



Neutrosophic Fuzzy Threshold Graph

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Abstract: The aim of this paper is to introduce the extension of intuitionistic fuzzy threshold graph to Neutrosophic field of approach. A new structure called Neutrosophic fuzzy threshold graph (NFTG) have been portrayed and the Neutrosophic fuzzy alternating 4-cycle have been explained using suitable examples. Two parameters namely, Neutrosophic fuzzy threshold dimension and Neutrosophic fuzzy threshold partition number have been illustrated with examples. Theorems based on proposed concepts were proved and one application has been discussed to insist the utility of Neutrosophic fuzzy threshold graph in resource management technique.

Keywords: threshold graph; Neutrosophic graph; threshold dimension; partition number; fuzzy alternating 4- cycle; fuzzy threshold graph.

1. Introduction

A graphical structure using fuzzy concept was first introduced in 1975[8]. Professor A Rosenfeld proposed the concept when he studied about uncertainty of systems. This new approach attracts several researchers as it is applicable to almost all real world problems where uncertainty plays a major role. The term Intuitionistic fuzzy set came into existence in 1986 and it was proposed by professor K. Atanassov [2,3]. The graphical representation of Intuitionistic fuzzy set was framed by Akram M., Akmal R [1] and they elaborate the work with basic operations of set theory as well as using matrix representations. Each vertex and edges of such graphs were labeled with two values that define the membership and non-membership degrees.

Adding one more parameter called indeterminacy value along with the existing two parameters of Intuitionistic fuzzy , a new concept called Neutrosophic fuzzy set was introduced by professor Florentin Smarandache [9] and he developed the graphical structures[10,11]. The term Threshold graph was coined by V. Chvatal and P. L. Hammer in 1973[4] while they do research on packaging problems and later in 1985 E. T. Ordman utilized the concept in resource allocation techniques[7]. In 2015, Sovan Samanta and Madhumangal Pal, professors from Vidyasagar University framed fuzzy threshold graphs[12] and it was generalized to intuitionistic threshold graphs by professors Lanzhen Yang and Hua Mao in 2019[6].

Contribution

Our country is facing challenges regarding Covid-19 vaccine, since we have limited number of doses. Optimum allocation should be needed to meet the needs. The challenges of supply, storage, and delivery of vaccines must take place under strict sanitary conditions is unavoidable too. It is also difficult to reach some remote areas or minorities due to the unavailability of storage requirements and safe delivery. To identify the effective allocation of the COVID-19 vaccine for priority groups, decision-makers must involve experts from multiple fields to get benefit from their experiences in setting priorities and principle guidelines. The neutrosophic, like other fields, contributed to the understanding and analyzing COVID-19 pandemic too. In this paper, we extend the concept of threshold graph by embedding the neutrosophic set properties and we will prove some basic theorems related to the concept. We also define two parameters called threshold dimension and threshold partition number of Neutrosophic fuzzy threshold graphs.

Motivation

A neutrosophic set plays an important role in uncertainty modeling. The development of uncertainty theory plays a fundamental role in formulation of real-life scientific mathematical model, structural modeling in engineering field, medical diagnoses problem etc. In this current decade, researchers have exposed their considerations to make progress with the theories related to neutrosophic area and constantly try to endorse its sufficient scope applications in dissimilar branches of neutrosophic domain. However, our main objective is to support the theory efficiently with these following points.

- 1. Introduction of Neutrosophic fuzzy threshold graph.
- 2. Extension of fuzzy threshold graphs and its concepts using neutrosophic fuzzy graph.
- 3. Application of neutrosophic fuzzy threshold graph in optimum resource allocation.

2. Preliminaries

This section gives a brief preview about the existing concepts which will be utilized in section 3.

Definition 2.1: A graph G = (P, Q) is called a fuzzy graph if there exist function $\mu_P : \mathbf{V}^* \to [0, 1]$ and $\mu_Q : \mathbf{V}^* \times \mathbf{V}^* \to [0, 1]$ called membership function such that for all $(\mathbf{v}_i, \mathbf{v}_j)$ in $\mathbf{V}^* \times \mathbf{V}^*$, $\mu_Q (\mathbf{v}_i, \mathbf{v}_j) \leq \min \{ \mu_Q (\mathbf{v}_i), \mu_Q (\mathbf{v}_j) \}$, where $\mathbf{V}^* = \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \cdots, \mathbf{v}_r \}$ is the vertex set of G.

