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Algebraic Structures Of Neutrosophic Soft Sets

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Abstract: In this paper, we study the algebraic operations of neutrosophic soft sets and their basic properties associated with these opertaions. And also define the associativity and distributivity of these operations. We discusse different algebraic structures, such as monoids, semiring and lattices, of neutrosophic soft sets.

Keywords: Soft sets, Neutrosophic soft sets, monoid, semiring, lattices.

1 Introduction

During recent years soft set theory has gained popularity among the researchers due to its applications in various areas. Number of publications related to soft sets has risen exponentially. Theory of soft sets is proposed by Moldtsov in [16]. Basic aim of this theory is to introduce a mathematical model with enough parameters to handle uncertainty. Prior to soft set theory, probability theory, fuzzy set theory, rough set theory and interval mathematics were common tools to discuss uncertainty. But unfortunately difficulties were attached with these theories, for details see [11, 16]. As mentioned above soft set theory has enough number of parameters, so it is free from difficulties associated with other theories. Soft set theory has been applied to various fields very successfully.
The concept of neutrosophic set was introduced by Smarnandache [20]. The traditional neutrosophic sets is characterized by the truth value, indeterminate value and false value. Neutrosophic set is a mathematically tool for handling problems involving imprecise, indeterminacy inconsistent data and inconsistent information which exits in belief system.
Maji et al. proposed the concept of "Fuzzy Soft Sets" [13] and later on applied the theories in decision making problem [14, 15]. Different algebraic structures and their applications have also been studied in soft and fuzzy soft context [2, 19]. In [12] Maji proposed the concept of "Neutrosophic soft set" and applied the theories in decision making problem. Later Broumi and Smarandache defined the concepts of interval valued neutrosophic soft set and inituitionistic neutrosophic soft set in [3, 5]. Recently Sahin and Kucuk applied the concept of neutrosophic soft set in decision making problems [17,18]. Different algebraic structures and their application can be study in neutrosophic soft set context [4,7, 8, 9, 10]. In this paper we define some new operations on the neutrosophic soft set and modified results and laws are established. And also define the associativity and distributivity of these operations. The paper is organized in five sections. First we have given preliminaries on the theories of soft sets and neutrosophic sets. Section 3 completely describes for what new and modified operations define on neutrosophic soft set. In section 4 we have used new and modified definitions and operations to discuss the properties of associativity and distributivity of these operations for neutrosophic soft sets. Counter examples are provided to show the converse of proper inclusion is not true in general. In section 5 , monoids, semiring and lattices of neutrosophic soft sets associated with new operations have been determined completely.

2 Preliminaries

In this section we present the theory of neutrosophic sets and soft sets, taken from [1,20]， and some definitions and notions about algebraic structures are given.
Let X be a universe of discourse and a neutrosophic

set A on X is defined as
$A=\{\langle x,\ T_A(x),\ I_A(x),\ F_A(x)\rangle,\ x\in X\}$
where $T,\ I,\ F\ :X\to\ ]^-0,1^+[$ and
$^-0\le T_A(x)+I_A(x)+F_A(x)\le 3^+.$
Philosophical point of view, neutrosophic set takes the value from real standard or non standard subsets of $]^-0,1^+[$. But it is difficult to use neutrosophic set with value from real standard or non standard subsets of $]^-0,1^+[$ in real life application like scientific and engineering problems.

Definition 2.1:

A neutrosophic set A is contained in another neutrosophic set B *i.e.* $A\subseteq B$ if
$T_A(x)\le T_B(x),\ I_A(x)\le I_A(x),\ F_A(x)\ge F_B(x)$
$\forall\ x\in X.$

***Example 2.2*:**

Mr. X and his father wants to purchase a laptop. They have their expectations and perceptions. Based on these, they identify three criteria x_1, x_2, x_3 which are as follows
x_1=Performance, x_2=Size of laptop, x_3=Price of laptop
It may be assumed that the values of x_1, x_2, x_3 are in $[0,\ 1]$. The buyer consults with experts and also collects data from his own survey. The experts may impose their opinion in three components viz, the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set of $X=\{x_1,x_2,x_3\}$ such that

$$A=\left\{\begin{array}{l}\langle x_1,0.8,0.4,0.5\rangle,\langle x_2,0.7,0.2,0.4\rangle,\\ \langle x_3,0.8,0.3,0.4\rangle\end{array}\right\}.$$

Where the degree of goodness of performance (x_1) is 0.8 degree of indeterminacy of performance (x_1) is 0.4 and the degree of poorness of performance is 0.5 etc.

