A Note On Neutrosophic Chaotic Continuous Functions

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Abstract. Many real time problems are based on uncertainity and chaotic environment. To demonstrate this ambiguous suituation more precisely we intend to amalgamate the ideas of chaos theory and neutrosophy. Neutrosophy is a flourishing arena which conceptualizes the notions of true, falsity and indeterminacy attributes of an event. Chaos theory is another branch which brings out the concepts of periodic point, orbit and sensitive of a set. Hence in this paper we focus on the introducing the idea of chaotic periodic points, orbit sets, sensitive functions under neutrosophic settings. We start with defining a neutrosophic chaotic space and enlist its properties, As a futher extension we coin neutrosophic chaotic continuous functions and discuss its charaterizations and their interrelationships. We have also illustrated the above said concepts with suitable examples.

Keywords: Neutrosophic periodic points, neutrosophic orbit sets, neutrosophic chaotic sets, neutrosophic sensitive functions, neutrosophic orbit extremally disconnected spaces.

1 Introduction

The introduction of the idea of fuzzy set was introduced in the year 1965 by Zadeh[16]. He proposed that each element in a fuzzy set has a degree of membership. Following this concept K.Atanassov[1,2,3] in 1983 introduced the idea of intuitionistic fuzzy set on a universe X as a generalization of fuzzy set. Here besides the degree of membership a degree of non-membership for each element is also defined. Smarandache[11,12] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. The significance of neutrosophy is that it finds and indispensible place in decision making. Several authors[7, 8, 9, 10] have done remarkable achievements in this area. One of the prime discoveries of the 20th century which has been widely investigated with significant progress and achievements is the theory of Chaos and fractals.It has become an exciting emerging interdisciplinary area in which a broad spectrum of technologies and methodologies have emerged to deal with large-scale, complex and dynamical systems and problems. In 1989, R.L. Deveney[4] defined chaotic function in general metric space. A breakthrough in the conventional general topology was intiated by T. Thrivikraman and P.B. Vinod Kumar[15] by defining Chaos and fractals in general topological spaces. M. Kousalyaparasakthi, E. Roja, M.K. Uma[6] introduced the above said idea to intuitionistic chaotic continuous functions. Tethering around this concept we introduce neutrosophic periodic points, neutrosophic orbit sets, neutrosophic sensitive functions, neutrosophic clopen chaotic sets and neutrossophic chaos spaces. The concepts of neutrosophic chaotic continuous functions, neutrosophic chaotic* continuous functions, neutrosophic chaotic** continuous functions, neutrosophic chaotic*** continuous functions are introduced and studied. Some interrelation are discussed with suitable examples. Also the concept of neutrosophic orbit extremally disconnected spaces, neutrosophic chaotic extremally disconnected spaces, neutrosophic orbit irresolute function are discussed.

2 Preliminaries

2.1 Definition [12]

Let X be a non empty set. A neutrosophic set (NS for short) V is an object having the form V = <x, V^1, V^2, V^3> where V^1, V^2, V^3 represent the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X to the set V.

2.2 Definition [12]

Let X be a non empty set, U = <x, U^1, U^2, U^3> and V = <x, V^1, V^2, V^3> be neutrosophic sets on X, and let {V_i: i ∈ J} be an arbitrary family of neutrosophic sets in X, where V^i = <x, V^1, V^2, V^3>
(i) U ⊆ V ⇔ $U^1 \subseteq V^1$, $U^2 \supseteq V^2$ and $U^3 \supseteq V^3$

(ii) U = V ⇔ U ⊆ V and V ⊆ U.
(iii) $\overline{V}$ = <x, V^3, V^2, V^1>
(iv)U∩V=<x, $U^1 \cap V^1$, $U^2 \cup V^2$, $U^3 \cup V^3$>
(v) U∪V=<x, $U^1 \cup V^1$, $U^2 \cap V^2$, $U^3 \cap V^3$>
(vi) $\cup V_i$ = <x, $\cup V_i^1$, $\cap V_i^2$, $\cap V_i^3$>
(vii) $\cap V_i$ = <x, $\cap V_i^1$, $\cup V_i^2$, $\cup V_i^3$>
(viii)U − V = U ∩ $\overline{V}$.
(ix) φ_N = <x, φ, X, X>; X_N = <x, X, φ, φ >.

2.3 Definition [14]

A neutrosophic topology (NT for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:
(i) φ_N, $X_N \in \tau$.
(ii) $T_1 \cap T_2 \in \tau$ for any $T_1, T_2 \in \tau$.
(iii) $\cup T_i \in \tau$ for any arbitrary family $\{T_i : i \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is called a neutrosophic open set (NOS for short) in X. The complement V of a neutrosophic open set V is called a neutrosophic closed set (NCS for short) in X.

2.4 Definition [14]

Let (X, τ) be a neutrosophic topological space and V = <X, V_1, V_2, V_3> be a set in X. Then the closure and interior of V are defined by
Ncl(V) = ∩{M : M is a neutrosophic closed set in X and V ⊆ M},
Nint(V) = ∪{N : N is a neutrosophic open set in X and N ⊆ V}.
It can be also shown that Ncl(V) is a neutrosophic closed set and Nint(V) is a neutrosophic open set in X, and V is a neutrosophic closed set in X iff Ncl(V) = V; and V is a neutrosophic open set in X iff Nint(V) = V.
Where Ncl - neutrosophic closure and Nint – neutrosophic interior

