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Soft Interval –Valued Neutrosophic Rough Sets

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Abstract: In this paper, we first defined soft interval-valued neutrosophic rough sets(SIVN- rough sets for short) which combines interval valued neutrosophic soft set and rough sets and studied some of its basic properties. This concept is an extension of soft interval valued intuitionistic fuzzy rough sets(SIVIF- rough sets). Finally an illustartive example is given to verfy the developped algorithm and to demonstrate its practicality and effectiveness.

Keywords: Interval valued neutrosophic soft sets, rough set, soft Interval valued neutrosophic rough sets

1. Introduction

In 1999, Florentin Smarandache introduced the concept of neutrosophic set (NS) [13] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The concept of neutrosophic set is the generalization of the classical sets, conventional fuzzy set [27], intuitionistic fuzzy set [24] and interval valued fuzzy set [45] and so on. A neutrosophic sets is defined on universe U. x= x(T, I, F) ∈ A with T, I and F being the real standard or non –standard subset of $]0^{-},1^{+}[$, T is the degree of truth membership of A, I is the degree of indeterminacy membership of A and F is the degree of falsity membership of A. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and false –membership are independent.

Recently, works on the neutrosophic set theory is progressing rapidly. M. Bhowmik and M. Pal [28, 29] defined the concept "intuitionistic neutrosophic set". Later on A. A. Salam and S. A.Alblowi [1] introduced another concept called "generalized neutrosophic set". Wang et al [18] proposed another extension of neutrosophic set called "single valued neutrosophic sets". Also, H.Wang et al. [17] introduced the notion of interval valued neutrosophic sets (IVNSs) which is an instance of neutrosophic set. The IVNSs is characterized by an interval membership degree,interval indeterminacy degree and interval non-membership degree. K.Geogiev [25] explored some properties of the neutrosophic logic and proposed a general simplification of the neutrosophic sets into a subclass of theirs, comprising of elements of R^3. Ye [20, 21] defined similarity measures between interval neutrosophic sets and their multicriteria decision-making method. P. Majumdar and S.K. Samant [34] proposed some types of similarity and entropy of neutrosophic sets. S.Broumi and F. Smarandache [38,39,40] proposed several similarity measures of neutrosophic sets. P. Chi and L. Peid [33] extended TOPSIS to interval neutrosophic sets.

In 1999, Molodtsov [8]initiated the concept of soft set theory as proposed a new mathematical for dealing with uncertainties. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applied to many different fields including game theory, operations research, Riemmann integration, Perron integration. Recently, I. Deli [10] combined the concept of soft set and interval valued neutrosophic sets together by introducing anew concept called " interval valued neutrosophic soft sets" and gave an application of interval valued neutrosophic soft sets in decision making. This concept generalizes the concept of the soft sets, fuzzy soft sets [35], intuitionistic fuzzy soft sets [36], interval valued intuitionistic fuzzy soft sets [22], the concept of neutrosophic soft sets [37] and intuitionistic neutrosophic soft sets [41].

The concept of rough set was originally proposed by Pawlak [50] as a formal tool for modeling and processing incomplete information in information systems. Rough set theory has been conceived as a tool to conceptualize, organize and analyze various types of data, in particular, to deal with inexact, uncertain or vague knowledge in applications related to artificial intelligence technique. Therefore, many models have been built upon different aspect, i.e, universe, relations, object, operators by many scholars [6, 9, 23, 48, 49, 51] such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough, rough intuitionistic fuzzy set, intuitionistic fuzzy rough sets [26]. The rough sets has been successfully applied in many fields such as attribute reduction [19, 30, 31, 46], feature selection [11, 18, 44], rule extraction [5, 7, 12, 47] and so on. The rough sets theory approximates any subset of objects of the universe by two sets, called the lower and upper approximations. The lower approximation of a given set is the union of all the equivalence classes which are subsets of the set, and the upper approximation is the union of all the equivalence classes which have a non empty intersection with the set.

Moreover, many new rough set models have also been established by combining the Pawlak rough set with other uncertainty theories such as soft set theory. Feng et al [14] provided a framework to combine fuzzy sets, rough sets, and soft sets all together, which gives rise to several interesting new concepts such as rough soft sets, soft rough sets, and soft rough fuzzy sets. The combination of hybrid structures of soft sets and rough sets models was also discussed by some researchers [15,32,43]. Later on, J. Zhang, L. Shu, and S. Liao [22] proposed the notions of soft rough intuitionistic fuzzy sets and intuitionistic fuzzy soft rough sets, which can be seen as two new generalized soft rough set models, and investigated some properties of soft rough intuitionistic fuzzy sets and intuitionistic fuzzy soft rough sets in detail. A.Mukherjee and A. Saha [3] proposed the concept of interval valued intuitionistic fuzzy soft rough sets. Also A. Saha and A. Mukherjee [4] introduced the concept of Soft interval valued intuitionistic fuzzy rough sets.

More recently, S.Broumi et al. [42] combined neutrosophic sets with rough sets in a new hybrid mathematical structure called “rough neutrosophic sets” handling incomplete and indeterminate information . The concept of rough neutrosophic sets generalizes rough fuzzy sets and rough intuitionistic fuzzy sets. Based on the equivalence relation on the universe of discourse, A. Mukherjee et al. [3] introduced soft lower and upper approximation of interval valued intuitionistic fuzzy set in Pawlak’s approximation space. Motivated by the idea of soft interval valued intuitionistic fuzzy rough sets introduced in [4], we extend the soft interval intuitionistic fuzzy rough to the case of an interval valued neutrosophic set. The concept of soft interval valued neutrosophic rough set is introduced by coupling both the interval valued neutrosophic soft sets and rough sets.

The paper is structured as follows. In Section 2, we first recall the necessary background on soft sets, interval neutrosophic sets, interval neutrosophic soft sets, rough set, rough neutrosophic sets and soft interval valued intuitionistic fuzzy rough sets. Section 3 presents the concept of soft interval neutrosophic rough sets and examines their respective properties. Section 4 presents a multiciteria group decision making scheme under soft interval –valued neutrosophic rough sets. Section 5 presents an application of multiciteria group decision making scheme regarding the candidate selection problem . Finally we concludes the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of soft sets, interval neutrosophic setsinterval neutrosophic soft set, rough set, rough neutrosophic sets and soft interval valued intuitionistic fuzzy rough sets. For more details the reader may refer to [4, 8, 10, 13, 17, 50, 42].

Definition 2.1 [13] : Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x: $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x)$>,x $\in$ U}, where the functions $\boldsymbol{\mu_A(x)}$, $\boldsymbol{\nu_A(x)}$, $\boldsymbol{\omega_A(x)}$: U→]$^-$0,1$^+$[define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x $\in$ X to the set A with the condition.

$$^-0 \leqslant sup\ \mu_A(x)+ \sup\nu_A(x)+ sup\omega_A(x)_) \leqslant 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-$0,1$^+$[. So instead of]$^-$0,1$^+$[we need to take the interval [0,1] for technical applications, because]$^-$0,1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.3 [13]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeterminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x) \in$ int([0 ,1]).