Definition 2.3:

Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denote the power set of U and let A be a non-empty subset of E. A pair $(F,\ A)$ is called soft set over U, where F is mapping given by $F\ :\ A\to P(U).$

Definition 2.4:

For two soft sets $(F,\ A)$ and $(G,\ B)$ over a common universe U, we say that $(F,\ A)$ is a soft subset of $(G,\ B)$ if
$(i)\quad A\subseteq B,$
$(ii)\quad F(e)\subseteq G(e)\ \ \forall\ \ e\in A.$
We write $(F,\ A)\subseteq(G,\ B).$

Definition 2.5:

Two soft sets $(F,\ A)$ and $(G,\ B)$ over a common universe U are said to be soft equal if $(F,\ A)$ is a soft subset of $(G,\ B)$ and $(G,\ B)$ is a soft subset of $(F,\ A).$

Definition 2.6:

Extended union of two soft sets $(F,\ A)$ and $(G,\ B)$ over the common universe U is the soft set $(H,\ C)$, where $C=A\cup B$ and for all $e\in C,$

$$H(e)=\begin{cases}F(e) & \text{if } e\in A-B\\ G(e) & \text{if } e\in B-A\\ F(e)\cup G(e) & \text{if } e\in A\cap B.\end{cases}$$

We write $(F,\ A)\cup_{\mathcal{E}}(G,\ B)=(H,\ C).$
Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over the same universe U, such that $A\cap B\neq\emptyset$. The restricted union of $(F,\ A)$ and $(G,\ B)$ is denoted by $(F,\ A)\cup_{\mathcal{R}}(G,\ B)$ and is defined as $(F,\ A)\cup_{\mathcal{R}}(G,\ B)=(H,\ C),$ where $C=A\cap B$ and for all $e\in C,$ $H(e)=F(e)\cup G(e).$

2.7 Definition:
The extended intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\mathbf{E}} (G, B) = (H, C)$.
Let (F, A) and (G, B) be two soft sets over the same universe U, such that $A \cap B \neq \Phi$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_{\mathbf{R}} (G, B)$ and is defined as $(F, A) \cap_{\mathbf{R}} (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

A semigroup $(S,*)$ is a non-empty set with an associative binary operation $*$. We use usual algebraic practice and write xy instead of $x * y$. If there exists an element e in S such that $ex = xe = x$ for all x in S then we say that S is a monoid and e is called the identity element. An element $x \in S$ is called idempotent if $xx = x$. If every element of S is idempotent then we say that S is idempotent.
A semiring is an algebraic structure consisting of a non-empty set R together with two associative binary operations, addition "+" and multiplication "." such that "." distributes over "+" from both sides. Semirings which are regarded as a generalization of rings. By a hemiring, we mean a semiring with a zero and with a commutative addition.
A Lattice $(L,\vee,\wedge)$ is a non-empty set with two binary operations $\vee$ and $\wedge$ such that
(1) $(L,\vee)$ is a commutative, idempotent semigroup,
(2) $(L,\wedge)$ is a commutative, idempotent semigroup,
(3) Absorption laws $a \vee (a \wedge b) = a$ and $a \wedge (a \vee b) = a$ hold for all $a, b \in L$.
If a lattice has identity elements with respect to both the operations then we say that it is bounded. Usually identity element of L with respect to operation $\wedge$ is denoted by 0 and whereas the identity element with respect to binary operation $\vee$ is denoted by 1. If a lattice L has identities and for each $a \in L$ there exists an element b such that $a \wedge b = 0$ and $a \vee b = 1$, then L is called complemented. If distributive laws hold in a lattice then it is called a distributive lattice.