2.5 Definition [5]

(a) If V = <y,V^1,V^2,V^3> is a neutrosophic set in Y , then the preimage of V under f, denoted by $f^{-1}(V)$, is the neutrosophic set in X defined by $f^{-1}(V)$ = <x,$f^{-1}(V^1)$,$f^{-1}(V^2)$,$f^{-1}(V^3)$>.
(b) If U = <x,U^1,U^2,U^3> is a neutrosophic set in X, then the image of U under f, denoted by f(U), is the neutrosophic set in Y defined by f(U) = <y,f(U^1),f(U^2),Y-f(X-U^3)> where

$$f(U^1)=\begin{cases} \sup_{x\in f^{-1}(y)} U^1 & if \;\; f^{-1}(y)\neq \phi \\ 0 & otherwise \end{cases}$$

$$f(U^2)=\begin{cases} \sup_{x\in f^{-1}(y)} U^2 & if \;\; f^{-1}(y)\neq \phi \\ 0 & otherwise \end{cases}$$

$$Y\text{-}f(X\text{-}U^3)=\begin{cases} \inf_{x\in f^{-1}(y)} U^3 & if \;\; f^{-1}(y)\neq \phi \\ 1 & otherwise \end{cases}$$

2.6 Definition [13]
Let (X, τ) and (Y,σ) be any two neutrosophic topological spaces and let f : X → Y be a function. Then f is said to be continuous if and only if the preimage of each neutrosophic set in σ is a neutrosophic set in τ.

2.7 Definition [13]
Let (X, τ) and (Y,σ) be two neutrosophic topological spaces and let f : (X, τ) → (Y,σ) be a function. Then f is said to be open iff the image of each neutrosophic set in τ is a neutrosophic set in σ.

2.8 Definition [4]
Orbit of a point x in X under the mapping f is $O_f(x)$={x, f(x), $f^2(x)$,...}

2.9 Definition [4]
x in X is called a periodic point of f if $f^n(x)$ =x, for some $n \in Z^+$. Smallest of these n is called period of x.

2.10 Definition [4]

f is sensitive if for each $\delta>0\ \exists$ (a) $\varepsilon>0$ (b) $y\in X$ and (c) $n\in Z_+\ \ni d(x,y)<\delta$ and $d(f^n(x),f^n(y))>\varepsilon$.

2.11 Definition [4]

f is chaotic on (X,d) if (i) Periodic points of f are dense in X (ii) Orbit of x is dense in X for some x in X and (iii) f is sensitive.

2.12 Definition [15]

Let (X, τ) be a topological space and f : (X, τ) → (X, τ) be continuous map. Then f is sensitive at x ∈ X if given any open set U containing x ∃ (i) y ∈ U (ii) $n\in Z^+$ and (iii) an open set V ∍ $f^n(x)\in V$, $f^n(y)\notin cl(V)$. We say that f is sensitive on a F if f|F is sensitive at every point of F.

2.13 Definition [15]

Let (X, τ) be a topological space and F ∈ K(X). Let f : F → F be a continuous. Then f is chaotic on F if

(i) $cl(O_f(x))$ = F for some x ∈ F.
(ii) periodic points of f are dense in F.
(iii) f ∈ S(F).

2.14 Definition [15]

(i) C(F) = {f : F → F | f is chaotic on F} and (ii) CH(X) = {F ∈ NK(X) | C(F) ≠ φ}.

2.15 Definition [15]

A topological space (X, τ) is called a chaos space if CH(X) ≠ φ. The members of CH(X) are called chaotic sets.

3 Characterizations of neutrosophic chaotic continuous functions

3.1 Definition

Let (X, τ) be a neutrosophic topological space and $V=<X,V^1,V^2,V^3>$ be a neutrosophic set of X.
(i) Ncl(V) denotes neutrosophic closure of V.
(ii) Nint(V) denotes neutrosophic interior of V.
(iii) NK(X) denotes the collection of all non empty neutrosophic compact sets of X.
(iv) clopen denotes closed and open

3.2 Definition

Let (X, τ) be a neutrosophic topological space. An orbit of a point x in X under the function f : (X, τ) → (X, τ) is denoted and defined as $O_f(x) = \{x,f^1(x),f^2(x),...f^n(x)\}$ for x ∈ X and $n\in Z^+$.

3.3 Example

Let X = {p,q,r}. Let f : X → X be a function defined by f(p) = q, f(q) = r, and f(r) = p. If n = 1, then the orbit points $O_f(p)$ = {p,q}, $O_f(q)$ = {q,r} and $O_f(r)$ = {p,r}. If n = 2, then the orbit points $O_f(p)$ = X, $O_f(q)$ = X and $O_f(r)$ = X.

3.4 Definition

Let (X, τ) be a neutrosophic topological space. A neutrosophic orbit set in X under the function f : (X, τ) → (X, τ) is denoted and defined as $NO_f(x) = <x,O_{fT}(x),O_{fI}(x),O_{fF}(x)>$ for x ∈ X.

3.5 Example

Let X = {p,q,r,s}. Let f : X → X be a function defined by f(p) = <q,s,q>, f(q) = <s,p.r>, f(r) = <p,q,s> and f(s) = <r,r,p>. If n = 1, then the neutrosophic orbit sets $NO_f(p)$ = <x,{p,q},{p,s},{p,q}>, $NO_f(q)$ = <x,{q,s},{q,p},{q,r}>, $NO_f(r)$ = <x,{p,r},{q,r},{r,s}> and $NO_f(s)$ = <x,{r,s},{r,s},{p,s}>. If n = 2, then the neutrosophic orbit sets $NO_f(p)$ = <x,{p,q,s},{p,r,s},{p,q,r}>, $NO_f(q)$ = <x,{q,r,s},{p,q,s},{q,r,s}>, $NO_f(r)$ = <x,{p,q,r},{p,q,r},{p,r,s}> and $NO_f(s)$ =<x,{p,r,s},{q,r,s},{p,q,s}>. If n = 3, then the neutrosophic orbit sets $NO_f(a)$ = <x,X,X,X>, $NO_f(b)$ =<x,X,X,X>, $NO_f(c)$ = <x,X,X,X>and $NO_f(d)$ =<x,X,X,X>.