For two IVNS, A_{IVNS}= {<x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | x ∈ X } (2)

And B_{IVNS}= {<x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | x ∈ X } the two relations are defined as follows:

(1)$A_{IVNS} \subseteq B_{IVNS}$if and only if $\mu_A^L(x) \le \mu_B^L(x)$, $\mu_A^U(x) \le \mu_B^U(x)$,$\nu_A^L(x) \ge \nu_B^L(x)$, $\omega_A^U(x) \ge \omega_B^U(x)$, $\omega_A^L(x) \ge \omega_B^L(x)$,$\omega_A^U(x) \ge \omega_B^U(x)$.

(2)$A_{\text{IVNS}} = B_{\text{IVNS}}$ if and only if , $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any $x \in X$

The complement of A_{IVNS} is denoted by A^o_{IVNS} and is defined by

A^o_{IVNS}={ <x , $[\omega^L_A(x), \omega^U_A(x)]$, $[1-\nu^U_A(x), 1-\nu^L_A(x)]$, $[\mu^L_A(x), \mu^U_A(x)]$ | x ∈ X }

A∩B ={ <x , $[\min(\mu^L_A(x),\mu^L_B(x)), \min(\mu^U_A(x),\mu^U_B(x))]$, $[\max(\nu^L_A(x),\nu^L_B(x))$, $\max(\nu^U_A(x),\nu^U_B(x)]$, $[\max(\omega^L_A(x),\omega^L_B(x))$, $\max(\omega^U_A(x),\omega^U_B(x))]$ >: x ∈ X }

A∪B ={ <x , $[\max(\mu^L_A(x),\mu^L_B(x)), \max(\mu^U_A(x),\mu^U_B(x))]$, $[\min(\nu^L_A(x),\nu^L_B(x)),\min(\nu^U_A(x),\nu^U_B(x)]$, $[\min(\omega^L_A(x),\omega^L_B(x))$, $\min(\omega^U_A(x),\omega^U_B(x))]$ >: x ∈ X }

As an illustration, let us consider the following example.

Example 2.4. Assume that the universe of discourse U={x_1, x_2, x_3}, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval valued neutrosophic set (IVNS) of U, such that, A = {< x_1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x_2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x_3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is [0.3, 0.4], degree of indeterminacy of capability is[0.5, 0.6] and degree of falsity of capability is [0.4, 0.5] etc.

Definition 2.5 . [8]

Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K : A → P(U).

As an illustration, let us consider the following example.

Example 2.6 . Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:

A={e_1,e_2,e_3,e_4,e_5};

K(e_1) = {h_2, h_3, h_5}, K(e_2) = {h_2, h_4}, K(e_3) = {h_1}, K(e_4) = U, K(e_5) = {h_3, h_5}.

Definition 2.7. [10]

Let U be an initial universe set and A ⊂ E be a set of parameters. Let IVNS (U) denote the set of all interval valued neutrosophic subsets of U. The collection (K, A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K: A → IVNS(U).

The interval valued neutrosophic soft set defined over an universe is denoted by IVNSS.

Here,

1. ϒ is an ivn-soft subset of Ψ, denoted by ϒ ⋐ Ψ, if K(e) ⊆L(e) for all e∈E.
2. ϒ is an ivn-soft equals to Ψ, denoted by ϒ = Ψ, if K(e)=L(e) for all e∈E.
3. The complement of ϒ is denoted by $ϒ^c$, and is defined by $ϒ^c$ = {(x, K^o (x)): x∈E}
4. The union of ϒ and Ψ is denoted by ϒ ∪̃ Ψ, if K(e) ∪L(e) for all e∈E.
5. The intersection of ϒand Ψ is denoted by ϒ ∩̃ Ψ,if K(e) ∪ L(e) for all e∈E.

Example 2.8 :

Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is an interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = { beautiful, costly, moderate, expensive }. In this case, to define an interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are four houses in the universe U given by, U = {h_1,h_2,h_3,h_4 } and the set of parameters A = {e_1,e_2,e_3}, where each e_i is a specific criterion for houses:

e_1 stands for 'beautiful',

e_2 stands for 'costly',

e_3 stands for 'moderate',

Suppose that,

K(beautiful)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >}

.K(costly)={< h_1,[0.3, 0.6], [0.2 0.7], [0.1, 0.4]>,< h_2,[0.3, 0.5], [0.6 ,0.8], [0.2, 0.6] >, < h_3,[0.3, 0.7],[0.1 ,0.3],[0.3, 0.6] >,< h_4,[0.6 ,0.8],[0.2, 0.4],[0.2, 0.5 >}

K(moderate)={< h_1,[0.5, 0.8], [0.4, 0.7], [0.3, 0.6]>,< h_2,[0.3, 0.5], [0.7 ,0.9], [0.2, 0.4] >, < h_3,[0.1, 0.7],[0.3 ,0.3],[0.3, 0.6] >,< h_4,[0.3,0.8],[0.2, 0.4],[0.3, 0.6] >}.

Defintion.2.9 [50]
Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R_*X and $\boldsymbol{R}^*X$ and are defined by

$R_*X=\{x \in U: [x]_R \subseteq X\}$,

$R^*X=\{ x \in U: [x]_R \cap X \neq \emptyset\}$.

Now if $R_*X = R^* X$, then X is called definable; otherwise X is called a rough set.

Definition 2.10 [42]
Let U be a non-null set and R be an equivalence relation on U. Let F be neutrosophic set in U with the membership function μ_F, indeterminacy function ν_F and non-membership function ω_F. Then, the lower and upper rough approximations of F in (U, R) are denoted by $\underline{R}$ (F) and $\overline{R}$(F) and respectively defined as follows:

$\overline{R}(F) =\{ <x, \mu_{\overline{R}(F)}(x), \nu_{\overline{R}(F)}(x), \omega_{\overline{R}(F)}(x)> \mid x\in U\}$,

$\underline{R}(F) =\{ <x, \mu_{\underline{R}(F)}(x), \nu_{\underline{R}(F)}(x), \omega_{\underline{R}(F)}(x)> \mid x\in U\}$,

Where:

$\mu_{\overline{R}(F)}(x) =\vee_{y \in [x]_R} \mu_F(y)$, $\nu_{\overline{R}(F)}(x)=\wedge_{y \in [x]_R} \nu_F(y)$, $\omega_{\overline{R}(F)} =\wedge_{y \in [x]_R} \omega_F(y)$,

$\mu_{\underline{R}(F)}(x)=\wedge_{y \in [x]_R} \mu_F(y)$, $\nu_{\underline{R}(F)}(x)=\vee_{y \in [x]_R} \nu_F(y)$, $\omega_{\underline{R}(F)} =\vee_{y \in [x]_R} \omega_F(y)$,

It is easy to observe that $\overline{R}$(F) and $\underline{R}$(F) are two neutrosophic sets in U, thus NS mapping $\overline{R}$, $\underline{R}$:R(U) → R(U) are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair ($\underline{R}$(F), $\overline{R}$(F)) is called the rough neutrosophic set.

