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Cauchy Single-Valued Neutrosophic Numbers and Their Applications in MAGDM

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Abstract. The neutrosophic sets and numbers have an important role in modeling the problems. Recently, studies on neutrosophic numbers and single-valued neutrosophic numbers which is a subclass of neutrosophic numbers have increased, rapidly. Cauchy distribution is an important concept in the statistic. In this paper, the notion of Cauchy single-valued neutrosophic numbers (CSVNNs) and $\alpha-$cuts are introduced based on the Cauchy distribution formula. Summation, multiplication, and division operations between two CSVNNs are defined and given related examples. Also, the score functions of CSVNNs, arithmetic and geometric aggregation operators of them are described. Based on the defined new concepts, a multi-attribute group decision-making method is developed. Finally, to illustrate how the proposed method works, an application of the proposed method in the selection of a project to be supported and funded is developed. In this method, for each of the criteria, different score functions are determined by using the aggregation operators and score functions of the CSVNNs. Then, evaluations of the decision-makers are transformed into new values under the derived score functions for the decision. In applications, in general, decision-makers assign to criteria some values between 0 and 1 directly. In the method proposed in this paper, weights of the criteria are considered as the different functions. Therefore this method presents a more general perspective on decision-making problems.

Keywords: Single-valued neutrosophic set; Cauchy Single-valued neutrosophic number; decision-making; aggregation operators.

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1. **Introduction**

The fuzzy set theory is a notable theory put forward by Zadeh [38] as a useful tool for decision-making problems and as a generalization of classical sets. After introducing fuzzy sets, many researchers needed to study many generalizations of fuzzy sets in order to model the problems they encountered. The best known of these are intuitionistic fuzzy set defined by Atanassov [1], Pythagorean Fuzzy set introduced by Yager [35], Picture fuzzy set introduced by Cuong [11, 12], q-rung orthopair fuzzy set defined by Yager [36] set, spherical fuzzy sets proposed by Gundogdu and Kahraman [15], T-spherical fuzzy sets introduced by [20] and neutrosophic set (NS) by Smarandache [32]. An NS is described by three mappings defined from a non-empty set to a real standard or non-standard subset of $]^{-}0, 1^{+}[$. These functions are called truth, indeterminacy, and falsity functions and are represented by notations T, I, and F, respectively. The basis of the NS is based on neutrosophy which is a branch of philosophy. Neutrosophic sets have a very important role in modeling and solving decision-making problems. However, real standard or nonstandard subsets of $]^{-}0, 1^{+}[$ are not useful in modeling real-life problems. Therefore, Wang et al. [34] revealed the notion of a single-valued neutrosophic (SVN) set (SVNS) identified by three functions which are defined from a nonempty set into the unit interval $[0, 1]$. Many researcher studied on SVN number (SVNN) [4, 9, 13, 14, 16] and applications in decision-making (DM) based on similarity measures, distance measures, entropy and aggregation operators [2, 3, 5–8, 10, 17, 18, 22–30, 37, 39, 46–48]. In addition, after the definition of hypersoft sets [33] as a generalization of soft sets [21], Martin and Smarandache [19] combined the hypersoft sets of neutrosophic sets and introduced the concept of neutrosophic hypersoft set as a generalization of hypersoft sets. Recently, studies related to the neutrosophic hypersoft sets and hypersoft sets have been rapidly increasing. Some of them are aggregation operators [41], interval-valued neutrosophic hypersoft set [42], corrlation coefficient of interval-valued neutrosophic hypersoft set [43] Pythagorean fuzzy hypersoft set [44], correlation coefficient of neutrosophic hypersoft set [31], neutrosophic hypersoft matrices [45].

In 2018, Karaaslan [16] defined the Gaussian SVNNs and developed a multi-attribute decision-making method under the Gaussian SVN environment. He also presented an application of the proposed method in order to illustrate the progress of the developed method. The following points motivate us to present this paper:

- Single-valued trapezoidal neutrosophic number (SVTrNN), single-valued triangular neutrosophic numbers (SVTNN) and GSVNNs are important tools to model decision-making problems involving indeterminate, and inconsistent data. SVTrNN and SVTNN are expressed by partial functions involving straight line. However, sometimes indeterminate and inconsistent data may not be expressed linearly. Therefore, In order to represent nonlinear states, we introduce a new concept of neutrophic numbers based on the Cauchy distribution.
- In MADMPs, weights of the attributes are determined as real values between 0 and 1 such that their summation is equal to 1. For weights of the attributes, different functions are not considered. In this paper, we consider different score functions for each

attribute according to the common opinions of the decision-makers. Thus, developing a more flexible decision-making approach is aimed.

Following are the contributions of this article:

- The concept of CSVNNs is defined. Also, $\alpha-$cut and arithmetic operations of CSVNNs are introduced, and some results are obtained related to $\alpha-$cut of CSVNNs.
- The score functions of CSVNNs and their aggregation operators are defined.
- Based on novel definitions and operations introduced in this paper, a multi-attribute group decision-making method is proposed and given an illustrative example in order to explain the process of the proposed method.

This paper is organized as follows:In section 2, some basic concepts are recalled. In section 3, The concept of CSVNNs, α-cuts of CSVNNs, arithmetic operations between two CSVNNs, score function of CSVNN, and arithmetic and geometric aggregarin operators of them are defined and given examples of them. In section 4, a MAGDM method is developed and presented an illustrative example to show the working of the proposed method.

2. **Preliminaries**

In this section, some basic definitions related to neutrosophic sets are recalled.

Definition 2.1. [32] Let $\mathbb{X} \neq \emptyset$. Then, a neutrosophic set $\tilde{\mathfrak{A}}$ on $\mathbb{X}$ is a set of quadruplets, defined by

$$\tilde{\mathfrak{A}} = \left\{ \langle \theta, \tilde{\mathfrak{A}}_t(\theta), \tilde{\mathfrak{A}}_i(\theta), \tilde{\mathfrak{A}}_f(\theta)) \rangle : \theta \in \mathbb{X} \right\}.$$

Here $\tilde{\mathfrak{A}}_t, \tilde{\mathfrak{A}}_i, \tilde{\mathfrak{A}}_f : \mathbb{X} \to]^-0, 1^+[$ called truth, indeterminacy and falsity membership functions (MF) of the neutrosophic set $\tilde{\mathfrak{A}}$, respectively and ${}^-0 \leq \tilde{\mathfrak{A}}_t(\theta) + \tilde{\mathfrak{A}}_i(\theta) + \tilde{\mathfrak{A}}_f(\theta) \leq 3^+$.

Definition 2.2. [34] Let $\mathbb{X} \neq \emptyset$. Then, a single-valued neutrosophic set $(SVNS)$ $\hat{\mathfrak{A}} = \left\{ \langle \theta, \hat{\mathfrak{A}}_t(\theta), \hat{\mathfrak{A}}_i(\theta), \hat{\mathfrak{A}}_f(\theta) \rangle : \theta \in \mathbb{X} \right\}$ is defined as follows:

If $\mathbb{X}$ is continuous, an $SVNS$ $\hat{\mathfrak{A}}$ can be expressed by

$$\hat{\mathfrak{A}} = \int_{\mathbb{X}} \left\langle \hat{\mathfrak{A}}_t(\theta), \hat{\mathfrak{A}}_i(\theta), \hat{\mathfrak{A}}_f(\theta) \right\rangle / \theta, \text{ for all } \theta \in \mathbb{X}.$$

If $\mathbb{X}$ is crisp set, an $SVNS$ $\hat{\mathfrak{A}}$ can be expressed by

$$\hat{\mathfrak{A}} = \sum_{\theta} \left\langle \hat{\mathfrak{A}}_t(\theta), \hat{\mathfrak{A}}_i(\theta), \hat{\mathfrak{A}}_f(\theta) \right\rangle / \theta, \text{ for all } \theta \in \mathbb{X}.$$

Note that $0 \leq \hat{\mathfrak{A}}_t(\theta) + \hat{\mathfrak{A}}_i(\theta) + \hat{\mathfrak{A}}_f(\theta) \leq 3$ for all $\theta \in \mathbb{X}$. For convenience, an SVNN is denoted by $\hat{\mathfrak{A}} = \langle \hat{\mathfrak{A}}_t, \hat{\mathfrak{A}}_i, \hat{\mathfrak{A}}_f \rangle$.

3. Cauchy Single-valued Neutrosophic Number

In this part, we define the Cauchy fuzzy number (CFN) and its α -cuts. Then we introduce the concept of Cauchy single-valued number number by similar way.

3.1. *Cauchy Fuzzy Number*

Definition 3.1. [40] A fuzzy number is said to be Cauchy fuzzy number $\mathbb{A} = CFN(\mathfrak{p}, \mathfrak{q})$ whose membership function of is given by

$$\mu_{\mathbb{A}}(\theta) = \frac{1}{1 + \left(\frac{\theta - \mathfrak{p}}{\mathfrak{q}}\right)^2}.$$

Definition 3.2. Let us consider membership function of $\mathbb{A} = CFN(\mathfrak{p}, \mathfrak{q})$ as follows:

$$\mu_{\mathbb{A}}(\theta) = \frac{1}{1 + \left(\frac{\theta - \mathfrak{p}}{\mathfrak{q}}\right)^2}.$$

The α-cut set of $CFN(\mathfrak{p}, \mathfrak{q})$ is defined as follows:

$$\mathbb{A}_\alpha = \left[\mathfrak{p} - \mathfrak{q}\sqrt{\frac{1-\alpha}{\alpha}}, \mathfrak{p} + \mathfrak{q}\sqrt{\frac{1-\alpha}{\alpha}}\right]$$

3.2. *Cauchy Single-valued Neutrosophic Number*

Definition 3.3. A Cauchy single-valued neutrosophic number (CSVNN) is defined by truth, indeterminacy and falsity MFs as follows:

$$\begin{aligned}
\wp(\theta_t) &= \frac{1}{1 + \left(\dfrac{\theta_t - \mathfrak{p}_t}{\mathfrak{q}_t}\right)^2}, \\
\wp(\theta_i) &= 1 - \frac{1}{1 + \left(\dfrac{\theta_i - \mathfrak{p}_i}{\mathfrak{q}_i}\right)^2} = \frac{\left(\dfrac{\theta_i - \mathfrak{p}_i}{\mathfrak{q}_i}\right)^2}{1 + \left(\dfrac{\theta_i - \mathfrak{p}_i}{\mathfrak{q}_i}\right)^2}, \\
\wp(\theta_f) &= 1 - \frac{1}{1 + \left(\dfrac{\theta_f - \mathfrak{p}_f}{\mathfrak{q}_f}\right)^2} = \frac{\left(\dfrac{\theta_f - \mathfrak{p}_f}{\mathfrak{q}_f}\right)^2}{1 + \left(\dfrac{\theta_f - \mathfrak{p}_f}{\mathfrak{q}_f}\right)^2},
\end{aligned}$$

respectively.

