45}^{(3)}, F_{45}^{(3)})$ | $r_{45}^{(4)} = (T_{45}^{(4)}, I_{45}^{(4)}, F_{45}^{(4)})$ |
| | A ₅ | G (0.70,0.25,0.30) | VG (0.80,0.15,0.20) | VG (0.80,0.15,0.20) | VG (0.80,0.15,0.20) |
| | | $r_{55}^{(1)} = (T_{55}^{(1)}, I_{55}^{(1)}, F_{55}^{(1)})$ | $r_{55}^{(2)} = (T_{55}^{(2)}, I_{55}^{(2)}, F_{55}^{(2)})$ | $r_{55}^{(3)} = (T_{55}^{(3)}, I_{55}^{(3)}, F_{55}^{(3)})$ | $r_{55}^{(4)} = (T_{55}^{(4)}, I_{55}^{(4)}, F_{55}^{(4)})$ |

Table 4. Importance and Weights of Decision-Makers

| | DM ₁ | DM ₂ | DM ₃ | DM ₄ |
|-----------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| Linguistic Variables | VI(0.90,0.10,0.10) | I (0.75,0.25,0.20) | M (0.50,0.50,0.50) | UI (0.35,0.75,0.80) |
| Weights | $(T_1^{dm}, I_1^{dm}, F_1^{dm})$ | $(T_2^{dm}, I_2^{dm}, F_2^{dm})$ | $(T_3^{dm}, I_3^{dm}, F_3^{dm})$ | $(T_4^{dm}, I_4^{dm}, F_4^{dm})$ |
| | $\lambda_{DM_1} = 0.37045$ | $\lambda_{DM_2} = 0.31508$ | $\lambda_{DM_3} = 0.20580$ | $\lambda_{DM_4} = 0.10867$ |

Step 2: Computation of Aggregated Single Valued Neutrosophic Decision Matrix (ASVNDM)

To find the ASVNDM not only the weights of the DMs, but the alternative ratings are also required.

The alternative ratings, according to the DMs given in the following table.

Now by using Equation 3, alternative ratings $r_{ij}^{(k)}$ and the DM weights λ_k we get

$$r_{ij} = \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \lambda_3 r_{ij}^{(3)} \oplus \dots \oplus \lambda_l r_{ij}^{(l)}$$

$$r_{ij} = (1 - \prod_{k=1}^l (1 - T_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^l (I_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^l (F_{ij}^{(k)})^{\lambda_k})$$

where $i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4, 5$ and $(l = 4)$.

For $i = j = 1$ and $l = 4$

$$r_{11} = \lambda_1 r_{11}^{(1)} \oplus \lambda_2 r_{11}^{(2)} \oplus \lambda_3 r_{11}^{(3)} \oplus \dots \oplus \lambda_l r_{11}^{(l)}$$

$$r_{11} = (1 - \prod_{k=1}^4 (1 - T_{11}^{(k)})^{\lambda_k}, \prod_{k=1}^4 (I_{11}^{(k)})^{\lambda_k}, \prod_{k=1}^4 (F_{11}^{(k)})^{\lambda_k})$$

$$r_{11} = (1 - (1 - T_{11}^{(1)})^{\lambda_1} (1 - T_{11}^{(2)})^{\lambda_2} (1 - T_{11}^{(3)})^{\lambda_3} (1 - T_{11}^{(4)})^{\lambda_4}, (I_{11}^{(1)})^{\lambda_1} (I_{11}^{(2)})^{\lambda_2} (I_{11}^{(3)})^{\lambda_3} (I_{11}^{(4)})^{\lambda_4}, (F_{11}^{(1)})^{\lambda_1} (F_{11}^{(2)})^{\lambda_2} (F_{11}^{(3)})^{\lambda_3} (F_{11}^{(4)})^{\lambda_4})$$

$$r_{11} = (1 - ((1 - 0.8)^{0.37045} (1 - 0.6)^{0.31508} (1 - 0.8)^{0.20580} (1 - 0.7)^{0.10867}), ((0.15)^{0.37045} (0.35)^{0.31508} (0.15)^{0.20580} (0.25)^{0.10867}), ((0.20)^{0.37045} (0.40)^{0.31508} (0.20)^{0.20580} (0.30)^{0.10867}))$$