3.6 Definition

Let (X, τ) be a neutrosophic topological space and f : (X, τ) → (X, τ) be a neutrosophic continuous function. Then f is said to be neutrosophic sensitive at x ∈ X if given any neutrosophic open set $U=<x,U^1,U^2,U^3>$ containing x ∃ a neutrosophic open set $V=<x,V^1,V^2,V^3>$ ∍ $f^n(x)\in V$, $f^n(y)\notin Ncl(V)$ and y ∈ U, $n\in Z^+$. We say that f is neutrosophic sensitive on a neutrosophic compact set $F=<x,F^1,F^2.F^3>$ if f|F is neutrosophic sensitive at every point of F.

3.7 Example

Let X = {p,q,r,s}. Then the neutrosophic sets P, Q, R and S are defined by P = <x,{p,r,s},{p,q,r},{p,r,s}>, Q = <x,{r,s},{p,r},{p,s}>, R = <x,{r,s},{p,q,r},{p,r,s}> and S = <x,{p,r,s},{p,r},{p,s}>. Then the family τ = $\{X_N,\varphi_N,P,Q,R,S\}$ is neutrosophic topology on X. Clearly, (X, τ) is an neutrosophic topological space. Let f : (X,

$\tau) \rightarrow (X, \tau)$ be a function defined by f(p) = <r,q,s> f(q) = <s,s,r>, f(r) =< q,p,p> and f(s) = <p,r,q>. Let x = p and y = r. If n = 1,3,5, then the neutrosophic open set P = <x,{p,r,s},{p,q,r},{p,r,s}> containing x there exists an neutrosophic open set R = <x,{r,s},{p,q,r},{p,r,s}> such that $f^n(x) \in R, f^n(y) \notin Ncl(R)$ and $y \in P$. Hence the function f is called neutrosophic sensitive.

3.8 Notation

Let (X, τ) be a neutrosophic topological space. Let $F = \langle x,F^1,F^2,F^3\rangle \subseteq X_N$ then $S(F) = \langle x,S(F)^1,S(F)^2,S(F)^3\rangle$ where $S(F)^1$ = {f | f is neutrosophic sensitive on F}, $S(F)^2$ = {f | f is indeterminacy neutrosophic sensitive on F} and $S(F)^3$ = {f | f is not neutrosophic sensitive on F}.

3.9 Definition

Let (X, τ) be a two neutrosophic topological space. Let $f : (X, \tau) \rightarrow (X, \tau)$ be a function. A neutrosophic periodic set is denoted and defined as $NP_f(x) = \langle x,\{x \in X \mid f^n_T(x) = x\},\{x \in X \mid f^n_I(x) = x\},\{x \in X \mid f^n_F(x) = x\}\rangle$

3.10 Example

Let X = {p,q,r}. Let $f : X \rightarrow X$ be a function defined by f(p) = <p,q,r>, f(q) = <r,p,q> and f(r) = <q,r,p>. If n = 1, then the neutrosophic periodic set $NP_f(p)$ = <x,{p},{q},{r}>, $NP_f(q)$ = <x,{r},{p},{q}>and $NP_f(q)$ = <x,{q},{r},{p}>. If n = 2, then the neutrosophic periodic sets $NP_f(p)$ = <x,{p},{p},{p}>, $NP_f(q)$ = <x,{q},{q},{q}> and $NP_f(r)$ = <x,{r},{r},{r}>.

3.11 Definition

Let (X, τ) be a neutrosophic topological space. A neutrosophic set $V = \langle X,V^1,V^2,V^3\rangle$ of X is said to be a neutrosophic dense in X, if Ncl(V) = X.

3.12 Definition

Let (X, τ) be a neutrosophic topological space and $F = \langle x,F^1,F^2,F^3\rangle \in NK(X)$. Let $f : F \rightarrow F$ be a neutrosophic continuous function. Then f is said to be neutrosophic chaotic on F if

(i) $Ncl(NO_f(x)) = F$ for some $x \in F$.
(ii) neutrosophic periodic points of f are neutrosophic dense in F. That is, $Ncl(NP_f(x)) = F$.
(iii) $f \in S(F)$.

3.13 Notation

Let (X, τ) be a neutrosophic topological space then $C(F) = \langle x,C(F)^1,C(F)^2,C(F)^3\rangle$ where $C(F)^1 = \{f : F \rightarrow F \mid$ f is neutrosophic chaotic on F}, $C(F)^2 = \{f : F \rightarrow F \mid$ f is indeterminacy neutrosophic chaotic on F},and $C(F)^3 = \{f : F \rightarrow F \mid$ f is not neutrosophic chaotic on F}.

3.14 Notation

Let (X, τ) be a neutrosophic topological space then $CH(X) = \{F = \langle x,F^1,F^2,F^3\rangle \in NK(X) \mid C(F) \neq \varphi\}$.

3.15 Definition

A neutrosophic topological space (X, τ) is called a neutrosophic chaos space if $CH(X) \neq \varphi$. The members of CH(X) are called neutrosophic chaotic sets.

3.16 Definition

Let (X, τ) be a neutrosophic topological space. A neutrosophic set $V = \langle x,V^1,V^2,V^3\rangle$ is neutrosophic clopen if it is both neutrosophic open and neutrosophic closed.

3.17 Definition

Let (X, τ) be a neutrosophic topological space.

(i) A neutrosophic open orbit set is a neutrosophic set which is both neutrosophic open and neutrosophic orbit.
(ii) A neutrosophic closed orbit set is a neutrosophic set which is both neutrosophic closed and neutrosophic orbit.
(iii) A neutrosophic clopen orbit set is a neutrosophic set which is both neutrosophic clopen and neutrosophic orbit.

3.18 Definition

Let (X, τ) be a neutrosophic topological space.

(i) A neutrosophic open chaotic set is a neutrosophic set which is both neutrosophic open and neutrosophic chaotic.
(ii) A neutrosophic closed chaotic set is a neutrosophic set which is both neutrosophic closed and neutrosophic chaotic.
(iii) A neutrosophic clopen chaotic set is a neutrosophic set which is both neutrosophic clopen and neutrosophic chaotic.