A CSVNN is denoted by $\tilde{\mathbb{A}} = CSVNN\big((\mathfrak{p}_t, \mathfrak{q}_t), (\mathfrak{p}_i, \mathfrak{q}_i), (\mathfrak{p}_f, \mathfrak{q}_f)\big)$. Set of all CSVNNs over $\mathbb{X}$ is denoted by $\mathcal{CSVNN}(\mathbb{X})$.

Example 3.4. Let $\tilde{\mathbb{A}} = CSVNN\big((0.8, 0.1), (0.7, 0.3), (0.5, 0.2)\big)$ be CSVNN. Truth, indeterminacy, and falsity MFs of CSVNN are shown in Fig 1.

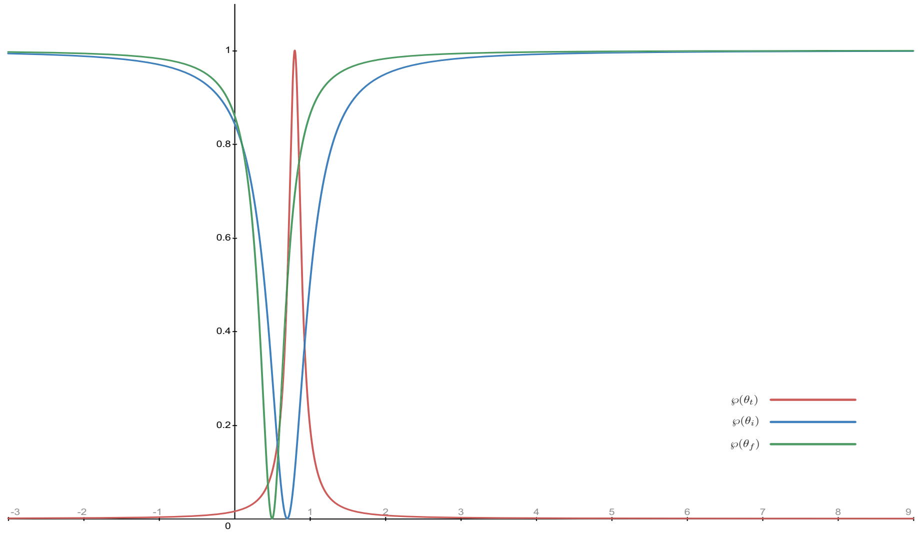

FIGURE 1. CSVNN $\tilde{\mathbb{A}}$

Definition 3.5. Let truth, indeterminacy and falsity MFs of CSVNN $\tilde{\mathbb{A}}$ be given as follows:

$$\begin{aligned}
\wp(\theta_t) &= \frac{1}{1+\left(\dfrac{\theta_t-\mathfrak{p}_t}{\mathfrak{q}_t}\right)^2} \\
\wp(\theta_i) &= 1-\frac{1}{1+\left(\dfrac{\theta_i-\mathfrak{p}_i}{\mathfrak{q}_i}\right)^2} \\
\wp(\theta_f) &= 1-\frac{1}{1+\left(\dfrac{\theta_f-\mathfrak{p}_f}{\mathfrak{q}_f}\right)^2}
\end{aligned}$$

respectively.

Then, α-cuts of above functions can be expressed as follows:

$$\begin{aligned}
\tilde{\mathbb{A}}_{t_\alpha} &= \left[\mathfrak{p}_t-\mathfrak{q}_t\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_t+\mathfrak{q}_t\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right], \\
\tilde{\mathbb{A}}_{i_\alpha} &= \left[\mathfrak{p}_i-\mathfrak{q}_i\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_i+\mathfrak{q}_i\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right], \\
\tilde{\mathbb{A}}_{f_\alpha} &= \left[\mathfrak{p}_f-\mathfrak{q}_f\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_f+\mathfrak{q}_f\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],
\end{aligned}$$

respectively.

3.3. *Arithmetic operations of CSVNNs*

Let $\tilde{\mathbb{A}} = CSVNN\big((\mathfrak{p}_{\tilde{\mathbb{A}}_t}, \mathfrak{q}_{\tilde{\mathbb{A}}_t}), (\mathfrak{p}_{\tilde{\mathbb{A}}_i}, \mathfrak{q}_{\tilde{\mathbb{A}}_i}), (\mathfrak{p}_{\tilde{\mathbb{A}}_f}, \mathfrak{q}_{\tilde{\mathbb{A}}_f})\big)$ and $\tilde{\mathbb{B}} = CSVNN\big((\mathfrak{p}_{\tilde{\mathbb{B}}_t}, \mathfrak{q}_{\tilde{\mathbb{B}}_t}), (\mathfrak{p}_{\tilde{\mathbb{B}}_i}, \mathfrak{q}_{\tilde{\mathbb{B}}_i}), (\mathfrak{p}_{\tilde{\mathbb{B}}_f}, \mathfrak{q}_{\tilde{\mathbb{B}}_f})\big)$ be two CSVNNs. Then, α-cuts ($\alpha \in (0,1]$) of them are as follows:

$$
\begin{aligned}
\tilde{\mathbb{A}}_{t_\alpha} &= \left[\mathfrak{p}_{\tilde{\mathbb{A}}_t} - \mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{A}}_t} + \mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{A}}_{i_\alpha} &= \left[\mathfrak{p}_{\tilde{\mathbb{A}}_i} - \mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{A}}_i} + \mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{A}}_{f_\alpha} &= \left[\mathfrak{p}_{\tilde{\mathbb{A}}_f} - \mathfrak{q}_{\tilde{\mathbb{A}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{A}}_f} + \mathfrak{q}_{\tilde{\mathbb{A}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{\mathbb{B}}_{t_\alpha} &= \left[\mathfrak{p}_{\tilde{\mathbb{B}}_t} - \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{B}}_t} + \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{B}}_{i_\alpha} &= \left[\mathfrak{p}_{\tilde{\mathbb{B}}_i} - \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{B}}_i} + \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{B}}_{f_\alpha} &= \left[\mathfrak{p}_{\tilde{\mathbb{B}}_f} - \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{B}}_f} + \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right]
\end{aligned}
$$

respectively.

By using α-cuts of CSVNNs $\tilde{\mathbb{A}}$ and $\tilde{\mathbb{B}}$, arithmetic operations between CSVNN $\tilde{\mathbb{A}}$ and CSVNN $\tilde{\mathbb{B}}$ are defined as follows:

(1) **Addition:** By using interval arithmetic, we have

$$
\begin{aligned}
\tilde{\mathbb{A}}_{t_\alpha} + \tilde{\mathbb{B}}_{t_\alpha} &= \left[(\mathfrak{p}_{\tilde{\mathbb{A}}_t} + \mathfrak{p}_{\tilde{\mathbb{B}}_t}) - (\mathfrak{q}_{\tilde{\mathbb{A}}_t} + \mathfrak{q}_{\tilde{\mathbb{B}}_t})\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}, (\mathfrak{p}_{\tilde{\mathbb{A}}_t} + \mathfrak{p}_{\tilde{\mathbb{B}}_t}) + (\mathfrak{q}_{\tilde{\mathbb{A}}_t} + \mathfrak{q}_{\tilde{\mathbb{B}}_t})\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{A}}_{i_\alpha} + \tilde{\mathbb{B}}_{i_\alpha} &= \left[(\mathfrak{p}_{\tilde{\mathbb{A}}_i} + \mathfrak{p}_{\tilde{\mathbb{B}}_i}) - (\mathfrak{q}_{\tilde{\mathbb{A}}_i} + \mathfrak{q}_{\tilde{\mathbb{B}}_i})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, (\mathfrak{p}_{\tilde{\mathbb{A}}_i} + \mathfrak{p}_{\tilde{\mathbb{B}}_i}) + (\mathfrak{q}_{\tilde{\mathbb{A}}_i} + \mathfrak{q}_{\tilde{\mathbb{B}}_i})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{A}}_{f_\alpha} + \tilde{\mathbb{B}}_{f_\alpha} &= \left[(\mathfrak{p}_{\tilde{\mathbb{A}}_f} + \mathfrak{p}_{\tilde{\mathbb{B}}_f}) - (\mathfrak{q}_{\tilde{\mathbb{A}}_f} + \mathfrak{q}_{\tilde{\mathbb{B}}_f})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, (\mathfrak{p}_{\tilde{\mathbb{A}}_f} + \mathfrak{p}_{\tilde{\mathbb{B}}_f}) + (\mathfrak{q}_{\tilde{\mathbb{A}}_f} + \mathfrak{q}_{\tilde{\mathbb{B}}_f})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right].
\end{aligned}
$$

