



Pairwise Neutrosophic b -Continuous Function in Neutrosophic Bitopological Spaces

Binod Chandra Tripathy¹ and Suman Das^{2,*}

^{1,2}Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.
E-mail:¹tripathybc@gmail.com, tripathybc@yahoo.com and ²sumandas18842@gmail.com,
suman.mathematics@tripurauniv.in

*Correspondence: sumandas18842@gmail.com

Abstract: The main focus of this article is to procure the notions of pairwise neutrosophic continuous and pairwise neutrosophic b -continuous mappings in neutrosophic bitopological spaces. Then, we formulate some results on them via neutrosophic bitopological spaces.

Keywords: Neutrosophic Topology; Neutrosophic Bitopology; Pairwise Neutrosophic b -Interior; Pairwise Neutrosophic b -Closure; Pairwise Neutrosophic Continuous.

1. Introduction

Zadeh [31] presented the notions of fuzzy set (in short FS) in the year 1965. Afterwards, Chang [4] applied the idea of topology on fuzzy sets and introduced the fuzzy topological space. In the year 2017, Dutta and Tripathy [15] studied on fuzzy b - θ open sets via fuzzy topological space. Later on, Smarandache [23] grounded the idea of neutrosophic set (in short N-set) in the year 1998, as an extension of the concept of intuitionistic fuzzy set (in short IF-set) [3], where every element has three independent membership values namely truth, indeterminacy, and false membership values respectively. Afterwards, Salama and Alblowi [21] applied the notions of topology on N-sets and introduced neutrosophic topological space (in short NT-space) by extending the notions of fuzzy topological spaces. Salama and Alblowi [22] also defined generalized N-set and introduced the concept of generalized NT-space. Later on, Arokiarani et al. [2] introduced the ideas of neutrosophic point and studied some functions in neutrosophic topological spaces. The notions of neutrosophic pre-open (in short NP-O) and neutrosophic pre-closed (in short NP-C) sets via NT-spaces are studied by Rao and Srinivasa [20]. The idea of b -open sets via topological spaces was established by Andrijevic [1]. Afterwards, Ebenanjar et al. [16] presents the concept of neutrosophic b -open set (in short N- b -O-set) via NT-spaces. In the year 2020, Das and Pramanik [8] presents the generalized neutrosophic b -open sets in NT-spaces. The notions of neutrosophic Φ -open set and neutrosophic Φ -continuous functions via NT-spaces was also presented by Das and Pramanik [9]. The concept of neutrosophic simply soft open set in neutrosophic soft topological space was studied by Das and Pramanik [10]. In the year 2021, Das and Tripathy [14] presented the notions of neutrosophic simply b -open set via NT-spaces. In the year 2020, Das and Tripathy [12] grounded the notions of neutrosophic multiset and applied topology on it. In the year 2021, Das et al. [5] studied the concept of quadripartitioned neutrosophic topological spaces. The notion of bitopological space was introduced by Kelly [17] in the year 1963. In the year 2011, Tripathy and Sarma [26] studied on b -locally open sets via bitopological spaces. The idea of pairwise b -locally

open and b -locally closed functions in bitopological spaces was studied by Tripathy and Sarma [27]. Tripathy and Sarma [28] also studied on weakly b -continuous mapping via bitopological spaces in the year 2013. Later on, the concept of generalized b -closed sets in ideal bitopological spaces was studied by Tripathy and Sarma [29]. Afterwards, Tripathy and Debnath [25] presented the notions of fuzzy b -locally open sets in fuzzy bitopological space. Thereafter, Ozturk and Ozkan [19] introduced the idea of neutrosophic bitopological space (in short NBi-T-space) in the year 2019. Recently, Das and Tripathy [13] presented the idea of pairwise N- b -O-sets and studied their different properties.

The main focus of this article is to procure the notions of pairwise τ_{ij} -neutrosophic- b -interior (in short P- τ_{ij} -N b -int), pairwise τ_{ij} -neutrosophic- b -closure (in short P- τ_{ij} -N b -cl), pairwise neutrosophic continuous mapping (in short P-N-C-mapping), pairwise neutrosophic b -continuous mapping (in short pairwise N- b C-mapping) via NBi-T-spaces.

2. Preliminaries and Definitions:

The notion of N-set is defined as follows:

Let X be a fixed set. Then, an N-set [23] L over X is denoted as follows:
 $L = \{(t, T_L(t), I_L(t), F_L(t)) : t \in X\}$, where $T_L, I_L, F_L : X \rightarrow [0,1]$ are called the truth-membership, indeterminacy-membership and false-membership functions and $0 \leq T_L(t) + I_L(t) + F_L(t) \leq 3$, for all $t \in X$.

The neutrosophic null set (0_N) and neutrosophic whole set (1_N) over a fixed set X are defined as follows:

- (i) $0_N = \{(t, 0, 0, 1) : t \in X\}$;
- (ii) $1_N = \{(t, 1, 0, 0) : t \in X\}$.

The N-sets 0_N and 1_N also has three other representations. They are given below:

$$0_N = \{(t, 0, 0, 0) : t \in X\} \ \& \ 1_N = \{(t, 1, 1, 1) : t \in X\};$$

$$0_N = \{(t, 0, 1, 0) : t \in X\} \ \& \ 1_N = \{(t, 1, 0, 1) : t \in X\};$$

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Let $p, q, r \in [0,1]$. An neutrosophic point (in short N-point) [2] $x_{p,q,r}$ is an N-set over X given by

$$x_{p,q,r}(y) = \begin{cases} (p, q, r), & \text{if } x = y, \\ (0, 0, 1), & \text{if } x \neq y, \end{cases}$$

where p, q, r denotes the truth, indeterminacy and false membership value of $x_{p,q,r}$.

The notion of NT-space is defined as follows:

A family τ of N-sets over X is called an [21] neutrosophic topology (in short N-topology) on X if the following axioms hold:

- (i) $0_N, 1_N \in \tau$;
- (ii) $L_1, L_2 \in \tau \Rightarrow L_1 \cap L_2 \in \tau$;
- (iii) $\cup L_i \in \tau$, for every $\{L_i : i \in \Delta\} \subseteq \tau$, where Δ is the support set.