Definition 2.11[4] . Let us consider an interval-valued intuitionstic fuzzy set σ defined by

σ = {x, $\mu_\sigma(x)$, $\nu_\sigma(x)$: x ∈ U} where $\mu_\sigma(x)$, $\nu_\sigma(x)$, ∈ int ([0, 1]) for each x ∈ U and

$0 \le \mu_\sigma(x)+ \nu_\sigma(x) \le 1$

Now Let Θ=(f,A) be an interval-valued intuitionstic fuzzy soft set over U and the pair SIVIF= (U, Θ) be the soft interval-valued intuitionistic fuzzy approximation space. Let f:A→ $IVIFS^U$ be defined f(a) ={ x, $\mu_{f(a)}(x)$, $\nu_{f(a)}(x)$: x ∈ U } for each a ∈ A. Then , the lower and upper soft interval-valued intuitionistic fuzzy rough approximations of σ with respect to SIVIF are denoted by $\downarrow Apr_{SIVIF}(\sigma)$ and $\uparrow Apr_{SIVIF}(\sigma)$ respectively, which are interval valued intuitionistic fuzzy sets in U given by:

$\downarrow Apr_{SIVIF}(\sigma)$ **={< x,**
[$\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_\sigma(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_\sigma(x)]$, **[** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_\sigma(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_\sigma(x)]$>: x ∈ U }

$\uparrow Apr_{SIVIF}(\sigma)$ **={< x, [** $\wedge_{a \in A}(\inf \mu_{f(a)}(x) \vee \inf \mu_\sigma(x))$, $\wedge_{a \in A}(\sup \mu_{f(a)}(x) \vee \sup \mu_\sigma(x)]$, **[** $\wedge_{a \in A}(\inf \nu_{f(a)}(x) \wedge \inf \nu_\sigma(x))$, $\wedge_{a \in A}(\sup \nu_{f(a)}(x) \wedge \sup \nu_\sigma(x)]$ >: x ∈ U}

The operators $\downarrow Apr_{SIVIF}(\sigma)$ and $\uparrow Apr_{SIVIF}(\sigma)$ are called the lower and upper soft interval-valued intuitionistic fuzzy rough approximation operators on interval valued intuitionistic fuzzy sets. If $\downarrow Apr_{SIVIF}(\sigma) = \uparrow Apr_{SIVIF}(\sigma)$, then σ is said to be soft interval valued intuitionistic fuzzy definable; otherwise is called a soft interval valued intuitionistic fuzzy rough set.

Example 3.3 . Let U={x, y) and A={a, b}. Let (f, A) be an interval –valued intuitionstic fuzzy soft set over U where f:A→ $IVIFS^U$ be defined

f(a)= **{** <x,[0.2, 0. 5], [0.3, 0.4]>, <y, [0.6, 0.7],[0.1, 0.2] >}

f(b)= **{** <x,[0.1, 0. 3], [0.4, 0.5>, <y, [0.5, 0.8],[0.1, 0.2] >}

Let σ = **{** <x,[0.3, 0.4], [0.3, 0.4]>, <y, [0.2, 0.4],[0.4, 0.5] >}. Then

$\downarrow Apr_{SIVIF}(\sigma)$= **{** <$x$,[0.1, 0.3],[0.3, 0.4] >, <y,[0.2, 0.4],[0.4, 0.5]>}

$\uparrow Apr_{SIVIF}(\sigma)$ = **{** <x,[0.3, 0.4],[0.3, 0.4] >, <y,[0.5, 0.7],[0.1, 0.2]>}. Then σ is a soft interval-valued intuitionstic fuzzy rough set.

3. Soft Interval Neutrosophic Rough Set.

A. Saha and A. Mukherjee [4] used the interval valued intuitioinstic fuzzy soft set to granulate the universe of discourse and obtained a mathematical model called soft interval –valued intuitionistic fuzzy rough set. Because the soft interval –valued intuitionistic fuzzy rough set cannot deal with indeterminate and inconsistent data, in this section, we attempt to develop an new concept called soft interval –valued neutrosophic rough sets.

Definition 3.1. Let us consider an interval-valued neutrosophic set σ defined by

σ = {x, $\mu_\sigma(x)$, $\nu_\sigma(x)$, $\omega_\sigma(x)$: x ∈ U} where $\mu_\sigma(x)$, $\nu_\sigma(x)$, $\omega_\sigma(x)$ ∈ int ([0, 1]) for each x ∈ U and

$$0 \le \mu_\sigma(x)+ \nu_\sigma(x) + \omega_\sigma(x) \le 3$$

Now Let Θ=(f,A) be an interval-valued neutrosophic soft set over U and the pair SIVN= (U, Θ) be the soft interval-valued neutrosophic approximation space.
Let f:A→ $IVNS^U$ be defined f(a) ={ x, $\mu_{f(a)}(x)$, $\nu_{f(a)}(x)$, $\omega_{f(a)}(x)$: x ∈ U } for each a ∈ A. Then , the lower and upper soft interval-valued neutrosophic rough

approximations of σ with respect to SIVN are denoted by $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ and $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ respectively, which are interval valued neutrosophic sets in U given by:

$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ **={<x,**
[$\wedge_{a\in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma}(x))$, $\wedge_{a\in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma}(x)]$, **[** $\wedge_{a\in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma}(x))$, $\wedge_{a\in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma}(x)]$, **[** $\wedge_{a\in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x))$, $\wedge_{a\in A}(\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma}(x)]$>: x ∈ U }

$\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ **={< x, [** $\wedge_{a\in A}(\inf \mu_{f(a)}(x) \vee \inf \mu_{\sigma}(x))$, $\wedge_{a\in A}(\sup \mu_{f(a)}(x) \vee \sup \mu_{\sigma}(x)]$, **[** $\wedge_{a\in A}(\inf \nu_{f(a)}(x) \wedge \inf \nu_{\sigma}(x))$, $\wedge_{a\in A}(\sup \nu_{f(a)}(x) \wedge \sup \nu_{\sigma}(x)]$,
[$\wedge_{a\in A}(\inf \omega_{f(a)}(x) \wedge \inf \omega_{\sigma}(x))$, $\wedge_{a\in A}(\sup \omega_{f(a)}(x) \wedge \sup \omega_{\sigma}(x)]$>: x ∈ U}

The operators $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ and $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ are called the lower and upper soft interval-valued neutrosophic rough approximation operators on interval valued neutrosophic sets. If $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) = \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$, then σ is said to be soft interval valued neutrosophic definable; otherwise is called a soft interval valued neutrosophic rough set.

Remark 3.2: it is to be noted that if $\mu_{\sigma}(x)$, $\nu_{\sigma}(x)$, $\omega_{\sigma}(x) \in$ int ([0, 1]) and $0 \le \mu_{\sigma}(x)+\nu_{\sigma}(x)+\omega_{\sigma}(x) \le 1$, then soft interval valued neutrosophic rough sets becomes soft interval valued intuitionistic fuzzy rough sets.