Truth, indeterminacy, and falsity MFs of $\tilde{\mathbb{A}}+\tilde{\mathbb{B}}$ can be expressed as follows:

$$\wp_{(\tilde{\mathbb{A}}+\tilde{\mathbb{B}})}(\theta_t) = \frac{1}{1+\left(\frac{\theta_t-(\mathfrak{p}_{\tilde{\mathbb{A}}t}+\mathfrak{p}_{\tilde{\mathbb{B}}t})}{(\mathfrak{q}_{\tilde{\mathbb{A}}t}+\mathfrak{q}_{\tilde{\mathbb{B}}t})}\right)^2}$$

$$\wp_{(\tilde{\mathbb{A}}+\tilde{\mathbb{B}})}(\theta_i) = 1-\frac{1}{1+\left(\frac{\theta_i-(\mathfrak{p}_{\tilde{\mathbb{A}}i}+\mathfrak{p}_{\tilde{\mathbb{B}}i})}{(\mathfrak{q}_{\tilde{\mathbb{A}}i}+\mathfrak{q}_{\tilde{\mathbb{B}}i})}\right)^2}$$

$$\wp_{(\tilde{\mathbb{A}}+\tilde{\mathbb{B}})}(\theta_f) = 1-\frac{1}{1+\left(\frac{\theta_f-(\mathfrak{p}_{\tilde{\mathbb{A}}f}+\mathfrak{p}_{\tilde{\mathbb{B}}f})}{(\mathfrak{q}_{\tilde{\mathbb{A}}f}+\mathfrak{q}_{\tilde{\mathbb{B}}f})}\right)^2}$$

(2) **Substraction:** By using interval arithmetic, we have

$$\tilde{\mathbb{A}}_{t_\alpha}-\tilde{\mathbb{B}}_{t_\alpha} = \left[(\mathfrak{p}_{\tilde{\mathbb{A}}_t}-\mathfrak{p}_{\tilde{\mathbb{B}}_t})-(\mathfrak{q}_{\tilde{\mathbb{A}}_t}-\mathfrak{q}_{\tilde{\mathbb{B}}_t})\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}},(\mathfrak{p}_{\tilde{\mathbb{A}}_t}-\mathfrak{p}_{\tilde{\mathbb{B}}_t})+(\mathfrak{q}_{\tilde{\mathbb{A}}_t}-\mathfrak{q}_{\tilde{\mathbb{B}}_t})\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right],$$

$$\tilde{\mathbb{A}}_{i_\alpha}-\tilde{\mathbb{B}}_{i_\alpha} = \left[(\mathfrak{p}_{\tilde{\mathbb{A}}_i}-\mathfrak{p}_{\tilde{\mathbb{B}}_i})-(\mathfrak{q}_{\tilde{\mathbb{A}}_i}-\mathfrak{q}_{\tilde{\mathbb{B}}_i})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}},(\mathfrak{p}_{\tilde{\mathbb{A}}_i}-\mathfrak{p}_{\tilde{\mathbb{B}}_i})+(\mathfrak{q}_{\tilde{\mathbb{A}}_i}-\mathfrak{q}_{\tilde{\mathbb{B}}_i})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],$$

$$\tilde{\mathbb{A}}_{f_\alpha}-\tilde{\mathbb{B}}_{f_\alpha} = \left[(\mathfrak{p}_{\tilde{\mathbb{A}}_f}-\mathfrak{p}_{\tilde{\mathbb{B}}_f})-(\mathfrak{q}_{\tilde{\mathbb{A}}_f}-\mathfrak{q}_{\tilde{\mathbb{B}}_f})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}},(\mathfrak{p}_{\tilde{\mathbb{A}}_f}-\mathfrak{p}_{\tilde{\mathbb{B}}_f})+(\mathfrak{q}_{\tilde{\mathbb{A}}_f}-\mathfrak{q}_{\tilde{\mathbb{B}}_f})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right].$$

Truth, indeterminacy, and falsity MFs of CSVNN $\tilde{\mathbb{A}}-\tilde{\mathbb{B}}$ can be expressed as follows:

$$\wp_{(\tilde{\mathbb{A}}-\tilde{\mathbb{B}})}(\theta_t) = \frac{1}{1+\left(\frac{\theta_t-(\mathfrak{p}_{\tilde{\mathbb{A}_t}}-\mathfrak{p}_{\tilde{\mathbb{B}_t}})}{(\mathfrak{q}_{\tilde{\mathbb{A}_t}}-\mathfrak{q}_{\tilde{\mathbb{B}_t}})}\right)^2},$$

$$\wp_{(\tilde{\mathbb{A}}-\tilde{\mathbb{B}})}(\theta_i) = 1-\frac{1}{1+\left(\frac{\theta_i-(\mathfrak{p}_{\tilde{\mathbb{A}_i}}-\mathfrak{p}_{\mathbb{B}_i})}{(\mathfrak{q}_{\tilde{\mathbb{A}_i}}-\mathfrak{q}_{\tilde{\mathbb{B}_i}})}\right)^2},$$

$$\wp_{(\tilde{\mathbb{A}}-\tilde{\mathbb{B}})}(\theta_f) = 1-\frac{1}{1+\left(\frac{\theta_f-(\mathfrak{p}_{\tilde{\mathbb{A}_f}}-\mathfrak{p}_{\tilde{\mathbb{B}_f}})}{(\mathfrak{q}_{\tilde{\mathbb{A}_f}}-\mathfrak{q}_{\tilde{\mathbb{B}_f}})}\right)^2},$$

respectively.

(3) **Multiplication:** Let

$$\tilde{\mathbb{A}}_{t_\alpha} = \left[\tilde{\mathbb{A}}^L_{t_\alpha},\tilde{\mathbb{A}}^U_{t_\alpha}\right] = \left[\mathfrak{p}_{\tilde{\mathbb{A}}_t}-\mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}},\mathfrak{p}_{\tilde{\mathbb{A}}_t}+\mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right],$$

$$\tilde{\mathbb{A}}_{i_\alpha} = \left[\tilde{\mathbb{A}}^L_{i_\alpha},\tilde{\mathbb{A}}^U_{i_\alpha}\right] = \left[\mathfrak{p}_{\tilde{\mathbb{A}}_i}-\mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}},\mathfrak{p}_{\tilde{\mathbb{A}}_i}+\mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],$$

$$\tilde{\mathbb{A}}_{f_\alpha} = \left[\tilde{\mathbb{A}}^L_{f_\alpha},\tilde{\mathbb{A}}^U_{f_\alpha}\right] = \left[\mathfrak{p}_{\tilde{\mathbb{A}}_f}-\mathfrak{q}_{\tilde{\mathbb{A}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}},\mathfrak{p}_{\tilde{\mathbb{A}}_f}+\mathfrak{q}_{\tilde{\mathbb{A}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],$$

and

$$\begin{aligned}
\tilde{\mathbb{B}}_{t_\alpha} &= \left[\tilde{\mathbb{B}}^L_{t_\alpha}, \tilde{\mathbb{B}}^U_{t_\alpha}\right] = \left[\mathfrak{p}_{\tilde{\mathbb{B}}_t} - \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{B}}_t} + \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{B}}_{i_\alpha} &= \left[\tilde{\mathbb{B}}^L_{i_\alpha}, \tilde{\mathbb{B}}^U_{i_\alpha}\right] = \left[\mathfrak{p}_{\tilde{\mathbb{B}}_i} - \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{B}}_i} + \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],\\
\tilde{\mathbb{B}}_{f_\alpha} &= \left[\tilde{\mathbb{B}}^L_{f_\alpha}, \tilde{\mathbb{B}}^U_{f_\alpha}\right] = \left[\mathfrak{p}_{\tilde{\mathbb{B}}_f} - \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \mathfrak{p}_{\tilde{\mathbb{B}}_f} + \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right],
\end{aligned}$$

Then,

$$\begin{aligned}
\tilde{\mathbb{A}}_{t_\alpha}\tilde{\mathbb{B}}_{t_\alpha} &= \left[\min\{\tilde{\mathbb{A}}^L_{t_\alpha}\tilde{\mathbb{B}}^L_{t_\alpha}, \tilde{\mathbb{A}}^L_{t_\alpha}\tilde{\mathbb{B}}^U_{t_\alpha}, \tilde{\mathbb{A}}^U_{t_\alpha}\tilde{\mathbb{B}}^L_{t_\alpha}, \tilde{\mathbb{A}}^U_{t_\alpha}\tilde{\mathbb{A}}^U_{t_\alpha}\}, \max\{\tilde{\mathbb{A}}^L_{t_\alpha}\tilde{\mathbb{B}}^L_{t_\alpha}, \tilde{\mathbb{A}}^L_{t_\alpha}\tilde{\mathbb{B}}^U_{t_\alpha}, \tilde{\mathbb{A}}^U_{t_\alpha}\tilde{\mathbb{B}}^L_{t_\alpha}, \tilde{\mathbb{A}}^U_{t_\alpha}\tilde{\mathbb{A}}^U_{t_\alpha}\}\right],\\
\tilde{\mathbb{A}}_{i_\alpha}\tilde{\mathbb{B}}_{i_\alpha} &= \left[\min\{\tilde{\mathbb{A}}^L_{i_\alpha}\tilde{\mathbb{B}}^L_{i_\alpha}, \tilde{\mathbb{A}}^L_{i_\alpha}\tilde{\mathbb{B}}^U_{i_\alpha}, \tilde{\mathbb{A}}^U_{i_\alpha}\tilde{\mathbb{B}}^L_{i_\alpha}, \tilde{\mathbb{A}}^U_{i_\alpha}\tilde{\mathbb{A}}^U_{i_\alpha}\}, \max\{\tilde{\mathbb{A}}^L_{i_\alpha}\tilde{\mathbb{B}}^L_{i_\alpha}, \tilde{\mathbb{A}}^L_{i_\alpha}\tilde{\mathbb{B}}^U_{i_\alpha}, \tilde{\mathbb{A}}^U_{i_\alpha}\tilde{\mathbb{B}}^L_{i_\alpha}, \tilde{\mathbb{A}}^U_{i_\alpha}\tilde{\mathbb{A}}^U_{i_\alpha}\}\right],\\
\tilde{\mathbb{A}}_{f_\alpha}\tilde{\mathbb{B}}_{f_\alpha} &= \left[\min\{\tilde{\mathbb{A}}^L_{f_\alpha}\tilde{\mathbb{B}}^L_{f_\alpha}, \tilde{\mathbb{A}}^L_{f_\alpha}\tilde{\mathbb{B}}^U_{f_\alpha}, \tilde{\mathbb{A}}^U_{f_\alpha}\tilde{\mathbb{B}}^L_{f_\alpha}, \tilde{\mathbb{A}}^U_{f_\alpha}\tilde{\mathbb{A}}^U_{f_\alpha}\}, \max\{\tilde{\mathbb{A}}^L_{f_\alpha}\tilde{\mathbb{B}}^L_{f_\alpha}, \tilde{\mathbb{A}}^L_{f_\alpha}\tilde{\mathbb{B}}^U_{f_\alpha}, \tilde{\mathbb{A}}^U_{f_\alpha}\tilde{\mathbb{B}}^L_{f_\alpha}, \tilde{\mathbb{A}}^U_{f_\alpha}\tilde{\mathbb{A}}^U_{f_\alpha}\}\right].
\end{aligned}$$

