Some Basic Concepts of Neutrosophic Soft Block Matrices

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Abstract: The main focus of this article is to discuss the concept of neutrosophic sets, neutrosophic soft sets , neutrosophic soft matrices theory and neutrosophic soft block matrix which are very useful and applicable in various situations involving uncertainties and imprecisions. Here different types of neutrosophic soft block matrices are studied and some operations on it along with some associated properties are discussed in details.

Keywords: Neutrosophic sets; neutrosophic soft sets; neutrosophic soft matrix, neutrosophic soft block matrix

1. Introduction

In real life situations, most of the problems in economics, social science , environmental science and in many other cases information are vague, imprecise and insufficient. Fuzzy set [1], intuitionistic fuzzy set [2] etc are used as the tool to deal with such uncertainties.

Later on Molodtsov[3], pointed out that these theories have their own difficulties and as such the novel concept of soft set theory was initiated. The theory of soft set has rich potential for solving problems in economics, social science and medical science etc. Maji *.et.al* [4, 5] have studied the theory of fuzzy soft set. Maji. *et. al* [6], have extended the theory of fuzzy soft set to intuitionistic fuzzy soft sets.

Smarandhache [7], introduced the concept of neutrosiphic sets as a mathematical tool to deal with some situations involving impreciseness, inconsistencies and interminancy. It is expected that neutrosophic sets will produce more accurate result than those obtained by using fuzzy sets or intuitionistic fuzzy sets. Maji. *et. al* [8], have extended the theory of neutrosophic set to neutrosophic soft set. Maji. *et. al* [9] applied the theory of neutrosophic soft set in decision making process.

In recent years several mathematicians have used this concept in different mathematical structures, which can be seen in the works of Deli *et.al* [10-12]. Later this very concept has been modified by Deli and Broumi [13] in developing the basic of neutrosophic soft matrices and its successful utilization in decision making process. The concept of intuitionistic neutrosophic sets was developed by Broumi and Smarandache [14] and some of its properties were discussed. Bera and Mahapatra [15] studied some algebraic structure of neutrosophic soft set. Various decision making algorithms over neutrosophic soft set theory have been developed in the literature of neutrosophc set theory. Many researchers for example [16]have worked on applying neutrosophic sets invarious decision making processes. Many new development regarding neutrosophic sets and neutrosophic soft matrices are found in the works of [17-23] Neutrosophic soft block matrix is a neutrosophic soft matrix which is defined using smaller neutrosophic soft matrices. Some authors, as for example[24, 25] have discussed neutrosophic soft block matrices.

The main focus of this article is to deal with the concept of various types of neutrosophic soft block matrices and some operations on these matrices. The next section briefly introduces some definitions related to neutrosopphic set, neutrosophic soft set, neutrosophic soft matrix, neutrosophic soft block matrices and so on. Section 3 defines operations on neutrosophic soft matrices etc. Section 4 presents some special form of neutrosophic soft block matrices and some related properties.

2. Preliminaries (proposed work with more details)

Some basic definitions that are useful in subsequent sections of this article are discussed in this section.

Definition 2.1: Neutrosophic sets (Smarandache, 2005)

Let U be the universe of discourse, The neutrosophic set A on the universe of discourse U is defined as $A = \{< T_A(x), I_A(x), F_A(x) >: x \in U\}$, where the characteristic functions $T, I, F : U \to [0,1]$ and $^-0 \le T + I + F \le 3^+$; T,I,F are neutrosophic components which defines the degree of membership, the degree of interminancy and the degree of non membership respectively.

Definition 2.2: Neutrosophic soft set (Maji, 2013)

Let U be an initial universe set and E is the set of parameters. Suppose P (U) denotes the collection of all neutrosophic subsets of U. Let $A \subseteq E$. A pair (F, E) is called neutrosophic soft set over U where F is a mapping given by $F : E \to P(U)$

Definition 2.3: Neutrosophic soft Matrix (Deli.*et.al*, 2015)

Let $U = \{u_1, u_2, u_3, ...u_m\}$ be the universe set and $E = \{x_1, x_2, x_3, ...x_n\}$ be the set of parameters. Let $A \subseteq E$. The set (F, A) is a neutrosophic soft set over U. Then the subset of UXE is uniquely defined by $R_A = \{(u, e) : e \in A, u \in F_A(e)\}$ which is a relation form of (F, E). Now the relation RA is characterized by truth membership function $T_{R_A} : U \times E \to [0,1]$, interminancy membership function $I_{R_A} : U \times E \to [0,1]$ and falsity membership function $F_{R_A} : U \times E \to [0,1]$

where $T_{R_A}(u,e)\in[0,1]$, $I_{R_A}(u,e)\in[0,1]$ and $F_{R_A}(u,e)\in[0,1]$ indicates truthfulness, interminancy and falsity.