Example 3.3 . Let U={x, y) and A={a, b}. Let (f, A) be an interval –valued neutrosophic soft se over U where f:A→ $IVNS^{U}$ be defined

f(a)= **{**<x,[0.2, 0. 5],[0.3, 0.4],[0.4, 0.5]>,<y,[0.6, 0.7],[0.1, 0.2],[0.3 0.4]>}
f(b)=**{**<x,[0.1, 0. 3],[0.4, 0.5],[0.1, 0.2]>, <y,[0.5, 0.8],[0.1, 0.2],[0.1 0.2]>}
Let σ =**{**<x,[0.3, 0.4],[0.3, 0.4],[0.1, 0.2]>, <y,[0.2, 0.4],[0.4, 0.5],[0.2 0.3]>}. Then

$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$= **{** <$x$,[0.1, 0.3],[0.3, 0.4],[0.1, 0.2]>, <y,[0.2, 0.4],[0.4, 0.5],[0.2, 0.3]>}
$\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma)$ = **{** <x,[0.3, 0.4],[0.3, 0.4],[0.1, 0.2]>, <y,[0.5, 0.7],[0.1, 0.2],[0.1, 0.2]>}. Then σ is a soft interval-valued neutrosophic rough set.

Theorem 3.4

Let Θ=(f,A) be an interval-valued neutrosophic soft set over U and SIVN= (U, Θ) be the soft interval-valued neutrosophic approximation space. Then for $\sigma, \lambda \in IVNS^{U}$, we have

1) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\emptyset) = \emptyset = \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\emptyset)$
2) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(U) = U = \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(U)$
3) $\sigma \subseteq \lambda \Longrightarrow \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$
4) $\sigma \subseteq \lambda \Longrightarrow \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \subseteq \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$
5) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.
6) $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda) \subseteq \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$**.**
7) $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cup \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda) \subseteq \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cup \lambda)$.
8) $\uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cup \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda) \subseteq \uparrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cup \lambda)$

Proof .(1)-(4) are straight forward.
(5) We have
σ={<x,[$\inf \mu_{\sigma}(x)$, $\sup \mu_{\sigma}(x)$], [$\inf \nu_{\sigma}(x)$, $\sup \nu_{\sigma}(x)$], [$\inf \omega_{\sigma}(x)$, $\sup \omega_{\sigma}(x)$]>:x∈ U},
λ ={<x,[$\inf \mu_{\lambda}(x)$, $\sup \mu_{\lambda}(x)$], [$\inf \nu_{\lambda}(x)$, $\sup \nu_{\lambda}(x)$], [$\inf \omega_{\lambda}(x)$, $\sup \omega_{\lambda}(x)$]>:x∈ U}
and

$\sigma \cap \lambda$= {<x,[$\inf \mu_{\sigma\cap\lambda}(x)$, $\sup \mu_{\sigma\cap\lambda}(x)$], [$\inf \nu_{\sigma\cap\lambda}(x)$, $\sup \nu_{\sigma\cap\lambda}(x)$], [$\inf \omega_{\sigma\cap\lambda}(x)$, $\sup \omega_{\sigma\cap\lambda}(x)$]>:x∈ U},
Now
$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cap \lambda)$=**{< x, [** $\wedge_{a\in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma\cap\lambda}(x))$, $\wedge_{a\in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma\cap\lambda}(x)]$,
[$\wedge_{a\in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma\cap\lambda}(x))$, $\wedge_{a\in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma\cap\lambda}(x)]$, **[** $\wedge_{a\in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma\cap\lambda}(x))$, $\wedge_{a\in A}(\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma\cap\lambda}(x)]$>: x ∈ U }
=**{< x, [** $\wedge_{a\in A}(\inf \mu_{f(a)}(x) \wedge \min(\inf \mu_{\sigma}(x), \inf \mu_{\lambda}(x))$, $\wedge_{a\in A}(\sup \mu_{f(a)}(x) \wedge \min(\sup \mu_{\sigma}(x), \sup \mu_{\lambda}(x))]$,
[$\wedge_{a\in A}(\inf \nu_{f(a)}(x) \vee \max(\inf \nu_{\sigma}(x), \inf \nu_{\lambda}(x)))$, $\wedge_{a\in A}(\sup \nu_{f(a)}(x) \vee \max(\sup \nu_{\sigma}(x), \sup \nu_{\lambda}(x))]$,
[$\wedge_{a\in A}(\inf \omega_{f(a)}(x) \vee \max(\inf \omega_{\sigma}(x), \inf \omega_{\lambda}(x)))$, $\wedge_{a\in A}(\sup \omega_{f(a)}(x) \vee \max(\sup \omega_{\sigma}(x), \sup \omega_{\lambda}(x)]$>: x ∈ U }

Now $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cap \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.

= **{< x, [min (** $\wedge_{a\in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma}(x))$, $\wedge_{a\in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\lambda}(x))$), **min (** $\wedge_{a\in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma}(x))$, $\wedge_{a\in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\lambda}(x))$)] ,[**max (** $\wedge_{a\in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma}(x))$, $\wedge_{a\in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\lambda}(x))$), **max(** $\wedge_{a\in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma}(x))$, $\wedge_{a\in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\lambda}(x))$)], [**max (** $\wedge_{a\in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x))$

, $\wedge_{a \in A}$ (inf $\omega_{f(a)}$(x) ∨ inf ω_{λ} (x))), **max(** $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) ∨ sup ω_{σ} (x)) , $\wedge_{a \in A}$ (sup $\omega_{f(a)}$(x) ∨ sup ω_{λ} (x)))]> : x∈ U}.

Since $\min(\inf \mu_{\sigma} (y), \inf\mu_{\lambda} (y)) \leq \inf \mu_{\sigma}(y)$
and $\min(\inf \mu_{\sigma} (y), \inf\mu_{\lambda} (y)) \leq \inf \mu_{\lambda}(y)$
we have
$\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \min(\inf \mu_{\sigma} (x), \inf \mu_{\lambda}(x)) \leq \wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma} (x))$
and $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \min(\inf \mu_{\sigma} (x), \inf \mu_{\lambda}(x)) \leq \wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_{\lambda} (x))$

Hence $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \min(\inf \mu_{\sigma} (x), \inf \mu_{\lambda}(x)) \leq$ **min** $(\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma} (x)), \wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_{\lambda} (x)))$

Similarly
$\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \min(\sup \mu_{\sigma} (x), \sup \mu_{\lambda}(x)) \leq$ **min** $(\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma} (x)), \wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_{\lambda} (x)))$
Again since
$\max(\inf \nu_{\sigma} (y), \inf\nu_{\lambda} (y)) \geq \inf \nu_{\sigma}(y)$
and $\max(\inf \nu_{\sigma} (y), \inf\nu_{\lambda} (y)) \geq \inf \nu_{\lambda}(y)$

we have
$\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \max(\inf \nu_{\sigma} (x), \inf \nu_{\lambda}(x)) \geq \wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma} (x))$
and $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \max(\inf \nu_{\sigma} (x), \inf \nu_{\lambda}(x)) \geq \wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_{\lambda} (x))$

Hence $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \max(\inf \nu_{\sigma} (x), \inf \nu_{\lambda}(x)) \geq$ **max** $(\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma} (x)), \wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_{\lambda} (x)))$

Similarly
$\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \max(\sup \nu_{\sigma} (x), \sup \nu_{\lambda}(x)) \geq$ **max** $(\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma} (x)), \wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_{\lambda} (x)))$