If $\tilde{\mathbb{A}}_{t_\alpha}, \tilde{\mathbb{A}}_{i_\alpha}, \tilde{\mathbb{A}}_{f_\alpha}, \tilde{\mathbb{B}}_{t_\alpha}, \tilde{\mathbb{B}}_{i_\alpha}$ and $\tilde{\mathbb{B}}_{f_\alpha} \subseteq \mathbb{R}^+$, then we can directly write it as follows:

$$\begin{aligned}
\tilde{\mathbb{A}}_{t_\alpha}\tilde{\mathbb{B}}_{t_\alpha} &= \left[\left(\mathfrak{p}_{\tilde{\mathbb{A}}_t} - \mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right).\left(\mathfrak{p}_{\tilde{\mathbb{B}}_t} - \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right), \left(\mathfrak{p}_{\tilde{\mathbb{A}}_t} + \mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right).\left(\mathfrak{p}_{\tilde{\mathbb{B}}_t} + \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\tfrac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right)\right],\\
\tilde{\mathbb{A}}_{i_\alpha}\tilde{\mathbb{B}}_{i_\alpha} &= \left[\left(\mathfrak{p}_{\tilde{\mathbb{A}}_i} - \mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right).\left(\mathfrak{p}_{\tilde{\mathbb{B}}_i} - \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right), \left(\mathfrak{p}_{\tilde{\mathbb{A}}_i} + \mathfrak{q}_{\tilde{\mathbb{A}}_i})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right)\left(\mathfrak{p}_{\tilde{\mathbb{B}}_i} + \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right)\right],\\
\tilde{\mathbb{A}}_{f_\alpha}\tilde{\mathbb{B}}_{f_\alpha} &= \left[\left(\mathfrak{p}_{\tilde{\mathbb{A}}_f} - \mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right).\left(\mathfrak{p}_{\tilde{\mathbb{B}}_f} - \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right), \left(\mathfrak{p}_{\tilde{\mathbb{A}}_f} + \mathfrak{q}_{\tilde{\mathbb{A}}_f})\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right)\left(\mathfrak{p}_{\tilde{\mathbb{B}}_f} + \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\tfrac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}\right)\right],
\end{aligned}$$

(4) **Division:**

$$\begin{aligned}
\frac{\tilde{\mathbb{A}}_{t\alpha}}{\tilde{\mathbb{B}}_{t\alpha}} &= \left[\frac{(\mathfrak{p}_{\tilde{\mathbb{A}}_t} - \mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}})}{(\mathfrak{p}_{\tilde{\mathbb{B}}_t} - \mathfrak{q}_{\tilde{\mathbb{B}}_t}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}})}, \frac{(\mathfrak{p}_{\tilde{\mathbb{A}}_t} + \mathfrak{q}_{\tilde{\mathbb{A}}_t}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}})}{((\mathfrak{p}_{\tilde{\mathbb{B}}_t} + \mathfrak{q}_{\tilde{\mathbb{B}}_t})\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}})}\right],\ 0 \notin \tilde{\mathbb{B}}_{t\alpha}\\
\frac{\tilde{\mathbb{A}}_{i\alpha}}{\tilde{\mathbb{B}}_{i\alpha}} &= \left[\frac{(\mathfrak{p}_{\tilde{\mathbb{A}}_i} - \mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}{(\mathfrak{p}_{\tilde{\mathbb{B}}_i} - \mathfrak{q}_{\tilde{\mathbb{B}}_i}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}, \frac{(\mathfrak{p}_{\tilde{\mathbb{A}}_i} + \mathfrak{q}_{\tilde{\mathbb{A}}_i}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}{((\mathfrak{p}_{\tilde{\mathbb{B}}_i} + \mathfrak{q}_{\tilde{\mathbb{B}}_i})\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}\right],\ 0 \notin \tilde{\mathbb{B}}_{i\alpha}\\
\frac{\tilde{\mathbb{A}}_{f\alpha}}{\tilde{\mathbb{B}}_{f\alpha}} &= \left[\frac{(\mathfrak{p}_{\tilde{\mathbb{A}}_f} - \mathfrak{q}_{\tilde{\mathbb{A}}_f}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}{(\mathfrak{p}_{\tilde{\mathbb{B}}_f} - \mathfrak{q}_{\tilde{\mathbb{B}}_f}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}, \frac{(\mathfrak{p}_{\tilde{\mathbb{A}}_f} + \mathfrak{q}_{\tilde{\mathbb{A}}_f}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}{((\mathfrak{p}_{\tilde{\mathbb{B}}_f} + \mathfrak{q}_{\tilde{\mathbb{B}}_f})\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}})}\right],\ 0 \notin \tilde{\mathbb{B}}_{f\alpha}
\end{aligned}$$

Example 3.6. Let us consider CSVNNs $\tilde{\mathbb{A}} = CSVNN((0.42, 0.68), (0.54, 0.32), (0.69, 0.21))$ and $\tilde{\mathbb{B}} = CSVNN((0.73, 0.32), (0.64, 0.23), (0.57, 0.41))$. The graphics of $\tilde{\mathbb{A}}$ and $\tilde{\mathbb{B}}$ are depicted in Figures (2) and (3).

Truth, indeterminacy, and falsity MFs of CSVNNs $\tilde{\mathbb{A}} + \tilde{\mathbb{B}}$ and $\tilde{\mathbb{A}} - \tilde{\mathbb{B}}$ are obtained as follows:

$$\wp_{(\tilde{\mathbb{A}}+\tilde{\mathbb{B}})}(\theta_t) = \frac{1}{1 + \left(\dfrac{\theta_t - 1.15}{1.0}\right)^2},$$

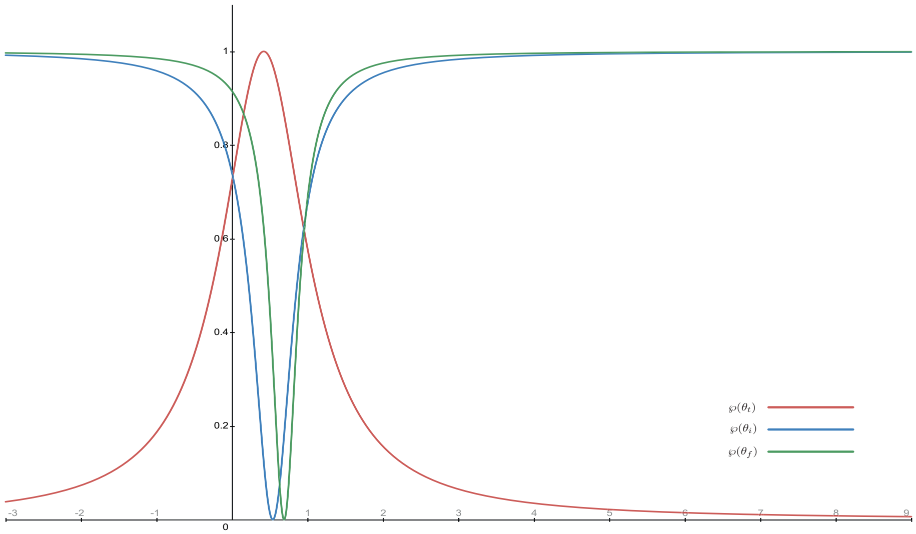

FIGURE 2. CSVNN $\tilde{\mathbb{A}}$

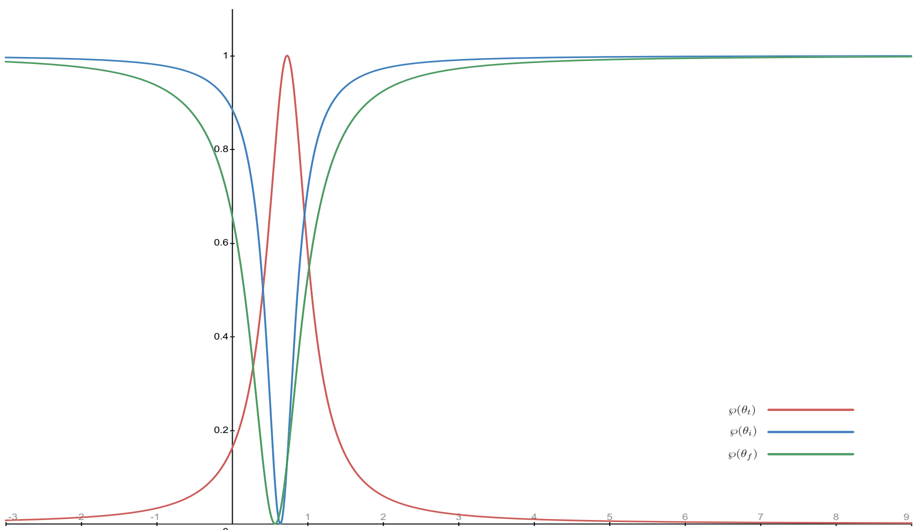

FIGURE 3. CSVNN $\tilde{\mathbb{B}}$

$$\wp_{(\tilde{\mathbb{A}}+\tilde{\mathbb{B}})}(\theta_i) = 1 - \frac{1}{1+\left(\frac{\theta_i - 1.18}{0.55}\right)^2},$$

