Tangent Similarity Measure Based MADM-Strategy under SVPNS-Environment

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Abstract: The main focus of this article is to procure a new similarity measure namely tangent similarity measure for single valued pentapartitioned neutrosophic sets (SVPNS). We formulate some results on tangent similarity measure of similarities between two SVPNSs. Then, we develop a SVPNS-MADM (SVPNS-Multi-Attribute-Decision-Making) model under the SVPNS environment based on the tangent similarity measure. Further, we validate our proposed SVPNS-MADM model by giving a numerical example.

Keywords: MADM; Pentapartitioned Neutrosophic Set; Tangent Similarity.

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1. Introduction:

In the year 1965, Zadeh [37] introduced the concept of fuzzy set (FS) theory to deal with the uncertainty events. Afterwards, Atanassov [1] exended the concept of FS by introducing the notions of intuitionistic fuzzy set (IFS). In the year 2011, Pramanik and Mukhopadhyaya [28] proposed a MADM approach based on grey relational analysis under intuitionistic fuzzy set-environment. In the year 2014, Mondal et al. [20] developed a MADM-strategy to select the quality brick under intuitionistic fuzzy environment. In the year 1998, Smarandache [30] grounded the idea of neutrosophic set (NS) by extending the notion of fuzzy set (FS) and intuitionistic fuzzy set (IFS) to deal with the uncertainty events having indeterminacy. In an NS, every element has three independent components namely truth, indeterminacy, and false membership values. Thereafter, Salama and Alblowi [29] applied the notions of topology on NSs and introduced the concept of neutrosophic topological space (NTS). Later on, many researchers around the globe gives their contributions ([6], [7], [12], etc.] in the area of NTS. Indeterminacy membership plays an important role in multi-attribute-decision-making problems of real world. In the year 2010, Wang et al. [31]

introduced the idea of SVNS (single valued neutrosophic set), which is a subclass of NS. One can represent indeterminate and incomplete information which makes trouble to take decision/selection in the real world by using a SVNS. The SVNS is more capable to deal with this situation. Later on, Mondal et al. [21] studied the role of neutrosophic logic in data mining process in the year 2016. Afterwards, many researchers of different countries studied SVNS for the formation of MADM model/algorithm in different branches of real-world such as medical diagnosis, educational problem, social problems, decision-making problems, conflict resolution, image processing, etc. Thereafter, many researchers (Biswas et al. [2], Das et al. [5], Mondal & Pramanik [16], Pramanik et al. [23], Pramanik et al. [24], etc.) used SVNS in their MCDM (multi-criteria-decision-making) models. Later on, Ye [32-35], Ye & zhang [36], Mondal & Pramanik [17], Mondal et al. [18], Mondal et al. [19] etc. established several MADM models based on similarity measures under the SVNS-environment / interval valued neutrosophic set-environment / rough neutrosophic set-environment. Pramanik et al. [25] proposed a MADM-approach under the single valued neutrosophic soft expert set environment in the year 2015. In the year 2020, Mukherjee and Das [22] presented the notions of neutrosophic bipolar vague soft set and proposed a MADM-strategy.

In the year 2020, Mallick and Pramanik [15] grounded the notions of single valued pentapartitioned neutrosophic set (SVPNS) by splitting indeterminacy into three independent components namely contradiction, ignorance, and unknown-membership. Later on, Das et al. [4] introduced the notions of pentapartitioned neutrosophic Q-ideals of Q-algebra in the year 2021. Recently, Das and Tripathy [13] applied the idea of topology on SVPNSs and defined pentapartitioned neutrosophic topological space.

In this article, we proposed a SVPNS-MADM model based on tangent similarity measure under the SVPNS environment. Also, we validate our model by a numerical example.

The rest of the paper has been split into following sections:

Section 2 recalls some relevant definitions, properties, and operations on SVPNSs. Section 3 presents the tangent similarity measure of similarities between two SVPNSs. We formulate some results on tangent similarity measure under SVPNS environment. In section 4, we present a SVPNS-MADM strategy based on tangent similarity measure under the SVPNS environment. In section 5, we have validated our proposed MADM model by a real world numerical example. Section 6 represents the concluding remarks of our work done in this study.

2. Some Relevant Definitions:

In this section, we give some basic definitions and results those are relevant to the main results of this article.