Definition 2.2: A graph G = (P, Q) is called a neutrosophic fuzzy graph (NFG) with Vertex set $V^* = \{v_1, v_2, v_3, \dots, v_r\}$, whose membership, non-member ship and Indeterminacy functions satisfy the following conditions:

(i) $\mu_p : V^* \to [0, 1], \upsilon_p : V^* \to [0, 1]$ and $\sigma_p : V^* \to [0, 1]$ denote the degree of truth-membership function, falsity-membership function and indeterminacy-membership function of the vertex $v_i \in V^*$ respectively, and $0 \leq \mu_p(v) + \upsilon_p(v) + \sigma_p(v) \leq 3$, $\forall v \in V^*$ (i=1,2,3...r).

(ii) $\mu_Q : \mathbf{V}^* \times \mathbf{V}^* \to [0, 1], \nu_Q : \mathbf{V}^* \times \mathbf{V}^* \to [0, 1] \text{ and } \sigma_Q : \mathbf{V}^* \times \mathbf{V}^* \to [0, 1] \text{ denote}$ the degree of truth-membership function, falsity-membership function and indeterminacymembership function of the edge (v_i, v_j) respectively such that $\mu_Q(\mathbf{v}_i, \mathbf{v}_j) \leq \min \left\{ \mu_Q(\mathbf{v}_i), \ \mu_Q(\mathbf{v}_j) \right\},$

$$\sigma_{\varrho}(\mathbf{v}_{i}, \mathbf{v}_{j}) \leq \min \left\{ \sigma_{\varrho}(\mathbf{v}_{i}), \sigma_{\varrho}(\mathbf{v}_{j}) \right\} \\ \nu_{\varrho}(\mathbf{v}_{i}, \mathbf{v}_{j}) \leq \max \left\{ \nu_{\varrho}(\mathbf{v}_{i}), \nu_{\varrho}(\mathbf{v}_{j}) \right\}$$

and $0 \leq \mu_o(\mathbf{v}_i, \mathbf{v}_i) + \upsilon_o(\mathbf{v}_i, \mathbf{v}_i) + \sigma_o(\mathbf{v}_i, \mathbf{v}_i) \leq 3$ for every $(\mathbf{v}_i, \mathbf{v}_i)$,

where the sets *P* and *Q* be the Neutrosophic fuzzy subsets defined on V^* and E^* respectively[5].

Definition 2.3: The vertex cardinality of a Neutrosophic fuzzy graph, G = (P, Q) denoted by $|V|_{N}$ and

it is defined as
$$|\mathbf{V}|_{N} = \sum_{\mathbf{v}\in\mathbf{V}} \frac{1+\mu_{P}(\mathbf{v})+\sigma_{P}(\mathbf{v})-\upsilon_{P}(\mathbf{v})}{3}$$

Definition 2.4: The stability number of a Neutrosophic fuzzy graph G = (P, Q) is defined as the order of largest stable set of G and it is denoted by $\zeta(G)$.

3. Neutrosophic fuzzy threshold graphs

In this section we introduce the definition of Neutrosophic fuzzy threshold graph (NFTG), Neutrosophic fuzzy threshold dimension $\eta(G)$ and Neutrosophic fuzzy threshold partition number $\eta_n(G)$ and we prove some theorem based on the concepts stated.

Definition 3.1: A graph G = (P, Q) is called a Neutrosophic fuzzy threshold graph (NFTG) if there exist $\tau_1 > 0$, $\tau_2 > 0$ and $\tau_3 > 0$ such that

$$\sum_{u \in U} \mu_P(u) \le \tau_1 , \sum_{u \in U} \left(1 - \upsilon_P(u) \right) \le \tau_2 \text{ and } \sum_{u \in U} \sigma_P(u) \le \tau_3$$
(1)

Provided that $U \subseteq V^*$ is an independent set in *G*. NFTG is generally denoted as $G = (P, Q; \tau_1, \tau_2, \tau_3)$.