If

$(T_{A_{mn}}, I_{A_{mn}}, F_{A_{mn}})$=($T_A(u_m,e_n)$, $I_A(u_m,e_n)$, $F_A(u_m,e_n)$) and a_{ij}=($T_A(u_i,e_j)$, $I_A(u_i,e_j)$, $F_A(u_i,e_j)$) we can define a matrix $\tilde{A}_{ij}=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$

This is called neutrosophic soft matrix of order mxn corresponding to the neutrosophioc soft set

(F,E) over U.

Definition 2.4: Triangular neutrosophic soft matrix

Triangular neutrosophic soft matrix is a special type of square neutrosophic soft matrix. A square neutrosophic soft matrix is called lower triangular if all the entries $a_{ij}=(0,0,1)$ for $i<j$, i,j=1, 2, 3,…..,n and upper triangular if $a_{ij}=(0,0,1)$ for $i>j$, i,j=1, 2, 3, ……,n.

Definition 2.5: Toeplitz neutrosophic soft matrix

Toeplitz neutrosophic soft matrix is a square neutrosophic soft matrix of the form

$$\tilde{A}=\begin{bmatrix} (T^A{}_{11},I^A{}_{11},F_{11}{}^A) & (T^A{}_{12},I^A{}_{12},F_{12}{}^A) & (T^A{}_{13},I^A{}_{13},F_{13}{}^A) & (T^A{}_{14},I^A{}_{14},F_{14}{}^A) \\ (T^A{}_{21},I^A{}_{21},F_{21}{}^A) & (T^A{}_{11},I^A{}_{11},F_{11}{}^A) & (T^A{}_{21},I^A{}_{21},F_{21}{}^A) & (T^A{}_{13},I^A{}_{13},F_{13}{}^A) \\ (T^A{}_{31},I^A{}_{31},F_{31}{}^A) & (T^A{}_{21},I^A{}_{21},F_{21}{}^A) & (T^A{}_{11},I^A{}_{11},F_{11}{}^A) & (T^A{}_{21},I^A{}_{21},F_{21}{}^A) \\ (T^A{}_{41},I^A{}_{41},F_{41}{}^A) & (T^A{}_{31},I^A{}_{31},F_{31}{}^A) & (T^A{}_{21},I^A{}_{21},F_{21}{}^A) & (T^A{}_{11},I^A{}_{11},F_{11}{}^A) \end{bmatrix}$$

Definition 2.6: Zero neutrosophic soft matrix

Zero neutrosophic soft matrix is neutrosophic soft matrix in which all the entries are of the form (0,0,1).

Definition 2.7: Tridiagonal neutrosophic soft matrix

Neutrosophic soft tridiagonal matrix is another special neutrosophic soft matrix which has non zero entries in the lower diagonal, main diagonal and upper diagonal and all other entries being (0.0.1). That is a Neutrosophic soft tridiagonal matrix A has the form

$\tilde{A}=\begin{bmatrix} B_1 & C_1 & \dots & 0 \\ A_1 & B_2 & C_2\dots & 0 \\ 0 & A_2 & B_3 & C_3 \\ 0 & 0 & A_3 & B_4 \end{bmatrix}$ where A_i, B_i, C_i are non zero entries in the lower, main and upper diagonal respectively.

Definition 2.8:Neutrosofic soft block matrix (Uma.*et.al*,2017 & Dhar, 2020)

A neutrosophic soft block matrix or a partitioned matrix is a neutrosophic soft matrix that is interpretated as having been broken into sections called blocks or submatrices. A neutrosophic soft block matrix can be visualized as the original neutrosophic soft matrix by drawing lines parallel to its rows and columns. These sub-matrices may be considered as the elements of the original matrices. Any neutrosophic soft matrix can be interpreted as a neutrosophic soft block matrix in one or more ways, with each interpretation defined by how its rows and columns are partitioned.

For example

$$\tilde{A}=\begin{bmatrix} (T^{A}_{11},I^{A}_{11},F_{11}^{A}) & (T^{A}_{12},I^{A}_{12},F_{12}^{A}) & : & (T^{A}_{13},I^{A}_{13},F_{13}^{A}) & (T^{A}_{14},I^{A}_{14},F_{14}^{A}) \\ \cdots & \cdots & & \cdots & \cdots \\ (T^{A}_{21},I^{A}_{21},F_{21}^{A}) & (T^{A}_{22},I^{A}_{22},F_{22}^{A}) & : & (T^{A}_{23},I^{A}_{23},F_{23}^{A}) & (T^{A}_{24},I^{A}_{24},F_{24}^{A}) \\ (T^{A}_{31},I^{A}_{31},F_{31}^{A}) & (T^{A}_{32},I^{A}_{32},F_{32}^{A}) & : & (T^{A}_{33},I^{A}_{33},F_{33}^{A}) & (T^{A}_{34},I^{A}_{34},F_{34}^{A}) \end{bmatrix}$$