Definition 2.1. [15] Let L be a fixed set. Then P, a SVPNS over L is denoted as follows:

$P=\{(\kappa,\Delta_P(\kappa),\Gamma_P(\kappa),\Pi_P(\kappa),\Omega_P(\kappa),\Phi_P(\kappa)): u \in L\}$, where Δ_P, Γ_P, Π_P, Ω_P, $\Phi_P : L\rightarrow]0,1[$ denotes the truth, contradiction, ignorance, unknown and falsity membership functions respectively. So

$$0 \le \Delta_P(\kappa)+\Gamma_P(\kappa)+\Pi_P(\kappa)+\Omega_P(\kappa)+\Phi_P(\kappa) \le 5.$$

Definition 2.2. [15] The absolute SVPNS (1_{PN}) and the null PNS (0_{PN}) over L are defined by

(i) $1_{PN} = \{(\kappa,1,1,0,0,0): \kappa \in L\}$;

(ii) $0_{PN} = \{(\kappa,0,0,1,1,1): \kappa \in L\}$.

Definition 2.3.[15] Let $X=\{(\kappa,\Delta_X(\kappa),\Gamma_X(\kappa),\Pi_X(\kappa),\Omega_X(\kappa),\Phi_X(\kappa)): q \in L\}$ and $Y = \{(\kappa,\Delta_Y(\kappa),\Gamma_Y(\kappa),\Pi_Y(\kappa),\Omega_Y(\kappa),\Phi_Y(\kappa)): \kappa \in L\}$ be two SVPNSs over L. Then, $X \subseteq Y \Leftrightarrow \Delta_X(\kappa) \leq \Delta_Y(\kappa)$, $\Gamma_X(\kappa) \leq \Gamma_Y(\kappa)$, $\Pi_X(\kappa) \geq \Pi_Y(\kappa)$, $\Omega_X(\kappa) \geq \Omega_Y(\kappa)$, $\Phi_X(\kappa) \geq \Phi_Y(\kappa)$, for all $\kappa \in L$.

Definition 2.4.[15] Let $X=\{(\kappa,\Delta_X(\kappa),\Gamma_X(\kappa),\Pi_X(\kappa),\Omega_X(\kappa),\Phi_X(\kappa)): \kappa \in L\}$ and $Y=\{(\kappa,\Delta_Y(\kappa),\Gamma_Y(\kappa),\Pi_Y(\kappa),\Omega_Y(\kappa))): \kappa \in L\}$ be two SVPNSs over L. Then, $X \cup Y=\{(\kappa,$ max$\{\Delta_X(\kappa),\Delta_Y(\kappa)\}$, max$\{\Gamma_X(\kappa),\Gamma_Y(\kappa)\}$, min$\{\Pi_X(\kappa),\Pi_X(\kappa)\}$, min$\{\Omega_X(\kappa),\Omega_X(\kappa)\}$, min$\{\Phi_X(\kappa),\Phi_X(\kappa)\}): \kappa \in L\}$.

Definition 2.5.[15] Let $X=\{(\kappa,\Delta_X(\kappa),\Gamma_X(\kappa),\Pi_X(\kappa),\Omega_X(\kappa),\Phi_X(\kappa)): \kappa \in L\}$ and $Y=\{(\kappa,\Delta_Y(\kappa),\Gamma_Y(\kappa),\Pi_Y(\kappa),\Omega_Y(\kappa),\Phi_Y(\kappa)): \kappa \in L\}$ be two SVPNSs over L. Then, $X \cap Y=\{(\kappa,$ min$\{\Delta_X(\kappa),\Delta_Y(\kappa)\}$, min$\{\Gamma_X(\kappa),\Gamma_Y(\kappa)\}$, max$\{\Pi_X(\kappa),\Pi_X(\kappa)\}$, max$\{\Omega_X(\kappa),\Omega_X(\kappa)\}$, max$\{\Phi_X(\kappa),\Phi_X(\kappa)\}): \kappa \in L\}$.

Definition 2.6.[15] Let $X=\{(\kappa,\Delta_X(\kappa),\Gamma_X(\kappa),\Pi_X(\kappa),\Omega_X(\kappa),\Phi_X(\kappa)): \kappa \in W\}$ be a SVPNS over L. Then, the complement of X is defined by $X^c=\{(\kappa,\Phi_X(\kappa),\Omega_X(\kappa),1-\Pi_X(\kappa),\Gamma_X(\kappa),\Delta_X(\kappa)): \kappa \in L\}$.