Then, (X, τ) is called an NT-space. Each element of τ is an neutrosophic open set (in short NO-set). If L is an NO-set in (X, τ) , then L^c is called an neutrosophic closed set (in short NC-set).

The notion of NBI-T-space is defined as follows:

Let τ_1 and τ_2 be two different N-topologies on X . Then, (X, τ_1, τ_2) is [19] called an NBI-T-space. An N-set L is called a pairwise NO-set in (X, τ_1, τ_2) , if there exist an NO-set L_1 in τ_1 and an NO-set L_2 in τ_2 such that $L=L_1 \cup L_2$. The complement of L i.e., L^c is called a pairwise neutrosophic closed set (in short pairwise NC-set) in (X, τ_1, τ_2) .

Remark 2.1.[13] In an NBI-T-space (X, τ_1, τ_2) , every τ_i -NO-set is a pairwise τ_{ij} -NO-set.

Remark 2.2. Let G be an N-set over X and (X, τ_1, τ_2) be an NBI-T-space. Then, we shall use the following notations throughout the article:

(i) $N_{cl}^i(G)$ = Neutrosophic closure of G in (X, τ_i) ($i=1, 2$);

(ii) $N_{int}^i(G)$ = Neutrosophic interior of G in (X, τ_i) ($i=1, 2$).

Definition 2.1.[13] Let (X, τ_1, τ_2) be an NBI-T-space. Then, P is called a

(i) τ_i -neutrosophic semi-open set (in short τ_i -NSO-set) if and only if $P \subseteq N_{cl}^i N_{int}^i(P)$;

(ii) τ_i -neutrosophic pre-open set (in short τ_i -NPO-set) if and only if $P \subseteq N_{int}^i N_{cl}^i(P)$;

(iii) τ_i -neutrosophic b -open set (in short τ_i -N-bO-set) if and only if $P \subseteq N_{cl}^i N_{int}^i(P) \cup N_{int}^i N_{cl}^i(P)$.

Remark 2.3.[13] Let (X, τ_1, τ_2) be an NBI-T-space. Then, an N-set P over X is called a τ_i -neutrosophic b -closed set (in short τ_i -N-bC-set) if and only if P^c is a τ_i -N-bO-set.

Proposition 2.1.[13] In an NBI-T-space (X, τ_1, τ_2) , if P is τ_i -NSO-set (τ_i -NPO-set), then P is a τ_i -N-bO-set.

Proposition 2.2.[13] Let (X, τ_1, τ_2) be an NBI-T-space. Then, the union of any two τ_i -N-bO-sets is a τ_i -N-bO-set.

Definition 2.2.[13] Let (X, τ_1, τ_2) be an NBI-T-space. Then, P is called a

(i) τ_{ij} -neutrosophic semi-open set (in short τ_{ij} -NSO-set) if and only if $P \subseteq N_{cl}^i N_{int}^j(P)$;

(ii) τ_{ij} -neutrosophic pre-open set (in short τ_{ij} -NPO-set) if and only if $P \subseteq N_{int}^j N_{cl}^i(P)$;

(iii) τ_{ij} -neutrosophic b -open set (in short τ_{ij} -N-bO-set) if and only if $P \subseteq N_{cl}^i N_{int}^j(P) \cup N_{int}^j N_{cl}^i(P)$.

Remark 2.4.[13] An N-set L over X is called a τ_{ij} -neutrosophic b -closed set (in short τ_{ij} -N-bC-set) if and only if L^c is a τ_{ij} -N-bO-set in (X, τ_1, τ_2) .

Theorem 2.1.[13] Let (X, τ_1, τ_2) be an NBI-T-space. Then, every τ_{ij} -NSO-set (τ_{ij} -NPO-set) is a τ_{ij} -N-bO-set.

Definition 2.3.[13] An N-set L is called a pairwise τ_{ij} -NPO-set (pairwise τ_{ij} -NSO-set) in an NBI-T-space (X, τ_1, τ_2) if $L=K \cup M$, where K is a τ_{ij} -NPO-set (τ_{ij} -NSO-set) and M is a τ_{ji} -NPO-set (τ_{ji} -NSO-set) in (X, τ_1, τ_2) .

Definition 2.4.[13] An N-set L is called a pairwise τ_{ij} -N-bO-set in a NBI-T-space (X, τ_1, τ_2) if $L=K \cup M$, where K is a τ_{ij} -N-bO-set and M is a τ_{ji} -N-bO-set in (X, τ_1, τ_2) . If L is a pairwise τ_{ij} -N-bO-set in (X, τ_1, τ_2) , then L^c is called a pairwise τ_{ij} -neutrosophic- b -closed set (in short pairwise τ_{ij} -N-bC-set) in (X, τ_1, τ_2) .

Lemma 2.1.[13] In an NBI-T-space (X, τ_1, τ_2) , every pairwise τ_{ij} -NPO-set (pairwise τ_{ij} -NSO-set) is a pairwise τ_{ij} -N-bO-set.

Proposition 2.3.[13] Let (X, τ_1, τ_2) be an NBI-T-space. Then, the union of two pairwise τ_{ij} -N-bO-set in (X, τ_1, τ_2) is also a pairwise τ_{ij} -N-bO-set.

Theorem 2.2. Let (X, τ_1, τ_2) be an NBI-T-space. Then, the union of two pairwise τ_{ij} -NSO-set in (X, τ_1, τ_2) is also a pairwise τ_{ij} -NSO-set.

Proof. Let L and M be two pairwise τ_{ij} -NSO-sets in an NBI-T-space (X, τ_1, τ_2) . So, one can write $L=L_1 \cup L_2$ and $M=M_1 \cup M_2$, where L_1, M_1 are τ_{ij} -NSO-sets and L_2, M_2 are τ_{ij} -NSO-sets in (X, τ_1, τ_2) . Since, L_1 and M_1 are τ_{ij} -NSO-sets, so $L_1 \subseteq N_{cl}^i N_{int}^j(L_1)$ and $M_1 \subseteq N_{cl}^i N_{int}^j(M_1)$. Further, Since L_2 and M_2 are τ_{ij} -NSO-sets, so $L_2 \subseteq N_{cl}^j N_{int}^i(L_2)$, $M_2 \subseteq N_{cl}^j N_{int}^i(M_2)$.