3.19 Definition

Let (X, τ) and (X,σ) be any two neutrosophic chaos spaces. A function $f : (X, \tau) \rightarrow (X,\sigma)$ is said to be neutrosophic chaotic continuous if for each periodic point x ∈ X and each neutrosophic clopen chaotic set F = <x,F^1,F^2,F^3> of f(x) ∃ a neutrosophic open orbit set $NO_f(x)$ of the periodic point x ∍ f($NO_f(x)$) ⊆ F.

3.20 Example

Let X={p,q,r,s}. Then the neutrosophic sets M,N,O,P,Q and R are defined by M=<x,{q,r},{r},{p,r}>, N=<x,{p},{p,q},{p,s}>, O=<x,{p,q,r},φ,{p}>, P=<x,φ,{p,q,r},{p,r,s}>, Q=<x,{p,q,r},{r},{p}>, R=<x,{p},{r},{p,q,r}>. Let τ={X_N,$φ_N$,M,N,O,P} and σ = {X_N,$φ_N$,Q,R} be a neutrosophic topologies on X. Clearly (X, τ) and (X,σ) be any two neutrosophic chaos spaces. The function f : (X, τ) → (X, σ) is defined by f(p) = <p,q,s> f(q) = <r,s,r>, f(r) =< q,r,p> and f(s) = <s,p,q>. Now the function f is called neutrosophic chaotic continuous.

3.21 Theorem

Let (X, τ) and (X,σ) be any two neutrosophic chaos spaces. Let f : (X, τ) → (X,σ) be a function. Then the following statements are equivalent:

(i) f is neutrosophic chaotic continuous.
(ii) Inverse image of every neutrosophic clopen chaotic set of (X,σ) is a neutrosophic open orbit set of (X, τ).
(iii) Inverse image of every neutrosophic clopen chaotic set of (X,σ) is a neutrosophic clopen orbit set of (X, τ).

Proof

(i)⇒ (ii) Let F = <x,F^1,F^2,F^3> be a neutrosophic clopen chaotic set of (X, σ) and the periodic point x ∈ $f^{-1}(F)$. Then f(x) ∈ F. Since f is neutrosophic chaotic continuous, ∃ a neutrosophic open orbit set $NO_f(x)$ of (X, τ) ∍ x ∈ $NO_f(x)$, f($NO_f(x)$) ⊆ F. That is, x ∈ $NO_f(x)$ ⊆ $f^{-1}(F)$. Now, $f^{-1}(F)$ = ∪{$NO_f(x)$: x ∈ $f^{-1}(F)$}. Since $f^{-1}(F)$ is union of neutrosophic open orbit sets. Therefore, $f^{-1}(F)$ is an neutrosophic open orbit set.

(ii) ⇒ (iii) Let F be a neutrosophic clopen chaotic set of (X, σ). Then X − F is also a neutrosophic clopen chaotic set, By (ii) $f^{-1}(X - F)$ is neutrosophic open orbit in (X, τ). So X − $f^{-1}(F)$ is a neutrosophic open orbit set in (X, τ). Hence, $f^{-1}(F)$ is neutrosophic closed orbit in (X, τ). By (ii), $f^{-1}(F)$ is a neutrosophic open orbit set of (X, τ). Therefore, $f^{-1}(F)$ is both neutrosophic open orbit and neutrosophic closed orbit in (X, τ). Hence, $f^{-1}(F)$ is a neutrosophic clopen orbit set of (X, τ).

(iii) ⇒ (i) Let x be a periodic point, x ∈ X and F be a neutrosophic clopen chaotic set containing f(x) then $f^{-1}(F)$ is a neutrosophic open orbit set of (X, τ) containing x and f($f^{-1}(F)$) ⊆ F. Hence, f is neutrosophic chaotic continuous.

3.22 Definition

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. A function f : (X, τ) → (X, σ) is said to be neutrosophic chaotic* continuous if for each periodic point x ∈ X and each neutrosophic closed chaotic set F containing f(x), ∃ neutrosophic open orbit set $NO_f(x)$ containing x ∍ f(Ncl($NO_f(x)$)) ⊆ F.

3.23 Theorem

A neutrosophic chaotic continuous function is a neutrosophic chaotic* continuous function.

Proof Since f is a neutrosophic chaotic continuous function, F is a neutrosophic clopen chaotic set containing f(x), ∃ a neutrosophic open orbit set $NO_f(x)$ containing x ∍ f($NO_f(x)$) ⊆ F. Then $f^{-1}(F)$ is a neutrosophic clopen chaotic set of (X, σ). By (iii) of Theorem 3.21., $f^{-1}(F)$ is a neutrosophic clopen orbit set in (X, τ). Therefore, F is a neutrosophic closed chaotic set containing f(x) and $f^{-1}(F)$ is a neutrosophic open orbit set ∍ f($f^{-1}(F)$) ⊆ F. Since $f^{-1}(F)$ is neutrosophic closed orbit set, Ncl($f^{-1}(F)$) = $f^{-1}(F)$. This implies that, f(Ncl($f^{-1}(F)$)) ⊆ F. Hence, f is a neutrosophic chaotic* continuous function.

3.24 Remark

The converse of Theorem 3.23. need not be true as shown in Example 3.25.