$$\wp_{(\tilde{\mathbb{A}}+\tilde{\mathbb{B}})}(\theta_f) = 1 - \frac{1}{1+\left(\frac{\theta_f - 1.26}{0.62}\right)^2},$$

and

$$\wp_{(\tilde{\mathbb{A}}-\tilde{\mathbb{B}})}(\theta_t) = \frac{1}{1+\left(\frac{\theta_t - (-0.31)}{0.36}\right)^2},$$

$$\wp_{(\tilde{\mathbb{A}}-\tilde{\mathbb{B}})}(\theta_i) = 1 - \frac{1}{1+\left(\frac{\theta_i-(-0.1)}{0.09}\right)^2},$$

$$\wp_{(\tilde{\mathbb{A}}-\tilde{\mathbb{B}})}(\theta_f) = 1 - \frac{1}{1+\left(\frac{\theta_f-0.12}{(-0.2)}\right)^2}.$$

Figures (4) and (5) show the graphical representations of CSVNNs $\tilde{\mathbb{A}}+\tilde{\mathbb{B}}$ and $\tilde{\mathbb{A}}-\tilde{\mathbb{B}}$.

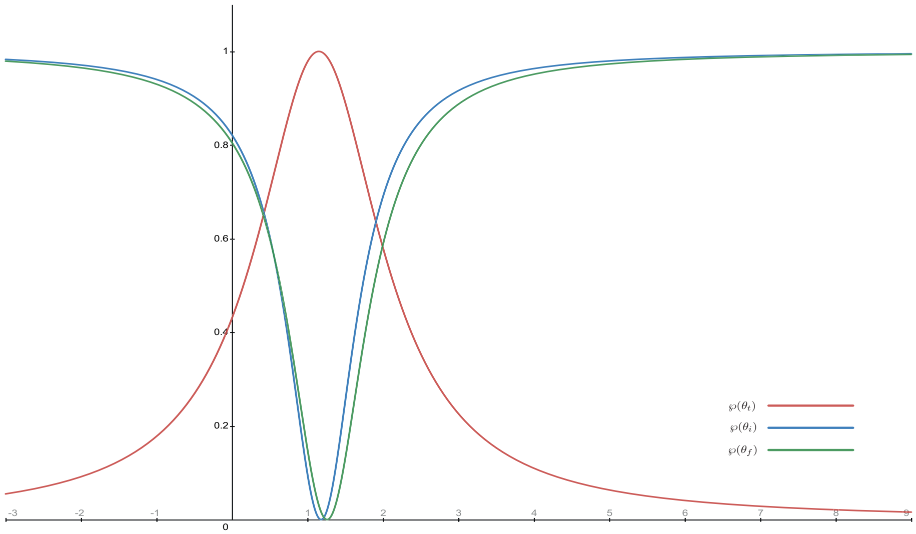

Figure 4. CSVNN $\tilde{\mathbb{A}}+\tilde{\mathbb{B}}$

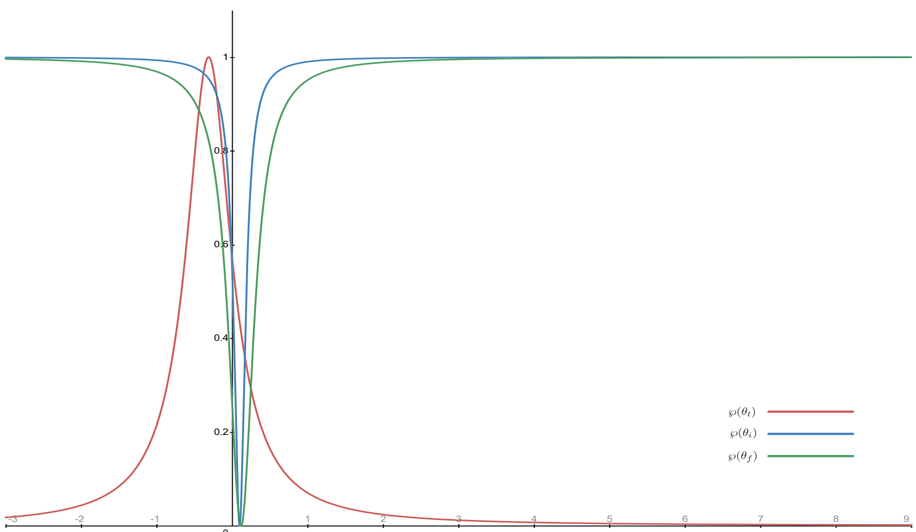

Figure 5. CSVNN $\tilde{\mathbb{A}}-\tilde{\mathbb{B}}$

Definition 3.7. Let $\tilde{\mathbb{A}} = ((\mathfrak{p}_t,\mathfrak{q}_t),(\mathfrak{p}_i,\mathfrak{q}_i),(\mathfrak{p}_f,\mathfrak{q}_f))$ be CSVNN. Then, score function of CSVNN $\tilde{\mathbb{A}}$, denoted by $S(\tilde{\mathbb{A}})$, is defined as follows:

$$S(\tilde{\mathbb{A}}) = \frac{1}{3}\Big(\frac{\mathfrak{q}_t^2}{\mathfrak{q}_t^2 + (\theta_t - \mathfrak{p}_t)^2} + \frac{\mathfrak{q}_i^2}{\mathfrak{q}_i^2 + (\theta_i - \mathfrak{p}_i)^2} + \frac{\mathfrak{q}_f^2}{\mathfrak{q}_f^2 + (\theta_f - \mathfrak{p}_f)^2}\Big)$$

Note that score functions are functions depending on neutrosophic variables $\langle \theta_t, \theta_i, \theta_f \rangle$. Furthermore, score functions of CSVNNs can be changed according to CSVNN.

Example 3.8. Let us consider CSVNNs $\tilde{\mathbb{A}} = ((0.5, 0.3), (0.7, 0.2), (0.3, 0.5))$ and $\tilde{\mathbb{B}} = ((0.6, 0.4), (0.2, 0.6), (0.5, 0.7))$. Then, score functions of CSVNNs $\tilde{\mathbb{A}}$ and $\tilde{\mathbb{B}}$ are obtained as follows:

$$S(\tilde{\mathbb{A}}) = \frac{1}{3}\Big(\frac{0.09}{0.09 + (\theta_t - 0.5)^2} + \frac{0.04}{0.04 + (\theta_i - 0.7)^2} + \frac{0.25}{0.25 + (\theta_t - 0.3)^2}\Big)$$

and

$$S(\tilde{\mathbb{B}}) = \frac{1}{3}\Big(\frac{0.16}{0.16 + (\theta_t - 0.6)^2} + \frac{0.36}{0.36 + (\theta_i - 0.4)^2} + \frac{0.49}{0.49 + (\theta_t - 0.5)^2}\Big).$$

If we consider SVN value $\langle 0.5, 0.6, 0.7 \rangle$, then score values of this SVN according to score functions of $\tilde{\mathbb{A}}$ and $\tilde{\mathbb{B}}$ are obtained as follows:

$$S(\tilde{\mathbb{A}})(\langle 0.5, 0.6, 0.7 \rangle) = 0.803 \ and \ S(\tilde{\mathbb{B}})(\langle 0.5, 0.6, 0.7 \rangle) = 0.923$$

Definition 3.9. Let $\mathbb{X}$ be a nonempty set and $\sum^n = \Big\{\Psi_k = \Big((\mathfrak{p}_{t_k}, \mathfrak{q}_{t_k}), (\mathfrak{p}_{i_k}, \mathfrak{q}_{i_k}), (\mathfrak{p}_{f_k}, \mathfrak{q}_{f_k})\Big) : (k = 1, 2, ..., n)\Big\}$ be a set of CSVNNs of which weight vector $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_n)^T$ such that $\varsigma_k > 0$, $\sum_{k=1}^{u} \varsigma_k = 1$. Then, CSVN weighted arithmetic aggregation (CSVNWAA) operator is defined by a mapping $CSVNWAA : \sum^n \to \mathcal{CSVNN}(\mathbb{X})$, where

$$CSVNWAA(\Psi_1, \Psi_2, ..., \Psi_n) = \bigoplus_{k=1}^{n} \varsigma_k \Psi_k.$$