Now, $L \cup M = (L_1 \cup L_2) \cup (M_1 \cup M_2) = (L_1 \cup M_1) \cup (L_2 \cup M_2)$.

$$\begin{aligned} \text{Therefore, } L_1 \cup M_1 &\subseteq N_{cl}^i N_{int}^j(L_1) \cup N_{cl}^i N_{int}^j(M_1) \\ &= N_{cl}^i(N_{int}^j(L_1) \cup N_{int}^j(M_1)) \\ &\subseteq N_{cl}^i N_{int}^j(L_1 \cup M_1). \end{aligned}$$

This implies, $L_1 \cup M_1$ is a τ_{ij} -NSO-set in (X, τ_1, τ_2) .

Similarly, it can be established that $L_2 \cup M_2$ is a τ_{ij} -NSO-set in (X, τ_1, τ_2) . Therefore, $L \cup M$ is a pairwise τ_{ij} -NSO-set in (X, τ_1, τ_2) . Hence, the union of two pairwise τ_{ij} -NSO-set in (X, τ_1, τ_2) is again a pairwise τ_{ij} -NSO-set in (X, τ_1, τ_2) .

Theorem 2.4. Let (X, τ_1, τ_2) be an NBI-T-space. Then, the union of two pairwise τ_{ij} -NPO-set in (X, τ_1, τ_2) is a pairwise τ_{ij} -NPO-set.

Proof. Let L and M be two pairwise τ_{ij} -NPO-sets in an NBI-T-space (X, τ_1, τ_2) . So, one can write $L=L_1 \cup L_2$ and $M=M_1 \cup M_2$, where L_1, M_1 are τ_{ij} -NPO-sets and L_2, M_2 are τ_{ij} -NPO-sets in (X, τ_1, τ_2) . Since, L_1 and M_1 are τ_{ij} -NPO-sets, so $L_1 \subseteq N_{int}^j N_{cl}^i(L_1)$ and $M_1 \subseteq N_{int}^j N_{cl}^i(M_1)$. Further, since L_2 and M_2 are τ_{ij} -NPO-sets, so $L_2 \subseteq N_{int}^i N_{cl}^j(L_2)$ and $M_2 \subseteq N_{int}^i N_{cl}^j(M_2)$.

Now, $L \cup M = (L_1 \cup L_2) \cup (M_1 \cup M_2) = (L_1 \cup M_1) \cup (L_2 \cup M_2)$.

$$\begin{aligned} \text{Therefore, } L_1 \cup M_1 &\subseteq N_{int}^j N_{cl}^i(L_1) \cup N_{int}^j N_{cl}^i(M_1) \\ &= N_{int}^j(N_{cl}^i(L_1) \cup N_{cl}^i(M_1)) \\ &\subseteq N_{int}^j N_{cl}^i(L_1 \cup M_1). \end{aligned}$$

This implies, $L_1 \cup M_1$ is a τ_{ij} -NPO-set in (X, τ_1, τ_2) . Similarly, it can be established that $L_2 \cup M_2$ is a τ_{ij} -NPO-set in (X, τ_1, τ_2) . Therefore, $L \cup M$ is a pairwise τ_{ij} -NPO-set in (X, τ_1, τ_2) . Hence, the union of two pairwise τ_{ij} -NPO-sets in (X, τ_1, τ_2) is again a pairwise τ_{ij} -NPO-set.

3. Pairwise b -Continuous Function:

In this section, we procure the notions of pairwise b -continuous functions via neutrosophic bitopological space and formulate some results on it.

Definition 3.1. Let (X, τ_1, τ_2) be an NBI-T-space. Then, the pairwise τ_{ij} -neutrosophic- b -interior (in short $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}$) of an N-set L is the union of all pairwise τ_{ij} - N - b O-sets contained in L , i.e. $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) = \cup\{K: K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{O-set in } X \text{ and } K \subseteq L\}$.

Clearly, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)$ is the largest pairwise τ_{ij} - N - b O-set which contained in L .

Definition 3.2. Let (X, τ_1, τ_2) be an NBI-T-space. Then, the pairwise τ_{ij} -neutrosophic- b -closure (in short $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}$) of an N-set L is the intersection of all pairwise τ_{ij} - N - b C-sets containing L , i.e. $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L) = \cap\{K: K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{C-set in } X \text{ and } L \subseteq K\}$.

Clearly, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)$ is the smallest pairwise τ_{ij} - N - b C-set which containing L .

Theorem 3.1. Let L and K be two neutrosophic subsets of an NBI-T-space (X, τ_1, τ_2) . Then,

- (i) $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(0_N) = 0_N$, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(1_N) = 1_N$;
- (ii) $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) \subseteq L$;
- (iii) $L \subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) \subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M)$;

(iv) $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=L$ if L is a pairwise $\tau_{ij}\text{-}N\text{-}bO$ -set.

Proof. (i) Straight forward.

(ii) By Definition 3.1, we have $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}$. Since, each $K\subseteq L$, so $\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}\subseteq L$, i.e. $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq L$. Therefore, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq L$.

(iii) Let L and M be two neutrosophic subset of an NBI-T-space (X, τ_1, τ_2) such that $L\subseteq M$.

$$\begin{aligned} \text{Now, } P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) &= \cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\} \\ &\subseteq \cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq M\} \quad [\text{since } L\subseteq M] \\ &= P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M) \end{aligned}$$

$$\Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M).$$

Therefore, $L\subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M)$.

(iv) Let L be a pairwise $\tau_{ij}\text{-}N\text{-}bO$ -set in an NBI-T-space (X, τ_1, τ_2) .