3.25 Example

Let X={p,q,r,s}. Then the neutrosophic sets M,N,O,P,Q,R,S and T are defined by M=<x,{p,r},{q,r},{r}>, N=<x,{r},{q},{p,q,r}>, O=<x,{r},{q,r},{p,q,r}>, P=<x,{p,r},{q},{r}>, Q=<x,{p,q,s},{q,s},{p,r}>, R=<x,{q,s},{p,q},{q,r}>, S=<x,{q,s},{p,q,s},{p,q,r}> and T=<x,{p,q,s},{q},{r}>. Let τ={X_N,$φ_N$,M,N,O,P} and σ = {X_N,$φ_N$,Q,R,S,T} be a neutrosophic topologies on X. Clearly (X, τ) and (X,σ) be any two neutrosophic chaos spaces. The function f : (X, τ) → (X, σ) is defined by f(p) = <q,p,s> f(q) = <s,r,p>, f(r) =< p,q,r> and f(s) = <r,s,q>. Now the function f is neutrosophic chaotic* continuous but not neutrosophic chaotic continuous. Hence, neutrosophic chaotic* continuous function need not be neutrosophic chaotic continuous function.

3.26 Definition

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. A function f : (X, τ) → (X, σ) is said to be neutrosophic chaotic** continuous if for each periodic point x ∈ X and each neutrosophic closed chaotic set F of f(x), ∃ a neutrosophic open orbit set $NO_f(x)$ of the periodic point x ∍ f($NO_f(x)$) ⊆ Nint(F).

3.27 Theorem

A neutrosophic chaotic continuous function is a neutrosophic chaotic** continuous function.

Proof Since f is a neutrosophic chaotic continuous function, F is a neutrosophic clopen chaotic set containing f(x), $\exists$ a neutrosophic open orbit set $NO_f(x)$ containing x $\ni$ $f(NO_f(x)) \subseteq F$. Since F is a neutrosophic open orbit set in (X, σ), F = Nint(F). This implies that, $f(NO_f(x)) \subseteq$ Nint(F). Hence, f is an neutrosophic chaotic** continuous function.

3.28 Remark

The converse of Theorem 3.27 need not be true as shown in the Example 3.29.

3.29 Example

Let X={p,q,r,s}. Then the neutrosophic sets M,N,O,P,Q,R,S and T are defined by M=<x,{q,r},{r},{p,r}>, N=<x,{p,s},{p,q},{p,q}>,O=<x,φ,{p,q,r},{p,q,r}>,P=<x,X,φ,{p}>,Q=<x,{p,q,r},{r},{p,s}>,R=<x,{q},{q,r},{p,r}>, S=<x,{p,q,r},{r},{p}> and T=<x,{q},{r},{p,r,s}>. Let $\tau=\{X_N,\varphi_N,M,N,O,P\}$ and $\sigma = \{X_N,\varphi_N,Q,R,S,T\}$ be a neutrosophic topologies on X. Clearly (X, τ) and (X,σ) be any two neutrosophic chaos spaces. The function f : (X, τ) → (X, σ) is defined by f(p) = <p,q,s> f(q) = <r,s,r>, f(r) =< q,r,p> and f(s) = <s,p,q>. Now the function f is neutrosophic chaotic** continuous but not neutrosophic chaotic continuous. Hence, neutrosophic chaotic** continuous function need not be neutrosophic chaotic continuous function.

3.30 Definition

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. A function f : (X, τ) → (X, σ) is said to be a neutrosophic chaotic*** continuous if for each periodic point x ∈ X and each neutrosophic closed chaotic set F of f(x) $\exists$ a neutrosophic clopen orbit set $NO_f(x)$ of the periodic point x $\ni$ $f(Nint(NO_f(x))) \subseteq F$.

3.31 Theorem

A neutrosophic chaotic continuous function is a neutrosophic chaotic*** continuous function.

Proof Since f is a neutrosophic chaotic continuous function, F is a neutrosophic clopen chaotic set containing f(x), $\exists$ a neutrosophic open orbit set $NO_f(x)$ containing x $\ni$ $f(NO_f(x)) \subseteq F$. This implies that, $NO_f(x) \subseteq f^{-1}(F)$. Then, $f^{-1}(F)$ is a neutrosophic clopen chaotic set of (X, σ). By (iii) of Theorem 3.21, $f^{-1}(F)$ is a neutrosophic clopen orbit set in (X, τ). Therefore, F is a neutrosophic closed chaotic set containing f(x) and $f^{-1}(F)$ is a neutrosophic open orbit set $\ni$ $f(f^{-1}(F)) \subseteq F$. Since $f^{-1}(F)$ is neutrosophic open orbit set, $Nint(f^{-1}(F)) = f^{-1}(F)$. This implies that, $f(Nint(f^{-1}(F))) \subseteq F$. Hence, f is a neutrosophic chaotic*** continuous function.

3.32 Remark

The converse of Theorem 3.31 need not be true as shown in the Example 3.33.