The above neutrosophic soft matrix can be represented as

$$\tilde{A}=\begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{bmatrix}$$

where

$$\tilde{P}_{11}=\begin{bmatrix} (T^{A}_{11},I^{A}_{11},F^{A}_{11}) & (T^{A}_{12},I^{A}_{12},F^{A}_{12}) \end{bmatrix}$$

$$\tilde{P}_{12}=\begin{bmatrix} (T^{A}_{13},I^{A}_{13},F^{A}_{13}) & (T^{A}_{14},I^{A}_{14},F^{A}_{14}) \end{bmatrix}$$

$$\tilde{P}_{21}=\begin{bmatrix} (T^{A}_{21},I^{A}_{21},F^{A}_{21}) & (T^{A}_{22},I^{A}_{22},F^{A}_{22}) \\ (T^{A}_{31},I^{A}_{31},F^{A}_{31}) & (T^{A}_{32},I^{A}_{32},F^{A}_{32}) \end{bmatrix}, \quad \tilde{P}_{22}=\begin{bmatrix} (T^{A}_{23},I^{A}_{23},F^{A}_{23}) & (T^{A}_{24},I^{A}_{24},F^{A}_{24}) \\ (T^{A}_{33},I^{A}_{33},F^{A}_{33}) & (T^{A}_{34},I^{A}_{34},F^{A}_{34}) \end{bmatrix}$$

Then

$\tilde{A}=\begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{bmatrix}$ is an example of neutrosophic soft block matrix or neutrosophic soft partitioned matrix.

So the neutrosophic soft matrix A is partitioned by the dotted lines dividing the neutrosophic soft matrix into neutrosophic soft sub-matrices $\tilde{P}_{11}, \tilde{P}_{12}, \tilde{P}_{21}, \tilde{P}_{22}$. The neutrosophic soft matrix $\tilde{A}$ can be partitioned in several ways.

Definition 2.9: Square neutrosophic soft block matrix

If the number of rows and the number of columns of a neutrosophic soft blocks are equal then the matrix is said to be square neutrosophic soft block matrix.

$$\tilde{A}=\begin{bmatrix} (T_{11}^A,I_{11}^A,F_{11}^A) & (T_{12}^A,I_{12}^A,F_{12}^A) & : & (T_{13}^A,I_{13}^A,F_{13}^A) & (T_{14}^A,I_{14}^A,F_{14}^A) & : & (T_{15}^A,I_{15}^A,F_{15}^A) & (T_{16}^A,I_{16}^A,F_{16}^A) \\ (T_{21}^A,I_{21}^A,F_{21}^A) & (T_{22}^A,I_{22}^A,F_{22}^A) & : & (T_{23}^A,I_{23}^A,F_{23}^A) & (T_{24}^A,I_{24}^A,F_{24}^A) & : & (T_{25}^A,I_{25}^A,F_{25}^A) & (T_{25}^A,I_{25}^A,F_{25}^A) \\ \ldots & \ldots & .. & \ldots & \ldots & \ldots & \ldots & \ldots \\ (T_{31}^A,I_{31}^A,F_{31}^A) & (T_{32}^A,I_{32}^A,F_{32}^A) & : & (T_{33}^A,I_{33}^A,F_{33}^A) & (T_{34}^A,I_{34}^A,F_{34}^A) & : & (T_{35}^A,I_{35}^A,F_{35}^A) & (T_{36}^A,I_{36}^A,F_{36}^A) \\ (T_{41}^A,I_{41}^A,F_{41}^A) & (T_{42}^A,I_{42}^A,F_{42}^A) & : & (T_{43}^A,I_{43}^A,F_{43}^A) & (T_{44}^A,I_{44}^A,F_{44}^A) & : & (T_{45}^A,I_{45}^A,F_{45}^A) & (T_{46}^A,I_{46}^A,F_{46}^A) \end{bmatrix}$$

or

$$\tilde{A}=\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{A}_{13} \\ \tilde{A}_{21} & \tilde{A}_{22} & \tilde{A}_{23} \end{bmatrix}$$

is a square fuzzy block matrix since all $\tilde{A}_{ij}$'s are square blocks.