Now, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}$. Since, L is a pairwise $\tau_{ij}\text{-}N\text{-}bO$ -set in (X, τ_1, τ_2) , so L is the largest pairwise $\tau_{ij}\text{-}N\text{-}bO$ -set in (X, τ_1, τ_2) , which is contained in L . Therefore, $\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}=L$. This implies, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=L$.

Theorem 3.2. Let L and K be two neutrosophic subsets of an NBI-T-space (X, τ_1, τ_2) . Then,

(i) $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(0_N)=0_N$ & $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(1_N)=1_N$;

(ii) $L\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)$;

(iii) $L\subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M)$;

(iv) $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=L$ if L is a pairwise $\tau_{ij}\text{-}N\text{-}bC$ -set.

Proof. (i) Straightforward.

(ii) It is clear that $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=\cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}$.

Since, each $L\subseteq K$, so $L\subseteq \cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}$, i.e. $L\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)$.

(iii) Let L and M be two neutrosophic subset of an NBI-T-space (X, τ_1, τ_2) such that $L\subseteq M$.

$$\begin{aligned} \text{Now, } P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L) &= \cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\} \\ &\subseteq \cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } M\subseteq K\} \quad [\text{since } L\subseteq M] \\ &= P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M) \end{aligned}$$

$$\Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M).$$

Therefore, $L\subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M)$.

(iv) Let L be a pairwise $\tau_{ij}\text{-}N\text{-}bC$ -set in an NBI-T-space (X, τ_1, τ_2) . Now, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=\cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}$. Since, L is a pairwise $\tau_{ij}\text{-}N\text{-}bC$ -set in a (X, τ_1, τ_2) , so L is the smallest pairwise $\tau_{ij}\text{-}N\text{-}bC$ -set, which contains L . This implies, $\cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}=L$. Therefore, $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=L$.

Proposition 3.3. Let L be a neutrosophic subset of an NBI-T-space (X, τ_1, τ_2) . Then,

(i) $[P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)]^c = P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L^c)$;

(ii) $[P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)]^c = P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L^c)$.

Proof. (i) Let (X, τ_1, τ_2) be an NBI-T-space. Let $L=\{(w, T_L(w), I_L(w), F_L(w)): w\in X\}$ be an neutrosophic subset of (X, τ_1, τ_2) .

$$\begin{aligned} \text{Now, } P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) &= \cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\} \\ &= \{(w, \vee T_{L_p}(w), \wedge I_{L_p}(w), \wedge F_{L_p}(w)): w\in X\}, \end{aligned}$$

where L_p is a pairwise $\tau_{ij}\text{-}N\text{-}bO$ -set in X such that $L_p\subseteq L$, for each $p\in\Delta$.

This implies, $[P-\tau_{ij}\text{-}N_{b\text{-}int}(L)]^c = \{(w, \wedge T_{L_p}(w), \vee I_{L_p}(w), \vee F_{L_p}(w)) : w \in X\}$.

Here $\wedge T_{L_p}(w) \leq T_L(w), I_{L_p}(w) \geq I_L(w), F_{L_p}(w) \geq F_L(w)$, for each $w \in X$.

Therefore, $P-\tau_{ij}\text{-}N_{b\text{-}int}(L^c) = \{(w, \wedge T_{L_p}(w), \vee I_{L_p}(w), \vee F_{L_p}(w)) : w \in X\}$

$$= \cap \{L_p : p \in \Delta \text{ and } L_p \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{C}\text{-set in } X \text{ such that } L^c \subseteq L_p\}$$

Hence, $[P-\tau_{ij}\text{-}N_{b\text{-}int}(L)]^c = P-\tau_{ij}\text{-}N_{b\text{-}cl}(L^c)$.

(ii) Let (X, τ_1, τ_2) be an NBi-T-space and $L = \{(w, T_L(w), I_L(w), F_L(w)) : w \in X\}$ be a N-set over X . Then,

$P-\tau_{ij}\text{-}N_{b\text{-}cl}(L) = \cap \{K : K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{C}\text{-set in } X \text{ and } L \subseteq K\}$

$$= \{(w, \wedge T_{L_p}(w), \vee I_{L_p}(w), \vee F_{L_p}(w)) : w \in X\},$$

where L_p is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$ in X such that $L \subseteq L_p$, for each $p \in \Delta$.

This implies, $[P-\tau_{ij}\text{-}N_{b\text{-}cl}(L)]^c = \{(w, \vee T_{L_p}(w), \wedge I_{L_p}(w), \wedge F_{L_p}(w)) : w \in X\}$.

Here, $\vee T_{L_p}(w) \geq T_L(w), \wedge I_{L_p}(w) \leq I_L(w), \wedge F_{L_p}(w) \leq F_L(w)$, for each $w \in X$.

Therefore, $P-\tau_{ij}\text{-}N_{b\text{-}int}(L^c) = \{(w, \vee T_{L_p}(w), \wedge I_{L_p}(w), \wedge F_{L_p}(w)) : w \in X\}$

$$= \cup \{L_p : p \in \Delta \text{ and } L_p \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{O}\text{-set in } X \text{ such that } L_p \subseteq L^c\}.$$

Hence, $[P-\tau_{ij}\text{-}N_{b\text{-}cl}(L)]^c = P-\tau_{ij}\text{-}N_{b\text{-}int}(L^c)$.

Theorem 3.1. Let (X, τ_1, τ_2) be an NBi-T-space. Then, the neutrosophic null set (0_N) and the neutrosophic whole set (1_N) are both $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ and $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$.

Proof. Let (X, τ_1, τ_2) be an NBi-T-space. Now, $N_{cl}^i N_{int}^j(0_N) \cup N_{int}^j N_{cl}^i(0_N) = N_{cl}^i(0_N) \cup N_{int}^j(0_N) = 0_N \cup 0_N = 0_N$. Therefore, $0_N \subseteq 0_N = N_{cl}^i N_{int}^j(0_N) \cup N_{int}^j N_{cl}^i(0_N)$. Hence, the neutrosophic null set (0_N) is a $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$.

Similarly, it can be established that the neutrosophic null set (0_N) is a $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$.

Further, one can show that the neutrosophic whole set (1_N) are both $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ and $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$.