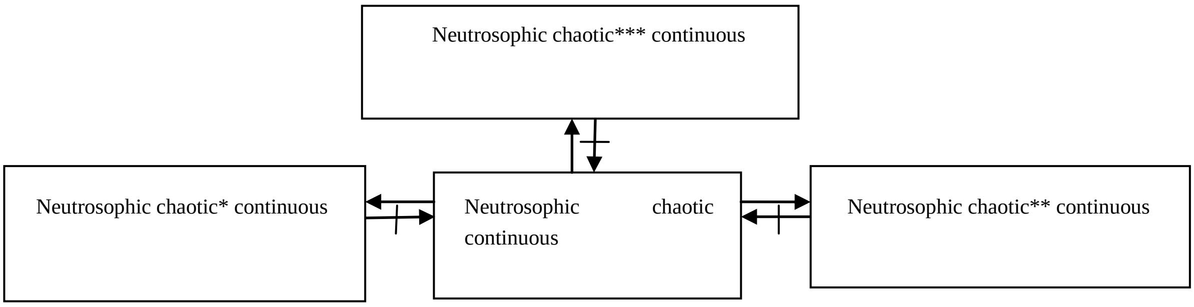
3.33 Example

Let X={p,q,r,s}. Then the neutrosophic sets M,N,O,P,Q,R,S and T are defined by M=<x,{q,r},{r},{p,r}>, N=<x,{p,r},{r},{q,r}>, O=<x,{p,q,r},{r},{r}>, P=<x,{r},{r},{p,q,r}>, Q=<x,{p,q,r},{q,r},{p,s}>, R=<x,{q,r},{p,q},{r,s}>, S=<x, {p,q,r},{r},{p}> and T=<x,{q},{r},{p,r,s}>. Let $\tau=\{X_N,\varphi_N,M,N,O,P\}$ and $\sigma = \{X_N,\varphi_N,Q,R,S,T\}$ be a neutrosophic topologies on X. Clearly (X, τ) and (X,σ) be any two neutrosophic chaos spaces. The function f : (X, τ) → (X, σ) is defined by f(p) = <p,q,s> f(q) = <r,s,r>, f(r) =< q,r,p> and f(s) = <s,p,q>. Now the function f is neutrosophic chaotic*** continuous but not neutrosophic chaotic continuous. Hence, neutrosophic chaotic*** continuous function need not be neutrosophic chaotic continuous function.

3.34 Remark

The interrelation among the functions introduced are given clearly in the following diagram.

Figure 1:



4 Properties of neutrosophic chaotic continuous functions

4.1 Definition

A neutrosophic chaos space (X, τ) is said to be a neutrosophic orbit extremally disconnected space if the neutrosophic closure of every neutrosophic open orbit set is neutrosophic open orbit.

4.2 Theorem

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. If f : (X, τ) → (X, σ) is a neutrosophic chaotic continuous function and (X, τ) is a neutrosophic orbit extremally disconnected space then f is a neutrosophic chaotic* continuous function.

Proof Let x be a periodic point and x ∈ X. Since f is neutrosophic chaotic continuous, F = $<x,F^1,F^2,F^3>$ is a neutrosophic clopen chaotic set of (X, σ), ∃ a neutrosophic open orbit set $NO_f(x)$ of (X, τ) containing x ∍ $f(NO_f(x)) \subseteq F$. Therefore, $NO_f(x)$ is a neutrosophic open orbit set $NO_f(x)$ of (X, τ). Since (X, τ) is neutrosophic orbit extremally disconnected, $Ncl(NO_f(x))$ is a neutrosophic open orbit set. Therefore, F is a neutrosophic closed chaotic set containing f(x) ∃ a neutrosophic open orbit set $Ncl(NO_f(x))$ ∍ $f(Ncl(NO_f(x))) \subseteq F$. Hence, f is neutrosophic chaotic* continuous.

4.3 Definition

A neutrosophic chaos space (X, τ) is said to be neutrosophic chaotic 0- dimensional if it has a neutrosophic base consisting of neutrosophic clopen chaotic sets.

4.4 Theorem

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. Let f : (X, τ) → (X, σ) be a neutrosophic chaotic*** continuous function. If (X, σ) is neutrosophic chaotic 0-dimensional then f is a neutrosophic chaotic continuous function.

Proof Let the periodic point x ∈ X. Since (X, σ) is neutrosophic chaotic 0-dimensional, ∃ a neutrosophic clopen chaotic set F = $<x,F^1,F^2,F^3>$ in (X, σ). Since f is a neutrosophic chaotic*** continuous function, ∃ a neutrosophic clopen orbit set $NO_f(x)$ ∍ $f(Nint(NO_f(x))) \subseteq F$. Since $NO_f(x)$ is a neutrosophic open orbit set, $Nint(NO_f(x) = NO_f(x)$. This implies that, $f(NO_f(x)) \subseteq F$. Therefore, f is neutrosophic chaotic continuous.

4.5 Definition

A neutrosophic chaos space (X, τ) is said to be a neutrosophic orbit connected space if X_N cannot be expressed as the union of two neutrosophic open orbit sets $NO_f(x)$ and $NO_f(y)$, x,y ∈ X of (X, τ) with $NO_f(x) \cap NO_f(y) \neq \varphi_N$.

4.6 Definition

A neutrosophic chaos space (X, τ) is said to be a neutrosophic chaotic connected space if X_N cannot be expressed as the union of two neutrosophic open chaotic sets U = $<x,U^1,U^2,U^3>$ and V = $<x,V^1,V^2,V^3>$ of (X, τ) with $U \cap V \neq \varphi_N$.

4.7 Theorem

A neutrosophic chaotic continuous image of a neutrosophic orbit connected space is a neutrosophic chaotic connected space.

Proof Let (X, σ) be neutrosophic chaotic disconnected. Let $F_1 = <x, F_1^{\ 1}, F_1^{\ 2}, F_1^{\ 3}>$ and $F_2 = <x, F_2^{\ 1}, F_2^{\ 2}, F_2^{\ 3}>$ be a neutrosophic chaotic disconnected sets of (X, σ). Then $F_1 \neq \varphi_N$ and $F_2 \neq \varphi_N$ are neutrosophic clopen chaotic sets in (X, σ) and $Y_N = F_1 \cup F_2$ where $F_1 \cap F_2 = \varphi_N$. Now, $X_N = f^{-1}(Y_N) = f^{-1}(F_1 \cup F_2) = f^{-1}(F_1) \cup f^{-1}(F_2)$.Since f is neutrosophic chaotic continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are neutrosophic open orbit sets in (X, τ). Also $f^{-1}(F_1) \cap f^{-1}(F_2) = \varphi_N$. Therefore, (X, τ) is not neutrosophic orbit connected. Which is a contradiction. Hence, (X, σ) is neutrosophic chaotic connected.

4.8 Theorem

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. If f : (X, τ) → (X, σ) is a neutrosophic chaotic continuous function and $NO_f(x)$ is neutrosophic open orbit set then the restriction $f|NO_f(x) : NO_f(x) \to (X, \sigma)$ is neutrosophic chaotic continuous.