Definition 2.10: Rectangular neutrosophic soft block matrix

If the number of rows and the number of columns of blocks are unequal then the matrix is said to be rectangular neutrosophic soft block matrix .For example

$$\tilde{A}=\begin{bmatrix} (T^B{}_{11},I^B{}_{11},F_{11}{}^B) & (T^B{}_{12},I^B{}_{12},F_{12}{}^B) & : & (T^B{}_{13},I^B{}_{13},F_{13}{}^B) & (T^B{}_{14},I^B{}_{14},F_{14}{}^B) \\ \ldots & \ldots & & \ldots & \ldots \\ (T^B{}_{21},I^B{}_{21},F_{21}{}^B) & (T^B{}_{22},I^B{}_{22},F_{22}{}^B) & : & (T^B{}_{23},I^B{}_{23},F_{23}{}^B) & (T^B{}_{24},I^B{}_{24},F_{24}{}^B) \\ (T^B{}_{31},I^B{}_{31},F_{31}{}^B) & (T^B{}_{32},I^B{}_{32},F_{32}{}^B) & : & (T^B{}_{33},I^B{}_{33},F_{33}{}^B) & (T^B{}_{34},I^B{}_{34},F_{34}{}^B) \end{bmatrix}$$

Is a rectangular neutrosophic soft block matrix.

3. **Operations on Neutrosophic Soft matrices**

3.1 Addition of neutrosophic soft matrices

Let $\tilde{A}=[(T_{ij}^A,I_{ij}^A,F_{ij}^A)]$, $\tilde{B}=[(T_{ij}^B,I_{ij}^B,F_{ij}^B)]$ be two neutrosophic soft matrices. Then the max-min product of the two neutrosophic soft matricesA and B is denoted as $A+B$ is defined as

$\tilde{A}+\tilde{B}=[\max(T_{ij}^A,T_{ij}^B),\min(I_{ij}^A,I_{ij}^B),\min(F_{ij}^A,F_{ij}^B)]$ for all i and j.

3.2 Max-min product of neutrosophic soft matrices

Let $\tilde{A}=[(T_{ij}^A,I_{ij}^A,F_{ij}^A)]$, $\tilde{B}=[(T_{ij}^B,I_{ij}^B,F_{ij}^B)]$ be two neutrosophic soft matrices. Then the max-min product of the two neutrosophic soft matricesA and B is denoted as $\tilde{A}\tilde{B}$ is defined as

$\tilde{A}\tilde{B}=[\max\min(T_{ij}^A,T_{ij}^B),\min\max(I_{ij}^A,I_{ij}^B),\min\max(F_{ij}^A,F_{ij}^B)]$ for all i and j.

3.3 Transpose of neutrosophic soft matrices

Let $\tilde{A}=[(T_{ij}^{A}, I_{ij}^{A}, F_{ij}^{A})]$ be two neutrosophic soft matrices. Then the transpose of this neutrosophic soft block matrix will be defined by denoted by $\tilde{A}^{T}$ and is defined by $\tilde{A}^{T}=[(T_{ji}^{A}, I_{ji}^{A}, F_{ji}^{A})]$

3.4 Addition of neutrosophic soft block matrices

Let $\tilde{A}=\begin{bmatrix}\tilde{A}_{11} & : & \tilde{A}_{12}\\ ..... & : & .....\\ \tilde{A}_{21} & : & \tilde{A}_{22}\end{bmatrix}$ and $\tilde{B}=\begin{bmatrix}\tilde{B}_{11} & : & \tilde{B}_{12}\\ ..... & : & .....\\ \tilde{B}_{21} & : & \tilde{B}_{22}\end{bmatrix}$ be two neutrosophic soft block matrices in which the corresponding blocks are conformable for addition, then the addition of two neutrosophic soft block matrices can be defined as

$$\tilde{A}+\tilde{B}=\begin{bmatrix}\tilde{A}_{11}+\tilde{B}_{11} & : & \tilde{A}_{12}+\tilde{B}_{12}\\ ..... & : & .....\\ \tilde{A}_{21}+\tilde{B}_{21} & : & \tilde{A}_{22}+\tilde{B}_{22}\end{bmatrix}$$

3.5 Multiplication of neutrosophic soft block matrices

Let $\tilde{A},\tilde{B}$ be two neutrosophic soft block matrices which can be represented by

$$\tilde{A}=\begin{bmatrix}\tilde{A}_{11} & : & \tilde{A}_{12}\\ ..... & : & .....\\ \tilde{A}_{21} & : & \tilde{A}_{22}\end{bmatrix},\ \tilde{B}=\begin{bmatrix}\tilde{B}_{11} & : & \tilde{B}_{12}\\ ..... & : & .....\\ \tilde{B}_{21} & : & \tilde{B}_{22}\end{bmatrix}$$

Then the product of two neutrosophic soft block matrices will be denoted by $\tilde{A}\tilde{B}$ and is defined by

$$\tilde{A}\tilde{B}=\begin{bmatrix}\tilde{A}_{11}\tilde{B}_{11}+\tilde{A}_{12}\tilde{B}_{21} & : & \tilde{A}_{11}\tilde{B}_{12}+\tilde{A}_{12}\tilde{B}_{22}\\ ..... & : & .....\\ \tilde{A}_{21}\tilde{B}_{11}+\tilde{A}_{22}\tilde{B}_{21} & : & \tilde{A}_{21}\tilde{B}_{12}+\tilde{A}_{22}\tilde{B}_{22}\end{bmatrix}$$

provided that the blocks considered here are conformable for multiplication.