3. Tangent Similarity Measure under SVPNS Environment:

Definition 3.1. Suppose that $Y=\{(\kappa,\Delta_Y(\kappa),\Gamma_Y(\kappa),\Pi_Y(\kappa),\Omega_Y(\kappa),\Phi_Y(\kappa)): \kappa \in L\}$ and $R=\{(\kappa,\Delta_R(\kappa),\Gamma_R(\kappa),\Pi_R(\kappa),\Omega_R(\kappa),\Phi_R(\kappa)): \kappa \in L\}$ be two SVPNSs over a fixed set L. Then, the tangent similarity measure of similarities between Y and R is defined by:

$$T_{SVPNSM}(Y, R) = 1 - \frac{1}{n}\sum_{x \in L} tan[\frac{\pi}{12}[|\Delta_Y(\kappa)-\Delta_R(\kappa)|+|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|+|\Pi_Y(\kappa)-\Pi_R(\kappa)|+|\Omega_Y(\kappa)-\Omega_R(\kappa)|$$

$$+|\Phi_Y(\kappa)-\Phi_R(\kappa)|]].$$

(1)

Theorem 3.1. Suppose that $T_{SVPNSM}(Y, R)$ be the tangent similarity measure of similarities between the SVPNSs Y and R. Then, the following properties hold:

(i) $0 \leq T_{SVPNSM}(Y, R) \leq 1$;

(ii) $T_{SVPNSM}(Y, R) = T_{SVPNSM}(R, Y)$;

(iii) $T_{SVPNSM}(Y, R)=1$ if and only if $Y=R$.

Proof. (i) It is known that, the tangent function is monotonic increasing in the interval $[0, \pi/4]$. It is also lies in the interval $[0, 1]$. Therefore, $0 \leq T_{SVPNSM}(Y, R) \leq 1$.

(ii) From Definition 3.1., we have,

$T_{SVPNSM}(Y, R)$

$=1-\frac{1}{n}\sum_{\kappa \in L} tan[\frac{\pi}{12}[|\Delta_Y(\kappa)-\Delta_R(\kappa)|+|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|+|\Pi_Y(\kappa)-\Pi_R(\kappa)|+|\Omega_Y(\kappa)-\Omega_R(\kappa)|+|\Phi_Y(\kappa)-\Phi_R(\kappa)|]]$.

$=1-\frac{1}{n}\sum_{\kappa \in L} tan[\frac{\pi}{12}[|\Delta_R(\kappa)-\Delta_Y(\kappa)|+|\Gamma_R(\kappa)-\Gamma_Y(\kappa)|+|\Pi_R(\kappa)-\Pi_Y(\kappa)|+|\Omega_R(\kappa)-\Omega_Y(\kappa)|+|\Phi_R(\kappa)-\Phi_Y(\kappa)|]]$

$= T_{SVPNSM}(R, Y)$.

Therefore, $T_{SVPNSM}(Y, R) = T_{SVPNSM}(R, Y)$.

(iii) Let Y and R be two SVPNSs over L such that $Y=R$. Therefore, $\Delta_Y(\kappa)=\Delta_R(\kappa)$, $\Gamma_Y(\kappa)=\Gamma_R(\kappa)$, $\Pi_Y(\kappa)=\Pi_R(\kappa)$, $\Omega_Y(\kappa)=\Omega_R(\kappa)$, and $\Phi_Y(\kappa)=\Phi_R(\kappa)$, for all $\kappa \in L$. This implies, $|\Delta_Y(\kappa)-\Delta_R(\kappa)|=0$, $|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|=0$, $|\Pi_Y(\kappa)-\Pi_R(\kappa)|=0$, $|\Omega_Y(\kappa)-\Omega_R(\kappa)|=0$ and $|\Phi_Y(\kappa)-\Phi_R(\kappa)|=0$, for all $\kappa \in L$. Hence, $T_{SVPNSM}(Y, R)=1-\frac{1}{n}\sum_{x \in L} tan(0)=1$.

Conversely, let T$_{SVPNSM}$(Y, R)=1. Therefore, $|\Delta_Y(\kappa)-\Delta_R(\kappa)|$=0, $|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|$=0, $|\Pi_Y(\kappa)-\Pi_R(\kappa)|$=0, $|\Omega_Y(\kappa)-\Omega_R(\kappa)|$=0, and $|\Phi_Y(\kappa)-\Phi_R(\kappa)|$=0, for all $\kappa\in L$. This implies, $\Delta_Y(\kappa)=\Delta_R(\kappa)$, $\Gamma_Y(\kappa)=\Gamma_R(\kappa)$, $\Pi_Y(\kappa)=\Pi_R(\kappa)$, $\Omega_Y(\kappa)=\Omega_R(\kappa)$, and $\Phi_Y(\kappa)=\Phi_R(\kappa)$, for all $\kappa\in L$. Hence, Y=R.

Theorem 3.2. Let Y, R and C be three SVPNSs over L. If $Y \subseteq R \subseteq C$, then T$_{SVPNSM}$(Y, R) $\geq$ T$_{SVPNSM}$(Y, C) and T$_{SVPNSM}$(R, C) $\geq$ T$_{SVPNSM}$(Y, C).