Proof Let F = $<x,F^1,F^2,F^3>$ be a neutrosophic clopen chaotic set in (X, σ). Then, $(f|NO_f(x))^{-1}(F) = f^{-1}(F) \cap NO_f(x)$. Since f is neutrosophic chaotic continuous, $f^{-1}(F)$ is neutrosophic open orbit in (X, τ) and $NO_f(x)$ is a neutrosophic open orbit set. This implies that, $f^{-1}(F) \cap NO_f(x)$ is a neutrosophic open orbit set. Therefore, $(f|NO_f(x))^{-1}(F)$ is neutrosophic open orbit in (X, τ). Hence, $f|NO_f(x)$ is neutrosophic chaotic continuous.

4.9 Definition

Let (X, τ) be a neutrosophic chaos space. If a family $\{NO_f(x_i) : i \in J\}$ of neutrosophic open orbit set in (X, τ) satisfies the condition $\cup NO_f(x_i) = X_N$, then it is called a neutrosophic open orbit cover of (X, τ).

4.10 Theorem

Let $\{NO_f(x)_\gamma : \gamma \in \Gamma\}$ be any neutrosophic open orbit cover of a neutrosophic chaos space (X, τ). A function f : (X, τ) → (X, σ) is a neutrosophic chaotic continuous function if and only if the restriction $f|NO_f(x)_\gamma : NO_f(x)_\gamma \to (X, \sigma)$ is neutrosophic chaotic continuous for each γ ∈ Γ.

Proof Let γ be an arbitrarily fixed index and $NO_f(x)_\gamma$ be a neutrosophic open orbit set of (X, τ). Let the periodic point $x \in NO_f(x)_\gamma$ and F = $<x,F^1,F^2,F^3>$ is neutrosophic clopen chaotic set containing $(f|NO_f(x)_\gamma)(x) = f(x)$. Since f is neutrosophic chaotic continuous there exists a neutrosophic open orbit set $NO_f(x)$ containing x such that

$f(NO_f(x)) \subseteq F$. Since $(NO_f(x)_\gamma)$ is neutrosophic open orbit cover in (X, τ), $x \in NO_f(x) \cap NO_f(x)_\gamma$ and $(f|NO_f(x)_\gamma)(NO_f(x) \cap (NO_f(x)_\gamma) = f(NO_f(x) \cap (NO_f(x)_\gamma)) \subset f(NO_f(x) \subset F$. Hence $f|NO_f(x)_\gamma$ is a neutrosophic chaotic continuous function. Conversely, let the periodic point $x \in X$ and F be a neutrosophic chaotic set containing f(x). There exists an $\gamma \in \Gamma$ such that $x \in NO_f(x)_\gamma$. Since $(f|NO_f(x)_\gamma) : NO_f(x)_\gamma \to (X, \sigma)$ is neutrosophic chaotic continuous, there exists a $NO_f(x) \in NO_f(x)_\gamma$ containing x such that $(f|NO_f(x)_\gamma)(NO_f(x)) \subseteq F$. Since $NO_f(x)$ is neutrosophic open orbit in (X, τ), $f(NO_f(x)) \subseteq F$. Hence, f is neutrosophic chaotic continuous.

4.11 Theorem

If a function $f : (X, \tau) \to \Pi (X, \sigma)_\lambda$ is neutrosophic chaotic continuous then $P_\lambda \circ f : (X, \tau) \to (X, \sigma)_\lambda$ is neutrosophic chaotic continuous for each $\lambda \in \Lambda$, where P_λ is the projection of $\Pi (X, \sigma)_\lambda$ onto $(X, \sigma)_\lambda$.

Proof Let $F_\lambda = <x, F_\lambda^1, F_\lambda^2, F_\lambda^3>$ be any neutrosophic clopen chaotic set of $(X, \sigma)_\lambda$. Then $P_\lambda^{-1}(F_\lambda)$ is a neutrosophic clopen chaotic set in $\Pi (X, \sigma)_\lambda$ and hence $(P_\lambda \circ f)^{-1}(F_\lambda) = f^{-1}(P_\lambda^{-1}(F_\lambda))$ is a neutrosophic open orbit set in (X, τ). Therefore, $P_\lambda \circ f$ is neutrosophic chaotic continuous.

4.12 Theorem

If a function $f : \Pi (X, \tau)_\lambda \to \Pi (X, \sigma)_\lambda$ is neutrosophic chaotic continuous then $f_\lambda : (X, \tau)_\lambda \to (X, \sigma)_\lambda$ is a neutrosophic chaotic continuous function for each $\lambda \in \Lambda$.

Proof Let $F_\lambda = <x, F_\lambda^1, F_\lambda^2, F_\lambda^3>$ be any neutrosophic clopen chaotic set of $(X, \sigma)_\lambda$. Then $P_\lambda^{-1}(F_\lambda)$ is neutrosophic clopen chaotic in $\Pi (X, \sigma)_\lambda$ and $f^{-1}(P_\lambda^{-1}(F_\lambda)) = f_\lambda^{-1}(F_\lambda) \times \Pi \{(X, \tau)_\alpha : \alpha \in \Lambda - \{\lambda\}\}$. Since f is neutrosophic chaotic continuous, $f^{-1}(P_\lambda^{-1}(F_\lambda))$ is a neutrosophic open orbit set in $\Pi (X, \tau)_\lambda$. Since the projection P_λ of $\Pi (X, \tau)_\lambda$ onto $(X, \tau)_\lambda$ is a neutrosophic open function, $f_\lambda^{-1}(F_\lambda)$ is neutrosophic open orbit in $(X, \tau)_\lambda$. Hence, f_λ is neutrosophic chaotic continuous.

4.13 Definition

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. A function $f : (X, \tau) \to (X, \sigma)$ is said to be neutrosophic chaotic irresolute if for each neutrosophic clopen chaotic set $F = <x,F^1,F^2,F^3>$ in (X, σ), $f^{-1}(A)$ is a neutrosophic clopen chaotic set of (X, τ).