$$r_{11} = (0.740, 0.207, 0.260)$$

Similarly, we can find other values

- $r_{21} = (0.711, 0.237, 0.289)$
- $r_{31} = (0.593, 0.373, 0.407)$
- $r_{41} = (0.661, 0.288, 0.339)$
- $r_{51} = (0.706, 0.241, 0.294)$
- $r_{12} = (0.682, 0.268, 0.318)$
- $r_{22} = (0.676, 0.275, 0.324)$
- $r_{32} = (0.681, 0.275, 0.324)$
- $r_{42} = (0.619, 0.342, 0.381)$
- $r_{52} = (0.695, 0.253, 0.305)$
- $r_{13} = (0.505, 0.392, 0.429)$
- $r_{23} = (0.773, 0.176, 0.227)$
- $r_{33} = (0.603, 0.359, 0.397)$
- $r_{43} = (0.661, 0.288, 0.339)$
- $r_{53} = (0.693, 0.255, 0.307)$
- $r_{14} = (0.605, 0.359, 0.395)$
- $r_{24} = (0.748, 0.203, 0.252)$
- $r_{34} = (0.600, 0.350, 0.400)$
- $r_{44} = (0.542, 0.443, 0.458)$
- $r_{54} = (0.693, 0.339, 0.307)$
- $r_{15} = (0.614, 0.349, 0.386)$
- $r_{25} = (0.697, 0.257, 0.303)$
- $r_{35} = (0.656, 0.299, 0.344)$
- $r_{45} = (0.548, 0.431, 0.452)$
- $r_{55} = (0.768, 0.181, 0.232)$

Table 5. Aggregated Single Valued Neutrosophic Decision Matrix $D = [r_{ij}]_{5 \times 4}$

| | X_1 | X_2 | X_3 | X_4 | X_5 |
|-------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| A_1 | $r_{11} = (0.740, 0.207, 0.260)$ | $r_{12} = (0.682, 0.268, 0.318)$ | $r_{13} = (0.505, 0.392, 0.429)$ | $r_{14} = (0.605, 0.359, 0.395)$ | $r_{15} = (0.614, 0.349, 0.386)$ |
| A_2 | $r_{21} = (0.711, 0.237, 0.289)$ | $r_{22} = (0.676, 0.275, 0.324)$ | $r_{23} = (0.773, 0.176, 0.227)$ | $r_{24} = (0.748, 0.203, 0.252)$ | $r_{25} = (0.697, 0.257, 0.303)$ |

| | | | | | |
|----|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| A3 | $r_{31} = (0.593, 0.373, 0.407)$ | $r_{32} = (0.681, 0.275, 0.324)$ | $r_{33} = (0.603, 0.359, 0.397)$ | $r_{34} = (0.600, 0.350, 0.400)$ | $r_{35} = (0.656, 0.299, 0.344)$ |
| A4 | $r_{41} = (0.661, 0.288, 0.339)$ | $r_{42} = (0.619, 0.342, 0.381)$ | $r_{43} = (0.661, 0.288, 0.339)$ | $r_{43} = (0.661, 0.288, 0.339)$ | $r_{45} = (0.548, 0.431, 0.452)$ |
| A5 | $r_{51} = (0.706, 0.241, 0.294)$ | $r_{52} = (0.695, 0.253, 0.305)$ | $r_{53} = (0.693, 0.255, 0.307)$ | $r_{54} = (0.693, 0.339, 0.307)$ | $r_{55} = (0.768, 0.181, 0.232)$ |

Step 3: Computation of the weights of the criteria

The individual weights given by each DM is given in Table 6.

Table 6. Weights of alternatives determined by the DMs $w_j^{(k)} = (T_j^{(k)}, I_j^{(k)}, F_j^{(k)})$