Theorem 3.2. In an NBi-T-space (X, τ_1, τ_2) , every $\tau_i\text{-}N\text{O}\text{-set}$ is a $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$.

Proof. Let L be a $\tau_i\text{-}N\text{O}\text{-set}$ in an NBi-T-space (X, τ_1, τ_2) . Therefore, $N_{int}^i(L) = L$. Now, $L \subseteq N_{cl}^j(L) = N_{cl}^j N_{int}^i(L)$. This implies, $L \subseteq N_{cl}^j N_{int}^i(L) \cup N_{int}^i N_{cl}^j(L)$. Hence, L is a $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ in (X, τ_1, τ_2) .

Theorem 3.3. In an NBi-T-space (X, τ_1, τ_2) ,

- (i) every $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$;
- (ii) every $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ is a pairwise $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$;
- (iii) every $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$ is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$;
- (iv) every $\tau_{ji}\text{-}N\text{-}b\text{C}\text{-set}$ is a pairwise $\tau_{ji}\text{-}N\text{-}b\text{C}\text{-set}$.

Proof. (i) Let L be a $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ in an NBi-T-space (X, τ_1, τ_2) . Then, L can be expressed as $L = L \cup 0_N$, where L is a $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ and 0_N is a $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ in (X, τ_1, τ_2) . This implies, L is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ in (X, τ_1, τ_2) .

(ii) Straightforward.

(iii) Let L be a $\tau_{ij}\text{-}N\text{C}\text{-set}$ in an NBi-T-space (X, τ_1, τ_2) . Then, L can be expressed as $L = L \cap 1_N$, where L is a $\tau_{ij}\text{-}N\text{C}\text{-set}$ and 1_N is a $\tau_{ji}\text{-}N\text{C}\text{-set}$ in (X, τ_1, τ_2) . This implies, L is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$ in (X, τ_1, τ_2) .

(iv) Straightforward.

Theorem 3.4. In an NBi-T-space (X, τ_1, τ_2) , every $\tau_i\text{-}N\text{O}\text{-set}$ is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$.

Proof. Let L be a $\tau_i\text{-}N\text{O}\text{-set}$ in an NBi-T-space (X, τ_1, τ_2) . By Theorem 3.2., it is clear that L is a $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$. Further, by Theorem 3.3., it is clear that L is a pairwise $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$.

Theorem 3.5. Let (X, τ_1, τ_2) be an NBi-T-space. Then, 0_N and 1_N are both pairwise $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ and pairwise $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$.

Proof. Let (X, τ_1, τ_2) be an NBi-T-space. One can write $0_N = A \cup B$, where $A = 0_N$ is a τ_{ij} -N-*b*O-set and $B = 0_N$ is a τ_{ji} -N-*b*O-set in (X, τ_1, τ_2) . This implies, 0_N is a pairwise τ_{ij} -N-*b*O-set in (X, τ_1, τ_2) .

Similarly, it can be established that 0_N is a pairwise τ_{ji} -N-*b*O-set in (X, τ_1, τ_2) .

Again, one can write $1_N = L \cup M$, where $L = 1_N$ is a τ_{ij} -N-*b*O-set and $M = 1_N$ is a τ_{ji} -N-*b*O-set in (X, τ_1, τ_2) . This implies, 1_N is a pairwise τ_{ij} -N-*b*O-set in (X, τ_1, τ_2) .

Similarly, it can be also established that 1_N is a pairwise τ_{ji} -N-*b*O-set in (X, τ_1, τ_2) .

Theorem 3.6. Let (X, τ_1, τ_2) be an NBi-T-space. Then, both 0_N and 1_N are pairwise τ_{ij} -N-*b*C-set and pairwise τ_{ji} -N-*b*C-set.

Proof. By Theorem 3.5, it is clear that 0_N is both pairwise τ_{ij} -N-*b*O-set and pairwise τ_{ji} -N-*b*O-set. Hence, its complement 1_N is both pairwise τ_{ij} -N-*b*C-set and pairwise τ_{ji} -N-*b*C-set.

Similarly, from Theorem 3.5, it is clear that 1_N is both pairwise τ_{ij} -N-*b*O-set and pairwise τ_{ji} -N-*b*O-set. Hence, its complement 0_N is both pairwise τ_{ij} -N-*b*C-set and pairwise τ_{ji} -N-*b*C-set.

Remark 3.1. Throughout the article, we denote τ_{ij}^b as a collection of all pairwise τ_{ij} -N-*b*O-sets and τ_{ij}^c as a collection of all pairwise τ_{ij} -N-*b*C-sets in (X, τ_1, τ_2) . The collection τ_{ij}^b forms a neutrosophic supra topology on X .

Definition 3.3. Let (X, τ_1, τ_2) and (Y, δ_1, δ_2) be two NBi-T-spaces. Then, an one to one and onto mapping $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is called a

- (i) pairwise neutrosophic semi continuous mapping (in short P-NS-C-mapping) if and only if $\xi^{-1}(L)$ is a τ_i -NSO-set in X , whenever L is a pairwise δ_{ij} -NO-set in Y .
- (ii) pairwise neutrosophic pre continuous mapping (in short P-NP-C-mapping) if and only if $\xi^{-1}(L)$ is a τ_i -NPO-set in X , whenever L is a pairwise δ_{ij} -NO-set in Y .
- (iii) pairwise neutrosophic continuous mapping (in short P-N-C-mapping) if and only if $\xi^{-1}(L)$ is a τ_i -NO-set in X , whenever L is a pairwise δ_{ij} -NO-set in Y .
- (iv) pairwise neutrosophic *b*-continuous mapping (in short P-N-*b*-C-mapping) if and only if $\xi^{-1}(L)$ is a τ_i -N-*b*O-set in X , whenever L is a pairwise δ_{ij} -NO-set in Y .

Theorem 3.7. Let (X, τ_1, τ_2) and (Y, δ_1, δ_2) be two NBi-T-spaces. Then, every P-N-C-mapping from (X, τ_1, τ_2) to (Y, δ_1, δ_2) is a P-NP-C-mapping (P-NS-C-mapping).