Theorem 3.10. *Let $\mathbb{X}$ be a nonempty set and $\sum^n = \Big\{\Psi_k = \Big((\mathfrak{p}_{t_k}, \mathfrak{q}_{t_k}), (\mathfrak{p}_{i_k}, \mathfrak{q}_{i_k}), (\mathfrak{p}_{f_k}, \mathfrak{q}_{f_k})\Big) : (k = 1, 2, ..., n)\Big\}$ be a set of CSNNs of which weight vector $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_n)^T$ such that $\varsigma_k > 0$, $\sum_{k=1}^{u} \varsigma_k = 1$.*
Then, aggregated value of set using CSNNWAA is a CSVNN defined as follows:

$$\begin{aligned} CSNNWAA(\Psi_1, \Psi_2, ..., \Psi_n) &= \bigoplus_{k=1}^{n} (\varsigma_k \Psi_k) \\ &= \Big((1 - \prod_{k=1}^{n} (1 - \tilde{\mathfrak{p}}_{t_k})^{\varsigma_k}, 1 - \prod_{k=1}^{n} (1 - \tilde{\mathfrak{q}}_{t_k})^{\varsigma_k}), \\ &\quad (\prod_{k=1}^{n} (\tilde{\mathfrak{p}}_{i_k})^{\varsigma_k}, \prod_{k=1}^{n} (\tilde{\mathfrak{q}}_{i_k})^{\varsigma_k}), (\prod_{k=1}^{n} (\tilde{\mathfrak{p}}_{f_k})^{\varsigma_k}, \prod_{k=1}^{n} (\tilde{\mathfrak{p}}_{f_k})^{\varsigma_k})\Big). \end{aligned} \tag{1}$$

Here there are two cases for $\mathfrak{p}_{\Delta_k}$ and $\mathfrak{q}_{\Delta_k}$ ($\Delta \in \{t, i, f\}$) and ($k = 1, 2, ..., n$). If one of $\mathfrak{p}_{\Delta_k}$ and $\mathfrak{q}_{\Delta_k}$ is greater then 1, then the following formula are used $\tilde{\mathfrak{p}}_{\Delta_k} = \frac{\mathfrak{p}_{\Delta_k}}{\sqrt{\sum_{k=1}^{n} \mathfrak{p}_{\Delta_k}^2}}$, $\tilde{\mathfrak{q}}_{\Delta_k} = \frac{\mathfrak{p}_{\Delta_k}}{\sqrt{\sum_{k=1}^{n} \mathfrak{p}_{\Delta_k}^2}}$, If $\mathfrak{p}_{\Delta_k}$ and $\mathfrak{q}_{\Delta_k}$ ($\Delta \in \{t, i, f\}$) ($k = 1, 2, ..., n$) are in interval $[0, 1]$, then $\mathfrak{p}_{\Delta_k}$ and $\mathfrak{q}_{\Delta_k}$ ($\Delta \in \{t, i, f\}$) and ($k = 1, 2, ..., n$) are used directly in $CSNNWAA(\Psi_1, \Psi_2, ..., \Psi_n)$ and other aggregation operations defined in the next .

Proof. The proof can be easily made based on aggregation operations of the SVNNs. Therefore, it is omitted. □

Definition 3.11. Let $\mathbb{X}$ be a nonempty set and $\sum^n = \Big\{\Psi_k = \Big((\mathfrak{p}_{t_k}, \mathfrak{q}_{t_k}), (\mathfrak{p}_{i_k}, \mathfrak{q}_{i_k}), (\mathfrak{p}_{f_k}, \mathfrak{q}_{f_k})\Big) : (k = 1, 2, ..., n)\Big\}$ be a set of CSVNNs of which weight vector $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_n)^T$ such that $\varsigma_k > 0$, $\sum_{k=1}^{u} \varsigma_k = 1$. Then, CSVN weighted geometric aggregation operator (CSNNWGA) operator is defined by a mapping $CSVNWGA : \sum^n \to \mathcal{CSVNN}(\mathbb{X})$, where

$$CSVNWGA(\Psi_1, \Psi_2, ..., \Psi_n) = \bigotimes_{k=1}^{n} \Psi_k^{\varsigma_k}$$

Theorem 3.12. *Let $\mathbb{X}$ be a universe and $\sum^n = \Big\{\Psi_k = \Big((\mathfrak{p}_{t_k}, \mathfrak{q}_{t_k}), (\mathfrak{p}_{i_k}, \mathfrak{q}_{i_k}), (\mathfrak{p}_{f_k}, \mathfrak{q}_{f_k})\Big) : (k = 1, 2, ..., n)\Big\}$ be a collection of CSNNs of which weight vector $\varsigma = (\varsigma_1, \varsigma_2, ..., \varsigma_n)^T$ such that $\varsigma_k > 0$, $\sum_{k=1}^{u} \varsigma_k = 1$.*
Then, aggregated value of set using CSVNWGA is a CSVNN defined as follows:

$$\begin{aligned} CSNNWGA(\Psi_1, \Psi_2, ..., \Psi_n) &= \bigotimes_{k=1}^{n} \Psi_k^{\varsigma_k} \\ &= \Big((\prod_{k=1}^{n} (\tilde{\mathfrak{p}}_{t_k})^{\varsigma_k}, \prod_{k=1}^{n} (\tilde{\mathfrak{q}}_{t_k})^{\varsigma_k}), (1 - \prod_{k=1}^{n} (1 - \tilde{\mathfrak{p}}_{i_k})^{\varsigma_k}, \\ &\quad 1 - \prod_{k=1}^{n} (1 - \tilde{\mathfrak{q}}_{i_k})^{\varsigma_k}), (1 - \prod_{k=1}^{n} (1 - \tilde{\mathfrak{p}}_{f_k})^{\varsigma_k}, 1 - \prod_{k=1}^{n} (1 - \tilde{\mathfrak{p}}_{f_k})^{\varsigma_k}\Big) \end{aligned} \tag{2}$$

Proof. The proof can be easily made based on aggregation operations of the SVNNs. Therefore, it is omitted. □

4. Multi-attribute group decision making method under CSVN environment

In the following table, beginning data and some notation are shown:

Notaions	Explanation
$\varepsilon = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$	Set of alternatives
$\Omega = \{\Omega_1, \Omega_2, ..., \Omega_k\}$	Set of attributes
$\delta = \{\delta_1, \delta_2, ..., \delta_m\}$	Set of decision makers
τ_{ij}	Evaluation of the criteria Ω_j made by decision maker δ_i
$\beta = (\beta_1, \beta_2, ..., \beta_m)$	weight vector of decision-makers

Table 1. Notation table for MAGDM method

4.1. *Decision making method*

Steps of the proposed method are explained as follows:

Step 1: Constructing decision-criteria (DA) matrix. In this step, each of decision makers $\delta_i, (i = 1, 2, ..., m)$ whose weight vector $\beta = (\beta_1, \beta_2, ..., \beta_m)$ such that $\beta_s > 0 (s = 1, 2, ..., m)$ and $\sum_{s=1}^{m} \beta_s = 1$, evaluates the attributes $\Omega_j, (j = 1, 2, ..., k)$ by CSVNNs and DA matrix is constructed as follows:

$$DA = \begin{bmatrix} \tau_{11} & \tau_{12} & \cdots & \tau_{1k} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{m1} & \tau_{m2} & \cdots & \tau_{mk} \end{bmatrix}_{m \times k}.$$

Here each of $\tau_{ij} = ((\mathfrak{p}_{t_{ij}}, \mathfrak{q}_{t_{ij}}), (\mathfrak{p}_{i_{ij}}, \mathfrak{q}_{i_{ij}}), (\mathfrak{p}_{f_{ij}}, \mathfrak{q}_{f_{ij}}))$ is a CSVNN.

Step 2: Finding the aggregation of attributes. By adapting Eqs. 1 and 2 for weigh vector $\beta = (\beta_1, \beta_2, ..., \beta_m)$ of decision-makers, aggregated weight for each attribute is calculated the following formula

$$\begin{aligned} Agg_A(\Omega_j) &= CSVNWAA(\tau_{1j}, \tau_{2j}, ..., \tau_{nj}) = \bigoplus_{k=1}^{m} \beta_k \tau_{kj} \\ &= \Big((1 - \prod_{k=1}^{m} (1 - \tilde{\mathfrak{p}}_{t_{kj}})^{\beta_k}, 1 - \prod_{k=1}^{m} (1 - \tilde{\mathfrak{q}}_{t_{kj}})^{\beta_k}), \\ &\quad (\prod_{k=1}^{m} \tilde{\mathfrak{p}}_{i_{kj}}^{\beta_k}, \prod_{k=1}^{m} \tilde{\mathfrak{q}}_{i_{kj}}^{\beta_k}), (\prod_{k=1}^{m} \tilde{\mathfrak{p}}_{f_{kj}}^{\beta_k}, \prod_{k=1}^{m} \tilde{\mathfrak{q}}_{f_{kj}}^{\beta_k}) \Big) \end{aligned} \tag{3}$$

and

$$\begin{aligned} Agg_G(\Omega_j) &= CSVNWGA(\tau_{1j}, \tau_{2j}, ..., \tau_{nj}) = \bigoplus_{k=1}^{m} \tau_{kj}^{\beta_k} \\ &= \Big(\big(\prod_{k=1}^{m} \tilde{\mathfrak{p}}_{t_{kj}}^{\beta_k}, \prod_{k=1}^{m} \tilde{\mathfrak{q}}_{t_{kj}}^{\beta_k} \big), \big(\prod_{k=1}^{m} \tilde{\mathfrak{p}}_{i_{kj}}^{\beta_k}, \prod_{k=1}^{m} \tilde{\mathfrak{q}}_{i_{kj}}^{\beta_k} \big), \\ &\quad \big(1 - \prod_{k=1}^{m} (1 - \tilde{\mathfrak{p}}_{f_{kj}})^{\beta_k}, 1 - \prod_{k=1}^{m} (1 - \tilde{\mathfrak{q}}_{f_{kj}})^{\beta_k} \big) \Big) \end{aligned} \tag{4}$$

respectively.