3.5 Transpose of a neutrosophic soft block matrix

Let $\tilde{A}=\begin{bmatrix}\tilde{P}_{11} & \tilde{P}_{12}\\ \tilde{P}_{21} & \tilde{P}_{22}\end{bmatrix}$

be a neutrosophic soft block matrix, then the transpose of that neutrosophic soft block matrix is defined as

$$\tilde{A}^{T}=\begin{bmatrix}\tilde{P}_{11}^{\ T} & \tilde{P}_{12}^{\ T}\\ \tilde{P}_{21}^{\ T} & \tilde{P}_{22}^{\ T}\end{bmatrix}$$

4. Some special types of neutrosophic soft block matrices

In this section, the intention is to discuss about various types of neutrosophic soft block matrices and the associative properties.

4.1 Neutrosophic soft block triangular matrix

Neutrosophic soft block triangular matrix is a special type of square neutrotrosophic soft matrix. Neutrosophic block triangular matrices can be of two forms such as upper triangular or lower triangular.

4.1.1 Neutrosophic Soft Block upper triangular matrix

Neutrosophic soft block upper triangular matrix is a neutrosophic soft matrix of the form $\tilde{A} = \begin{bmatrix} \tilde{X} & \tilde{Y} \\ O & \tilde{W} \end{bmatrix}$ where $\tilde{X}$, $\tilde{Y}$ and $\tilde{W}$ are square neutrosophic soft matrices.

4.1.2 Neutrosophic Soft Block lower triangular matrix

Neutrosophic soft block lower triangular matrix is a neutrosophic soft matrix of the form $\tilde{A} = \begin{bmatrix} \tilde{X} & O \\ \tilde{Y} & \tilde{W} \end{bmatrix}$ where $\tilde{X}$, $\tilde{Y}$ and $\tilde{W}$ are square neutrosophic soft matrices.

4.1.3 Properties of Neutrosophic Soft Block triangular matrix

- Addition of two neutrosophic soft block upper triangular matrices of same order results in a neutrosophic soft block upper triangular matrix.
- Product of two neutrosophic soft block upper triangular matrices is again a neutrosophic soft block upper triangular matrix.
- Addition of two neutrosophic soft block lower triangular matrices results in a neutrosophic soft block upper triangular matrix.
- Multiplication of two neutrosophic soft block lower triangular matrices of same order is again a neutrosophic soft block lower triangular matrix.

4.2 Neutrosophic soft block diagonal matrix

Neutrosophic soft block diagonal matrix is a square neutrosophic soft block matrix in which the main diagonal blocks are square neutrosophic soft matrices and all off diagonal blocks are zero neutrosophic soft matrices. Neutrosophic soft block diagonal matrix A has the following form

$\tilde{A}=\begin{bmatrix} A_{11} & 0 & \ldots & 0 \\ 0 & A_{22} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & A_{nn} \end{bmatrix}$ where A_{ij} is a square neutrosophic soft block matrix for all i,j=1,2,3,…..,n.

4.3 Neutrosophic soft block quasidiagonal matrix

It is a neutrosophic soft block matrix whose diagonal blocks are square neutrosophic soft block matrices of different order and off diagonal blocks are zero neutrosophic soft block matrices. Thus

$\tilde{A}=\begin{bmatrix} D_1 & 0 & \ldots & 0 \\ 0 & D_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & D_n \end{bmatrix}$ is a quasidiagonal matrix whose diagonal blocks D_i, i=1,2,3,…..,n are square neutrosophic soft matrices of different orders.

4.4 Neutrosophic soft block tridiagonal matrix

Neutrosophic soft block tridiagonal matrix is another special neutrosophic soft block matrix which is just like the neutrosophic soft block diagonal matrix, a square neutrosophic soft matrix , having square neutrosophic soft matrices in the lower diagonal, main diagonal and upper diagonal with all other blocks being zero neutrosophic soft matrices. Neutrosophic soft block tridiagonal matrix A has the form

$\tilde{A}=\begin{bmatrix} B_1 & C_1 & \ldots & 0 \\ A_1 & B_2 & C_2\ldots & 0 \\ 0 & A_2 & B_3 & C_3 \\ 0 & 0 & A_3 & B_4 \end{bmatrix}$ where A_i, B_i, C_i are square neutrosophic soft block matrices in the lower diagonal, main diagonal and upper diagonal respectively.