Remark 3.1: The notion $U \subset V^*$ is an independent set in *G* is same as that the notion $U \subset V^*$ is an independent set in G^* . If $G = (P, Q; \tau_1, \tau_2, \tau_3)$ and $U \subseteq V^*$ is a dependent set in G, then we have at least one of the condition (3.1) does not hold. For this case,

$$\sum_{u \in U} \mu_P(u) > \tau_1 \text{ or } \sum_{u \in U} (1 - \upsilon_P(u)) > \tau_2 \text{ or } \sum_{u \in U} \sigma_P(u) > \tau_3.$$

Example 3.1: Let $G^* = (V^*, E^*)$ be a graph whose vertex and edge set is $V^* = \{m, n, o, p, q\}$ and $E^* = \{m, n, o, q, q, q, q\}$ and $E^* = \{m, n, q, q, q, q\}$ an $\{(m, n), (n, o), (o, p), (p, n), (n, q)\}$ respectively and the sets *P* and *Q* be the Neutrosophic fuzzy subsets defined on V* and E* respectively (see Table). Based on the data, the NFTG for this graph is given as G = (P, Q; 0.6, 0.8, 0.5).

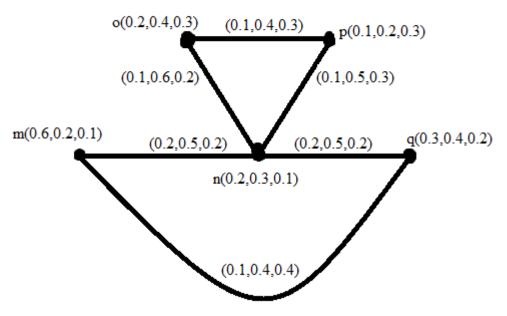


Figure 1. Neutrosophic fuzzy threshold graph

Table 1: Membership, Non-membership and Indeterminacy degrees for Vertices and Edges

Vertices	m	n	0	р	q	
$\mu_{\scriptscriptstyle P}$	0.6	0.2	0.2	0.1	0.3	
\mathcal{D}_P	0.2	0.3	0.4	0.2	0.4	
$\sigma_{\scriptscriptstyle P}$	0.1	0.1	0.2	0.3	0.2	
Edges	(m,n)	(n,o)	(o,p)	(p,n)	(m,q)	(n,q)
μ_Q	0.2	0.1	0.1	0.1	0.1	0.2
$ u_{\varrho}$	0.5	0.6	0.4	0.5	0.4	0.5
σ_{arrho}	0.2	0.2	0.3	0.3	0.4	0.2

Proposition 3.1: If $G^*=(V^*, E^*)$ is an underlying graph for a Neutrosophic fuzzy graph G = (P, Q) and if $W \subseteq V^*$ is an independent set in NFTG, $G = (P, Q; \tau_1, \tau_2, \tau_3)$ then, the cardinality of W satisfies the following relation:

$$\left|\mathbf{W}\right|_{N} \leq \sum_{w \in \mathbf{W}} \frac{\tau_{1} + \tau_{2} + \tau_{3}}{3}$$

Proof:

Given that $G = (P, Q; \tau_1, \tau_2, \tau_3)$ is and NFTG. Then by definition (3.1), we have

$$\sum_{w \in W} \mu_P(w) \le \tau_1, \sum_{w \in W} \left(1 - \upsilon_P(w) \right) \le \tau_2 \text{ and } \sum_{w \in W} \sigma_P(w) \le \tau_3$$
(2)

If *r* denotes the number of vertices in W, then

$$\sum_{w \in W} (1 - \upsilon_p(w)) \le \tau_2 \quad \Rightarrow r - \sum_{w \in W} \upsilon_p(w) \le \tau_2$$
(3)

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Thus,

$$-\sum_{w\in W} \upsilon_p(w) \le \tau_2 - r \quad \Longrightarrow \sum_{w\in W} \upsilon_p(w) \ge r - \tau_2.$$

Now from definition (2.3), we have

$$\left|\mathbf{W}\right|_{N} = \sum_{w \in \mathbf{W}} \frac{1 + \mu_{P}\left(w\right) + \sigma_{P}\left(w\right) - \upsilon_{P}\left(w\right)}{3} \tag{4}$$

Substituting (2), (3) in (4) we get

$$\begin{split} \left|\mathbf{W}\right|_{N} &= \sum_{w \in \mathbf{W}} \frac{\mu_{P}\left(w\right)}{3} + \sum_{w \in \mathbf{W}} \frac{1 - \upsilon_{P}\left(w\right)}{3} + \sum_{w \in \mathbf{W}} \frac{\sigma_{P}\left(w\right)}{3} \\ &\leq \frac{1}{3} \left(\tau_{1} + \tau_{2} + \tau_{3}\right) \end{split}$$

Therefore, $\left|\mathbf{W}\right|_{N} \leq \sum_{w \in \mathbf{W}} \frac{\tau_{1} + \tau_{2} + \tau_{3}}{3}$.