Step 3: Obtaining score functions of aggregated attributes. By using Definition 3.7 , for $j = 1, 2, ..., k$ $S(Agg_A(\Omega_j))$ $(S(Agg_G(\Omega_j)))$ are found.

Step 4: Construction of evaluation matrices by decision-makers δ_s $(s = 1, 2, ..., m)$. For each of decision-makers, evaluation matrices denoted EM_s $(s = 1, 2, ..., k)$ are obtained as follows:

$$EM_s = \begin{bmatrix} \kappa_{11}^s & \kappa_{12}^s & \cdots & \kappa_{1k}^s \\ \kappa_{21}^s & \kappa_{22}^s & \cdots & \kappa_{2k}^s \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{n1}^s & \kappa_{n2}^s & \cdots & \kappa_{nm}^s \end{bmatrix}.$$

Here $\kappa_{ij}^s = \langle \mathfrak{t}_{ij}^s, \mathfrak{i}_{ij}^s, \mathfrak{f}_{ij}^s \rangle$ denotes an SVNN which implies evaluation of alternative ε_i according to criteria Ω_j made by the decision maker δ_s.

Step 5: Compiling the EM_s (s=1,2,...,k). Compiling matrix (CM) is obtained as follows:

$$CM = \begin{bmatrix} \cup_{s=1}^{k} \kappa_{11}^s & \cup_{s=1}^{k} \kappa_{12}^s & \cdots & \cup_{s=1}^{k} \kappa_{1k}^s \\ \cup_{s=1}^{k} \kappa_{21}^s & \cup_{s=1}^{k} \kappa_{22}^s & \cdots & \cup_{s=1}^{k} \kappa_{2k}^s \\ \vdots & \vdots & \ddots & \vdots \\ \cup_{s=1}^{k} \kappa_{n1}^s & \cup_{s=1}^{k} \kappa_{n2}^s & \cdots & \cup_{s=1}^{k} \kappa_{nm}^s \end{bmatrix}.$$

Here $\cup_{s=1}^{k} \kappa_{11}^s = \langle \vee_{s=1}^{k} \mathfrak{t}_{ij}^s, \wedge_{s=1}^{k} \mathfrak{i}_{ij}^s, \wedge_{s=1}^{k} \mathfrak{f}_{ij}^s \rangle$ where $\vee$ and $\wedge$ denote the maximum and minimum, respectively.

Step 6: Finding score matrix $SM = [\partial_{ij}]_{nk}$: By using score functions obtained from DA matrix in Step 2 for each attribute, score values of elements in CM matrix are found.

Step 7: Evaluation of the alternatives. For $i = 1, 2, ..., n$, grade of the alternative $\mathfrak{g}_i$ are calculated by

$$\mathfrak{g}_i = \frac{1}{k} \sum_{j=1}^{k} \partial_{ij}$$

Step 8: Choosing the optimum alternative: Alternatives are ordered according to grades of them and alternative having maximum grade is selected as an optimum alternative.

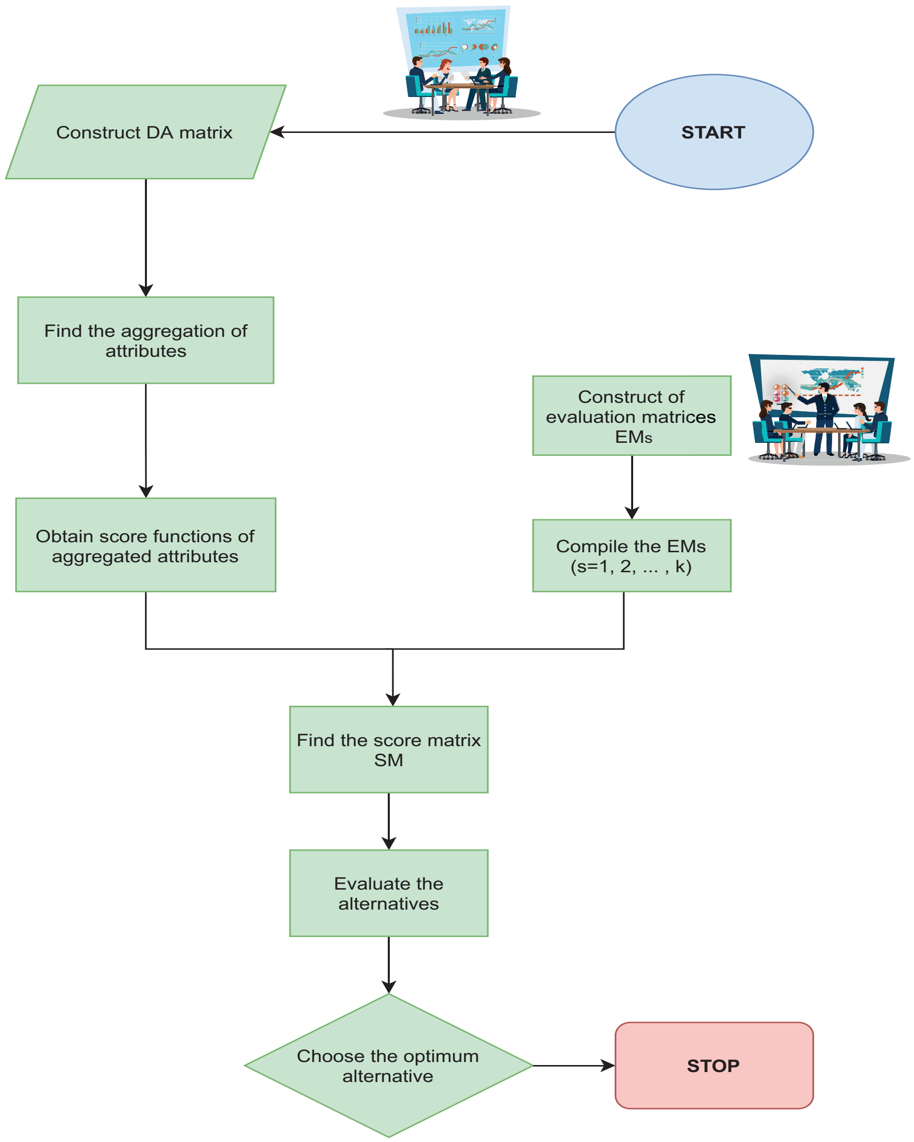

FIGURE 6. Flow chart of the proposed method

Flowchart of the algorithm is showed in Figure 6.

4.2. *Illustrative Example*

In this section, an example is provided to display the functioning of the developed decision-making method.

Example 4.1. Suppose that two projects are wanted to be selected among five projects to provide financial support. There are three experts $\delta = \{\delta_1, \delta_2, \delta_3\}$ with different academic qualifications in the panel. Their weight vector is $(0.5, 0.3, 0.2)$. These experts evaluate the project $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ and ε_5 according to four attributes Ω_1=Content, Ω_2=practicability, Ω_3=Originality and Ω_4= widespread impact.

The method proposed for choosing the best two projects that need financial support is applied as follows.

Step 1: Constructing decision-attribute (DA) matrix: Each of the experts evaluates the attribute under CSVN environment and they construct DA matrix as follows:

$$DA = \begin{bmatrix} ((0.4,0.2),(0.1,0.1),(0.6,0.2)) & ((0.7,0.1),(0.5,0.4),(0.7,0.3)) & ((0.6,0.2),(0.4,0.3),(0.5,0.1)) & ((0.8,0.1),(0.5,0.3),(0.6,0.3)) \\ ((0.7,0.1),(0.3,0.1),(0.8,0.3)) & ((0.9,0.6),(0.9,0.4),(0.6,0.4)) & ((0.9,0.8),(0.4,0.1),(0.8,0.6)) & ((0.1,0.1),(0.9,0.3),(0.9,0.2)) \\ ((0.8,0.6),(0.9,0.2),(0.7,0.3)) & ((0.9,0.2),(0.8,0.6),(0.7,0.5)) & ((0.4,0.2),(0.9,0.4),(0.7,0.4)) & ((0.7,0.6),(0.7,0.3),(0.7,0.5)) \end{bmatrix}$$

Step 2: By using Equation (4) the arithmetic aggregate of each column is obtained as follows:

$$Agg_A(\Omega_j) = \Big(((0.609,0.279),(0.216,0.115),(0.675,0.245)) \quad ((0.827,0.311),(0.655,0.434),(0.668,0.362)) \quad ((0.714,0.472),(0.470,0.229),(0.616,0.226)) \quad ((0.659,0.235),(0.638,0.300),(0.699,0.294)) \Big)$$

Step 3: By using Eq. 3.7, for $j = 1,2,...,k$ $S(Agg_A(\Omega_j))$ are found.

$$S(Agg_A(\Omega_1)) = \frac{1}{3}\Big(\frac{0.078}{0.078+(\theta_t-0.609)^2} + \frac{0.013}{0.013+(\theta_i-0.216)^2} + \frac{0.060}{0.060+(\theta_f-0.675)^2}\Big),$$

$$S(Agg_A(\Omega_2)) = \frac{1}{3}\Big(\frac{0.097}{0.097+(\theta_t-0.827)^2} + \frac{0.188}{0.188+(\theta_i-0.655)^2} + \frac{0.131}{0.131+(\theta_f-0.668)^2}\Big),$$

$$S(Agg_A(\Omega_3)) = \frac{1}{3}\Big(\frac{0.223}{0.223+(\theta_t-0.714)^2} + \frac{0.052}{0.052+(\theta_i-0.470)^2} + \frac{0.051}{0.051+(\theta_f-0.616)^2}\Big),$$

$$S(Agg_A(\Omega_4)) = \frac{1}{3}\Big(\frac{0.055}{0.055+(\theta_t-0.659)^2} + \frac{0.090}{0.090+(\theta_i-0.638)^2} + \frac{0.087}{0.087+(\theta_f-0.699)^2}\Big).$$