Proof. Let L be a pairwise δ_{ij} -NO-set in (Y, δ_1, δ_2) . Since, $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N-C-mapping from (X, τ_1, τ_2) to (Y, δ_1, δ_2) , so $\xi^{-1}(L)$ is a τ_i -NO-set in (X, τ_1, τ_2) . It is known that every τ_i -NO-set is a τ_i -NPO-set (τ_i -NSO-set). Therefore, $\xi^{-1}(L)$ is a τ_i -NPO-set (τ_i -NSO-set) in (X, τ_1, τ_2) . Hence, $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-NP-C-mapping (P-NS-C-mapping).

Theorem 3.8. Let (X, τ_1, τ_2) and (Y, δ_1, δ_2) be two NBi-T-spaces. Then, every P-NS-C-mapping (P-NP-C-mapping) from (X, τ_1, τ_2) to (Y, δ_1, δ_2) is a P-N-*b*-C-mapping.

Proof. Let L be a pairwise δ_{ij} -NO-set in (Y, δ_1, δ_2) . Since, $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-NS-C-mapping (P-NP-C-mapping) from (X, τ_1, τ_2) to (Y, δ_1, δ_2) , so $\xi^{-1}(L)$ is a τ_i -NSO-set (τ_i -NPO-set) in (X, τ_1, τ_2) . It is known that, every τ_i -NSO-set (τ_i -NPO-set) is a τ_i -N-*b*O-set. Therefore, $\xi^{-1}(L)$ is a τ_i -N-*b*O-set in (X, τ_1, τ_2) . Hence, $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N-*b*-C-mapping.

Theorem 3.9. Let (X, τ_1, τ_2) and (Y, δ_1, δ_2) be two NBi-T-spaces. Then, every P-N-C-mapping from (X, τ_1, τ_2) to (Y, δ_1, δ_2) is a P-N-*b*-C-mapping.

Proof. Let L be a pairwise δ_{ij} -NO-set in (Y, δ_1, δ_2) . Since, $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N-C-mapping from (X, τ_1, τ_2) to (Y, δ_1, δ_2) , so $\xi^{-1}(L)$ is a τ_i -NO-set in (X, τ_1, τ_2) . It is known that, every τ_i -NO-set is a

τ_i -N- b -O-set. Therefore, $\xi^{-1}(L)$ is a τ_i -N- b -O-set in (X, τ_1, τ_2) . Hence, $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a p-N- b -C-mapping.

Theorem 3.10. If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ and $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$ be two P-N-C-mapping, then the composition mapping $\chi \circ \xi: (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is also a P-N-C-mapping.

Proof. Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ and $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$ be two P-N-C-mappings. Let L be a pairwise θ_{ij} -NO-set in (Z, θ_1, θ_2) . Since, $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$ is a P-N-C-mapping, so $\chi^{-1}(L)$ is a δ_i -NO-set in Y . Since, $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N-C-mapping, so $\xi^{-1}(\chi^{-1}(L)) = (\chi \circ \xi)^{-1}(L)$ is a τ_i -NO-set in X .

Theorem 3.11. If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be an one to one and onto mapping between two NBi-T-spaces, then the following two are equivalent:

(i) ξ is a P-N- b -C-mapping.

(ii) $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(A))$, for every neutrosophic subset A of Y .

Proof. (i) \Rightarrow (ii)

Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a P-N- b -C-mapping. Let A be an neutrosophic subset of Y . Here, $P\text{-}\delta_{ij}\text{-}N_{int}(A)$ is a pairwise δ_{ij} -NO-set in Y and $P\text{-}\delta_{ij}\text{-}N_{int}(A) \subseteq A$. This implies, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq \xi^{-1}(A)$. By the hypothesis, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A))$ is a τ_i -N- b -O-set in X . Therefore, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A))$ is a τ_i -N- b -O-set in X such that $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq \xi^{-1}(A)$. It is known that $\tau_i\text{-}N_{b-int}(\xi^{-1}(A))$ is the largest τ_i -N- b -O-set in X , which is contained in $\xi^{-1}(A)$. Hence, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(A))$.

(ii) \Rightarrow (i)

Let A be a pairwise δ_{ij} -NO-set in (Y, δ_1, δ_2) . Therefore, $P\text{-}\delta_{ij}\text{-}N_{int}(A) = A$. By hypothesis, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(A))$. This implies, $\xi^{-1}(A) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(A))$. It is known that $\tau_i\text{-}N_{b-int}(\xi^{-1}(A)) \subseteq \xi^{-1}(A)$. Therefore, $\tau_i\text{-}N_{b-int}(\xi^{-1}(A)) = \xi^{-1}(A)$. Hence, $\xi^{-1}(A)$ is a τ_i -N- b -O-set in (X, τ_1, τ_2) . Therefore, ξ is a P-N- b -C-mapping from an NBi-T-space (X, τ_1, τ_2) to another NBi-T-space (Y, δ_1, δ_2) .

Theorem 3.12. An one to one and onto mapping $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N- b -C-mapping if and only if $P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi} (\tau_i\text{-}N_{b-int}(A))$, for every N-set A over X and $i, j = 1, 2$, and $i \neq j$.

Proof. Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a P-N- b -C-mapping. Let A be an N-set over X . Then, $\xi(A)$ is also an N-set over Y . By Theorem 3.11, we have $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A))) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(\xi(A)))$. This implies, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A))) \subseteq_{\tau_i} N_{b-int}(A)$. Hence, $P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi} (\tau_i\text{-}N_{b-int}(A))$. Therefore, $P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi} (\tau_i\text{-}N_{b-int}(A))$, for every N-set A over X and $i, j = 1, 2$; and $i \neq j$.

Conversely, let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a mapping between two NBi-T-spaces such that

$$P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi} (\tau_i\text{-}N_{b-int}(A)) \tag{1}$$

for every N-set A over X and $i, j = 1, 2$; and $i \neq j$.