Proof. Suppose that Y, R and C be three SVPNSs over L such that $Y\subseteq R\subseteq C$. Therefore, $\Delta_Y(\kappa)\leq\Delta_R(\kappa)$, $\Gamma_Y(\kappa)\leq\Gamma_R(\kappa)$, $\Pi_Y(\kappa)\geq\Pi_R(\kappa)$, $\Omega_Y(\kappa)\geq\Omega_R(\kappa)$, $\Phi_Y(\kappa)\geq\Phi_R(\kappa)$, $\Delta_R(\kappa)\leq\Delta_C(\kappa)$, $\Gamma_R(\kappa)\leq\Gamma_C(\kappa)$, $\Pi_R(\kappa)\geq\Pi_C(\kappa)$, $\Omega_R(\kappa)\geq\Omega_C(\kappa)$, $\Phi_R(\kappa)\geq\Phi_C(\kappa)$, $\Delta_Y(\kappa)\leq\Delta_C(\kappa)$, $\Gamma_Y(\kappa)\leq\Gamma_C(\kappa)$, $\Pi_Y(\kappa)\geq\Pi_C(\kappa)$, $\Omega_Y(\kappa)\geq\Omega_C(\kappa)$, $\Phi_Y(\kappa)\geq\Phi_C(\kappa)$, for all $\kappa\in L$.

We have,

$|\Delta_Y(\kappa)-\Delta_R(\kappa)|\leq|\Delta_Y(\kappa)-\Delta_C(\kappa)|$, $|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|\leq|\Gamma_Y(\kappa)-\Gamma_C(\kappa)|$, $|\Pi_Y(\kappa)-\Pi_R(\kappa)|\leq|\Pi_Y(\kappa)-\Pi_C(\kappa)|$, $|\Omega_Y(\kappa)-\Omega_R(\kappa)|\leq|\Omega_Y(\kappa)-\Omega_C(\kappa)|$, $|\Phi_Y(\kappa)-\Phi_R(\kappa)|\leq|\Phi_Y(\kappa)-\Phi_C(\kappa)|$, for all $\kappa\in L$.

Therefore,

T$_{SVPNSM}$(Y, R)

$=1- \frac{1}{n}\sum_{\kappa\in L} tan[\frac{\pi}{12}[|\Delta_Y(\kappa)-\Delta_R(\kappa)|+|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|+|\Pi_Y(\kappa)-\Pi_R(\kappa)|+|\Omega_Y(\kappa)-\Omega_R(\kappa)|+|\Phi_Y(\kappa)-\Phi_R(\kappa)|]]$

$\geq 1- \frac{1}{n}\sum_{\kappa\in L} tan[\frac{\pi}{12}[|\Delta_Y(\kappa)-\Delta_C(\kappa)|+|\Gamma_Y(\kappa)-\Gamma_C(\kappa)|+|\Pi_Y(\kappa)-\Pi_C(\kappa)|+|\Omega_Y(\kappa)-\Omega_C(\kappa)|+|\Phi_Y(\kappa)-\Phi_Y(\kappa)|]]$

$=$ T$_{SVPNSM}$(Y, C)

This implies, T$_{SVPNSM}$(Y, R) $\geq$ T$_{SVPNSM}$(Y, C).

Further, we have,

$|\Delta_R(\kappa)-\Delta_C(\kappa)|\leq|\Delta_Y(\kappa)-\Delta_C(\kappa)|$, $|\Gamma_R(\kappa)-\Gamma_C(\kappa)|\leq|\Gamma_Y(\kappa)-\Gamma_C(\kappa)|$, $|\Pi_R(\kappa)-\Pi_C(\kappa)|\leq|\Pi_Y(\kappa)-\Pi_C(\kappa)|$, $|\Omega_R(\kappa)-\Omega_C(\kappa)|\leq|\Omega_Y(\kappa)-\Omega_C(\kappa)|$, $|\Phi_R(\kappa)-\Phi_C(\kappa)|\leq|\Phi_Y(\kappa)-\Phi_C(\kappa)|$, for all $\kappa\in L$.

Therefore,

T$_{SVPNSM}$(R, C)

$=1- \frac{1}{n}\sum_{\kappa\in L} tan[\frac{\pi}{12}[|\Delta_R(\kappa)-\Delta_C(\kappa)|+|\Gamma_R(\kappa)-\Gamma_C(\kappa)|+|\Pi_R(\kappa)-\Pi_C(\kappa)|+|\Omega_R(\kappa)-\Omega_C(\kappa)|+|\Phi_R(\kappa)-\Phi_C(\kappa)|]]$

$\geq 1- \frac{1}{n}\sum_{\kappa\in L} tan[\frac{\pi}{12}[|\Delta_Y(\kappa)-\Delta_C(\kappa)|+|\Gamma_Y(\kappa)-\Gamma_C(\kappa)|+|\Pi_Y(\kappa)-\Pi_C(\kappa)|+|\Omega_Y(\kappa)-\Omega_C(\kappa)|+|\Phi_Y(\kappa)-\Phi_C(\kappa)|]]$

$=$ T$_{SVPNSM}$(Y, C)

Hence, T$_{SVPNSM}$(R, C) $\geq$ T$_{SVPNSM}$(Y, C).