Also, by using $CSVNWGA$ operator score functions of the attributes are obtained as follows:

$$S(Agg_G(\Omega_1)) = \frac{1}{3}\Big(\frac{0.041}{0.041+(\theta_t-0.543)^2} + \frac{0.015}{0.015+(\theta_i-0.462)^2} + \frac{0.063}{0.063+(\theta_f-0.693)^2}\Big),$$

$$S(Agg_G(\Omega_2)) = \frac{1}{3}\Big(\frac{0.039}{0.039+(\theta_t-0.794)^2} + \frac{0.200}{0.200+(\theta_i-0.743)^2} + \frac{0.141}{0.141+(\theta_f-0.673)^2}\Big),$$

$$S(Agg_G(\Omega_3)) = \frac{1}{3}\Big(\frac{0.049}{0.049+(\theta_t-0.588)^2} + \frac{0.072}{0.072+(\theta_i-0.581)^2} + \frac{0.122}{0.122+(\theta_f-0.657)^2}\Big),$$

$$S(Agg_G(\Omega_4)) = \frac{1}{3}\Big(\frac{0.020}{0.020 + (\theta_t - 0.417)^2} + \frac{0.090}{0.090 + (\theta_i - 0.721)^2} + \frac{0.102}{0.102 + (\theta_f - 0.751)^2}\Big).$$

Step 4: For each of decision-makers, evaluation matrices are obtained as follows:

$$EM_1 = \begin{pmatrix} \langle 0.6, 0.8, 0.3\rangle & \langle 0.2, 0.5, 0.4\rangle & \langle 0.5, 0.1, 0.6\rangle & \langle 0.6, 0.9, 0.4\rangle \\ \langle 0.8, 0.3, 0.1\rangle & \langle 0.2, 0.6, 0.8\rangle & \langle 0.7, 0.2, 0.8\rangle & \langle 0.4, 0.5, 0.5\rangle \\ \langle 0.4, 0.7, 0.2\rangle & \langle 0.1, 0.1, 0.2\rangle & \langle 0.9, 0.3, 0.7\rangle & \langle 0.3, 0.4, 0.2\rangle \\ \langle 0.2, 0.3, 0.9\rangle & \langle 0.4, 0.3, 0.9\rangle & \langle 0.3, 0.4, 0.9\rangle & \langle 0.8, 0.8, 0.3\rangle \\ \langle 0.5, 0.1, 0.9\rangle & \langle 0.8, 0.4, 0.5\rangle & \langle 0.2, 0.5, 0.1\rangle & \langle 0.7, 0.6, 0.9\rangle \end{pmatrix},$$

$$EM_2 = \begin{pmatrix} \langle 0.5, 0.1, 0.2\rangle & \langle 0.8, 0.8, 0.7\rangle & \langle 0.1, 0.6, 0.4\rangle & \langle 0.7, 0.4, 0.3\rangle \\ \langle 0.3, 0.2, 0.7\rangle & \langle 0.2, 0.6, 0.3\rangle & \langle 0.7, 0.4, 0.3\rangle & \langle 0.6, 0.8, 0.6\rangle \\ \langle 0.8, 0.6, 0.9\rangle & \langle 0.4, 0.7, 0.4\rangle & \langle 0.4, 0.5, 0.4\rangle & \langle 0.7, 0.6, 0.8\rangle \\ \langle 0.4, 0.7, 0.5\rangle & \langle 0.6, 0.3, 0.2\rangle & \langle 0.3, 0.6, 0.2\rangle & \langle 0.2, 0.7, 0.5\rangle \\ \langle 0.9, 0.3, 0.3\rangle & \langle 0.7, 0.5, 0.1\rangle & \langle 0.1, 0.8, 0.6\rangle & \langle 0.5, 0.2, 0.1\rangle \end{pmatrix},$$

and

$$EM_3 = \begin{pmatrix} \langle 0.7, 0.2, 0.9\rangle & \langle 0.2, 0.6, 0.7\rangle & \langle 0.1, 0.7, 0.8\rangle & \langle 0.7, 0.6, 0.6\rangle \\ \langle 0.6, 0.4, 0.3\rangle & \langle 0.1, 0.2, 0.9\rangle & \langle 0.6, 0.6, 0.6\rangle & \langle 0.6, 0.3, 0.1\rangle \\ \langle 0.5, 0.6, 0.5\rangle & \langle 0.6, 0.3, 0.3\rangle & \langle 0.4, 0.2, 0.1\rangle & \langle 0.4, 0.2, 0.3\rangle \\ \langle 0.7, 0.8, 0.6\rangle & \langle 0.7, 0.4, 0.5\rangle & \langle 0.5, 0.6, 0.3\rangle & \langle 0.4, 0.1, 0.4\rangle \\ \langle 0.2, 0.6, 0.8\rangle & \langle 0.9, 0.6, 0.3\rangle & \langle 0.6, 0.4, 0.2\rangle & \langle 0.8, 0.8, 0.6\rangle \end{pmatrix}.$$

Here $\kappa_{ij}^s = \langle \mathfrak{t}_{ij}^s, \mathfrak{i}_{ij}^s, \mathfrak{f}_{ij}^s\rangle$ denote a SVN number which implies evaluation of alternative ε_i according to criteria Ω_j $(j = 1, 2, 3, 4)$ made by the decision maker δ_s, $(s = 1, 2, 3)$.

Step 5: Compiling matrix (CM) is obtained as follows:

$$CM = \begin{bmatrix} \langle 0.7, 0.1, 0.2\rangle & \langle 0.8, 0.5, 0.4\rangle & \langle 0.5, 0.1, 0.4\rangle & \langle 0.7, 0.4, 0.2\rangle \\ \langle 0.8, 0.2, 0.1\rangle & \langle 0.8, 0.2, 0.3\rangle & \langle 0.7, 0.2, 0.3\rangle & \langle 0.6, 0.2, 0.1\rangle \\ \langle 0.8, 0.6, 0.2\rangle & \langle 0.6, 0.1, 0.3\rangle & \langle 0.9, 0.2, 0.1\rangle & \langle 0.7, 0.2, 0.2\rangle \\ \langle 0.7, 0.3, 0.5\rangle & \langle 0.9, 0.2, 0.1\rangle & \langle 0.5, 0.4, 0.1\rangle & \langle 0.8, 0.1, 0.3\rangle \\ \langle 0.9, 0.1, 0.3\rangle & \langle 0.9, 0.4, 0.1\rangle & \langle 0.9, 0.4, 0.1\rangle & \langle 0.8, 0.2, 0.1\rangle \end{bmatrix}.$$

Here $\cup_{s=1}^k \kappa_{11}^s = \langle \vee_{s=1}^k \mathfrak{t}_{ij}^s, \wedge_{s=1}^k \mathfrak{i}_{ij}^s, \wedge_{s=1}^k \mathfrak{f}_{ij}^s\rangle$ where $\vee$ and $\wedge$ denote the maximum and minimum, respectively.

Step 6: By using score functions obtained from DA matrix for each attribute, score values of elements in CM matrix are found by using CSVNWAA and CSVNWGAO operators as follows:

$$SM_A = \begin{bmatrix} 0.537 & 0.842 & 0.543 & 0.614 \\ 0.605 & 0.653 & 0.585 & 0.485 \\ 0.324 & 0.508 & 0.481 & 0.516 \\ 0.739 & 0.571 & 0.635 & 0.442 \\ 0.424 & 0.660 & 0.647 & 0.417 \end{bmatrix}.$$

and

$$SM_G = \begin{bmatrix} 0.311 & 0.808 & 0.583 & 0.307 \\ 0.237 & 0.635 & 0.539 & 0.274 \\ 0.342 & 0.445 & 0.316 & 0.235 \\ 0.537 & 0.492 & 0.611 & 0.215 \\ 0.212 & 0.568 & 0.435 & 0.188 \end{bmatrix},$$

respectively.

Step 7: Evaluation of the alternatives: For $i = 1, 2, 3, 4, 5$, grade of the projects ε_i are calculated by

$$\varepsilon_i = \frac{1}{k}\sum_{j=1}^{k} \partial_{ij}$$

by using matrices SM_A and SM_G

	ε_1	ε_2	ε_3	ε_4	ε_5
SM_A	0.634	0.582	0.457	0.597	0.537
SM_G	0.502	0.421	0.334	0.464	0.35

Step 8: Projects are ordered according to grades of them and two projects having maximum grade are selected as projects to be provided financial support. Then, according to SM_A and SM_G ranking order is as follows:

$$\varepsilon_1 > \varepsilon_4 > \varepsilon_2 > \varepsilon_5 > \varepsilon_3.$$

It is seen that same ranking order is obtained according to both of SM_A and SM_G. Therefore, the projects ε_1 , ε_4 are the projects to be provided the financial support.

5. Conclusion

Recently, SVNN has a very important place in modeling decision making problems. Many researchers have studied on the types of SVNNs. The best known of them are SVTNN and SVTrNNs. These numbers are SVNNs containing a maximum point and a flatness, respectively. They are represented by piecewise functions using lines. However, problems in daily life may not always follow a linear course. Therefore, in this study, the concept of CSVNN was defined based on the Cauchy distribution function. CSVNNs are important in that they are non-linear and a generalization of other neutrosophic numbers. Since the score functions defined in this study are defined separately for each CSVNN, it can be considered as a generalization of type-2 fuzzy structures that include modeling the uncertainty of the membership function. In future, many mathematical structures can be defined in the environment containing CSVNNs. In addition, distance measures and similarity measures between CSVNNs may be defined and integrated TOPSIS, VIKOR and other classical DMs under CSVN environment. its more advanced potential applications under such a flexible and versatile CSVN environment

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