Let A be an N-set over Y . Then, $\xi^{-1}(A)$ is an N-set over X . By putting $A = \xi^{-1}(A)$ in eq. (1), we have,

$$P\text{-}\delta_{ij}\text{-}N_{int}(\xi(\xi^{-1}(A))) \subseteq_{\xi} (\tau_i\text{-}N_{b-int}(\xi^{-1}(A)))$$

$$\Rightarrow P\text{-}\delta_{ij}\text{-}N_{int}(A) \subseteq_{\xi} (\tau_i\text{-}N_{b-int}(\xi^{-1}(A)))$$

$$\Rightarrow \xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(A)).$$

Therefore, $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i} N_{b-int}(\xi^{-1}(A))$, for every N-set A of Y . Hence, by Theorem 3.11., the mapping $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N- b -C-mapping.

Corollary 3.1. If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is an one to one and onto mapping from an NBi-T-space (X, τ_1, τ_2) to another NBi-T-space (Y, δ_1, δ_2) , then the following two are equivalent:

(i) ξ is a P-N-C-mapping.

(ii) $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(Q)) \subseteq_{\tau_i} N_{int}(\xi^{-1}(Q))$, for every N-set Q over Y .

Definition 3.4. Let (X, τ_1, τ_2) be an NBi-T-space. Let $x_{a,b,c}$ be an N-point in X . Then, an N-set Q over X is called a pairwise τ_{ij} -neutrosophic b -neighbourhood (in short P- τ_{ij} -N- b -nbd) of $x_{a,b,c}$, if there exist a pairwise τ_{ij} -N- b O-set U such that $x_{a,b,c} \in U \subseteq Q$.

Theorem 3.13. Let (X, τ_1, τ_2) be an NBi-T-space. An N-set Q over X is a pairwise τ_{ij} -N- b O-set if and only if Q is a P- τ_{ij} -N- b -nbd of all of its N-points.

Proof. Let Q be a pairwise τ_{ij} -N- b O-set in an NBi-T-space (X, τ_1, τ_2) . Let $x_{a,b,c}$ be an N-point in X such that $x_{a,b,c} \in Q$. Therefore, $x_{a,b,c} \in Q \subseteq Q$. This implies, Q is a P- τ_{ij} -N- b -nbd of $x_{a,b,c}$. Hence, Q is the P- τ_{ij} -N- b -nbd of all of its N-points.

Conversely, let Q be a P- τ_{ij} -N- b -nbd of all of its N-points. Assume that $x_{a,b,c}$ be an N-point in X , such that $x_{a,b,c} \in Q$. Therefore, there exist a pairwise τ_{ij} -N- b O-set G such that $x_{a,b,c} \in G \subseteq Q$.

Now, $Q = \cup_{x_{a,b,c} \in Q} x_{a,b,c} \subseteq \cup_{x_{a,b,c} \in Q} G \subseteq \cup_{x_{a,b,c} \in Q} Q = Q$. This implies, $Q = \cup_{x_{a,b,c} \in Q} G$, which is a pairwise τ_{ij} -N- b O-set. Therefore, Q is a pairwise τ_{ij} -N- b O-set in (X, τ_1, τ_2) .

Theorem 3.14. An one to one and onto mapping $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N- b -C-mapping if and only if for every N-point $x_{a,b,c} \in Y$ and for any P- δ_{ij} -N- b -nbd V of $x_{a,b,c}$ in Y , there exist a τ_i -neutrosophic- b -neighbourhood (in short τ_i -N- b -nbd) U of $\xi^{-1}(x_{a,b,c})$ in X such that $U \subseteq \xi^{-1}(V)$.

Proof. Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a P-N- b -C-mapping. Let $x_{a,b,c}$ be an N-point in Y and V be a P- δ_{ij} -N- b -nbd of $x_{a,b,c}$. Then, there exist a pairwise δ_{ij} -NO-set G in Y such that $x_{a,b,c} \in G \subseteq V$. This implies, $\xi^{-1}(x_{a,b,c}) \in \xi^{-1}(G) \subseteq \xi^{-1}(V)$. Since, $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N- b -C-mapping, so $\xi^{-1}(G)$ is a τ_i -N- b O-set in X . By taking $U = \xi^{-1}(G)$, we see that U is a τ_i -N- b O-set in X such that $\xi^{-1}(x_{a,b,c}) \in U \subseteq \xi^{-1}(V)$. Hence, $U = \xi^{-1}(G)$ is a τ_i -N- b -nbd of $\xi^{-1}(x_{a,b,c})$ and $U \subseteq \xi^{-1}(V)$.

Conversely, let for every N-point $x_{a,b,c} \in Y$ and for any P- δ_{ij} -N- b -nbd V of $x_{a,b,c}$ in Y , there exist a τ_i -N- b -nbd U of $\xi^{-1}(x_{a,b,c})$ in X such that $U \subseteq \xi^{-1}(V)$. Let G be a pairwise δ_{ij} -NO-set in Y and $x_{a,b,c} \in G$. By Theorem 3.13., G is a P- δ_{ij} -N- b -nbd of $x_{a,b,c}$. By hypothesis, there exists a τ_i -N- b -nbd H of $\xi^{-1}(x_{a,b,c}) \in X$ such that $\xi^{-1}(x_{a,b,c}) \in H \subseteq \xi^{-1}(G)$. This implies, $\xi^{-1}(G)$ is the τ_i -N- b -nbd of each of its N-points. Therefore, $\xi^{-1}(G)$ is a τ_i -N- b O-set in X . Hence, $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N- b -C-mapping.

Theorem 3.15. If $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a P-N- b -C-mapping and $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$ be a P-N-C-mapping, then the composition mapping $\chi \circ \xi: (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is a P-N- b -C-mapping.

Proof. Let $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be a P-N- b -C-mapping and $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$ be a P-N-C-mapping. Let L be a pairwise θ_{ij} -NO-set in (Z, θ_1, θ_2) . Since, $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$ is a P-N-C-mapping, so $\chi^{-1}(L)$ is a δ_i -NO-set in Y . Now, by Lemma 2.1., it is clear that $\chi^{-1}(L)$ is a pairwise δ_{ij} -NO-set in (Y, δ_1, δ_2) . Since, $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is a P-N- b -C-mapping, so $\xi^{-1}(\chi^{-1}(L)) = (\chi \circ \xi)^{-1}(L)$ is a τ_i -NO-set in X . Since, every τ_i -NO-set is a τ_i -N- b O-set, so $(\chi \circ \xi)^{-1}(L)$ is a τ_i -N- b O-set in X . Hence, $\chi \circ \xi: (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$ is a P-N- b -C-mapping.

