

Separation Axioms in Neutrosophic Crisp Topological Spaces

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Abstract. The main idea of this research is to define a new neutrosophic crisp points in neutrosophic crisp topological space namely [NCP_N], the concept of neutrosophic crisp limit point was defined using [NCR], with some of its properties the separation axioms τ - $\hat{\tau}$ -space $\tau = \{0, 1, 2\}$ were constructed in neutrosophic crisp topological space using [NCP] and examined the relationship between them in details
Keywords: Neutrosophic crisp topological spaces, neutrosophic crisp limit point, separation axioms.

Introduction

Smarandache [1,2,3] introduced the notions of neutrosophic theory and introduced the neutrosophic components $\{ \hat{a}, \bar{a}, \check{a} \}$ which represent the membership indeterminacy, and non membership values respectively, where $[0, 1]^*$ is a non standard unit interval. In [4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20] many scientists presented the concepts of the neutrosophic set theory in their works. Salama et al. [21,22] provided natural foundations to put mathematical treatments for the neutrosophic pervasively phenomenon in our real world and for building new branches of neutrosophic mathematics.

Salama et al [23,24] put some basic concepts of the neutrosophic crisp set and their operations and because of their wide applications and their great flexibility to solve the problem we used these concepts to define new types of neutrosophic points, that we called neutrosophic crisp points [NCP_N].

Finally, we used the points [NCP_N] to define the concept of neutrosophic crisp limit point, with some of its properties and to construct the separation axioms τ - $\hat{\tau}$ -space $\tau = \{0, 1, 2\}$ in neutrosophic crisp topological and examined the relationship between them in details.

Throughout this paper (NCTS) means a neutrosophic crisp topological space. Also, simply we denote neighborhood by \hat{S}_x .

1 Basic Concepts

1.1 Definition [25]

Let $\hat{\tau}$ be a non empty fixed set. A neutrosophic crisp set [NCS for short] B is an object having the form $B = \langle B_1, B_2, B_3 \rangle$ where B_1, B_2 and B_3 are subsets of $\hat{\tau}$.

1.2 Definition [25]

The object having the form $B = \langle B_1, B_2, B_3 \rangle$ is called:

1. A neutrosophic crisp set of Type 1 [NCS/Type1] if satisfying

$$B_1 \cap B_2 \subseteq \hat{\tau}, B_1 \cap B_3 \subseteq \hat{\tau} \text{ and } B_2 \cap B_3 \subseteq \hat{\tau}$$

A neutrosophic crisp set of Type 2 [NCS/Type2] if satisfying

$$B_1 \cap B_2 \subseteq \hat{\tau}, B_1 \cap B_3 \subseteq \hat{\tau} \text{ and } B_2 \cap B_3 \subseteq \hat{\tau}, B_1, B_2, B_3 \subseteq \hat{\tau}.$$

A neutrosophic crisp set of Type 3 [NCS/Type3] if satisfying

$$B_1 \cap B_2 \cap B_3 \subseteq \hat{\tau}, B_1, B_2, B_3 \subseteq \hat{\tau}$$

1.3 Definition [25]

Types of NCSs $\hat{\tau}_R \rightarrow \hat{\tau}_R$ in $\hat{\tau}$ as follows:

1. $\hat{\tau}_R$ may be defined in many ways as a NCS as follows:

1. Type1 : $\hat{\tau}_R = \langle \hat{\tau}, \hat{\tau}, \hat{\tau} \rangle$!

2. Type2: $\hat{\tau}_R = \langle \hat{\tau}, \hat{\tau}, \hat{\tau} \rangle$!

3. Type3: $\hat{\tau}_R = \langle \hat{\tau}, \hat{\tau}, \hat{\tau} \rangle$!

4. Type4: $\hat{\tau}_R = \langle \hat{\tau}, \hat{\tau}, \hat{\tau} \rangle$!

2. $\hat{\tau}_R$ may be defined in many ways as a NCS as follows:

1. Type1: $\hat{\tau}_R = \langle \hat{\tau}, \hat{\tau}, \hat{\tau} \rangle$!

- 2. Type2: $\hat{\delta}_R = \hat{\delta}, \hat{\delta}, \hat{\delta} !$
- 3. Type3: $\hat{\delta}_R = \hat{\delta}, \hat{\delta}, \hat{\delta} !$
- 4. Type4: $\hat{\delta}_R = \hat{\delta}, \hat{\delta}, \hat{\delta} !$