Proposition 3.2: A fuzzy threshold graph is a special case of Neutrosophic fuzzy threshold graph. **Proof:**

Let G = (P, Q) be a fuzzy threshold graph, then there exist $\tau_1 > 0$, such that $\sum_{u \in U} \mu_P(u) \le \tau_1$, where U

is an independent set contained in vertex set V of G. Since the non-membership and indeterminacy degree values for a fuzzy threshold graph is zero, we can choose $\tau_2 = r$ and $\tau_3 = 1$. Thus there exist $\tau_2 > 0$ and $\tau_3 > 0$ such that

$$\tau_1 > 0, \tau_2 > 0 \text{ and } \tau_3 > 0 \text{ such that}$$
$$\sum_{u \in U} \mu_p(u) \le \tau_1, \sum_{u \in U} (1 - \upsilon_p(u)) \le \tau_2 \text{ and } \sum_{u \in U} \sigma_p(u) \le \tau_3,$$

Where *r* denotes the number of vertices and hence $G = (P, Q; \tau_1, \tau_2, \tau_3)$ forms a Neutrosophic fuzzy threshold graph.

Definition 3.2: Let G = (P, Q) be a Neutrosophic fuzzy graph with $V^* = \{m, n, o, p\}$, we say that the four vertices constitute a Neutrosophic fuzzy alternating 4-cycle if it satisfies the following four conditions:

(i)
$$(\mu_{\varrho}(m,n), \nu_{\varrho}(m,n), \sigma_{\varrho}(m,n)) \neq (0,0,0)$$

(ii) $(\mu_{\varrho}(o,p), \nu_{\varrho}(o,p), \sigma_{\varrho}(o,p)) \neq (0,0,0)$
(iii) $(\mu_{\varrho}(m,o), \nu_{\varrho}(m,o), \sigma_{\varrho}(m,o)) = (0,0,0)$ and
(iv) $(\mu_{\varrho}(n,p), \nu_{\varrho}(n,p), \sigma_{\varrho}(n,p)) = (0,0,0).$

Remark 3.2: The following three graphs may form a sub graph for a Neutrosophic fuzzy alternating 4-cycle:

(i) A Neutrosophic fuzzy path P₄:

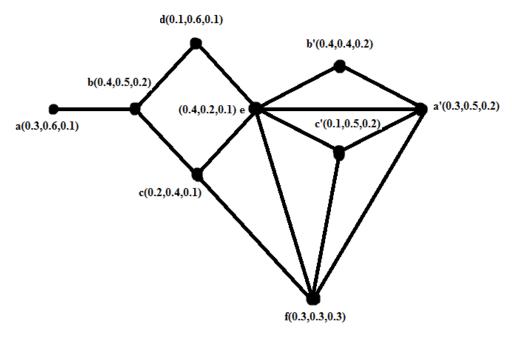
$$\left(\mu_{\varrho}(m, p), \upsilon_{\varrho}(m, p), \sigma_{\varrho}(m, p) \right) = (0, 0, 0) \text{ and}$$
$$\left(\mu_{\varrho}(n, o), \upsilon_{\varrho}(n, o), \sigma_{\varrho}(n, o) \right) \neq (0, 0, 0).$$
Or

- $(\mu_{\varrho}(n,o), \upsilon_{\varrho}(n,o), \sigma_{\varrho}(n,o)) = (0,0,0) \text{ and}$ $(\mu_{\varrho}(m,p), \upsilon_{\varrho}(m,p), \sigma_{\varrho}(m,p)) \neq (0,0,0).$
- (ii) A Neutrosophic fuzzy square C₄: $(\mu_Q(m, p), \nu_Q(m, p), \sigma_Q(m, p)) \neq (0, 0, 0)$ and $(\mu_Q(n, 0), \nu_Q(n, 0), \sigma_Q(n, 0)) \neq (0, 0, 0).$
- (iii) A Neutrosophic fuzzy matching 2K2:

$$(\mu_Q(m, p), \upsilon_Q(m, p), \sigma_Q(m, p)) = (0, 0, 0)$$
 and
 $(\mu_Q(n, o), \upsilon_Q(n, o), \sigma_Q(n, o)) = (0, 0, 0).$