| Criteria | DM1 | DM2 | DM3 | DM4 |
|---------------------|--|--|--|--|
| X1 (DELIVERY) | VI (0.90,0.10,0.10) $w_1^{(1)} = (T_1^{(1)}, I_1^{(1)}, F_1^{(1)})$ | VI (0.90,0.10,0.10) $w_1^{(2)} = (T_1^{(2)}, I_1^{(2)}, F_1^{(2)})$ | VI (0.90,0.10,0.10) $w_1^{(3)} = (T_1^{(3)}, I_1^{(3)}, F_1^{(3)})$ | I (0.75,0.25,0.20) $w_1^{(4)} = (T_1^{(4)}, I_1^{(4)}, F_1^{(4)})$ |
| X2 (QUALITY) | I (0.75,0.25,0.20) $w_2^{(1)} = (T_2^{(1)}, I_2^{(1)}, F_2^{(1)})$ | M (0.50,0.50,0.50) $w_2^{(2)} = (T_2^{(2)}, I_2^{(2)}, F_2^{(2)})$ | M (0.50,0.50,0.50) $w_2^{(3)} = (T_2^{(3)}, I_2^{(3)}, F_2^{(3)})$ | I (0.75,0.25,0.20) $w_2^{(4)} = (T_2^{(4)}, I_2^{(4)}, F_2^{(4)})$ |
| X3 (FLEXIBILITY) | VI (0.90,0.10,0.10) $w_3^{(1)} = (T_3^{(1)}, I_3^{(1)}, F_3^{(1)})$ | VI (0.90,0.10,0.10) $w_3^{(2)} = (T_3^{(2)}, I_3^{(2)}, F_3^{(2)})$ | I (0.75,0.25,0.20) $w_3^{(3)} = (T_3^{(3)}, I_3^{(3)}, F_3^{(3)})$ | VI (0.90,0.10,0.10) $w_3^{(4)} = (T_3^{(4)}, I_3^{(4)}, F_3^{(4)})$ |
| X4 (SERVICE) | I (0.75,0.25,0.20) $w_4^{(1)} = (T_4^{(1)}, I_4^{(1)}, F_4^{(1)})$ | I (0.75,0.25,0.20) $w_4^{(2)} = (T_4^{(2)}, I_4^{(2)}, F_4^{(2)})$ | M (0.50,0.50,0.50) $w_4^{(3)} = (T_4^{(3)}, I_4^{(3)}, F_4^{(3)})$ | UI (0.35,0.75,0.80) $w_4^{(4)} = (T_4^{(4)}, I_4^{(4)}, F_4^{(4)})$ |
| X5 (PRICE) | M (0.50,0.50,0.50) $w_5^{(1)} = (T_5^{(1)}, I_5^{(1)}, F_5^{(1)})$ | M (0.50,0.50,0.50) $w_5^{(2)} = (T_5^{(2)}, I_5^{(2)}, F_5^{(2)})$ | VI (0.90,0.10,0.10) $w_5^{(3)} = (T_5^{(3)}, I_5^{(3)}, F_5^{(3)})$ | VI (0.90,0.10,0.10) $w_5^{(4)} = (T_5^{(4)}, I_5^{(4)}, F_5^{(4)})$ |

By using the values from Table 6, the aggregated criteria weights are calculated as follows

$$w_j = (T_j, I_j, F_j) = \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \lambda_3 w_j^{(3)} \oplus \dots \oplus \lambda_l w_j^{(l)}$$

$$w_j = (1 - \prod_{k=1}^l (1 - T_j^{(k)})^{\lambda_k}, \prod_{k=1}^l (I_j^{(k)})^{\lambda_k}, \prod_{k=1}^l (F_j^{(k)})^{\lambda_k}) \text{ where } j = 1, 2, 3, 4, 5 \text{ and } (l = 4).$$

For j = 1 and l = 4

$$w_1 = \lambda_1 w_1^{(1)} \oplus \lambda_2 w_1^{(2)} \oplus \lambda_3 w_1^{(3)} \oplus \lambda_4 w_1^{(4)}$$

$$w_1 = (1 - \prod_{k=1}^4 (1 - T_1^{(k)})^{\lambda_k}, \prod_{k=1}^4 (I_1^{(k)})^{\lambda_k}, \prod_{k=1}^4 (F_1^{(k)})^{\lambda_k})$$

$$w_1 = (1 - (1 - T_1^{(1)})^{\lambda_1} (1 - T_1^{(2)})^{\lambda_2} (1 - T_1^{(3)})^{\lambda_3} (1 - T_1^{(4)})^{\lambda_4}, (I_1^{(1)})^{\lambda_1} (I_1^{(2)})^{\lambda_2} (I_1^{(3)})^{\lambda_3} (I_1^{(4)})^{\lambda_4}, (F_1^{(1)})^{\lambda_1} (F_1^{(2)})^{\lambda_2} (F_1^{(3)})^{\lambda_3} (F_1^{(4)})^{\lambda_4})$$

$$w_1 = (1 - ((1 - 0.9)^{0.37045} (1 - 0.9)^{0.31508} (1 - 0.9)^{0.20580} (1 - 0.75)^{0.10867}), ((0.10)^{0.37045} (0.10)^{0.31508} (0.10)^{0.20580} (0.25)^{0.10867}), ((0.10)^{0.37045} (0.10)^{0.31508} (0.10)^{0.20580} (0.20)^{0.10867}))$$

$$r_{11} = (0.740, 0.207, 0.260)$$

$$w_1 = (T_1, I_1, F_1) = (0.890, 0.110, 0.108)$$

Similarly, we can get other values

Therefore

$$W_{\{X_1, X_2, X_3, X_4\}} = \begin{bmatrix} (0.890, 0.110, 0.108)^T \\ (0.641, 0.359, 0.322) \\ (0.879, 0.121, 0.115) \\ (0.680, 0.325, 0.281) \\ (0.699, 0.301, 0.301) \end{bmatrix}$$