1.4 Definition [25]

Let $\hat{\delta}$ be a nonempty set and the NCSs C & D in the form $\langle C_1, C_2, C_3 \rangle, \langle D_1, D_2, D_3 \rangle$ then we may consider two possible definitions for subsets $C \cap D$, may be defined in two ways :

- 1. $C \cap D = \langle C_1 \cap D_1, C_2 \cap D_2 \text{ and } D_3 \cap C_3 \rangle$
- 2. $C \cap D = \langle C_1 \cap D_1, D_2 \cap C_2 \text{ and } D_3 \cap C_3 \rangle$

1.5 Definition [25]

Let $\hat{\delta}$ be a nonempty set and the NCSs C & D in the form $\langle C_1, C_2, C_3 \rangle, \langle D_1, D_2, D_3 \rangle$ then:

- 1. $C \cdot D$ may be defined in two ways as a NCS as follows :
 - x $C \cdot D = \langle C_1 \cdot D_1, [C_2, D_2], [C_3, D_3] \rangle$
 - x $C \cdot D = \langle [C_1, D_1], [C_2 \cdot D_2], [C_3, D_3] \rangle$
- 2. C, D may be defined in two ways as a NCS as follows:
 - x $C, D = \langle [C_1, D_1], [C_2, D_2], [C_3 \cdot D_3] \rangle$
 - x $C, D = \langle [C_1, D_1], [C_2 \cdot D_2], [C_3 \cdot D_3] \rangle$

1.6 Definition [25]

A neutrosophic crisp topology (NCT) on a nonempty set $\hat{\delta}$ is a family $\hat{\tau}$ of neutrosophic crisp subsets in $\hat{\delta}$ satisfying the following axioms:

- 1. $\hat{\tau}_R \hat{\delta}_R \in \hat{\tau}$
- 2. $\hat{\tau} \cdot \hat{\tau} \in \hat{\tau}$
- 3. The union of any number of sets in $\hat{\tau}$ belong to $\hat{\tau}$

The pair $(\hat{\delta}, \hat{\tau})$ is said to be a neutrosophic crisp topological space (NCTS) in $\hat{\delta}$. Moreover The elements in $\hat{\tau}$ are said to be neutrosophic crisp open sets (NCOS), a neutrosophic crisp set F is closed (NCCS) iff its complement F^c is an open neutrosophic crisp set.

1.7 Definition [25]

Let $\hat{\delta}$ be a nonempty set and the NCS D in the form $\langle D_1, D_2, D_3 \rangle$. Then $\hat{\tau}^a$ may be defined in three ways as a NCS as follows:

$$\hat{\tau}^a = \langle \hat{\tau}_a, \hat{\tau}_a \rangle, \hat{\tau}^a = \langle D_3, D_2, D_1 \rangle \text{ or } \hat{\tau}^a = \langle D_3, \hat{\tau}_a, D_1 \rangle$$

1.8 Definition [25]

Let $(\hat{\delta}, \hat{\tau})$ be neutrosophic crisp topological space (NCTS). A be neutrosophic crisp set then The intersection of any neutrosophic crisp closed sets contained in $\hat{\delta}$ is called neutrosophic crisp closure of A (NC-Cl(A) for short).

2 Neutrosophic crisp limit point :

In this section, we will introduce the neutrosophic crisp limit points with some of its properties. This work contains an adjustment for the above mentioned definitions 1.4 & 1.5, this was necessary to homogeneous suitable results for the upgrade of this research.

2.1 Definition

Let $\hat{\delta}$ be a nonempty set and the NCSs C & D in the form $\langle C_1, C_2, C_3 \rangle, \langle D_1, D_2, D_3 \rangle$ then the additional new ways for the intersection, union and inclusion between $f \in \hat{\tau}$

$$C \cdot D = \langle [C_1 \cdot D_1], [C_2 \cdot D_2], [C_3 \cdot D_3] \rangle$$

$$C, D = \langle [C_1, D_1], [C_2, D_2], [C_3, D_3] \rangle$$

$$C \cap D = \langle C_1 \cap D_1, C_2 \cap D_2 \text{ and } C_3 \cap D_3 \rangle$$

2.2 Definition

For all x, y, z belonging to a nonempty set δ . Then the neutrosophic crisp points related to xy, z are defined as follows

- $x \check{s}_R \{x\}, \hat{I}, \hat{I} >$, is called a neutrosophic crisp point (C_R) in δ .
- $x \check{y}_R \hat{I}, < \neq \hat{I} >$, is called a neutrosophic crisp point (C_R) in δ .
- $x \check{c}_R \hat{I}, \hat{I} < \neq \hat{I} >$, is called a neutrosophic crisp point ($C_{R'}$) in δ .