Definition 3.3: A Neutrosophic fuzzy threshold graph $G = (P, Q; \tau_1, \tau_2, \tau_3)$ is said to have a threshold dimension $\eta(G)$ if there exists $\eta(G)$ number of Neutrosophic fuzzy threshold sub graphs whose union covers the edge set E^{*} of $G = (P, Q; \tau_1, \tau_2, \tau_3)$, provided that such partition is minimal.

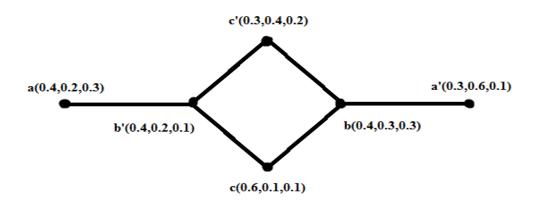
Definition 3.4: If $G_1,G_2,...,G_p$ is *p* Neutrosophic fuzzy threshold sub graphs whose union covers the edge set E* of a Neutrosophic fuzzy threshold graph $G = (P,Q; \tau_1, \tau_2, \tau_3)$ and does not have common arcs , then *p* is said to be the Neutrosophic fuzzy threshold partition number of G and it is denoted by $\eta_p(G)$.



For this graph, $\eta(G) = 2, \eta_p(G) = 3$

Figure 2. Threshold dimension and partition number of NFTG

Proposition 3.3: Let $G = (P, Q; \tau_1, \tau_2, \tau_3)$ be a Neutrosophic fuzzy threshold graph. Then its threshold dimension $\eta(G)$ satisfies the relation, $\eta(G) \le r - \zeta(G)$, where r denotes the number of vertices in V* of G. In particular if $G = (P, Q; \tau_1, \tau_2, \tau_3)$ is a triangular free graph, then $\eta(G) = \eta_p(G) = r - \zeta(G)$. **Proof:** Let U* be the stable set with maximum number of vertices of G. Then each star having center at $u \in V^* - U^*$ forms a Neutrosophic fuzzy threshold graph of G. The union of all such stars along with weak arcs of stable set U* forms a covering for the edge set E* of G. Therefore, $\eta(G) \leq |V^* - U^*|$. Given that r denotes the number of vertices in V* of G and by definition (2.4), we have $\eta(G) \leq r - \zeta(G)$. In particular if $G = (P, Q; \tau_1, \tau_2, \tau_3)$ is a triangular free graph, then each Neutrosophic fuzzy threshold graph of G is a star or a star with weak edge and therefore $\eta(G) = r - \zeta(G)$. Also such partition does not have common arcs among them. Hence, if G is triangle free then $\eta(G) = r - \zeta(G) = \eta_p(G)$.



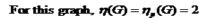


Figure 3. Threshold dimension and partition number of triangle free NFTG

S.No.	Neutrosophic fuzzy threshold graph	Intuitionistic Fuzzy threshold graph		
1	G*=(V*, E*) is an underlying graph for a Neutrosophic fuzzy graph $G = (P, Q)$ and if $W \subseteq V^*$ is an independent set in NFTG, $G = (P, Q; \tau_1, \tau_2, \tau_3)$ then, the cardinality of W satisfies the following relation:	Let $G = (P, Q; \tau_1, \tau_2)$ and $U \subseteq V$ be an independent set in Intuitionistic Fuzzy threshold graph G, then $ U _{IF} \leq \frac{\tau_1 + \tau_2}{3}$		
	$\left \mathbf{W}\right _{N} \leq \sum_{\mathbf{w}\in\mathbf{W}} \frac{\tau_{1} + \tau_{2} + \tau_{3}}{3}$			
2	A fuzzy threshold graph is a special case of Neutrosophic fuzzy threshold graph.	A fuzzy threshold graph is a special case of Intuitionistic Fuzzy threshold graph.		
3	Let $G = (P, Q; \tau_1, \tau_2, \tau_3)$ be a Neutrosophic fuzzy threshold graph. Then its threshold dimension $\eta(G)$ satisfies the relation, $\eta(G) \leq r - \zeta(G)$, where r denotes the number of vertices in V* of G. In particular if $G = (P, Q; \tau_1, \tau_2, \tau_3)$ is a triangular free graph, then $\eta(G) = \eta_p(G) = r - \zeta(G)$.	If $G = (P, Q; \tau_1, \tau_2)$ is a triangle free IFG, then $t(G)=t_P(G) = n - \alpha(G)$, where $\alpha(G)$ is the number of vertices of the maximum independent set of G, and n is the number of vertices of G.		