Step 4: Construction of Aggregated Weighted Single Valued Neutrosophic Decision Matrix (AWSVNDM)

After finding the weights of the criteria and the alternative ratings, the aggregated weighted single-valued neutrosophic ratings are calculated by using Equation 4 as follows:

$$r'_{ij} = (T'_{ij}, I'_{ij}, rF'_{ij}) = (T_{A_i}(x).T_j, I_{A_i}(x) + I_j - I_{A_i}(x).I_j, F_{A_i}(x) + F_j - F_{A_i}(x).F_j)$$

By using the above equation, we can get an aggregated weighted single-valued neutrosophic decision matrix.

Table 7. Aggregated Weighted Single Valued Neutrosophic Decision Matrix $R' = [r'_{ij}]_{5 \times 5}$

| | X_1 | X_2 | X_3 | X_4 | X_5 |
|-------|------------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| A_1 | $r'_{11} = (0.659, 0.294, 0.340)$ | $r'_{12} = (0.437, 0.531, 0.538)$ | $r'_{13} = (0.444, 0.466, 0.495)$ | $r'_{14} = (0.411, 0.567, 0.565)$ | $r'_{15} = (0.429, 0.545, 0.571)$ |
| A_2 | $r'_{21} = (0.633, 0.321, 0.366)$ | $r'_{22} = (0.433, 0.535, 0.542)$ | $r'_{23} = (0.679, 0.276, 0.316)$ | $r'_{24} = (0.509, 0.462, 0.462)$ | $r'_{25} = (0.487, 0.481, 0.513)$ |
| A_3 | $r'_{31} = (0.528, 0.442, 0.471)$ | $r'_{32} = (0.437, 0.535, 0.542)$ | $r'_{33} = (0.530, 0.437, 0.466)$ | $r'_{34} = (0.408, 0.561, 0.569)$ | $r'_{35} = (0.459, 0.510, 0.541)$ |
| A_4 | $r'_{41} = (0.588, 0.366, 0.410)$ | $r'_{42} = (0.397, 0.578, 0.580)$ | $r'_{43} = (0.581, 0.374, 0.415)$ | $r'_{44} = (0.037, 0.624, 0.610)$ | $r'_{45} = (0.383, 0.602, 0.617)$ |
| A_5 | $r'_{51} = (0.628, 0.324, 0.3700)$ | $r'_{52} = (0.445, 0.521, 0.529)$ | $r'_{53} = (0.609, 0.345, 0.387)$ | $r'_{54} = (0.471, 0.554, 0.502)$ | $r'_{55} = (0.537, 0.428, 0.463)$ |

Step 5: Computation of SVN-PIS and SVN-NIS

Since Delivery, Quality, Flexibility, and Services are benefit criteria that is why they are in the set

$$J_1 = \{X_1, X_2, X_3, X_4\}$$

whereas Price being the cost criteria, so it is in the set $J_2 = \{X_5\}$ SVN-PIS and SVN-NIS are calculated as,