The set of all neutrosophic crisp points : $C_R, C_R \acute{a} C_{R'}$; is denoted by C_R .

2.3 Definition

Let δ be a nonempty set and $x \acute{a} \check{y} \acute{a} \check{c}$. Then the neutrosophic crisp point:

- $x \check{s}_R$ is belonging to the neutrosophic crisp set B_{B_1, B_2, B_3} , denoted by $\check{s}_R \check{D}$, if $\check{s} \check{D}$ wherein \check{s}_R does not belong to the neutrosophic crisp set B denoted by $\check{s}_R \check{N}$, if $\check{s} \check{N}$
- $x \check{y}_R$ is belonging to the neutrosophic crisp set B_{B_1, B_2, B_3} , denoted by $\check{y}_R \check{D}$, if $\check{y} \check{D}$ In contrast \check{y}_R does not belong to the neutrosophic crisp set B , denoted by $\check{y}_R \check{N}$, if $\check{y} \check{N}$
- $x \check{c}_R$ is belonging to the neutrosophic crisp set B_{B_1, B_2, B_3} , denoted by $\check{c}_R \check{D}$, if $\check{c} \check{D}$ In contrast \check{c}_R does not belong to the neutrosophic crisp set B , denoted by $\check{c}_R \check{N}$, if $\check{c} \check{N}$

2.4 Remark

- If B_{B_1, B_2, B_3} is a NCS in a nonempty set δ then:
 $B \check{s}_R \check{L} O \check{s} \acute{a} \check{P} \check{B} \check{s}_R$ means that the component B doesn't contain \check{s}_R
- $B \check{y}_R \check{L} O \check{y} \acute{a} \check{P} \check{B} \check{y}_R$ means that the component B doesn't contain \check{y}_R
- $B \check{c}_R \check{L} O \check{c} \acute{a} \check{P} \check{B} \check{c}_R$ means that the component B doesn't contain \check{c}_R

2.5 Example

- If $B = O < f \acute{a} \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow$ is an NCS in $\delta \check{L} < f \acute{a} \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow$ then:
 $B \check{s}_R \check{L} O < \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \check{P} \check{B} \check{s}_R$
- $B \check{y}_R \check{L} O < f \acute{a} \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \check{P} \check{B} \check{y}_R$
- $B \check{c}_R \check{L} O < f \acute{a} \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \check{P} \check{B} \check{c}_R$

2.6 Remark

- If B_{B_1, B_2, B_3} is a NCS in a nonempty set δ then:
 $B \check{s}_R \check{L} O \check{s} \acute{a} \check{P} \check{B} \check{s}_R \check{L} O \check{s} \acute{a} \check{P} \check{B} \check{s}_R$
 $B \check{y}_R \check{L} O \check{y} \acute{a} \check{P} \check{B} \check{y}_R \check{L} O \check{y} \acute{a} \check{P} \check{B} \check{y}_R$
 $B \check{c}_R \check{L} O \check{c} \acute{a} \check{P} \check{B} \check{c}_R \check{L} O \check{c} \acute{a} \check{P} \check{B} \check{c}_R$

2.7 Definition

Let $(\delta \check{a} \check{y} \acute{a} \check{c})$ be NCTS $\acute{a} \check{D} C_R \langle \bullet \delta \acute{a} \rangle$ neutrosophic crisp set $B_{B_1, B_2, B_3} \check{D} \hat{I}$ is called neutrosophic crisp open nhd of $\check{y} \acute{a} \check{c}$ in $(\delta \check{a} \check{y} \acute{a} \check{c})$; if \check{D} .

2.8 Definition

Let $(\delta \check{a} \check{y} \acute{a} \check{c})$ be NCTS $\acute{a} \check{D} C_R \langle \bullet \delta \acute{a} \rangle$ neutrosophic crisp set $B_{B_1, B_2, B_3} \check{D} \hat{I}$ is called neutrosophic crisp nhd of $\check{y} \acute{a} \check{c}$ in $(\delta \check{a} \check{y} \acute{a} \check{c})$; if there is neutrosophic crisp open set A_{A_1, A_2, A_3} containing $\check{y} \acute{a} \check{c}$ such that C .