Definition 3.2. Suppose that, $Y=\{(\kappa,\Delta_Y(\kappa),\Gamma_Y(\kappa),\Pi_Y(\kappa),\Omega_Y(\kappa),\Phi_Y(\kappa)): \kappa\in L\}$ and $R=\{(\kappa,\Delta_R(\kappa),\Gamma_R(\kappa),\Pi_R(\kappa),\Omega_R(\kappa),\Phi_R(\kappa)): \kappa\in L\}$ be two SVPNSs over L. Then, the weighted tangent similarity measure of the similarities between two SVPNSs Y and R is defined by

T$_{WSVPNSM}$(Y, R) $= 1- \frac{1}{n}\sum_{\kappa\in L} w_\kappa tan[\frac{\pi}{12}[|\Delta_Y(\kappa)-\Delta_R(\kappa)|+|\Gamma_Y(\kappa)-\Gamma_R(\kappa)|+|\Pi_Y(\kappa)-\Pi_R(\kappa)|+|\Omega_Y(\kappa)-\Omega_R(\kappa)|$

$+|\Phi_Y(\kappa)-\Phi_R(\kappa)|]]$,   (2)

where, $\sum_{\kappa\in L} w_\kappa = 1$.

In view of Theorem 3.1. and Theorem 3.2., we formulate the following two Propositions.

Proposition 3.1. Suppose that $T_{WSVPNSM}(Y, R)$ be the weighted tangent similarity measure of similarities between the SVPNSs Y and R. Then,

(*i*) $0 \leq T_{WSVPNSM}(Y, R) \leq 1$;

(*ii*) $T_{WSVPNSM}(Y, R) = T_{WSVPNSM}(R, Y)$;

(*iii*) $T_{WSVPNSM}(Y, R) = 1$ if and only if $Y = R$.

Proposition 3.2. Assume that Y, R and C be three SVPNSs over L. If $Y \subseteq R \subseteq C$, then $T_{WSVPNSM}(Y, R) \geq T_{WSVPNSM}(Y, C)$ and $T_{WSVPNSM}(R, C) \geq T_{WSVPNSM}(Y, C)$.

4. SVPNS-MADM Strategy Based on Tangent Similarity Measure:

In this section, we develop a SVPNS-MADM model / algorithm under the SVPNS environment using the tangent similarity measure of similarities between two SVPNSs.

In our day to day life we face difficulty when we need to choose a suitable alternative from a set of possible alternatives. For that we should have to plan a strategy to take the appropriate decision.

Let $L = \{L_1, L_2,..., L_p\}$ be the family of possible alternatives. Let $A = \{A_1, A_2, ..., A_q\}$ be the family of attributes. Then, a group of DM (decision maker) together can give their evaluation information for each alternative L_i ($i = 1, 2,..., p$) against the attribute A_j ($j = 1, 2, ..., q$) by a SVPNS. Therefore, by using the whole evaluation information of all alternatives given by the decision makers, we can form a decision matrix.

The following are the steps of the proposed SVPNS-MADM:

Step-1: Decision Matrix Formation using SVPNS.

According to the decision makers evaluation information $E_{L_i} = \{(A_j, \Delta_{ij}(L_i, A_j), \Gamma_{ij}(L_i, A_j), \Pi_{ij}(L_i, A_j), \Omega_{ij}(L_i, A_j), \Phi_{ij}(L_i, A_j)): A_j \in A\}$ for each alternatives L_i against the attributes A_j ($j = 1, 2, ..., q$), we can build a decision matrix, where $(\Delta_{ij}(L_i, A_j), \Gamma_{ij}(L_i, A_j), \Pi_{ij}(L_i, A_j), \Omega_{ij}(L_i, A_j), \Phi_{ij}(L_i, A_j)) = (L_i, A_j)$ indicates the evaluation information of the alternative L_i ($i = 1, 2,..., p$) against the attribute A_j ($j = 1, 2, ..., q$).