Table 8. SVN-PIS and SVN-NIS

| SVN-PIS | SVN-NIS |
|--|--|
| $T_1^+ = \max \{0.659, 0.633, 0.528, 0.588, 0.628\} = 0.659$ | $T_1^- = \min \{0.659, 0.633, 0.528, 0.588, 0.628\} = 0.528$ |
| $I_1^+ = \min \{0.294, 0.321, 0.442, 0.366, 0.324\} = 0.294$ | $I_1^- = \max \{0.294, 0.321, 0.442, 0.366, 0.324\} = 0.442$ |
| $F_1^+ = \min \{0.340, 0.366, 0.471, 0.410, 0.370\} = 0.340$ | $F_1^- = \max \{0.340, 0.366, 0.471, 0.410, 0.370\} = 0.471$ |
| $T_2^+ = \max \{0.437, 0.433, 0.437, 0.397, 0.445\} = 0.445$ | $T_2^- = \min \{0.437, 0.433, 0.437, 0.397, 0.445\} = 0.397$ |
| $I_2^+ = \min \{0.531, 0.535, 0.535, 0.578, 0.521\} = 0.521$ | $I_2^- = \max \{0.531, 0.535, 0.535, 0.578, 0.521\} = 0.578$ |
| $F_2^+ = \min \{0.538, 0.542, 0.542, 0.580, 0.529\} = 0.529$ | $F_2^- = \max \{0.538, 0.542, 0.542, 0.580, 0.529\} = 0.580$ |
| $T_3^+ = \max \{0.444, 0.679, 0.530, 0.581, 0.609\} = 0.679$ | $T_3^- = \min \{0.444, 0.679, 0.530, 0.581, 0.609\} = 0.444$ |
| $I_3^+ = \min \{0.466, 0.276, 0.437, 0.374, 0.345\} = 0.276$ | $I_3^- = \max \{0.466, 0.276, 0.437, 0.374, 0.345\} = 0.466$ |
| $F_3^+ = \min \{0.495, 0.316, 0.466, 0.415, 0.387\} = 0.316$ | $F_3^- = \max \{0.495, 0.316, 0.466, 0.415, 0.387\} = 0.495$ |
| $T_4^+ = \max \{0.411, 0.509, 0.408, 0.037, 0.471\} = 0.509$ | $T_4^- = \min \{0.411, 0.509, 0.408, 0.037, 0.471\} = 0.037$ |
| $I_4^+ = \min \{0.567, 0.462, 0.561, 0.624, 0.554\} = 0.462$ | $I_4^- = \max \{0.567, 0.462, 0.561, 0.624, 0.554\} = 0.624$ |
| $F_4^+ = \min \{0.565, 0.462, 0.569, 0.610, 0.502\} = 0.462$ | $F_4^- = \max \{0.565, 0.462, 0.569, 0.610, 0.502\} = 0.610$ |
| $T_5^+ = \min \{0.429, 0.487, 0.459, 0.383, 0.537\} = 0.383$ | $T_5^- = \max \{0.429, 0.487, 0.459, 0.383, 0.537\} = 0.537$ |
| $I_5^+ = \max \{0.545, 0.481, 0.510, 0.602, 0.428\} = 0.602$ | $I_5^- = \min \{0.545, 0.481, 0.510, 0.602, 0.428\} = 0.428$ |
| $F_5^+ = \max \{0.571, 0.513, 0.541, 0.617, 0.463\} = 0.617$ | $F_5^- = \min \{0.571, 0.513, 0.541, 0.617, 0.463\} = 0.463$ |

$$A^+ = \left\{ \begin{matrix} (0.659, 0.294, 0.340), \\ (0.445, 0.521, 0.529), \\ (0.679, 0.276, 0.316), \\ (0.509, 0.462, 0.462), \\ (0.383, 0.602, 0.617) \end{matrix} \right\} \quad A^- = \left\{ \begin{matrix} (0.528, 0.442, 0.471), \\ (0.397, 0.578, 0.580), \\ (0.444, 0.466, 0.495), \\ (0.037, 0.624, 0.610), \\ (0.537, 0.428, 0.463) \end{matrix} \right\}$$

Step 6: Computation of Separation Measures

Normalized Euclidean Distance Measure is used to find the negative and positive separation measures d^+ and d^- respectively by using Equation 7, 8. Now for the SVN-PIS, we use

$$d_i^+ = \left(\frac{1}{3n} \sum_{j=1}^n \left[\left(T_{A_i.W}(x_j) - T_{A^*W}(x_j) \right)^2 + \left(I_{A_i.W}(x_j) - I_{A^*W}(x_j) \right)^2 + \left(F_{A_i.W}(x_j) - F_{A^*W}(x_j) \right)^2 \right] \right)^{0.5}$$

For $i = 1$ and $n = 5$

$$d_1^+ = \left(\frac{1}{3(5)} \sum_{j=1}^5 \left[\left(T_{A_1.W}(x_j) - T_{A^*W}(x_j) \right)^2 + \left(I_{A_1.W}(x_j) - I_{A^*W}(x_j) \right)^2 + \left(F_{A_1.W}(x_j) - F_{A^*W}(x_j) \right)^2 \right] \right)^{0.5}$$

$$d_1^+ = \left(\frac{1}{15} \left[\begin{matrix} \left(T_{A_1.W}(X_1) - T_{A^*W}(X_1) \right)^2 + \left(I_{A_1.W}(X_1) - I_{A^*W}(X_1) \right)^2 + \left(F_{A_1.W}(X_1) - F_{A^*W}(X_1) \right)^2 + \\ \left(T_{A_1.W}(X_2) - T_{A^*W}(X_2) \right)^2 + \left(I_{A_1.W}(X_2) - I_{A^*W}(X_2) \right)^2 + \left(F_{A_1.W}(X_2) - F_{A^*W}(X_2) \right)^2 + \\ \left(T_{A_1.W}(X_3) - T_{A^*W}(X_3) \right)^2 + \left(I_{A_1.W}(X_3) - I_{A^*W}(X_3) \right)^2 + \left(F_{A_1.W}(X_3) - F_{A^*W}(X_3) \right)^2 + \\ \left(T_{A_1.W}(X_4) - T_{A^*W}(X_4) \right)^2 + \left(I_{A_1.W}(X_4) - I_{A^*W}(X_4) \right)^2 + \left(F_{A_1.W}(X_4) - F_{A^*W}(X_4) \right)^2 + \\ \left(T_{A_1.W}(X_5) - T_{A^*W}(X_5) \right)^2 + \left(I_{A_1.W}(X_5) - I_{A^*W}(X_5) \right)^2 + \left(F_{A_1.W}(X_5) - F_{A^*W}(X_5) \right)^2 \end{matrix} \right] \right)^{0.5}$$