3 Neutrosophic soft set

Definition 3.1[12]:

Let U be an initial universe set and E be the set of all parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. A pair (F, A) is termed to be the neutrosophic soft set (NSS) over U, where F is mapping given by $F : A \to P(U)$.

Example 3.2:

Let U be the set of calculators under consideration and E is the set of parameters. Consider

$$U = \left\{ \begin{array}{l} c_1 = scientific,\ c_2 = programmable, \\ c_3 = four \text{ function} \end{array} \right\}$$

and

$$E = \{e_1 = \text{performance},\ e_2 = size,\ e_3 = price\}$$

suppose that

$$F(e_1) = \left\{ \begin{array}{l} \langle c_1, 0.7, 0.4, 0.5\rangle, \langle c_2, 0.8, 0.5, 0.3\rangle \\ , \langle c_3, 0.4, 0.6, 0.8\rangle \end{array} \right\},$$

$$F(e_2)=\left\{\begin{array}{l}\langle c_1,0.6,0.7,0.8\rangle,\langle c_2,0.2,0.4,0.7\rangle,\\ \langle c_3,0.8,0.4,0.6\rangle\end{array}\right\},$$

$$F(e_3)=\left\{\begin{array}{l}\langle c_1,0.8,0.1,0.7\rangle,\langle c_2,0.5,0.8,0.9\rangle,\\ \langle c_3,1,0.3,0.4\rangle\end{array}\right\}.$$

The neutrosophic soft set $(F,\ E)$ is a parametrized family $\{F(e_i),\ i=1,2,3\}$ of all neutrosophic sets of U and describes a collection of approximation of an object.

To store a neutrosophic soft set in computer, we could present it in the form of a table as shown below. In this table the entries are a_{ij} corresponding to the calculator c_i and the parameter e_j where

$$a_{ij}=\left(\begin{array}{c}\text{true - membership value of } c_i,\\ \text{indeterminacy - membership value of } c_i,\\ \text{falsity - membership value of } c_i\end{array}\right)$$

in $F(e_j)$. The neutrosophic soft set $(F,\ E)$ in tabular representation is as follow:

U	e_1 =performance	e_2 =size of calculator	e_3 =price of calculator
c_1	$\langle 0.7,0.4,0.5\rangle$	$\langle 0.6,0.7,0.8\rangle$	$\langle 0.8,0.1,0.7\rangle$
c_2	$\langle 0.8,0.5,0.3\rangle$	$\langle 0.2,0.4,0.7\rangle$	$\langle 0.5,0.8,0.9\rangle$
c_3	$\langle 0.4,0.6,0.8\rangle$	$\langle 0.8,0.4,0.6\rangle$	$\langle 1,0.3,0.4\rangle$

Tabular form of the neutrosophic soft set $(F,\ E)$.

Definition 3.3 [14]:

For two neutrosophic soft sets $(H,\ A)$ and $(G,\ B)$ over the common universe U . We say that $(H,\ A)$ is a neutrosophic soft subset of $(G,\ B)$ if

(i) $A\subseteq B,$

(ii) $T_{H(e)}(x)\leq T_{G(e)}(x),\quad I_{H(e)}(x)\leq I_{G(e)}(x),$
$F_{H(e)}(x)\geq F_{G(e)}(x)$

for all $e\in A$ and $x\in U$. We write $(H,A)\subseteq(G,B)$.

Definition 3.4:

For two neutrosophic soft sets $(H,\ A)$ and $(G,\ B)$ over the common universe U . We say that $(H,\ A)$ is a neutrosophic soft twisted subset of $(G,\ B)$ if

(i) $A\subseteq B$

(ii) $T_{H(e)}(x)\geq T_{G(e)}(x),\ I_{H(e)}(x)\geq I_{G(e)}(x),$
$F_{H(e)}(x)\leq F_{G(e)}(x)$

for all $e\in A$ and $x\in U$. We write $(H,\ A)\widehat{\subseteq}(G,\ B)$.

Definition 3.5:

(1) $(H,\ A)$ is called relative null neutrosophic soft set (with respect to parameter A), if
$T_{H(e)}(x)=0,\ I_{H(e)}(x)=0,\ F_{H(e)}(x)=1\ \forall\, e\in A$ and $x\in U$.
It is denoted by Φ_A.