The decision matrix (D^M) can be expressed as follows:

D^M	A_1	A_2	....	....	A_q
L_1	$(\Delta_{11}(L_1, A_1),\ \Gamma_{11}(L_1, A_1),$ $\Pi_{11}(L_1, A_1),\ \Omega_{11}(L_1, A_1),$ $\Phi(L_1, A_1))$	$(\Delta_{12}(L_1, A_2),\ \Gamma_{12}(L_1, A_2),$ $\Pi_{12}(L_1, A_2),\ \Omega_{12}(L_1, A_2),$ $\Phi_{12}(L_1, A_2))$	....	....	$(\Delta_{1q}(L_1, A_q),\ \Gamma_{1q}(L_1, A_q),$ $\Pi_{1q}(L_1, A_q),\ \Omega_{1q}(L_1, A_q),$ $\Phi_{1q}(L_1, A_q))$
L_2	$(\Delta_{21}(L_2, A_1),\ \Gamma_{21}(L_2, A_1),$ $\Pi(L_2, A_1),\ \Omega_{21}(L_2, A_1),$ $\Phi_{21}(L_2, A_1))$	$(\Delta_{22}(L_2, A_2),\ \Gamma_{22}(L_2, A_2),$ $\Pi_{22}(L_2, A_2),\ \Omega_{22}(L_2, A_2),$ $\Phi_{22}(L_2, A_2))$	......	....	$(\Delta_{2q}(L_2, A_q),\ \Gamma_{2q}(L_2, A_q),$ $\Pi_{2q}(L_2, A_q),\ \Omega_{2q}(L_2, A_q),$ $\Phi_{2q}(L_2, A_q))$
. . . .	. . . .	. . . .	. . . .	. . . .	. . . .

| L_p | $(\Delta_{p1}(L_p, A_1), \Gamma_{p1}(L_p, A_1),$ $\Pi_{p1}(L_p, A_1), \Omega_{p1}(L_p, A_1),$ $\Phi_{p1}(L_p, A_1))$ | $(\Delta_{p2}(L_p, A_2), \Gamma_{p2}(L_p, A_2),$ $\Pi_{p2}(L_p, A_2), \Omega_{p2}(L_p, A_2),$ $\Phi_{p2}(L_p, A_2))$ | …… ….. | …… …. | $(\Delta_{pq}(L_p, A_q), \Gamma_{pq}(L_p, A_q),$ $\Pi_{pq}(L_p, A_q), \Omega_{pq}(L_p, A_q),$ $\Phi_{pq}(L_p, A_q))$ |

Step-2. Determination of the Weights for each Attribute.

Determination of the value of weights for each attributes is an important task for any multi attribute decision making model. If the weights of the attributes are completely unknown in a MADM problem, then the decision makers can use the compromise function.

The compromise function of L is defined as follows:

$$\xi_j = \sum_{i=1}^{p} (3 + \Delta_{ij}(L_i, A_j) + \Gamma_{ij}(L_i, A_j) - \Pi_{ij}(L_i, A_j) - \Omega_{ij}(L_i, A_j) - \Phi_{ij}(L_i, A_j))/5. \tag{3}$$

Then, the weights of the j-th attribute is defined by $w_j = \dfrac{\xi_j}{\sum_{j=1}^{q} \xi_j}$ (4)

Here, $\sum_{j=1}^{q} w_j = 1$.

Step-3. Selection of the Benefit-type Attributes and Cost-type Attributes.

In any MADM problems, the attributes can be divided into two types namely benefit-type attribute and cost-type attribute. In our proposed SVPNS-MADM strategy, an ideal alternative can be defined by using a minimum operator for the cost-type attributes and maximum operator for the benefit-type attributes to determine the best value of each attribute among all alternatives.

The ideal alternative is defined as follows:

$$I = (C_1^+, C_2^+, C_3^+, \dots \dots, C_q^+) \tag{5}$$

When C_j (j=1, 2, …., q) is a benefit type of attribute, then

$C_j^+ = (\max \{\Delta_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, max $\{\Gamma_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, min $\{\Pi_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, min $\{\Omega_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, min $\{\Phi_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$). (6)

When C_j (j=1, 2, …., q) is a cost type of attribute, then

$C_j^+ = (\min \{\Delta_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, min $\{\Gamma_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, max $\{\Pi_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, max $\{\Omega_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$, max $\{\Phi_{ij}(L_i, A_j): i$=1, 2, 3, ….., $p\}$). (7)

Step-4. Determination of the Tangent Similarity Measure between the Ideal Alternative and Other Alternatives.

In this step, we calculate the tangent similarity measure of similarities between the ideal alternatives and the decision elements from the decision matrix by using eq. (1).

Step-5: Determination of the accumulated measure values.