$$d_1^+ = \left(\frac{1}{15} \left[\begin{matrix} (0.659 - 0.659)^2 + (0.294 - 0.294)^2 + (0.340 - 0.340)^2 + \\ (0.437 - 0.445)^2 + (0.531 - 0.521)^2 + (0.538 - 0.529)^2 + \\ (0.444 - 0.679)^2 + (0.466 - 0.276)^2 + (0.495 - 0.316)^2 + \\ (0.411 - 0.509)^2 + (0.567 - 0.462)^2 + (0.565 - 0.462)^2 + \\ (0.429 - 0.383)^2 + (0.545 - 0.602)^2 + (0.571 - 0.617)^2 \end{matrix} \right] \right)^{0.5}$$

$$d_1^+ = \left[\frac{1}{15} (0.000245 + 0.123366 + 0.031238 + 0.007481) \right]^{0.5}$$

$$d_1^+ = 0.1040$$

Similarly, we can find other separation measures.

Step 7: Computation of Relative Closeness Coefficient (RCC)

The RCC is calculated by using Equation 9.

$$RCC_i = \frac{d_i^-}{d_i^+ + d_i^-} ; i = 1, 2, 3, 4, 5$$

$$RCC_1 = \frac{d_1^-}{d_1^+ + d_1^-} = \frac{0.127532}{0.127532 + 0.104029} = 0.551$$

$$RCC_2 = 0.896$$

$$RCC_3 = 0.505$$

$$RCC_4 = 0.363$$

$$RCC_5 = 0.757$$

The separation measure and the value of relative closeness coefficient (RCC) expressed in the following Figure 2.

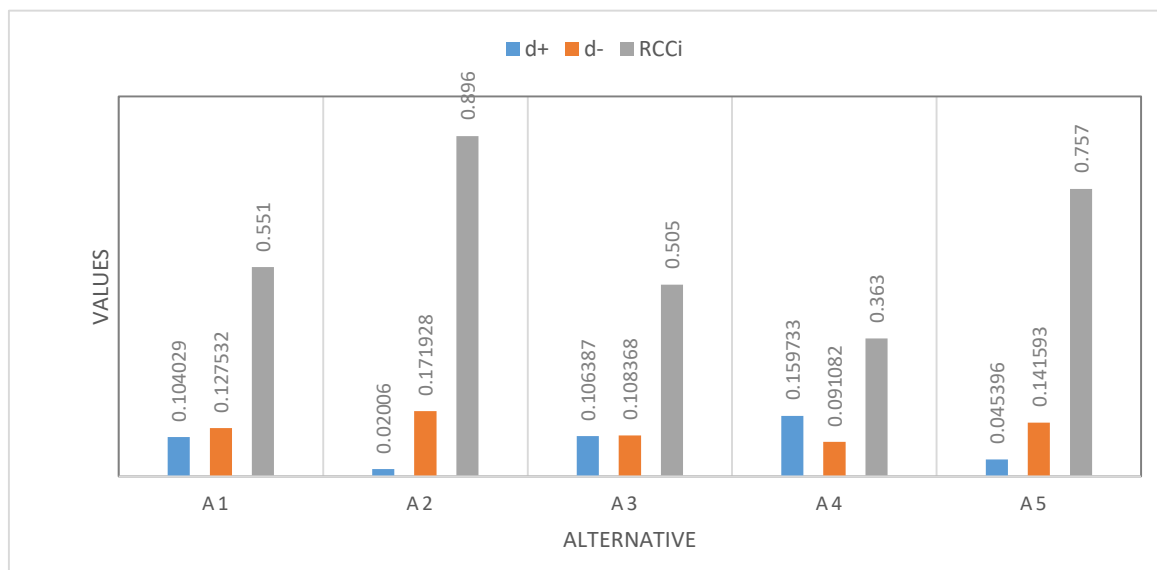


Figure 2. Separation measure and the RCC for each Alternative

Step 8: Ranking alternatives

From the above figure, we can see the RCC are ranked as follows

$$RCC_2 > RCC_5 > RCC_1 > RCC_3 > RCC_4 \Rightarrow A_2 > A_5 > A_1 > A_3 > A_4$$

By using the presented technique, we choose the best supplier for the production industry and observe that A2 is the best alternative.