4. Conclusion

In this article, we introduce the notion of pairwise neutrosophic- b -interior, pairwise neutrosophic- b -closure, pairwise neutrosophic b -continuous mapping, we prove some propositions and theorems on NBi-T-spaces. In the future, we hope that based on these notions in NBi-T-spaces, many new investigations can be carried out.

Conflict of Interest: The authors declare that they have no conflict of interest.

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References:

- [1]. Andrijevic, D. (1996). On b -open sets. *Matematički Vesnik*, 48, 59-64.
- [2]. Arokiarani, I., Dhavaseelan, R., Jafari, S., & Parimala, M. (2017). On some new notations and functions in neutrosophic topological spaces. *Neutrosophic Sets and Systems*, 16, 16-19.
- [3]. Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87-96.
- [4]. Chang, C.L. (1968). Fuzzy Topological Spaces. *Journal of Mathematical Analysis and Applications*, 24, 182-190.
- [5]. Das, S., Das, R., & Granados, C. (2021). Topology on Quadripartitioned Neutrosophic Sets. *Neutrosophic Sets and Systems*. (In Press).
- [6]. Das, S., Das, R., Granados, C., & Mukherjee, A. (2021). Pentapartitioned Neutrosophic Q -Ideals of Q -Algebra. *Neutrosophic Sets and Systems*, 41, 52-63.
- [7]. Das, S., Das, R., & Tripathy, B.C. (2020). Multi-criteria group decision making model using single-valued neutrosophic set. *LogForum*, 16 (3), 421-429.
- [8]. Das, S., & Pramanik, S. (2020). Generalized neutrosophic b -open sets in neutrosophic topological space, *Neutrosophic Sets and Systems*, 35, 522-530.
- [9]. Das, S., & Pramanik, S. (2020). Neutrosophic Φ -open sets and neutrosophic Φ -continuous functions. *Neutrosophic Sets and Systems*, 38, 355-367.
- [10]. Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. *Neutrosophic Sets and Systems*, 38, 235-243.
- [11]. Das, R., Smarandache, F., & Tripathy, B.C. (2020). Neutrosophic fuzzy matrices and some algebraic operation. *Neutrosophic Sets and Systems*, 32, 401-409.
- [12]. Das, R., & Tripathy B.C. (2020). Neutrosophic multiset topological space. *Neutrosophic Sets and Systems*, 35, 142-152.
- [13]. Das, S., & Tripathy, B.C. (2020). Pairwise neutrosophic- b -open set in neutrosophic bitopological spaces. *Neutrosophic Sets and Systems*, 38, 135-144.
- [14]. Das, S., & Tripathy, B.C. (2021). Neutrosophic simply b -open set in neutrosophic topological spaces. *Iraqi Journal of Science*, In Press.
- [15]. Dutta, A., & Tripathy, B.C. (2017). On fuzzy b - θ open sets in fuzzy topological space. *Journal of Intelligent and Fuzzy Systems*, 32 (1), 137-139.
- [16]. Ebenanjar, E., Immaculate, J., & Wilfred, C.B. (2018). On neutrosophic b -open sets in neutrosophic topological space. *Journal of Physics Conference Series*, 1139 (1), 012062.
- [17]. Kelly, J.C. (1963). Bitopological Spaces. *Proceedings of the London Mathematical Society*, 3 (1), 71-89.
- [18]. Mukherjee, A., & Das, R. (2020). Neutrosophic Bipolar Vague Soft Set and Its Application to Decision Making Problems. *Neutrosophic Sets and Systems*, 32, 410-424.
- [19]. Ozturk, T.Y., & Ozkan, A. (2019). Neutrosophic Bitopological Spaces. *Neutrosophic Sets and Systems*, 30, 88-97.

- [20]. Rao, V.V., & Srinivasa, R. (2017). Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology. *International Journal of Chemical Technology Research*, 10 (10), 449-458.
- [21]. Salama, A.A., & Alblowi, S.A. (2012a). Neutrosophic set and neutrosophic topological space. *ISOR Journal of Mathematics*, 3 (4), 31-35.
- [22]. Salama, A.A., & Alblowi, S.A. (2012b). Generalized Neutrosophic Set and Generalized Neutrosophic Topological Space. *Computer Science and Engineering*, 2, 129-132.
- [23]. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. *Rehoboth: American Research Press*.
- [24]. Smarandache, F. (2005). Neutrosophic set: a generalization of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 24, 287-297.
- [25]. Tripathy, B.C., & Debnath, S. (2015). On fuzzy b -locally open sets in bitopological spaces. *Songklanakarin Journal of Science and Technology*, 37 (1), 93-96.
- [26]. Tripathy, B.C., & Sarma, D.J. (2011). On b -Locally Open Sets in Bitopological Spaces. *Kyungpook Mathematical Journal*, 51, 429-433.
- [27]. Tripathy, B.C., & Sarma, D.J. (2012). On pairwise b -locally open and pairwise b -locally closed functions in bitopological spaces. *Tamkang Journal of Mathematics*, 43 (4), 533-539.
- [28]. Tripathy, B.C., & Sarma, D.J. (2013). On weakly b -continuous functions in bitopological spaces. *ActaScientiarum Technology*, 35 (3), 521-525.
- [29]. Tripathy, B.C., & Sarma, D.J. (2014). Generalized b -closed sets in Ideal bitopological spaces. *Proyecciones Journal of Mathematics*, 33 (3), 315-324.
- [30]. Tripathy, B.C., & Sarma, D.J. (2017). Pairwise generalized b - R_0 spaces in bitopological spaces. *Proyecciones Journal of Mathematics*, 36 (4), 589-600.
- [31]. Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8, 338-353.

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