(2) $(G,\ A)$ is called relative whole neutrosophic soft set (with respect to parameter A) if
$T_{G(e)}(x)=1,\ I_{G(e)}(x)=1,\ F_{G(e)}(x)=0\ \forall\, e\in A$ and $x\in U$.
It is denoted by $\cup_A$.

Similarly we define absolute neutrosophic soft set over U , and it is denoted by $\cup_E$, and null neutrosophic soft set over U, it is denoted by Φ_E.

Definition 3.6:

Let $E=\{e_1,e_2,e_3,\ldots e_n\}$ be a set of parameters. The not set of E is denoted by $\neg E$ and defined as $\neg E=\{\neg e_1,\neg e_2,\neg e_3,\ldots\neg e_n\}$, where $\neg\ e_i=$ not e_i, $\forall i$.

Definition 3.7 [14]:

Complement of a neutrosophic soft set $(G,\ A)$ denoted by $(G,\ A)^c$ and is defined as

$(G,\ A)^c = (G^c,\ \neg A)$ where
$G^c\ :\neg A \to P(U)$ is a mapping given by
$G^c(\neg e) =$ neutrosophic soft compliment with
$T_{G^c(\neg e)} = F_{G(e)},\quad I_{G^c(\neg e)} = I_{G(e)}$ and
$F_{G^c(\neg e)} = T_{G(e)}$.

Definition 3.8:

Let $(H,\ A)$ and $(G,\ B)$ be two NSS,s over the common universe U. Then the extended union of $(H,\ A)$ and $(G,\ B)$ is denoted by $(H,\ A)\cup_{\mathtt{E}}(G,\ B)$ and defined as $(H,\ A)\cup_{\mathtt{E}}(G,\ B) = (K,\ C)$, where $C = A\cup B,$ and the truth-membership, indeterminacy-membership and falsity-membership of $(K,\ C)$ are as follows

$$T_{K(e)}(x) = \begin{cases} T_{H(e)}(x) & \text{if } e\in A-B \\ T_{G(e)}(x) & \text{if } e\in B-A \\ \max\left(T_{H(e)}(x), T_{G(e)}(x)\right) & \text{if } e\in A\cap B \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{H(e)}(x) & \text{if } e\in A-B \\ I_{G(e)}(x) & \text{if } e\in B-A \\ \max\left(I_{H(e)}(x), I_{G(e)}(x)\right) & \text{if } e\in A\cap B \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{H(e)}(x) & \text{if } e\in A-B \\ F_{G(e)}(x) & \text{if } e\in B-A \\ \min\left(F_{H(e)}(x), F_{G(e)}(x)\right) & \text{if } e\in A\cap B \end{cases}$$

and the restricted union of $(H,\ A)$ and $(G,\ B)$ is denoted and defined as $(H,\ A)\cup_{\mathtt{R}}(G,\ B) = (K,\ C)$ where $C = A\cap B$ and

$$T_{K(e)}(x) = \max\left(T_{H(e)}(x), T_{G(e)}(x)\right) \text{ if } e\in A\cap B$$

$$I_{K(e)}(x) = \max(I_{H(e)}(x), I_{G(e)}(x)) \text{ if } e \in A\cap B$$

$$F_{K(e)}(x) = \min\left(F_{H(e)}(x), F_{G(e)}(x)\right) \text{ if } e\in A\cap B$$

If $A\cap B = \emptyset,$ then $(H,\ A)\cup_{\mathcal{R}}(G,\ B) = \Phi_{\Phi}.$

Definition 3.9:

Let $(H,\ A)$ and $(G,\ B)$ be two NSS,s over the common universe U. Then the extended intersection of $(H,\ A)$ and $(G,\ B)$ is denoted by $(H,\ A)\cap_{\mathtt{E}}(G,\ B)$ and defined as $(H,\ A)\cap_{\mathtt{E}}(G,\ B) = (K,\ C),$ where $C = A\cup B,$ and the truth-membership, indeterminacy-membership and falsity-membership of $(K,\ C)$ are as follows

$$T_{K(e)}(x) = \begin{cases} T_{H(e)}(x) & \text{if } e\in A-B \\ T_{G(e)}(x) & \text{if } e\in B-A \\ \min\left(T_{H(e)}(x), T_{G(e)}(x)\right) & \text{if } e\in A\cap B \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{H(e)}(x) & \text{if } e\in A-B \\ I_{G(e)}(x) & \text{if } e\in B-A \\ \min\left(I_{H(e)}(x), I_{G(e)}(x)\right) & \text{if } e\in A\cap B \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{H(e)}(x) & \text{if } e\in A-B \\ F_{G(e)}(x) & \text{if } e\in B-A \\ \max\left(F_{H(e)}(x), F_{G(e)}(x)\right) & \text{if } e\in A\cap B \end{cases}$$

and the restricted intersection of $(H,\ A)$ and $(G,\ B)$ is denoted and defined as $(H,\ A)\cap_{\mathtt{R}}(G,\ B) = (K,\ C)$ where $C = A\cap B$ and

$T_{K(e)}(x)=\min(T_{H(e)}(x),T_{G(e)}(x))$ if $e\in A\cap B$
$I_{K(e)}(x)=\min(I_{H(e)}(x),I_{G(e)}(x))$ if $e\in A\cap B$
$F_{K(e)}(x)=\max(F_{H(e)}(x),F_{G(e)}(x))$ if $e\in A\cap B$

If $A\cap B=\Phi$ then $(H,\ A)\cap_{\mathbf{R}}(G,\ B)=\Phi_{\Phi}$.

4 Distributive laws for neutrosophic soft sets

In this section we present distributive laws on the collection of neutrosophic soft set. It is interesting to see that the equality does not hold in some assertions and counter example is given to show it.

Let U be an initial universe and E be the set of parameters. We denote the collection as follows:

$\mathbf{NSS}(U)^E$: The collection of all neutrosophic soft sets over U.

$\mathbf{NSS}(U)_A$: The collection of all those neutrosophic soft sets over U with a fixed parameter set A.

Theorem 4.1:

Let $(H,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over the common universe U. Then

(i) $(H,\ A)\cup_R(H,\ A)=(H,\ A)$ and $(H,\ A)\cap_R(H,\ A)=(H,\ A)$,

(ii) $(H,A)\cap_R\Phi_A=\Phi_A$,

(iii) $(H,\ A)\cup_R\Phi_A=(H,\ A)$,

(iv) $(H,\ A)\cup_R\mathsf{U}_A=\mathsf{U}_A$,

(v) $(H,\ A)\cap_R\mathsf{U}_A=(H,\ A)$,

(vi) $((H,\ A)\cup_R(G,\ B))^c=(H,\ A)^c\cap_R(G,\ B)^c$,

(vii) $((H,\ A)\cap_R(G,\ B))^c=(H,\ A)^c\cup_R(G,\ B)^c$.

***Proof*:** Straightforward.

Remark 4.2:

Let $\alpha,\ \beta\in\{\cup_{\mathbf{R}},\cap_{\mathbf{R}},\cup_{\mathbf{E}},\cap_{\mathbf{E}}\}$, if

$(H,\ A)\alpha((G,\ B)\beta(K,\ C))=$
$((H,\ A)\alpha(G,\ B))\beta((H,\ A)\alpha(K,\ C))$

holds, then we have 1 otherwise 0 in the following table

	$\cap_{\mathbf{E}}$	$\cup_{\mathbf{E}}$	$\cap_{\mathbf{R}}$	$\cup_{\mathbf{R}}$
$\cap_{\mathbf{E}}$	1	0	0	1
$\cup_{\mathbf{E}}$	0	1	1	0
$\cap_{\mathbf{R}}$	1	1	1	1
$\cup_{\mathbf{R}}$	1	1	1	1

Distributi ve law for neutrosophic soft sets

Proofs in the cases where equality holds can be followed by definition of respective operations. For which the equality does not hold, see the following example.

Example 4.3:

Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word. Consider $U=\{h_1,h_2,h_3,h_4,h_5\}$ and $E=\{$ beautiful, wooden , costly, green surroundings, good repair, cheap, expensive $\}$.