5. Conclusion

In this paper, we studied the neutrosophic set and SVNss with some basic operations and developed the generalized neutrosophic TOPSIS by using single-valued neutrosophic numbers. By using crisp data, it is more difficult to solve decision-making problems in uncertain environments. Single valued neutrosophic sets can handle these limitations competently and provide the appropriate choice to decision-makers. We also developed the integrated model for neutrosophic TOPSIS. The closeness coefficient has been defined to compute the ranking of the alternatives by using an established approach under-considered environment. Moreover, for the justification of the proposed technique an illustrated example has been described for the selection of suppliers in the production industry. Consequently, relying upon the obtained results it can be confidently concluded that the proposed methodology indicates higher stability and usability for decision-makers in the DM process. Future research will surely concentrate upon presenting the TOPSIS technique based on correlation coefficient under-considered environment. The suggested approach can be applied to quite a lot of issues in real life, including the medical profession, robotics, artificial intelligence, pattern recognition, economics, etc.

Acknowledgment

This research is partially supported by a grant of National Natural Science Foundation of China (11971384).

References

1. L. A. Zadeh, Fuzzy Sets, *Information and Control*, 8(1965) 338–353.

2. I. B. Turksen, Interval Valued Fuzzy Sets Based on Normal Forms, *Fuzzy Sets and Systems*, 20(1986) 191–210.
3. K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy Sets Syst*, 20(1986) 87–96.
4. F. Smarandache, Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis, *Rehoboth Am. Res. Press* 1998.
5. H. Wang, F. Smarandache, R. Sunderraman, Y. Q. Zhang, Single valued neutrosophic sets. *Multisp. Multistructure*, 4(2010) 410–413.
6. F. Smarandache, A geometric interpretation of the neutrosophic set - A generalization of the intuitionistic fuzzy set. *Proc. - 2011 IEEE Int. Conf. Granul. Comput. GrC 2011* (2011) 602–606.
7. G. Shahzadi, M. Akram, A. B. Saeid, An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis. *Neutrosophic Sets Syst*, 18(2017) 80–88.
8. S. Broumi, F. Smarandache, Single Valued Neutrosophic Soft Expert Sets and Their Application in Decision Making. *J. New Theory* (2015) 67–88.
9. I. Deli, Y. Şubaş, A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *Int. J. Mach. Learn. Cybern*, 8(2017) 1309–1322.
10. J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *Int. J. Gen. Syst*, 42(2013) 386–394.
11. J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst*, 26(2014) 2459–2466.
12. R. Şahin, M. Yigider, A multi-criteria neutrosophic group decision making method based TOPSIS for supplier selection. *Appl. Math. Inf. Sci*, 10(2016) 1843–1852.
13. C. Yoon, K. Hwang, Multiple Attribute Decision Making: Methods and Applications. *A State Art Surv*, 1(1981).
14. S. J. J. Chen, C. L. Hwang, *Chapter 5 Fuzzy Multiple Attribute Decision Making Methods*; Springer-Verlag Berlin Heidelberg, 1992.
15. C. T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets Syst*, 114(2000) 1–9.
16. M. Zulqarnain, F. Dayan, M. Saeed. TOPSIS Analysis for The Prediction of Diabetes Based on General Characteristics of Humans. *Int. J. Pharm. Sci. Res*, 9(2018) 2932-2938.
17. M. Zulqarnain, F. Dayan, Selection Of Best Alternative For An Automotive Company By Intuitionistic Fuzzy TOPSIS Method. *Int. J. Sci. Technol. Res*, 6(2017) 126–132.
18. M. Zulqarnain, F. Dayan, Choose Best Criteria for Decision Making Via Fuzzy Topsis Method. *Math. Comput. Sci*. 2(2017) 113-119.
19. R. M. Zulqarnain, S. Abdal, A. Maalik, B. Ali, Z. Zafar, M. I. Ahamad, S. Younas, , A. Mariam, F. Dayan, Application of TOPSIS Method in Decision Making Via Soft Set. *Biomed J Sci Tech Res*, 24(2020), 19208–19215.
20. S. Pramanik, P. Dey, B. Giri, TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-attribute Decision Making Problems. *Neutrosophic Sets Syst*, 10(2015) 88–95.
21. R. M. Zulqarnain, X. L. Xin, M. Saeed, F. Smarandache, N. Ahmad, Generalized Neutrosophic TOPSIS to Solve Multi-Criteria Decision-Making Problems, *Neutrosophic Sets and Systems*, 38(2020), 276-292.
22. R. M. Zulqarnain, M. Saeed, B. Ali, S. Abdal, M. Saqlain, M. I. Ahamad, Z. Zafar. Generalized Fuzzy TOPSIS to Solve Multi-Criteria Decision-Making Problems, *Journal of New Theory*, 32 (2020) 40-50.