To aggregate the similarity measures corresponding to each alternative we use the following accumulated measure function (AMF):

$$D_{AMF}^i = \sum_{j=1}^q w_j \text{T}_{\text{SVPNSM}} ((L_i, A_j), \ C_j^+) \tag{8}$$

where, $(L_i, A_j) = (\Delta_{ij}(L_i, A_j), \Gamma_{ij}(L_i, A_j), \Pi_{ij}(L_i, A_j), \Omega_{ij}(L_i, A_j), \Phi_{ij}(L_i, A_j))$.

Step-6: Ranking of the alternatives.

Ranking of alternatives is prepared based on the ascending order of accumulated measure values. The alternative associated with the highest accumulated measure value is the best suitable alternatives.

Step-7: End.

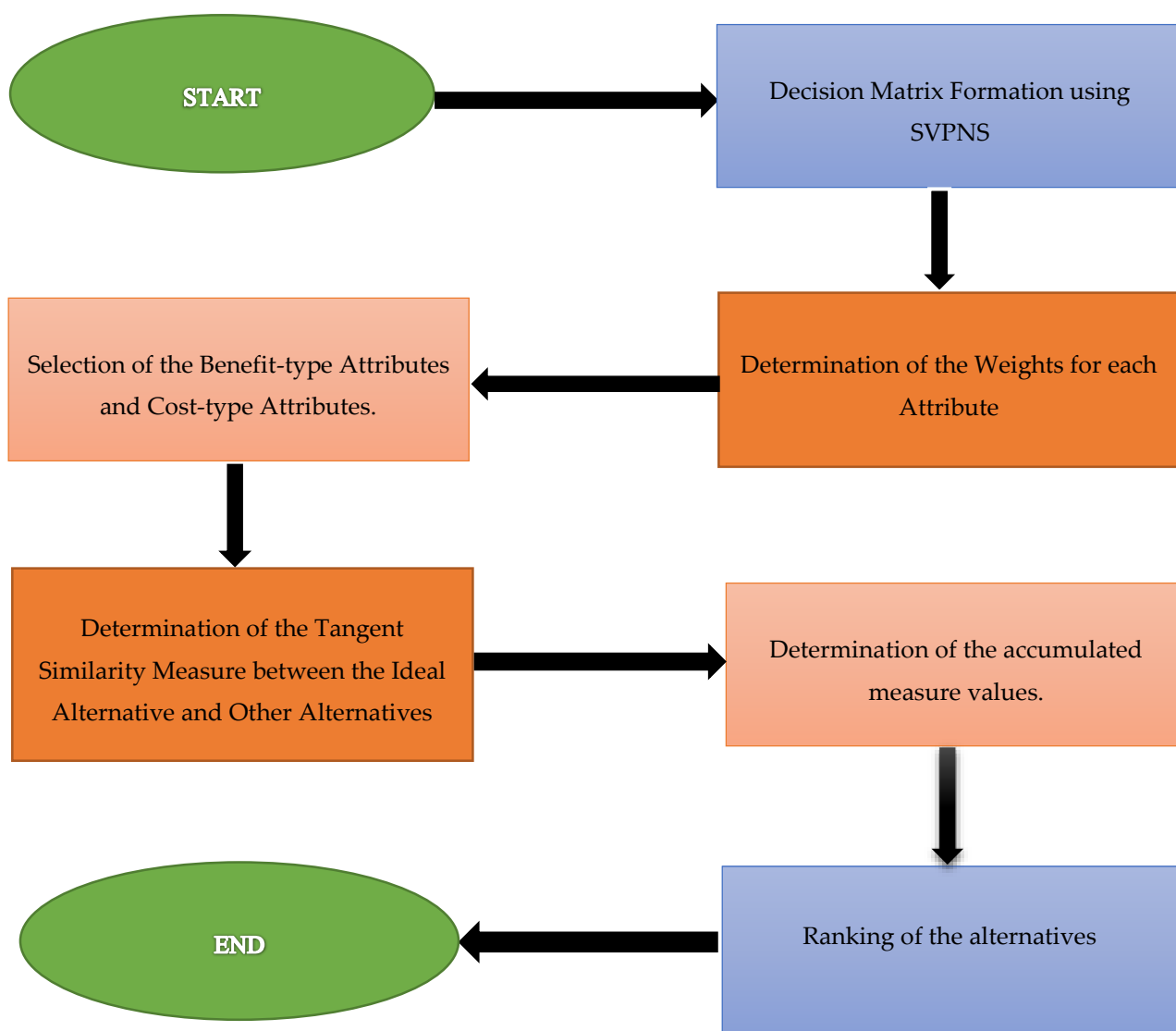**The flow chart of the proposed SVPNS-MADM strategy is given below:**

Figure- 1

5. Validation of the Proposed SVPNS-MADM Strategy.

In this section, we validate our proposed SVPNS-MADM strategy by a numerical example.