4.14 Theorem

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. If $f : (X, \tau) \to (X, \sigma)$ is a neutrosophic chaotic continuous function and $g : (X, \sigma) \to (X, \eta)$ is a neutrosophic chaotic irresolute function, then $g \circ f : (X, \tau) \to (X, \eta)$ is neutrosophic chaotic continuous.

Proof Let $F = <x,F^1,F^2,F^3>$ be a neutrosophic clopen set of (X, η). Since g is neutrosophic chaotic irresolute, $g^{-1}(F)$ is neutrosophic clopen chaotic set of (X, σ). Since f is neutrosophic chaotic continuous, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is a neutrosophic open orbit set of (X, τ) such that $f^{-1}(g^{-1}(F)) \subseteq F$. Hence $g \circ f$ is neutrosophic chaotic continuous.

4.15 Definition

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. A function $f : (X, \tau) \to (X, \sigma)$ is said to be neutrosophic orbit irresolute if for each neutrosophic open orbit set $NO_f(x)$ in (X, σ), $f^{-1}(NO_f(x))$ is a neutrosophic open orbit set of (X, τ).

4.16 Definition

Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. Let $f : (X, \tau) \to (X, \sigma)$ be a function. Then f is said to be a neutrosophic open orbit function if the image of every neutrosophic open orbit set in (X, τ) is neutrosophic open orbit in (X, σ).

4.17 Theorem

Let $f : (X, \tau) \to (X, \sigma)$ be neutrosophic orbit irresolute, surjective and neutrosophic open orbit function. Then $g \circ f : (X, \tau) \to (X, \eta)$ is neutrosophic chaotic continuous iff $g : (X, \sigma) \to (X, \eta)$ is neutrosophic chaotic continuous.

Proof Let $F_\lambda = <x, F_\lambda^1, F_\lambda^2, F_\lambda^3>$ be a neutrosophic clopen chaotic set of (X, η). Since g is neutrosophic chaotic continuous, $g^{-1}(F)$ is neutrosophic open orbit in (X, σ). Since f is neutrosophic orbit irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is neutrosophic open orbit in (X, τ). Hence $g \circ f$ is neutrosophic chaotic continuous. Conversely, let $g \circ f : (X, \tau) \to (X, \eta)$ be neutrosophic chaotic continuous function. Let F be a neutrosophic clopen chaotic set of (X, η), then $(g \circ f)^{-1}(F)$ is a neutrosophic open orbit set of (X, τ). Since f is neutrosophic open orbit and surjective, $f(f^{-1}(g^{-1}(F))$ is a neutrosophic open orbit set of (X, σ). Therefore, $g^{-1}(F)$ is a neutrosophic open orbit set in (X, σ). Hence, g is neutrosophic chaotic continuous.

Conclusion

In this paper, characterization of neutrosophic chaotic continuous functions are studied. Some interrelations are discussed with suitable examples. Also, neutrosophic orbit, extremally disconnected spaces and neutrosophic chaotic zero-dimensional spaces has been discussed with some interesting properties. This paper paves way in future to introduce and study the notions of neutrosophic orbit Co-kernal spaces, neutrosophic hardly open orbit spaces, neutrosophic orbit quasi regular spaces and neutrosophic orbit strongly complete spaces, neutrosophic orbit Co-kernal function, neutrosophic hardly open orbit function for which the above discussed set form the basis.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, in V.Sgurev, ed., VII ITKRS session, sofia (June 1983 central Sci. and Tech. Library, Bulg, Academy of sciences (1984)).
[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems 20(1986) 81-86.
[3] K.Atanassov, Review and new result on intuitionistic fuzzy sets, preprint Im-MFAIS-1-88, Sofia, 1988.
[4] R.L. Devaney, Introduction to chaotic dynamical systems, Addison-wesley.
[5] R. Dhavaseelan and S. Jafari, Generalized neutrosophic closed set, New Trends in Neutrosophic Theory and Applications, Vol II(2017), pp: 261-273.
[6] M. Kousalyaparasakthi, E. Roja and M.K. Uma, Intuitionistic chaotic continuous functions, Annals of Fuzzy Mathematics and Informatics, pp: 1-17 (2015).
[7] Mohamed Abdel-Basset, Victor chang, Abduallah Gamal and Florentin Smarandache, An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field, Computers in Industry **106**: 94-110, (2019).
[8] Mohamed Abdel-Basset, M. Saleh, Abduallah Gamal and Florentin Smarandache, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, Applied Soft Computing. **77:** 438-452, (2019).
[9] Mohamed Abdel-Basset, Gunasekaran Manogaran, Abduallah Gamal and Florentin Smarandache, A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection, Journal of Medical Systems **43**(2): 38, (2019).
[10] Mohamed Abdel-Basset, Gunasekaran Manogaran, Abduallah Gamal and Florentin Smarandache, A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria, Design Automation for Embedded Systems: 1-22, (2018).
[11] Florentin Smarandache, neutrosophy and neutrosophic Logic, First International conference on neutrosophy, Neutrosophic Logic, set, probability and statistics university of New mexico, Gallup NM 87301, USA(2002)
[12] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic set, Neutrosophic probability American Research press, Rehoboth NM, 1999.
[13] A.A. Salama, Florentin Smarandache and valeri kromov, Neutrosophic closed set and Neutrosophic continuous functions, Neutrosophic sets and systems Vol 4, 2014.
[14] A.A. Salama and S.A. Albowi, Neutrosophic set and neutrosophic topological spaces, ISORJ mathematics, Vol.(3), Issue(3), (2012) Pp-31-35.
[15] T. Thrivikraman and P.B. Vinod Kumar, On chaos and fractals in general topological spaces, Ph.D Thesis, Cochin University of Sciences and Technology,2001.
[16] L.A. Zadeh, Fuzzy sets, Information and control 8(1965) 338-353.

Received: March 29, 2019. Accepted: March 22, 2019.