Again since
$\max(\inf \omega_{\sigma} (y), \inf\omega_{\lambda} (y)) \geq \inf \omega_{\sigma}(y)$
And $\max(\inf \omega_{\sigma} (y), \inf\omega_{\lambda} (y)) \geq \inf \omega_{\lambda}(y)$

we have
$\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \max(\inf \omega_{\sigma} (x), \inf \omega_{\lambda}(x)) \geq \wedge_{a \in A} (\inf \nu_{\omega f(a)}(x) \vee \inf \omega_{\sigma} (x))$
and $\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \max(\inf \omega_{\sigma} (x), \inf \omega_{\lambda}(x)) \geq \wedge_{a \in A} (\inf \omega_{f(a)}(x) \wedge \inf \omega_{\lambda} (x))$

Hence
$\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \max(\inf \omega_{\sigma} (x), \inf \nu_{\lambda}(x)) \geq$ **max** $(\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma} (x)), \wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \inf \omega_{\lambda} (x)))$

Similarly
$\wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \max(\sup \omega_{\sigma} (x), \sup \omega_{\lambda}(x)) \geq$ **max** $(\wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma} (x)), \wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \sup \omega_{\lambda} (x)))$
Consequently,
$\downarrow Apr_{SIVN}(\sigma \cap \lambda) \subseteq \downarrow Apr_{SIVN}(\sigma) \cap \downarrow Apr_{SIVN}(\lambda)$.

(6) Proof is similar to (5).
(7) we have
σ={<x,[inf μ_{σ}(x), sup μ_{σ}(x)] ,[inf ν_{σ}(x), sup ν_{σ}(x)] ,[inf ω_{σ}(x), sup ω_{σ}(x)]>:x∈ U},
λ ={<x,[inf μ_{λ}(x), sup μ_{λ}(x)],[inf ν_{λ}(x), sup ν_{λ}(x)] ,[inf ω_{λ}(x), sup ω_{λ}(x)]>:x∈ U}
And

$\sigma \cup \lambda$= {<x,[$\inf \mu_{\sigma \cup \lambda}(x)$, $\sup \mu_{\sigma \cup \lambda}(x)$], [$\inf \nu_{\sigma \cup \lambda}(x)$, $\sup \nu_{\sigma \cup \lambda}(x)$], [$\inf \omega_{\sigma \cup \lambda}(x)$, $\sup \omega_{\sigma \cup \lambda}(x)$]>:x∈ U},
$\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma \cup \lambda)$=**{< x, [** $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma \cup \lambda}(x))$, $\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma \cup \lambda}(x)]$,
[$\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma \cup \lambda}(x))$, $\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma \cup \lambda}(x)]$, **[** $\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma \cup \lambda}(x))$,
$\wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma \cup \lambda}(x)]$>: x ∈ U }
=**{< x, [** $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \max(\inf \mu_\sigma (x), \inf \mu_\lambda(x))$, $\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \max(\sup \mu_\sigma (x), \sup \mu_\lambda(x))]$,
[$\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \min(\inf \nu_\sigma (x), \inf \nu_\lambda(x)))$, $\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \min(\sup \nu_\sigma (x), \sup \nu_\lambda(x))]$,
[$\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \min(\inf \omega_\sigma (x), \inf \omega_\lambda(x)))$, $\wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \min(\sup \omega_\sigma (x), \sup \omega_\lambda(x)]$>: x ∈ U }

Now $\downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\sigma) \cup \downarrow \mathrm{Apr}_{\mathrm{SIVN}}(\lambda)$.

= **{< x, [max (** $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_\sigma (x))$, $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_\lambda (x))$), **max(** $\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_\sigma (x))$, $\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_\lambda (x))$)], [**min (** $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_\sigma (x))$, $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_\lambda (x))$), **min (** $\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_\sigma (x))$, $\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_\lambda (x))$)], [**min (** $\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \inf \omega_\sigma (x))$, $\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \inf \omega_\lambda (x))$), **min (** $\wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \sup \omega_\sigma (x))$, $\wedge_{a \in A} (\sup \omega_{f(a)}(x) \vee \sup \omega_\lambda (x))$)]> : x∈ U}

Since $\max(\inf \mu_\sigma (y), \inf \mu_\lambda (y)) \geq \inf \mu_\sigma(y)$
and $\max(\inf \mu_\sigma (y), \inf \mu_\lambda (y)) \geq \inf \mu_\lambda(y)$
we have
$\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \max(\inf \mu_\sigma (x), \inf \mu_\lambda(x)) \geq \wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_\sigma (x))$
and $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \max(\inf \mu_\sigma (x), \inf \mu_\lambda(x)) \geq \wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_\lambda (x))$

Hence $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \max(\inf \mu_\sigma (x), \inf \mu_\lambda(x)) \geq$ **max** ($\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_\sigma (x))$, $\wedge_{a \in A} (\inf \mu_{f(a)}(x) \wedge \inf \mu_\lambda (x))$)

Similarly
$\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \max(\sup \mu_\sigma (x), \sup \mu_\lambda(x)) \geq$ **max** ($\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_\sigma (x))$, $\wedge_{a \in A} (\sup \mu_{f(a)}(x) \wedge \sup \mu_\lambda (x))$)
Again since
$\min(\inf \nu_\sigma (y), \inf \nu_\lambda (y)) \leq \inf \nu_\sigma(y)$
and $\min(\inf \nu_\sigma (y), \inf \nu_\lambda (y)) \leq \inf \nu_\lambda(y)$

we have
$\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \min(\inf \nu_\sigma (x), \inf \nu_\lambda(x)) \leq \wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_\sigma (x))$
and $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \min(\inf \nu_\sigma (x), \inf \nu_\lambda(x)) \leq \wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_\lambda (x))$

Hence $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \min(\inf \nu_\sigma (x), \inf \nu_\lambda(x)) \leq$ **min** ($\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_\sigma (x))$, $\wedge_{a \in A} (\inf \nu_{f(a)}(x) \vee \inf \nu_\lambda (x))$)

Similarly
$\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \min(\sup \nu_\sigma (x), \sup \nu_\lambda(x)) \leq$ **minx** ($\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_\sigma (x))$, $\wedge_{a \in A} (\sup \nu_{f(a)}(x) \vee \sup \nu_\lambda (x))$)

Again since
$\min(\inf \omega_\sigma (y), \inf \omega_\lambda (y)) \leq \inf \omega_\sigma(y)$
And $\min(\inf \omega_\sigma (y), \inf \omega_\lambda (y)) \leq \inf \omega_\lambda(y)$

we have
$\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \min(\inf \omega_\sigma (x), \inf \omega_\lambda(x)) \leq \wedge_{a \in A} (\inf \nu_{\omega f(a)}(x) \vee \inf \omega_\sigma (x))$
and $\wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \min(\inf \omega_\sigma (x), \inf \omega_\lambda(x) \leq \wedge_{a \in A} (\inf \omega_{f(a)}(x) \vee \inf \omega_\lambda (x))$

Hence $\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \min(\inf \omega_{\sigma}(x), \inf \nu_{\lambda}(x)) \leq \mathbf{min}(\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x)), \wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\lambda}(x)))$

Similarly

$\wedge_{a \in A}(\sup \omega_{f(a)}(x) \vee \min(\sup \omega_{\sigma}(x), \sup \omega_{\lambda}(x)) \leq \mathbf{min}(\wedge_{a \in A}(\sup \omega_{f(a)}(x) \wedge \sup \omega_{\sigma}(x)), \wedge_{a \in A}(\sup \omega_{f(a)}(x) \wedge \sup \omega_{\lambda}(x)))$

Consequently,

$\downarrow Apr_{SIVN}(\sigma) \cup \downarrow Apr_{SIVN}(\lambda) \subseteq \downarrow Apr_{SIVN}(\sigma \cup \lambda)$

(8) Proof is similar to (7).