2.9 Note

Every neutrosophic crisp open nhd of any point $\check{y} \acute{a} \check{c} \in C_R \langle \bullet \delta \acute{a} \rangle$ is neutrosophic crisp nhd of $\check{y} \acute{a} \check{c}$, but in general the inverses not true, the following example illustrates this fact.

2.10 Example

- If $\delta \check{L} \check{s} \acute{a} \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I} \check{L} \check{D}_R \hat{I} \acute{a} \acute{a} \acute{a} \check{y} \acute{a} \check{f} \Rightarrow$
- $A \check{s} \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I}, \hat{I} >, B \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I}, \hat{I} >, G \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I}, \hat{I} >$
- If we take $U \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I}$.
- Then $G \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I}, \hat{I} >$ is an open set containing $\check{L} \check{s}_R \check{y} \acute{a} \check{c} \check{y} \acute{a} \check{f} \Rightarrow \hat{I}, \hat{I} >$ and $G \in C$. That is U is a neutrosophic crisp nhd of $\check{y} \acute{a} \check{c}$ in $(\delta \check{a} \check{y} \acute{a} \check{c})$; while it is not a neutrosophic crisp open nhd of $\check{y} \acute{a} \check{c}$.

2.11 Definition

Let $(\mathcal{O}, \mathcal{A})$ be NCTS and $\langle B_1, B_2, B_3 \rangle$ be NCS of \mathcal{O} . A neutrosophic crisp point $P \in C_R$ in \mathcal{O} is called a neutrosophic crisp limit point of $\langle B_1, B_2, B_3 \rangle$ iff every neutrosophic crisp open set containing P must contains at least one neutrosophic crisp point of B different from P . It is easy to say that the point P is not neutrosophic crisp limit point of B if there is a neutrosophic crisp open set U of \mathcal{O} and $\hat{e} \in U$; $L \hat{I}_R$.

2.12 Definition

The set of all neutrosophic crisp limit points of a neutrosophic crisp set B is called neutrosophic crisp derived set of B , denoted by $\hat{e}B$; $L \hat{I}_R$.

2.13 Example

If $\mathcal{O} = L \langle \hat{a}, \hat{b} \rangle$, $\hat{I} = L \langle \hat{a}_R, \hat{a} \rangle = A \langle \hat{a}, \hat{a} \rangle$, $B = \langle \hat{a}, \hat{a} \rangle$, $G = \langle \hat{a}, \hat{a} \rangle$. If we take $D = \langle \hat{a}, \hat{a} \rangle$, Then $P = \langle \hat{a}, \hat{a} \rangle$ is the only neutrosophic crisp limit point of D . i.e. $\hat{e}D = \langle \hat{a}, \hat{a} \rangle$.

2.14 Remarks

x Let B be any neutrosophic crisp set of \mathcal{O} , If $P = \langle x \rangle, \hat{I} \in \hat{I}$ in any NCT space (\mathcal{O}, \hat{I}) , then $P \in \hat{e}B$; $L \hat{I}_R$.

x Let B be any neutrosophic crisp set of \mathcal{O} , the following facts is true:

$$\hat{e}(\hat{e}B) = \hat{e}B, \quad A \subseteq \hat{e}B, \quad \text{and sometimes } \hat{e}B = \hat{I}_R \text{ or } \hat{e}B = M \hat{I}_R$$

x In any NCT space (\mathcal{O}, \hat{I}) ; we have $\hat{e}B = \hat{I}_R$.

2.15 Theorem

Let $(\mathcal{O}, \mathcal{A})$ be NCTS and $\langle B_1, B_2, B_3 \rangle$ be a neutrosophic crisp set of \mathcal{O} , then B is neutrosophic crisp closed set (NCCS for short) iff $\hat{e}B = B$; C .

Proof

Let B be NCCS, then $\mathcal{O} \setminus B$ is neutrosophic crisp open set (NCOS for short) this implies that for each neutrosophic crisp point $\hat{e} \in \mathcal{O} \setminus B$, there is a neutrosophic crisp open set of P and $C = \mathcal{O} \setminus B$.

Since $\hat{e} \in \mathcal{O} \setminus B$; $L \hat{I}_C$ then \hat{e} is not neutrosophic crisp limit point of B , thus $\hat{e} \in L \hat{I}_C$, which implies that $\hat{e} \in \mathcal{O} \setminus B$; Hence $\hat{e}B = \mathcal{O} \setminus B$; C .