Suppose that $A=\{$ beautiful, wooden , costly, green surroundings $\}$ $B=\{$ costly, good repair, green surroundings $\}$ and $C=\{$ costly,good repair,beautiful $\}$. Let $(F,\ A)$, (G,B) and $(H,\ C)$ be the neutrosophic soft sets over U , which are defined as follows:

Neutrosophic soft set $(G,\ B)$

U	*costly*	*good repair*	*green surroundings*
h_1	$\langle 0.6,0.5,0.4\rangle$	$\langle 0.7,0.2,0.5\rangle$	$\langle 0.6,0.2,0.7\rangle$
h_2	$\langle 0.7,0.3,0.6\rangle$	$\langle 0.6,0.6,0.8\rangle$	$\langle 0.4,0.5,0.2\rangle$
h_3	$\langle 0.8,0.6,0.3\rangle$	$\langle 1,0.7,0.5\rangle$	$\langle 0.6,0.4,0.7\rangle$

Neutrosophic soft set $(F,\ A)$

U	beautiful	wooden	costly	green surroundings
h_1	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.1, 0.7\rangle$	$\langle 0.5, 0.3, 0.7\rangle$
h_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.5, 0.8, 0.9\rangle$	$\langle 0.6, 0.2, 0.3\rangle$
h_3	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.3, 0.4\rangle$	$\langle 0.7, 0.1, 0.5\rangle$

and Neutrosophic soft set $(H,\ C)$ is

U	costly	good repair	beautiful
h_1	$\langle 0.4, 0.4, 0.6\rangle$	$\langle 0.5, 0.7, 0.4\rangle$	$\langle 0.6, 0.8, 0.4\rangle$
h_2	$\langle 0.3, 0.7, 0.4\rangle$	$\langle 0.3, 0.7, 0.4\rangle$	$\langle 0.5, 0.3, 0.3\rangle$
h_3	$\langle 0.2, 0.4, 0.3\rangle$	$\langle 0.4, 0.5, 0.3\rangle$	$\langle 0.8, 0.5, 0.8\rangle$

Let

$(F,\ A)\cup_{\mathrm{E}}((G,\ B)\cup_{\mathrm{R}}(H,\ C))=(I,\ A\cup(B\cap C))$ and

$((F,\ A)\cup_{\mathrm{E}}(G,\ B))\cup_{\mathrm{R}}((F,\ A)\cup_{\mathrm{E}}(H,\ C))=(J,\ (A\cup B)\cap(A\cup C))$

Then

Neutrosophic soft set $(I,\ A\cup(B\cap C))$

U	beautiful	wooden	costly	green surroundings	good repair
h_1	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.7, 0.7, 0.4\rangle$
h_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.7, 0.8, 0.4\rangle$	$\langle 0.6, 0.2, 0.3\rangle$	$\langle 0.6, 0.7, 0.4\rangle$
h_3	$\langle 0.4, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.6, 0.3\rangle$	$\langle 0.7, 0.1, 0.5\rangle$	$\langle 1, 0.7, 0.3\rangle$

Neutrosophic soft set $(J,\ (A\cup B)\cap(A\cup C))$

U	beautiful	wooden	costly	green surroundings	good repair
h_1	$\langle 0.7, 0.8, 0.4\rangle$	$\langle 0.6, 0.7, 0.8\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.6, 0.3, 0.7\rangle$	$\langle 0.7, 0.7, 0.4\rangle$
h_2	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.2, 0.4, 0.7\rangle$	$\langle 0.7, 0.8, 0.4\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.6, 0.7, 0.4\rangle$
h_3	$\langle 0.8, 0.6, 0.8\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 1, 0.6, 0.3\rangle$	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 1, 0.7, 0.3\rangle$

Thus

$(F,\ A)\cup_{\mathrm{E}}((G,\ B)\cup_{\mathrm{R}}(H,\ C))\neq$
$((F,\ A)\cup_{\mathrm{E}}(G,\ B))\cup_{\mathrm{R}}((F,\ A)\cup_{\mathrm{E}}(H,\ C)).$