Theorem 3.5. Every soft interval-valued neutrosophic rough set is an interval valued neutrosophic soft set.

Proof. Let Θ=(f,A) be an interval-valued neutrosophic soft set over U and SIVN=(U, Θ) be the soft interval-valued neutrosophic approximation space. Let σ be a soft interval-valued neutrosophic rough set. Let us define an interval-valued neutrosophic set χ by:

$$\chi = \{(x, [\frac{\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma}(x))}{\wedge_{a \in A}(\inf \mu_{f(a)}(x) \vee \inf \mu_{\sigma}(x))}, \frac{\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma}(x))}{\wedge_{a \in A}(\sup \mu_{f(a)}(x) \vee \sup \mu_{\sigma}(x))}], [\frac{\wedge_{a \in A}(\inf \nu_{f(a)}(x) \wedge \inf \nu_{\sigma}(x))}{\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma}(x))}, \frac{\wedge_{a \in A}(\sup \nu_{f(a)}(x) \wedge \sup \nu_{\sigma}(x))}{\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma}(x))}], [\frac{\wedge_{a \in A}(\inf \omega_{f(a)}(x) \wedge \inf \omega_{\sigma}(x))}{\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x))}, \frac{\wedge_{a \in A}(\sup \omega_{f(a)}(x) \wedge \sup \mu\omega_{\sigma}(x))}{\wedge_{a \in A}(\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma}(x))}]): x \in U \}$$

Now, for $\theta \in [0, 1]$, we consider the following six sets:

$F_1(\theta)= \{ x \in U : \frac{\wedge_{a \in A}(\inf \mu_{f(a)}(x) \wedge \inf \mu_{\sigma}(x))}{\wedge_{a \in A}(\inf \mu_{f(a)}(x) \vee \inf \mu_{\sigma}(x))} \geq \theta\}$

$F_2(\theta)= \{ x \in U : \frac{\wedge_{a \in A}(\sup \mu_{f(a)}(x) \wedge \sup \mu_{\sigma}(x))}{\wedge_{a \in A}(\sup \mu_{f(a)}(x) \vee \sup \mu_{\sigma}(x))} \geq \theta\}$

$F_3(\theta)= \{ x \in U : \frac{\wedge_{a \in A}(\inf \nu_{f(a)}(x) \wedge \inf \nu_{\sigma}(x))}{\wedge_{a \in A}(\inf \nu_{f(a)}(x) \vee \inf \nu_{\sigma}(x))} \geq \theta\}$

$F_4(\theta)= \{ x \in U : \frac{\wedge_{a \in A}(\sup \nu_{f(a)}(x) \wedge \sup \nu_{\sigma}(x))}{\wedge_{a \in A}(\sup \nu_{f(a)}(x) \vee \sup \nu_{\sigma}(x))} \geq \theta\}$

$F_5(\theta)= \{ x \in U : \frac{\wedge_{a \in A}(\inf \omega_{f(a)}(x) \wedge \inf \omega_{\sigma}(x))}{\wedge_{a \in A}(\inf \omega_{f(a)}(x) \vee \inf \omega_{\sigma}(x))} \geq \theta\}$

$F_6(\theta)= \{ x \in U : \frac{\wedge_{a \in A}(\sup \omega_{f(a)}(x) \wedge \sup \mu\omega_{\sigma}(x))}{\wedge_{a \in A}(\sup \omega_{f(a)}(x) \vee \sup \omega_{\sigma}(x))} \geq \theta\}$

Then $\psi(\theta)$= { (x, [inf{ θ: x ∈ $F_1(\theta)$}, inf{ θ: x ∈ $F_2(\theta)$}], [inf{ θ: x ∈ $F_3(\theta)$}, inf{ θ: x ∈ $F_4(\theta)$}], [inf{ θ: x ∈ $F_5(\theta)$}, inf{ θ: x ∈ $F_6(\theta)$}]) :x ∈ U}is an interval –valued neutrosophic set over U for each $\theta \in [0, 1]$. Consequently (ψ, θ) is an interval-valued neutrosophic soft set over U.

4.A Multi-criteria Group Decision Making Problem

In this section, we extend the soft interval –valued intuitionistic fuzzy rough set based multi-criteria group decision making scheme [4] to the case of the soft interval-valued neutrosophic rough set.

Let U={$o_1, o_2, o_3,\ldots, o_r$} be a set of objects and E be a set of parameters and A = {$e_1, e_2, e_3,\ldots, e_m$} ⊆ E and S=(F, A) be an interval- neutrosophic soft set over U. Let us assume that we have an expert group G = {$T_1, T_2, T_3,\ldots, T_n$} consisting of n specialists to evaluate the objects in U. Each specialist will examine all the objects in U and will point out his/her evaluation result. Let X_i denote the primary evaluation result of the specialist T_i. It is easy to see that the primary evaluation result of the whole expert group G can be represented as an interval valued neutrosophic evaluation soft set S^* = (F^*, G) over U, where $F^*: G \longrightarrow IVNS^U$ is given by $F^*(T_i)= X_i$, for i=1,2,..n.

Now we consider the soft interval valued neutrosophic rough approximations of the specialist T_i's primary evaluation result X_i w.r.t the soft interval valued neutrosophic approximation space SIVN = (U, S). Then we obtain two other interval valued neutrosophic soft sets $\downarrow S^*$= ($\downarrow F^*$,G) and $\uparrow S^*$= ($\uparrow F^*$,G) over U, where $\downarrow S^*$: $G \longrightarrow IVNS^U$ is given by $\downarrow F^* =\downarrow X_i$ and $\uparrow F^*: G \longrightarrow IVNS^U$ is given by $\uparrow F^*(T_i)= =\uparrow X_i$, for i=1,2,..n. Here $\downarrow S^*$ can be considered as the evaluation result for the whole expert group G with 'low confidence', $\uparrow S^*$ can be considered as the evaluation result for the whole expert group G with 'high confidence' and S^* can be considered as the evaluation result for the whole expert group G with 'middle confidence' Let us define two interval valued neutrosophic sets $IVNS_{\downarrow S^*}$ **and** $IVNS_{\uparrow S^*}$ by

$IVNS_{\downarrow S^*}$ ={⟨o_k, [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\mu_{\downarrow F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\mu_{\downarrow F^*(T_j)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\nu_{\downarrow F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\nu_{\downarrow F^*(T_j)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\omega_{\downarrow F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\omega_{\downarrow F^*(T_j)}(o_k)$]>: $k = 1,2,..r$}