4.4.1 Properties of Neutrosophic soft block tridiagonal matrix

- Sum of two neutrosophic soft block tridiagonal matrices of same order is again a neutrosophic soft block tridiagonal matrix.

Let $\tilde{A}=\begin{bmatrix} B_1 & C_1 & \ldots & 0 \\ A_1 & B_2 & C_2\ldots & 0 \\ 0 & A_2 & B_3 & C_3 \\ 0 & 0 & A_3 & B_4 \end{bmatrix}$ and $\tilde{B}=\begin{bmatrix} D_1 & E_1 & \ldots & 0 \\ F_1 & D_2 & E_2\ldots & 0 \\ 0 & F_2 & D_3 & E_3 \\ 0 & 0 & F_3 & D_4 \end{bmatrix}$

Then from the definition of addition of two neutrosopic soft block matrices it can be obtained that

$$\tilde{A}+\tilde{B}=\begin{bmatrix} B_1+D_1 & C_1+E_1 & ... & 0 \\ A_1+F_1 & B_2+D_2 & C_2+E_2... & 0 \\ 0 & A_2+F_2 & B_3+D_3 & C_3+E_3 \\ 0 & 0 & A_3+F_3 & B_4+D_4 \end{bmatrix}$$

which is obviously a tridiagonal neutrosophic soft block matrix.

- Product of two neutrosophic soft block tridiagonal matrices is again a neutrosophic soft block up tridiagonal matrix.
- Transpose of neutrosophic soft block tridiagonal matrix is again a neutrosophic soft tridiagonal matrix.

Example: Let be $\tilde{A}$ neutrosophic soft block tridiagonal matrix $\tilde{A}=\begin{bmatrix} B_1 & C_1 & ... & 0 \\ A_1 & B_2 & C_2... & 0 \\ 0 & A_2 & B_3 & C_3 \\ 0 & 0 & A_3 & B_4 \end{bmatrix}$ then

$\tilde{A}^T=\begin{bmatrix} B_1 & A_1 & ... & 0 \\ C_1 & B_2 & A_2... & 0 \\ 0 & C_2 & B_3 & A_3 \\ 0 & 0 & C_3 & B_4 \end{bmatrix}$ which a neutrosophic soft block tridiagonal matrix is again

4.5 Neutrosophic soft block toeplitz matrix

Neutrosophic soft block tridiagonal matrix is another special neutrosophic soft block matrix, which contains blocks that are repeated down the diagonals of the matrix. The individual block elements of A_{ij} must also be Toeplitz matrices. Neutrosophic soft block toeplitz matrix $\tilde{A}$ has the form

$\tilde{A}=\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{11} & A_{12} & A_{13} \\ A_{31} & A_{21} & A_{11} & A_{12} \\ A_{41} & A_{31} & A_{21} & A_{11} \end{bmatrix}$ where A_{ij} are square neutrosophic soft block matrices respectively.

4.5.1 Properties of neutrosophic soft block toepliz matrix

- Addition of neutrosophic soft block toepliz matrices is again a neutrosophic soft block toepliz matrix provided the matrices are conformal for addition.

- Transpose of a neutrosophic soft block toepliz matrix is again a neutrosophic soft block toepliz matrix.

Example: If the above toepliz matrix A is considered then

$$\tilde{A}^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{11} & A_{21} & A_{31} \\ A_{13} & A_{12} & A_{11} & A_{21} \\ A_{14} & A_{13} & A_{12} & A_{11} \end{bmatrix}$$

this is again a neutrosophic soft toplitz matrix.

8.1.3 Transportation

If $\tilde{A}, \tilde{B}$ be two neutrosophic soft block toeplitz matrices, then $(\tilde{A}+\tilde{B})^T = \tilde{A}^T + \tilde{B}^T$

Let $\tilde{A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{11} & A_{12} & A_{13} \\ A_{31} & A_{21} & A_{11} & A_{12} \\ A_{41} & A_{31} & A_{21} & A_{11} \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{11} & B_{12} & B_{13} \\ B_{31} & B_{21} & B_{11} & B_{12} \\ B_{41} & B_{31} & B_{21} & B_{11} \end{bmatrix}$