Similarly we can show that

$(F,\ A)\cap_{\mathrm{E}}((G,\ B)\cap_{\mathrm{R}}(H,\ C))\neq$
$((F,\ A)\cap_{\mathrm{E}}(G,\ B))\cap_{\mathrm{R}}((F,\ A)\cap_{\mathrm{E}}(H,\ C)),$

and

$(F,\ A)\cup_{\mathrm{E}}((G,\ B)\cap_{\mathrm{E}}(H,\ C))\neq$
$((F,\ A)\cup_{\mathrm{E}}(G,\ B))\cap_{\mathrm{E}}((F,\ A)\cup_{\mathrm{E}}(H,\ C)),$
$(F,\ A)\cap_{\mathrm{E}}((G,\ B)\cup_{\mathrm{E}}(H,\ C))\neq$
$((F,\ A)\cap_{\mathrm{E}}(G,\ B))\cup_{\mathrm{E}}((F,\ A)\cap_{\mathrm{E}}(H,\ C)).$

5 Algebraic structures associated with neutrosophic soft sets

In this section, we initiate the study of algebraic structures associated with single and double binary operations, for the set of all neutrosophic soft sets over the universe U, and the set of all neutrosophic soft sets with a fixed set of parameters. Recall that, let U be an initial universe and E be the set of parameters. Then we have:

$\mathbf{NSS}(U)^E$: The collection of all neutrosophic soft sets over U.

$\mathbf{NSS}(U)_A$: The collection of all those neutrosophic soft sets over U with a fixed parameter set A.

5.1 Commutative monoids

From Theorem 4.1, it is clear that $(\mathbf{NSS}(U)^E,\ \alpha)$ are idempotent, commutative, semigroups for $\alpha\in\{\cup_{\mathcal{R}}, \cap_{\mathcal{R}}, \cup_{\varepsilon}, \cup_{\varepsilon}\}$.

(1) $(\mathbf{NSS}(U)^E,\ \cup_{\mathrm{R}})$ is a monoid with Φ_E as an identity element, $(\mathbf{NSS}(U)_A,\ \cup_{\mathrm{R}})$ is a subsemigroup of $(\mathbf{NSS}(U)^E,\ \cup_{\mathrm{R}})$.

(2) $(\mathbf{NSS}(U)^E,\ \cap_{\mathrm{R}})$ is a monoid with $\mathfrak{U}_E$ as an identity element, $(\mathbf{NSS}(U)_A,\ \cap_{\mathrm{R}})$ is a subsemigroup of $(\mathbf{NSS}(U)^E,\ \cap_{\mathrm{R}})$.

(3) $(\mathbf{NSS}(U)^E,\ \cup_{\mathrm{E}})$ is a monoid with Φ_{Φ} as an identity element, $(\mathbf{NSS}(U)_A,\ \cup_{\mathrm{E}})$ is a subsemigroup of $(\mathbf{NSS}(U)^E,\ \cup_{\mathrm{E}})$.

(4) $(\mathbf{NSS}(U)^E,\ \cap_{\mathrm{E}})$ is a monoid with Φ_{Φ} as an identity element, $(\mathbf{NSS}(U)_A,\ \cap_{\mathrm{E}})$ is a

subsemigroup of $\left(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{E}}\right)$.

5.2 Semirings:

(1) $\left(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{R}},\ \cap_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with U_E as an identity element.

(2) $\left(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{R}},\ \cup_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(3) $\left(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{R}},\ \cap_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(4) $\left(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{R}},\ \cup_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with Φ_E as an identity element.

(5) $\left(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{R}},\ \cup_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(6) $\left(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{R}},\ \cap_{\mathbf{E}}\right)$ is a commutative, idempotent semiring with Φ_Φ as an identity element.

(7) $\left(\mathbf{NSS}(U)^E,\ \cup_{\mathbf{E}},\ \cap_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with $\mathfrak{U}_E$ as an identity element.

(8) $\left(\mathbf{NSS}(U)^E,\ \cap_{\mathbf{E}},\ \cup_{\mathbf{R}}\right)$ is a commutative, idempotent semiring with Φ_E as an identity element.

5.3 Lattices:

In this subsection we study what type of lattice structure is associated with the neutrosophic soft sets.