Conversely, assume that $\hat{e}B = B$; implies that B is not neutrosophic crisp limit point of \mathcal{O} , hence, there is a neutrosophic crisp open set of P and $\hat{e} \in L \hat{I}_C$ which means that $C = \mathcal{O} \setminus B$; and since $\mathcal{O} \setminus B$ is a neutrosophic crisp open set Hence $\hat{e}B = \mathcal{O} \setminus B$; C .

2.16 Theorem

Let $(\mathcal{O}, \mathcal{A})$ be NCTS, B, G be neutrosophic crisp sets of \mathcal{O} , then the following properties hold:

- (1) $\hat{e}(B \cap G) = \hat{e}B \cap \hat{e}G$
- (2) If $C = \hat{e}B$, then $\hat{e}C = C$; $C = \hat{e}C$;
- (3) $\hat{e}(\hat{e}B) = \hat{e}B$; $C = \hat{e}C$; $\hat{e}C = C$;
- (4) $\hat{e}(\hat{e}B) = \hat{e}B$; $L \hat{I}_C = \hat{e}B$; $\hat{e}B = L \hat{I}_C$;

Proof (1) the proof is, directly.

Proof (2)

Assume that $C = \hat{e}B$; be a neutrosophic crisp set containing a neutrosophic crisp point $\hat{e} \in C$, then by definition 2.11, for each neutrosophic crisp open set of C , we have $\hat{e} \in M \hat{I}_C$ but $C = \hat{e}B$, hence $\hat{e} \in M \hat{I}_C$ this means that $\hat{e} \in B$; $\hat{e} \in C$;

Proof (3)

$$\text{Since } \hat{e} \in C, \text{ then by (2) } \hat{e} \in \hat{e}C; \quad C = \hat{e}C; \quad (1)$$

$$\hat{e} \in C, \text{ implies } \hat{e} \in \hat{e}C; \quad C = \hat{e}C; \quad (2)$$

From (1) & (2) $\hat{e} \in \hat{e}C$; $C = \hat{e}C$; $\hat{e}C = C$;

Proof (4)

Let $\hat{e} \in \mathcal{O} \setminus B$; $\hat{e} \in L \hat{I}_C$; then either $\hat{e} \in \mathcal{O} \setminus B$; and $\hat{e} \in L \hat{I}_C$; then there is a neutrosophic crisp open set of \hat{e} and $\hat{e} \in L \hat{I}_C$ and $\hat{e} \in L \hat{I}_C$ this implies that $\hat{e} \in L \hat{I}_C$; i.e. $\hat{e} \in \mathcal{O} \setminus B$; hence $\hat{e} \in \mathcal{O} \setminus B$; $\hat{e} \in L \hat{I}_C$; $\hat{e} \in \mathcal{O} \setminus B$;

X is a τ - \hat{I}_2 -space if $\hat{E}_{\check{R}_Y} \subseteq \check{R}_Y \cdot \hat{\delta}$ is a neutrosophic crisp open set in G_1, G_2 such that $\check{R}_Y \cdot G_1, \check{R}_Y \cdot G_2$ and $\check{R}_Y \cdot G_2, \check{R}_Y \cdot G_1$ with $G_1 \in G_2 \subseteq L \hat{I} \hat{a}$

3.2 Definition

A neutrosophic crisp topological space $(\hat{\delta}, \hat{a})$ is called:

- X is \hat{I}_0 -space if $(\hat{\delta}, \hat{a})$ is $\hat{\epsilon}$ - \hat{I}_0 -space, $\hat{\sigma}$ - \hat{I}_0 -space $f \cdot \dagger \tau$ - \hat{I}_0 -space
- X is \hat{I}_1 -space if $(\hat{\delta}, \hat{a})$ is $\hat{\epsilon}$ - \hat{I}_1 -space, $\hat{\sigma}$ - \hat{I}_1 -space $f \cdot \dagger \tau$ - \hat{I}_1 -space
- X is \hat{I}_2 -space if $(\hat{\delta}, \hat{a})$ is $\hat{\epsilon}$ - \hat{I}_2 -space, $\hat{\sigma}$ - \hat{I}_2 -space $f \cdot \dagger \tau$ - \hat{I}_2 -space

3.3 Remark

For a neutrosophic crisp topological space $(\hat{\delta}, \hat{a})$;

- X Every \hat{I}_0 -space is $\hat{\epsilon}$ - \hat{I}_0 -space
- X Every \hat{I}_0 -space is $\hat{\sigma}$ - \hat{I}_0 -space
- X Every \hat{I}_0 -space is τ - \hat{I}_0 -space

Proof the proof is directly from definition 3.2.

The inverse of remark 3.3 is not true the following example explain this state.