Then $\tilde{A}+\tilde{B} = \begin{bmatrix} A_{11}+B_{11} & A_{12}+B_{12} & A_{13}+B_{13} & A_{14}+B_{14} \\ A_{21}+B_{21} & A_{11}+B_{11} & A_{12}+B_{12} & A_{13}+B_{13} \\ A_{31}+B_{31} & A_{21}+B_{21} & A_{11}+B_{11} & A_{12}+B_{12} \\ A_{41}+B_{41} & A_{31}+B_{31} & A_{21}+B_{21} & A_{11}+B_{11} \end{bmatrix}$

Then $\tilde{A}^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{11} & A_{21} & A_{31} \\ A_{13} & A_{12} & A_{11} & A_{21} \\ A_{14} & A_{13} & A_{12} & A_{11} \end{bmatrix}$ and $\tilde{B}^T = \begin{bmatrix} B_{11} & B_{21} & B_{31} & B_{41} \\ B_{12} & B_{11} & B_{21} & B_{31} \\ B_{13} & B_{12} & B_{11} & B_{21} \\ B_{14} & B_{13} & B_{12} & B_{11} \end{bmatrix}$

$$(\tilde{A}+\tilde{B})^T = \begin{bmatrix} A_{11}+B_{11} & A_{21}+B_{21} & A_{31}+B_{31} & A_{41}+B_{41} \\ A_{12}+B_{21} & A_{11}+A_{11} & A_{21}+B_{21} & A_{31}+B_{31} \\ A_{13}+B_{13} & A_{12}+B_{12} & A_{11}+B_{11} & A_{21}+B_{21} \\ A_{14}+B_{14} & A_{13}+B_{13} & A_{12}+B_{12} & A_{11}+B_{11} \end{bmatrix} = \tilde{A}^T + \tilde{B}^T$$

4.6 Neutrosophic soft Block Circulant Matrix

A neutrosophic soft Block Circulant Matrix is a neutrosophic soft block matrix of the form

$$\tilde{A} = \begin{bmatrix} A_0 & A_1 & ..... & A_{m-1} \\ A_{m-1} & A_0 & ..... & A_{m-2} \\ \vdots & \vdots & ..... & \vdots \\ A_1 & A_2 & ...... & A_0 \end{bmatrix}$$

where $A_i\text{'}s$ are nxn arbitrary matrics.

4.6.1 **Properties of Neutrosophic soft Block Circulant Matrix**

If A and B be two neutrosophic block circulant matrices then A+B and AB is again a neutrosophic block circulant matrix. Again for block circulant matrices AB=BA.

4.7 Direct sum of neutrosophic soft block matrices.

If $\tilde{A}_{11}, \tilde{A}_{22}, \tilde{A}_{33}, \ldots\ldots, \tilde{A}_{rr}$ are square neutrosophic soft block matrices of order $m_1, m_2, m_3, \ldots\ldots, m_r$ respectively.

Then $$diag(\tilde{A}_{11}, \tilde{A}_{22}, \tilde{A}_{33}, \ldots\ldots, \tilde{A}_{rr}) = \begin{bmatrix} \tilde{A}_{11} & 0 & \ldots & 0 \\ 0 & \tilde{A}_{22} & \ldots & 0 \\ : & : & : & : \\ 0 & 0 & \ldots & \tilde{A}_{rr} \end{bmatrix}_{m_1+m_2+\ldots\ldots+m_r}$$

is called the direct sum of the square neutrosophic soft block matrix $\tilde{A}_{11}, \tilde{A}_{22}, \tilde{A}_{33}, \ldots\ldots, \tilde{A}_{rr}$ and it is expressed as $\tilde{A}_{11} \oplus \tilde{A}_{22} \oplus \tilde{A}_{33} \oplus \ldots\ldots \oplus \tilde{A}_{rr}$ of order $m_1 + m_2 + m_3 + \ldots\ldots + m_r$.

4.7.1 Properties of direct sum

The following algebraic properties are hold by neutrosophic soft block matrices:

- **Commutativity:** Let $\tilde{A}, \tilde{B}$ be two diagonal neutrosophic soft block matrices then

$$\tilde{A} \oplus \tilde{B} = \begin{bmatrix} \tilde{A} & : & 0 \\ \ldots\ldots & : & \ldots\ldots \\ 0 & : & \tilde{B} \end{bmatrix} \quad \text{and} \quad \tilde{B} \oplus \tilde{A} = \begin{bmatrix} \tilde{B} & : & 0 \\ \ldots\ldots & : & \ldots\ldots \\ 0 & : & \tilde{A} \end{bmatrix}$$

Thus $\tilde{A} \oplus \tilde{B} \neq \tilde{B} \oplus \tilde{A}$ and hence it can be concluded that the direct sum of two neutrosophic soft block matrices are not commutative.