23. P. K. Maji, Neutrosophic soft set. *Ann. Fuzzy Math. Informatics*, 5(2013) 157–168.
24. I. Deli, S. Broumi, Neutrosophic soft matrices and NSM-decision making. *J. Intell. Fuzzy Syst*, 28(2015) 2233–2241.
25. H. M. Balami, Neutrosophic soft set and its application in multicriteria decision making problems. *Ann. Fuzzy Math. Informatics*, 18(2019) 245–271.
26. F. Karaaslan, Neutrosophic soft sets with applications in decision making. *International Journal of Information Science and Intelligent System*, 4 (2015) 1–20.
27. I. Deli, S. Eraslan, N. Çağman, ivnpiv-Neutrosophic soft sets and their decision making based on similarity measure. *Neural Comput. Appl*, 29(2018) 187–203.
28. A. Elhassouny, F. Smarandache, Neutrosophic-simplified-TOPSIS multi-criteria decision-making using combined simplified-TOPSIS method and neutrosophics. *2016 IEEE Int. Conf. Fuzzy Syst. FUZZ-IEEE 2016*, (2016) 2468–2474.
29. J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems. *Appl. Math. Model*, 38(2014) 1170–1175.
30. F. Smarandache, Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA. 1998, pp 105.
31. A. Saha, S. Broumi, New Operators on Interval Valued Neutrosophic Sets, *Neutrosophic sets and systems*, 28 (2019) 128-137.
32. I. Deli, Some operators with IVGSVTrN-numbers and their applications to multiple criteria group decision making, *Neutrosophic sets and systems*, 25 (2019) 33-53.
33. H. Hashim, L. Abdullah, A. Al-Quran, Interval Neutrosophic Vague Sets, *Neutrosophic sets and systems*, 25 (2019) 66-75.
34. M. Abdel-Basset, R. Mohamed, K. Sallam, M. Elhoseny, A novel decision-making model for sustainable supply chain finance under uncertainty environment. *Journal of Cleaner Production*, 269(2020) 122324.
35. M. Abdel-Basset, W. Ding, R. Mohamed, N. Metawa, An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries. *Risk Manag*, 22(2020) 192–218.
36. M. Abdel-Basset, R. Mohamed, A. N. H. Zaied, A. Gamal, F. Smarandache, Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In *Optimization Theory Based on Neutrosophic and Plithogenic Sets*. Academic Press, (2020) 1-19.
37. N. A. Nabeeh, F. Smarandache, M. Abdel-Basset, H. A. El-Ghareeb, A. Aboelfetouh, An integrated neutrosophic-topsis approach and its application to personnel selection: A new trend in brain processing and analysis. *IEEE Access*, 7(2019) 29734-29744.
38. N. A. Nabeeh, M. Abdel-Basset, H. A. El-Ghareeb, A. Aboelfetouh, Neutrosophic multi-criteria decision making approach for iot-based enterprises. *IEEE Access*, 7(2019) 59559-59574.
39. M. Abdel-Basset, N. A. Nabeeh, H. A. El-Ghareeb, A. Aboelfetouh, Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. *Enterprise Information Systems*, 14(9-10) (2020) 1304-1324.
40. M. Abdel-Basset, M. Saleh, A. Gamal, F. Smarandache, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Appl. Soft Comput*, (2019) <https://doi.org/10.1016/j.asoc.2019.01.035>.
41. M. Abdel-Basset, G. Manogaran, A. Gamal, F. Smarandache, A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection. *J. Med. Syst*, 43 (2019). <https://doi.org/10.1007/s10916-019-1156-1>

42. Imtiaz. M., Saqlain. M. and Saeed. M. (2020). TOPSIS for Multi Criteria Decision Making in Octagonal Intuitionistic Fuzzy Environment by Using Accuracy Function, *Journal of New Theory*, **31**, 32-40.
43. Saqlain, M., Jafar, M. N., Riaz, M. (2020). A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection. *Neutrosophic Sets and Systems*, 32 : 307-316. Doi: 10.5281/zenodo.3723161
44. Riaz M., Saqlain M. and Saeed M. (2019). Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis, *Punjab University Journal of Mathematics*, **51(8)**: 111-126.

Received: Sep 8, 2020. Accepted: Feb 10, 2021