And

$IVNS_{\uparrow S^*}$ ={⟨o_k, **[**$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\mu_{\uparrow F^*(T_i)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\mu_{\uparrow F^*(T_i)}(o_k)$**]**, **[**$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\nu_{\uparrow F^*(T_i)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\nu_{\uparrow F^*(T_i)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{inf}\,\omega_{\uparrow F^*(T_i)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{sup}\,\omega_{\uparrow F^*(T_i)}(o_k)$]>: $k = 1,2,..r$}

Now we define another interval valued neutrosophic set $IVNS_{S^*}$ by

$IVNS_{S^*}$ ={⟨o_k, [$\frac{1}{n}\sum_{j=1}^{n} inf\, \mu_{F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} sup\, \mu_{F^*(T_j)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} inf\, \nu_{F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} sup\, \nu_{F^*(T_j)}(o_k)$], [$\frac{1}{n}\sum_{j=1}^{n} inf\, \omega_{F^*(T_j)}(o_k)$, $\frac{1}{n}\sum_{j=1}^{n} sup\, \omega_{F^*(T_j)}(o_k)$]>: $k = 1,2,..r$}

Then clearly,

$IVNS_{\downarrow S^*} \subseteq IVNS_{S^*} \subseteq IVNS_{\uparrow S^*}$

Let C={L (low confidence), M (middle confidence), H (high confidence)} be a set of parameters. Let us consider the interval valued neutrosophic soft set S^{**}= (f, C) over U, where f: $C \longrightarrow IVNS^U$ is given by f(L)= $IVNS_{\downarrow S^*}$, f(M)= $IVNS_{S^*}$, f(H)= $IVNS_{\uparrow S^*}$. Now given a weighting vector W= (ω_L , ω_M , ω_H) such that ω_L , ω_M , $\omega_H \in$ [0, 1], we define $\alpha: U \longrightarrow P(U) by\ \alpha(o_k) = \omega_L \diamond s_{f(L)}(o_k) + \omega_M \diamond s_{f(M)}(o_k) + \diamond s_{f(H)}(o_k)$, $o_k \in$ U ($\diamond$ represents ordinary multiplication) where

$$s_{f(L)}(o_k) = \frac{inf\mu_{\downarrow F^*(T_j)} + sup\mu_{\downarrow F^*(T_j)} - inf\, \nu_{\downarrow F^*(T_j)} . sup\, \nu_{\downarrow F^*(T_j)} - inf\omega_{\downarrow F^*(T_j)} . sup\omega_{\downarrow F^*(T_j)}}{2}$$

denotes the score function, the same as $s_{f(M)}(o_k)$ and $s_{f(H)}(o_k)$. Here $\alpha(o_k)$ is called the weighted evaluation value of the alternative $o_k \in$ U. Finally, we can select the object o_p =max{ $\alpha(o_k)$}:k=1,2,…,r} as the most preferred alternative.

Algorithm:

(1) Input the original description Interval valued neutrosophic soft set (F, A).
(2) Construct the interval valued neutrosophic evaluation soft set S^* =(F^*, G)
(3) Compute the soft interval valued neutrosophic rough approximations and then construct the interval valued neutrosophic soft sets $\downarrow S^*$ **and** $\uparrow S^*$
(4) Construct the interval valued neutrosophic $IVNS_{\downarrow S^*}$, $IVNS_{S^*}$, $IVNS_{\uparrow S^*}$
(5) Construct the interval valued neutrosophic soft set S^{**}.
(6) Input the weighting vector W and compute the weighted evaluation values of each alternative $\alpha(o_k)$ of each alternative $o_k \in$ U.
(7) Select the object o_p such that object o_p =max{ $\alpha(o_k)$}:k=1,2,…,r} as the most preferred alternative.

5.An illustrative example

The following example is adapted from **[4]** with minor changes.

Let us consider a staff selection problem to fill a position in a private company.

Let U = {c_1, c_2, c_3, c_4, c_5} is the universe set consisting of five candidates. Let us consider the soft set S=(F, A), which describes the "quality of the candidates", where A={e_1 (experience), e_2 (computer knowledge), e_3 (young and efficient), e_4 (good communication skill)}. Let the tabular representation of the interval valued neutrosophicsoft set (F, A) be:

	c_1	c_2	c_3	c_4	c_5
e_1	([.2, .3],[.4, .5],[.3, .4])	([.5, .7],[.1, .3],[.2, .3])	([.4, .5],[.2, .4],[.2, .5])	([.1, .2],[.1, .3],[.1, .2])	([.3, .5],[.3, .4],[.1, .2])
e_2	([.3, .6],[.1, .2],[.2, .3])	([.1, .3],[.2, .3],[.2, .4])	([.3, .6],[.2, .4],[.2, .4])	([.5, .6],[.2, .3],[.2, .4])	([.1, .3],[.3, .6],[.2, .5])
e_3	([.4, .5],[.2, .3],[.4, .5])	([.2, .4],[.2, .5],[.1, .2])	([1, .3],[.4, .6],[.3, .5])	([.3, .4],[.3, .4],[.4, .6])	([.4, .6],[.1, .3],[.2, .3])
e_4	([.2, .4],[.6, .7],[.6, .7])	([.6, .7],[.1, .2],[.4, .5])	([.3, .4],[.3, .4],[.1, .2])	([.2, .4],[.4, .6],[.1, .2])	([.5, .7],[.1, .2],[.1, .5])

Let G = {T_1, T_2, T_3, T_4, T_4} be the set of interviewers to judge the quality of the candidate in U. Now if X_i denote the primary evaluation result of the interviewer T_i (for i=1, 2, 3, 4,5), then the primary evaluation result of the whole expert group G can be represented as an interval valued neutrosophic evaluation soft set S^*= (F^*,G) over U, where $F^*: G \longrightarrow IVNS^U$ is given by F^* (T_i) = X_i for i=1, 2, 3, 4,5.

Let the tabular representation of S^* be given as:

	c_1	c_2	c_3	c_4	c_5
T_1	([.4, .6],[.4, .5],[.3, .4])	([.3, .4],[.1, .2],[.2, .3])	([.2, 3],[.2, .3],[.2, .5])	([.6, .8],[.1, .2],[.1, .2])	([.1, .4],[.2, .3],[.1, .2])
T_2	([.3, .5],[.2, .4],[.2, .3])	([.5, .7],[.1, .3],[.2, .4])	([.4, .6],[.1, .3],[.2, .4])	([.3, .5],[.1, .3],[.2, .4])	([.4, .5],[.2, .3],[.2, .5])
T_3	([.1, .3],[.5, .6],[.4, .5])	([.2, .3],[.4, .5],[.1, .2])	([.1, .4],[.2, .4],[.3, .5])	([.2, .3],[.5, .6],[.4, .6])	([.3, .6],[.2, .3],[.2, .3])
T_4	([.2, .3],[.3, .4],[.6, .7])	([.4, .7],[.1, .2],[.4, .5])	([.3, .5],[.4, .5],[.1, .2])	([.4, .5],[.2, .4],[.1, .2])	([.5, .7],[.1, .2],[.1, .5])
T_5	([.6, .7],[.1, .2],[.6, .7])	([.3, .5],[.3, .4],[.4, .6])	([.5, .6],[.3, .4],[.2, .3])	([.1, .3],[.3, .6],[.4, .6])	([.1, .2],[.6, .8],[.2, .5])

Let us choose P=(U, S) as the soft interval valued neutrosophic approximation space. Let us consider the interval valued neutrosophic evaluation soft sets.