Table 2. Comparison table on properties of NFTG and FTG

4. Resource Management Technique using Neutrosophic fuzzy threshold graph:

This section discusses the application of Neutrosophic Fuzzy Threshold graph in resource management technique.

Resource management plays a vital role in production field and it one of the emerging topics of optimization techniques. Neutrosophic Fuzzy Threshold graph find its unique way of solving the problems faced during best resource allocation. Let us elaborate one such application here.

Invention of Covid-19 vaccine is the big challenge faced by almost all countries of the world now. Several researches were effectively undertaken to reach the goal. India, which is in top second position of affected people count in the world, is at the last stage of testing and will soon release the vaccine for Covid-19. At the same time, being a developing country it does not have enough resource to supply the medicine to all people in the country immediately. Definitely, the resource controlling becomes necessary, so that the vaccine must reach the needed ones at proper time. Clinics must be situated at optimized places which ensure enough supply of medical resources to the cities. On the other hand, the medical resource couldn't get wasted by supplying it to the peoples with good immune and were in safe zone, who really doesn't need that. Let us analyze such situation using a Neutrosophic Fuzzy Threshold graph G= (P, Q), in which the vertices denote the cities and the clinic providing medical aids as given below:

Suppose that 3 clinics C1,C2 and C2 were supplying medical aids to peoples of six cities a, b, c, e, f and g. and The labeling values of Graph G=(P,Q) represents the requirement and supply of medical resource. For example,

- For the city g, μ_p(g) denotes the Covid-19 positive people who required immediate medicine, υ_p(g) denotes the people who were under safe zone and required medicine only for precaution and σ_p(g) denoted the people who were asymptotic and those were the ones who need medical attention so that they couldn't spread disease further.
- In case of clinics, $(\mu_P(C1), \upsilon_P(C1), \sigma_P(C1))$ denoted the supply, storage and the sudden unexpected demand of medical resources respectively.
- The meaning of triplet (μ_Q(g,C1), ν_Q(g,C1), σ_Q(g,C1)) is that, it is the actual amount of medical aids provided to the three categories of people in city from the clinic C1.

Since the medical resource utilized by the people is dominated by the one in the clinics, the Neutrosophic threshold dimension can easily be determined from the number of clinics. It is clear that the Neutrosophic threshold dimension of graph in Figure (4) is 3, that is we can induce three Neutrosophic fuzzy threshold sub graphs as given in figure (5, 6 & 7), where the triplet (τ_1, τ_2, τ_3) denotes the limitation of amount of medicine provided to three categories of people corresponding to the clinic C.

- Figure (5) gives the Neutrosophic fuzzy threshold graph with $(\tau_1 = 0.6, \tau_2 = 2.92, \tau_3 = 0.34)$, where the clinic C1 supplies 0.6 amount of medicine to the affected people in three cities {a,b,c}, where only $\mu_P(a) + \mu_P(b) + \mu_P(c) = 0.37$ is required , 0.34 amount of medicine to the one who were asymptotic and 0.08(3-2.92) amount to the people in safe zone, whose actual requirement is only 0.055.
- Figure (6) gives the Neutrosophic fuzzy threshold graph with $(\tau_1 = 0.62, \tau_2 = 4.93, \tau_3 = 0.2)$, where the clinic C2 supplies 0.62 amount of medicine to the affected people in five cities {b,c,e,f,g}, where only $\mu_p(b) + \mu_p(c) + \mu_p(e) + \mu_p(g) + \mu_p(f) = 0.6$ is required ,0.2 amount of medicine to the one who were asymptotic and 0.07(5-4.93) amount to the people in safe zone, whose actual requirement is only 0.06.