- **Associativity:** Let $\tilde{A}, \tilde{B}, \tilde{C}$ be three square neutrosophic soft block matrices. Then as obtained above

$$\tilde{A} \oplus \tilde{B} = \begin{bmatrix} \tilde{A} & : & 0 \\ \ldots\ldots & : & \ldots\ldots \\ 0 & : & \tilde{B} \end{bmatrix} = \tilde{D}(say)$$

Therefore

$$(\tilde{A} \oplus \tilde{B}) \oplus \tilde{C} = \tilde{D} \oplus \tilde{C} = \begin{bmatrix} \tilde{D} & 0 \\ 0 & \tilde{C} \end{bmatrix} = \begin{bmatrix} \tilde{A} & 0 & 0 \\ 0 & \tilde{B} & 0 \\ 0 & 0 & \tilde{C} \end{bmatrix} \text{ where } \tilde{A} \oplus \tilde{B} = \tilde{D}$$

Again

$$\tilde{A}\oplus(\tilde{B}\oplus\tilde{C})=\tilde{A}\oplus\tilde{E}=\begin{bmatrix}\tilde{A} & 0\\ 0 & \tilde{E}\end{bmatrix}=\begin{bmatrix}\tilde{A} & 0 & 0\\ 0 & \tilde{B} & 0\\ 0 & 0 & \tilde{C}\end{bmatrix} \text{ where } \tilde{B}\oplus\tilde{C}=\tilde{E}$$

Hence associative laws hold for neutrosophic soft block matrices.

4.8 Mixed sum of neutrosophic soft block matrices

Let $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ be four neutrosophic soft block matrices which are conformable for addition. Then

By the definitions of addition and direct sum of neutrosophic soft block matrices it can be obtained that

$$\tilde{A}\oplus\tilde{C}=\begin{bmatrix}\tilde{A} & : & 0\\ ..... & : & .....\\ 0 & : & \tilde{C}\end{bmatrix},\ \tilde{B}\oplus\tilde{D}=\begin{bmatrix}\tilde{B} & : & 0\\ ..... & : & .....\\ 0 & : & \tilde{D}\end{bmatrix} \text{and } (\tilde{A}\oplus\tilde{C})+(\tilde{B}\oplus\tilde{D})=\begin{bmatrix}\tilde{A}+\tilde{B} & : & 0\\ ..... & : & .....\\ 0 & : & \tilde{C}+\tilde{D}\end{bmatrix}$$

Then the following result holds:

$$(\tilde{A}+\tilde{B})\oplus(\tilde{C}+\tilde{D})=(\tilde{A}\oplus\tilde{C})+(\tilde{B}\oplus\tilde{D})$$

4.9 Multiplication of direct sum of neutrosophic soft block matrices

If $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ be four neutrosophic soft block matrices which are conformable for addition and multiplication then the mixed multiplication of direct sum is

$$(\tilde{A}\oplus\tilde{B})(\tilde{C}\oplus\tilde{D})=(\tilde{A}\tilde{C})\oplus(\tilde{B}\tilde{D})$$

By the definition of direct sum and multiplication of neutrosophic soft block matrices

$$(\tilde{A}\oplus\tilde{B})(\tilde{C}\oplus\tilde{D})=\begin{bmatrix}\tilde{A} & 0\\ 0 & \tilde{B}\end{bmatrix}\begin{bmatrix}\tilde{C} & 0\\ 0 & \tilde{D}\end{bmatrix}$$

$$=\begin{bmatrix}\tilde{A}\tilde{C} & 0\\ 0 & \tilde{B}\tilde{D}\end{bmatrix}=\tilde{A}\tilde{C}\oplus\tilde{B}\tilde{D}$$

- **Transposition**

If $\tilde{A},\tilde{B}$ be two neutrosophic soft block matrices then the transportation of the direct sum of A and B is

$$(\tilde{A}\oplus\tilde{B})^T=\begin{bmatrix}\tilde{A}^T & 0\\ 0 & \tilde{B}^T\end{bmatrix}=\tilde{A}^T\oplus\tilde{B}^T$$

5. Results

In the process it is found that different types of neutrosophic soft block matrices which are discussed here behave in the same way as the block matrices that exist in the literature.

6. Applications

Neutrosophic soft matrices having been broken into sections called blocks or partitioned are useful for cutting down calculations in the cases of problems which involves neutrosophic soft matrices.

7. Conclusions

Different types of neutrosophic soft block matrices as triangular, tridiagonal, quasidiagonal, circulant, toepliz are discussed. Some operations on neutrosophic soft block matrices which are also discussed in this article gives a clear indication that such operations produces almost similar results to those of classical matrices. Future research will be in the direction of finding determinants of neutrosophic soft block matrices.

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