$\downarrow S^* = (\downarrow F^*, G)$ and $\uparrow S^* = (\uparrow F^*, G)$ over U.
Then the tabular representation of these sets are:

$\downarrow S^* = (\downarrow F^*, G)$:

	c_1	c_2	c_3	c_4	c_5
T_1	([.2, .3],[.1, .2],[.3, .4])	([.1, .3],[.3, .4],[.2, .3])	([.1, .3],[.2, .4],[.2, .5])	([.1, .2],[.1, .3],[.1, .2])	([.1, .3],[.2, .4],[.1, .2])
T_2	([.2, .3],[.2, .4],[.2, .3])	([.1, .3],[.1, .3],[.2, .4])	([.1, 3],[.2, .4],[.2, .4])	([.1, .2],[.1, .3],[.2, .4])	([.1, .3],[.2, .3],[.2, .5])
T_3	([.1, .3],[.5, .6],[.4, .5])	([.1, .3],[.4, .5],[.1, .2])	([.1, .3],[.2, .4],[.3, .5])	([.1, .2],[.5, .6],[.4, .6])	([.1, .3],[.2, .3],[.2, .3])
T_4	([.2, .3],[.3, .4],[.6, .7])	([.1, .3],[.1, .2],[.4, .5])	([.1, .3],[.4, .5],[.1, .2])	([.1, .2],[.2, .4],[.1, .2])	([.1, .3],[.1, .2],[.1, .5])
T_5	([.2, .3],[.1, .2],[.6, .7])	([.1, .3],[.2, .5],[.4, .6])	([.1, .3],[.3, .4],[.2, .3])	([.1, .2],[.3, 6],[.4, .6])	([.1, .2],[.6, .8],[.2, .5])

$\uparrow S^* = (\uparrow F^*, G)$

	c_1	c_2	c_3	c_4	c_5
T_1	([.4, .6],[.1, .2],[.2, .3])	([.3, .4],[.1, .2],[.1, .2])	([.2, .3],[.2, .3],[.1, .2])	([.6, .8],[.1, .2],[.1, .2])	([.1, .4],[.1, .2],[.1, .2])
T_2	([.3, .5],[1, .2],[.2, .3])	([.5, .7],[.1, .2],[.1, .2])	([.4, .6],[.1, .3],[.1, .2])	([.3, .5],[.1, .3,[.1, .2])	([.4, .5],[.1, .2],[.1, .2])
T_3	([.2, .3],[.1, .2],[.2, .3])	([.2, .3],[.1, .2],[.1, .2])	([.1, .4],[.2, .4],[.1, .2])	([.2, .3],[.1 .3],[.1, .2])	([.3, .6],[.1, .2],[.1, .2])
T_4	([.2, .3],[.1, .2],[.2, .3])	([.4, .7],[.1, .2],[.1, .2])	([.3, .5],[.2, .4],[.1, .2])	([.4, .5],[.1, .3],[.1, .2])	([.5, .7],[.1, .2],[.1, .2])
T_5	([.6, .7],[.1, .2],[.2, .3])	([.3, .5],[.1, .2],[.1, .2])	([.5, .6],[.2, .4],[.1, .2])	([.1, .3],[.1, 3],[.1, .2])	([.1, .3],[.1, .2],[.1, .2])

Here, $\downarrow S^* \subseteq S^* \subseteq \uparrow S^*$

$IVNS_{\downarrow S^*}$ = **{** < c_1,[0.15, 0.35],[0.4, 0.625],[0.42, 0.52]> < c_2,[0.175, 0.325],[0.375, 0.575],[0.26, 0.4]>, < c_3,[0.175, 0.375],[0.375, 0.575],[0.2, 0.38]>, < c_4,[0.175, 0.375],[0.375, 0.575],[0.24, 0.4]>, < c_5,[0.175, 0.375],[0.375, 0.575],[0.16, 0.4]>}.

$IVNS_{\uparrow S^*}$= **{** < c_1,[0.575, 0.75],[0.125, 0.225],[0.2, 0.3]> < c_2,[0.575, 0.75],[0.125, 0.225], [0.1, 0.2]>, < c_3,[0.575, 0.725],[0.125, 0.225],[0.1, 0.2]>, < c_4,[0.525, 0.700],[0.125, 0.225],[0.1, 0.2]>, < c_5,[0.55, 0.700],[0.125, 0.225],[0.1, 0.2]>}.

$IVNS_{S^*}$= **{** < c_1,[0.25, 0.45],[0.375, 0.475],[0.42, 0.52]> < c_2,[0.375, 0.525],[0.225, 0.35], [0.26, 0.4]>, < c_3,[0.350, 0.525],[0.2, 0.4],[0.2, 0.38]>, < c_4,[0.4, 0.6],[0.20, 0.35],[0.24, 0.4]>, < c_5,[0.35, 0.55],[0.15, 0.375],[0.16, 0.4]>}.

Here, $IVNS_{\downarrow S^*} \subseteq IVNS_{S^*} \subseteq IVNS_{\uparrow S^*}$ **. Let** C={ L (low confidence), M (middle confidence),H(high confidence)} be a set of parameters. Let us consider the interval valued neutrosophic soft set S^{**}= (f, C) over U, where f: $C \longrightarrow IVNS^U$ is given by f(L) = $IVNS_{\downarrow S^*}$, f(M) = $IVNS_{S^*}$, f(H) = $IVNS_{\uparrow S^*}$. Now assuming the weighting vector W =(ω_L, ω_M, ω_H) such that ω_L= 0.7 ω_M=0.6, ω_H=0.8, we have ,

$\alpha(c_1)$ = 0.7 ⋄ 0.0158 +0.6 ⋄ 0.15174 +0.8 ⋄ 0.6184
=0.5968
$\alpha(c_2)$= 0.7 ⋄ 0.0901 +0.6 ⋄ 0.3586 +0.8 ⋄ 0.6384
= 0.7890
$\alpha(c_3)$= 0.7 ⋄ 0.1041 +0.6 ⋄ 0.3595 +0.8 ⋄ 0.6384
=0.7993
$\alpha(c_4)$= 0.7 ⋄ 0.1191 +0.6 ⋄ 0.4170 +0.8 ⋄ 0.6134
=0.8243
$\alpha(c_5)$= 0.7 ⋄ 0.1351 +0.6 ⋄ 0.3898 +0.8 ⋄ 0.600
=0.8093

Since max($\alpha(c_1)$, $\alpha(c_2)$, $\alpha(c_3)$, $\alpha(c_4)$, $\alpha(c_5)$} = 0.8243, so the candidate c_4 will be selected as the most preferred alternative.

5.Conclusions

In this paper we have defined, for the first time, the notion of soft interval valued neutrosophic rough sets which is a combination of interval valued neutrosophic rough sets and soft sets. We have studied some of their basic properties. Thus our work is a generalization of SIVIF-rough sets. We hope that this paper will promote the future study on soft interval valued neutrosophic rough sets to carry out a general framework for their application in practical life.

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