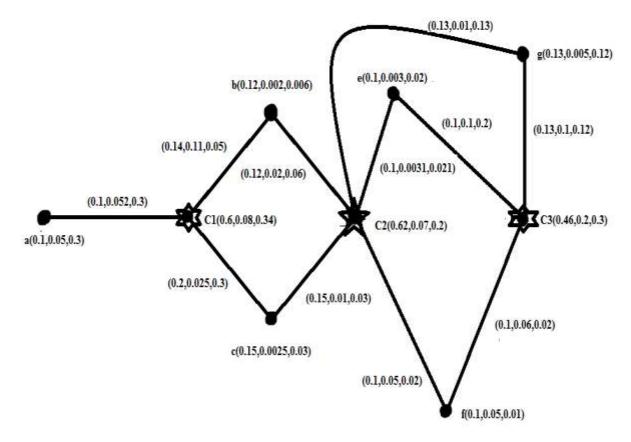
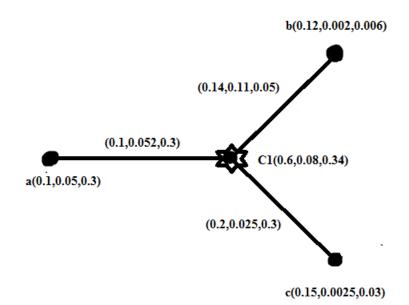
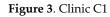


Figure 4. Neutrosophic Fuzzy threshold graph





• Figure (7) gives the Neutrosophic fuzzy threshold graph with $(\tau_1 = 0.46, \tau_2 = 2.8, \tau_3 = 0.3)$, where the clinic C3 supplies 0.46 amount of medicine to the affected people in three cities {e,f,g},

where only $\mu_P(e) + \mu_P(g) + \mu_P(f) = 0.33$ is required, 0.3 amount of medicine to the one who were asymptotic and 0.2(3-2.8) amount to the people in safe zone, whose actual requirement is only 0.058.

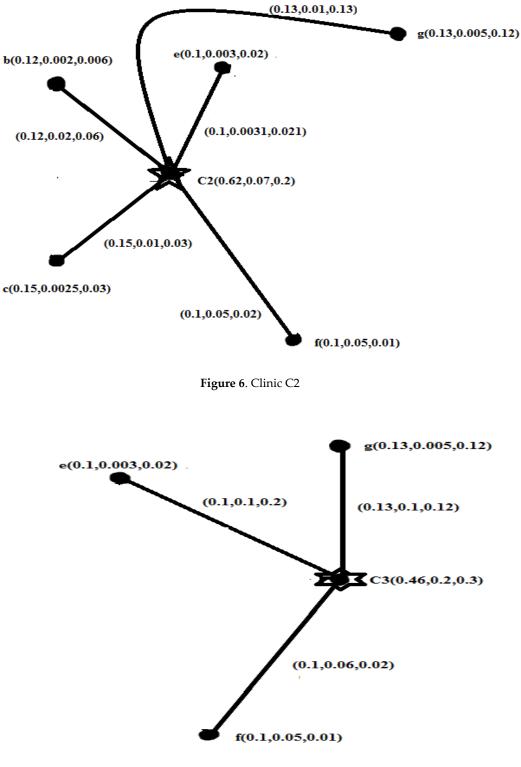


Figure 7. Clinic C3

Thus Neutrosophic fuzzy threshold graphs will give better comparison and proper details in resource analysis. Using Neutrosophic fuzzy threshold graph we can get more accurate results than intuitionistic fuzzy threshold graphs.

5. Conclusions

Neutrosophic graphs will give more accurate results in case of uncertainty. Even though, we obtain some basic information using intuitionistic fuzzy graphs, the value of indeterminacy will provide a clear cut results in resource allocation process. As an extension of intuitionistic fuzzy threshold graphs, Neutrosophic fuzzy threshold graph was introduced and Neutrosophic fuzzy alternating 4- cycle, threshold dimensions of Neutrosophic fuzzy threshold graphs were defined. We proved that Neutrosophic fuzzy threshold graph is the generalized case of Fuzzy threshold graph. Proper examples were given for each proposed concepts and theorems based on concepts were proved. We also gave one application that illustrates how Neutrosophic fuzzy threshold graph and threshold dimension were utilized in allocation of medical resource from clinics to people in cities.

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Conflicts of Interest

Authors declare that there is no conflict of interest.

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