3.4 Example

If $\hat{\delta} \subseteq L \hat{a} \Rightarrow \hat{\epsilon} \hat{I}_5 \hat{L} \hat{\delta} \hat{R} \hat{a} \hat{R} \hat{a} \Rightarrow \hat{\epsilon} \hat{I}_6 \hat{L} \hat{\delta} \hat{R} \hat{a} \hat{R} \hat{a} \Rightarrow \hat{\epsilon} \hat{I}_7 \hat{L} \hat{\delta} \hat{R} \hat{a} \hat{R} \hat{a} \Rightarrow A \{x\}, \hat{I}, \hat{I}\rangle, B \hat{I}, \{y\}, \hat{I}\rangle, G \hat{I}, \hat{I}, \{x\}\rangle$, Then $(\hat{\delta}, \hat{I}_5)$ is $\hat{\epsilon}$ - \hat{I}_0 -space but is not \hat{I}_0 -space $(\hat{\delta}, \hat{I}_6)$ is $\hat{\sigma}$ - \hat{I}_0 -space but is not \hat{I}_0 -space $(\hat{\delta}, \hat{I}_7)$ is τ - \hat{I}_0 -space but is not \hat{I}_0 -space

3.5 Remark

For a neutrosophic crisp topological space $(\hat{\delta}, \hat{a})$;

- X Every \hat{I}_1 -space is $\hat{\epsilon}$ - \hat{I}_1 -space
- X Every \hat{I}_1 -space is $\hat{\sigma}$ - \hat{I}_1 -space
- X Every \hat{I}_1 -space is τ - \hat{I}_1 -space

Proof the proof is directly from definition 3.2.

The inverse of remark (3.5) is not true as it is shown in the following example,

3.6 Example

If $\hat{\delta} \subseteq L \hat{a} \Rightarrow \hat{\epsilon} \hat{I}_5 \hat{L} \hat{\delta} \hat{R} \hat{a} \hat{R} \hat{a} \Rightarrow \hat{\epsilon} \hat{I}_6 \hat{L} \hat{\delta} \hat{R} \hat{a} \hat{R} \hat{a} \Rightarrow A \{x\}, \langle \hat{I}, \hat{I} \rangle, B \langle \hat{I}, \hat{I} \rangle, G \hat{I}, \hat{I}, \{x\}\rangle, F \hat{I}, \hat{I}, \{y\}\rangle$, Then $(\hat{\delta}, \hat{I}_5)$ is $\hat{\epsilon}$ - \hat{I}_1 -space but is not \hat{I}_1 -space $(\hat{\delta}, \hat{I}_6)$ is $\hat{\sigma}$ - \hat{I}_1 -space but is not \hat{I}_1 -space $(\hat{\delta}, \hat{I}_7)$ is τ - \hat{I}_1 -space but is not \hat{I}_1 -space

3.7 Remark

For a neutrosophic crisp topological space $(\hat{\delta}, \hat{a})$;

- X Every \hat{I}_2 -space is $\hat{\epsilon}$ - \hat{I}_2 -space
- X Every \hat{I}_2 -space is $\hat{\sigma}$ - \hat{I}_2 -space
- X Every \hat{I}_2 -space is τ - \hat{I}_2 -space

Proof the proof is directly from definition 3.2.

The inverse of remark (3.7) is not true as it is shown in the example (3.6).

3.8 Remark

For a neutrosophic crisp topological space $(\hat{\delta}, \hat{a})$;

- X Every \hat{I}_1 -space is \hat{I}_0 -space
- X Every \hat{I}_2 -space is \hat{I}_1 -space

Proof the proof is directly.

The inverse of remark (3.8) is not true as it is shown in the following example:

3.9 Example

If $L \hat{a} \Rightarrow \hat{I} \hat{L} \hat{\delta} \hat{R} \hat{a} \hat{R} \hat{a} \hat{R} \hat{a} = A \{x\}, \hat{I}, \hat{I}\rangle, B \hat{I}, \{y\}, \hat{I}\rangle, G \hat{I}, \hat{I}, \{x\}\rangle$, Then $(\hat{\delta}, \hat{I})$ is \hat{I}_0 -space but not \hat{I}_1 -space

Conclusion

- We defined a new neutrosophic crisp points in neutrosophic crisp topological space
- We introduced the concept of neutrosophic crisp limit point, with some of its properties
- We constructed the separation axioms T_i -space, $i = 0, 1, 2$ in neutrosophic crisp topological and examined the relationship between them in details

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