Example 5.1. <u>Selection of Plot to Build a New House in Urban Area.</u>

There are few other things come into account, when we are searching for a good plot to build our house. To choose a plot, lots of things we have to took in our consideration. Initially it is obscure, whether the place had good communication with the city, availability of well roads, gas-pipeline, water facility, electricity etc.

Most of the common problems to select a plot are:

(*i*) Price of the plot.

(*ii*) Well connectivity with necessary facilities.

(*iii*) Does the plot have significant slope or have to fill or cut the slope.

(*iv*) The buildings in neighbor plot, size, shape which also affect your disclosure to sunlight in your living area.

(*v*) Types of soil is also another important factor to build a house, to keep the house stable types of soil composition and reactive nature of soil are relevant. Normally, worth will high to build in more reactive area.

So, the selection of plot by a person can be considered as a MAMD problem. After initializing, the decision maker selects three major alternatives namely L_1, L_2 and L_3. For the selection of suitable place, the decision maker select four attributes such as A_1: Price of the place, A_2: Well connectivity with the other part of the city, A_3: Shape of the plot, A_4: Type of the soil of plot.

Then, the SVPNS-MADM strategy is presented as follows:

By using the evaluation information for all alternatives given by the decision makers, we prepare the decision matrix as follows:

Table-1:

	A_1	A_2	A_3	A_4
L_1	(0.8,0.3,0.2,0.4,0.3)	(0.7,0.3,0.5,0.2,0.4)	(0.8,0.2,0.3,0.4,0.2)	(0.9,0.2,0.3,0.4,0.3)
L_2	(0.9,0.1,0.4,0.2,0.3)	(0.8,0.3,0.5,0.4,0.3)	(0.7,0.2,0.2,0.2,0.3)	(0.8,0.1,0.2,0.4,0.3)
L_3	(0.8,0.4,0.3,0.2,0.2)	(0.9,0.1,0.5,0.2,0.2)	(0.6,0.1,0.2,0.3,0.1)	(0.8,0.2,0.2,0.4,0.5)

Now, we determine the weight of each attribute by using the eq. (3) and eq. (4). The weight vector for all attributes is given below.

(w_1, w_2, w_3, w_4) = (0.268097, 0.238606, 0.252011, 0.241287).

According to the expert opinion, we choose the attribute A_2, A_3, A_4 as benefit-types of attribute and the attributes A_1 as cost-type of attributes. Now, we choose the ideal alternative solution by using eq. (5), eq. (6), and eq. (7). The ideal solution *I* is given in the following table.

Table-2:

	A_1	A_2	A_3	A_4
L_1	(0.9,0.3,0.1,0.5,0.2)	(0.8,0.2,0.2,0.1,0.4)	(0.9,0.1,0.3,0.1,0.3)	(0.9,0.4,0.2,0.3,0.4)
L_2	(0.8,0.1,0.3,0.3,0.2)	(0.9,0.2,0.3,0.4,0.2)	(0.6,0.1,0.2,0.3,0.3)	(0.9,0.2,0.1,0.2,0.2)
L_3	(0.9,0.4,0.2,0.3,0.1)	(0.7,0.3,0.4,0.1,0.2)	(0.8,0.2,0.1,0.2,0.3)	(0.8,0.3,0.1,0.3,0.1)
I	(0.8,0.1,0.3,0.5,0.2)	(0.9,0.3,0.2,0.1,0.2)	(0.9,0.2,0.1,0.1,0.3)	(0.9,0.4,0.1,0.2,0.1)

After the formation of ideal alternative solution in Table-2, we determine the tangent similarity measure of similarities between the ideal alternative solution and the decision elements from table -1 by using eq. (1).

The aggregate tangent similarity measures corresponding to each alternative are given below:
D_{AMF}^1 = 0.70862, D_{AMF}^2 =0.889084, D_{AMF}^3 = 0.885916.

Therefore, $D_{AMF}^1 < D_{AMF}^3 < D_{AMF}^2$. This implies, the alternative *L₂* is the most suitable alternative (plot) for choosing to build a house.

6. Conclusions:

In the article, we have established a SVPNS-MADM strategy based on tangent similarity measure of similarities between two SVPNSs. We have also validated our proposed SVPNS-MADM strategy by solving an illustrative numerical example to demonstrate the effectiveness of the proposed SVPNS-MADM strategy.

The proposed SVPNS-MADM strategy can also be used to deal with the other decision-making problems such as tender selection [5], teacher selection [28], medical diagnosis [26, 27], weaver selection [14], brick selection [16, 20], logistic center location selection [23, 24], etc.

Conflict of Interest: The authors declare that they have no conflict of interest.

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