Remark 5.3.1:

Let $\alpha,\beta\in\{\cup_{\mathbf{R}},\cap_{\mathbf{R}},\cup_{\mathbf{E}},\cap_{\mathbf{E}}\}$. if the absorption law

$(F,\ A)\alpha((F,\ A)\beta(G,\ B))=(F,\ A)$

holds we write 1 otherwise 0 in the following table.

	$\cap_{\mathbf{E}}$	$\cup_{\mathbf{E}}$	$\cap_{\mathbf{R}}$	$\cup_{\mathbf{R}}$
$\cap_{\mathbf{E}}$	0	0	0	1
$\cup_{\mathbf{E}}$	0	0	1	0
$\cap_{\mathbf{R}}$	0	1	0	0
$\cup_{\mathbf{R}}$	1	0	0	0

Absorption law for neutrosophic soft sets

(1) $\left(\mathbf{NSS}(U)^E,\ \Phi_\Phi,\ \mathsf{U}_E,\ \cup_{\mathbf{R}},\ \cap_{\mathbf{E}}\right)$ and $\left(\mathbf{NSS}(U)^E,\ \mathsf{U}_E,\ \Phi_\Phi,\ \cap_{\mathbf{R}},\ \cup_{\mathbf{E}}\right)$ are lattices with $\left(\mathbf{NSS}(U)_A,\ \Phi_A,\ \mathsf{U}_A,\ \cup_{\mathbf{R}},\ \cap_{\mathbf{E}}\right)$ and $\left(\mathbf{NSS}(U)_A,\ \mathsf{U}_A,\ \Phi_A,\ \cap_{\mathbf{R}},\ \cup_{\mathbf{E}}\right)$ as their sublattices respectively.

(2) $\left(\mathbf{NSS}(U)^E,\ \Phi_\Phi,\ \mathsf{U}_E,\ \cup_{\mathbf{E}},\ \cap_{\mathbf{R}}\right)$ and $\left(\mathbf{NSS}(U)^E,\ \mathsf{U}_E,\ \Phi_\Phi,\ \cap_{\mathbf{R}},\ \cup_{\mathbf{E}}\right)$ are lattices with $\left(\mathbf{NSS}(U)_A,\ \Phi_A,\ \mathsf{U}_A,\ \cup_{\mathbf{E}},\ \cap_{\mathbf{R}}\right)$ and $\left(\mathbf{NSS}(U)_A,\ \mathsf{U}_A,\ \Phi_A,\ \cap_{\mathbf{R}},\ \cup_{\mathbf{E}}\right)$ as their sublattices respectively.

The above mentioned lattices and sublattices are bounded distributive lattices.

Proposition 5.3.2 :

For the lattice of neutrosophic soft set $\left(\mathbf{NSS}(U)^E,\ \Phi_\Phi,\ \mathsf{U}_E,\ \cup_{\mathbf{R}},\ \cap_{\mathbf{E}}\right)$ for any $(H,\ A)$ and $(G,\ B)\in\mathbf{NSS}(U)^E$, then

(1) $(H,\ A)\hat{\subset}(G,\ B)$ if and only if $(H,\ A)\cup_{\mathbf{R}}(G,\ B)=(H,\ A)$.

(2) $(H,\ A)\hat{\subset}(G,\ B)$ if and only if $(H,\ A)\cap_{\mathcal{E}}(G,\ B)=(G,\ B)$.

Proof: Straightforward.

Proposition 5.3.3:

For the lattice of neutrosophic soft set $\left(\mathbf{NSS}(U)^E,\ \Phi_\Phi,\ \mathsf{U}_E,\ \cup_{\mathbf{E}},\ \cap_{\mathbf{R}}\right)$ for any

$(H,\ A)$ and $(G,\ B) \in \mathbf{NSS}(U)^E$, then

(1) $(H,\ A) \subset (G,\ B)$ if and only if $(H,\ A) \cup_{\mathbf{E}} (G,\ B) = (G,\ B)$.

(2) $(H,\ A) \subset (G,\ B)$ if and only if $(H,\ A) \cap_{\mathbf{R}} (G,\ B) = (H,\ A)$.

Proof: Straightforward.

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