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Topology on Quadripartitioned Neutrosophic Sets

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Abstract: The focus of this paper is to introduce the notion of quadripartitioned neutrosophic topology (Q-NT) on quadripartitioned neutrosophic sets (Q-NS). In this paper, we define quadripartitioned neutrosophic closure, quadripartitioned neutrosophic interior operator of Q-NSs in quadripartitioned neutrosophic topological space (Q-NTS) and investigate several properties of them. Again, we introduce quadripartitioned neutrosophic semi-open (Q-NSO) set, quadripartitioned neutrosophic pre-open (Q-NPO) set, quadripartitioned neutrosophic *b*-open (Q-N-*b*-O) set, and quadripartitioned neutrosophic α-open (Q-Nα-O) set in Q-NTSs. Further, we furnish some suitable examples and prove some basic results on Q-NTS.

Keywords: *Quadripartitioned Neutrosophic Set*; *Q-NT*; *Q-NTS*; *Quadripartitioned-Neutrosophic Closure*; *Quadripartitioned-Neutrosophic Closure*; *Q-NPO*; *Q-Nα-O*.

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1. Introduction: In the year 2005, Smarandache [20] extended the concept of intuitionistic fuzzy set by introducing the notion of neutrosophic set (NS). Later on, many researchers use NS in their theoretical and practical research. In the year 2016, Chatterjee et. al. [4] grounded the idea of quadripartitioned neutrosophic set and defined several similarity measures between two quadripartitioned neutrosophic sets. The idea of neutrosophic topological space (NTS) was presented by Salama and Alblowi [18] in the year 2012. The neutrosophic semi-open mappings are studied by Arokiarani et. al. [2]. Afterwards, Iswaraya and Bageerathi [11] studied the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets. Pushpalatha and Nandhini [15] grounded the idea of neutrosophic generalized closed sets in NTSs. The notion of neutrosophic *b*-open sets in NTSs was presented by Ebenanjar et al. [10]. Rao and Srinivasa [17] grounded the concept of pre open set and pre closed set via neutrosophic topological spaces. Thereafter, Maheswari et. al. [13] studied the neutrosophic generalized *b*-closed sets in NTSs. In the year 2019, Mohammed Ali Jaffer and Ramesh [14] studied the concept of neutrosophic generalized pre-regular closed sets. The generalized neutrosophic *b*-open sets in NTSs was introduced by Das and Pramanik [6]. Das and Pramanik [7] also defined the neutrosophic Φ-open sets and neutrosophic Φ-continuous mappings via NTSs. Recently, Ramesh [16] presented the notion of Ngpr homomorphism via neutrosophic topological spaces. In this study, we introduce the notion of Q-NT and present the concept of quadripartitioned neutrosophic closure and quadripartitioned neutrosophic interior operator in Q-NTSs. It is just the beginning of a new notion. In the future, based on these notions and various open sets on Q-NTSs, many new investigations (compectness, paracompectness, separation axioms) can be easily done. Also, the researchers can use the quadripartitioned neutrosophic topological operators in the area of multi criteria decision making problems.

Research Gap: No investigation on Q-NTSs has been reported in the recent literature.

Motivation: To reduce the gap of research, we present the notion of Q-NTSs.

The remaining part of this paper has been splited into following four sections:
In section-2, we recall some relevant definitions, properties, results of NSs, Q-NSs. Section-3 is on the notion of quadripartitioned neutrosophic topological spaces. In this section, we give some basic definitions, theorems, and propositions on Q-NTSs. In section-4, we conclude our work done in this paper and stating the future scope of research.

2. Some Relevant Definitions:

Definition 2.1. [4] Assume that Ω be a fixed set. Then, a quadripartitioned neutrosophic set (Q-NS) P over Ω is defined by:
$P = \{(q, T_P(q), C_P(q), G_P(q), F_P(q)): q\in\Omega\}$, where $T_P(q)$, $C_P(q)$, $G_P(q)$, and $F_P(q)$ ($\in [0,1]$) are the truth, contradiction, ignorance, and falsity membership values of $q\in\Omega$. So, $0\leq T_P(q)+C_P(q)+G_P(q)+F_P(q) \leq 4$.

Example 2.1. Suppose $\Omega=\{u, v\}$. Then, $P= \{(u,0.5,0.6,0.3,0.6), (v,0.9,0.3,0.4,0.2)\}$ is a Q-NS over Ω.

Definition 2.2. [4] The absolute Q-NS (1_{QN}) and the null Q-NS (0_{QN}) over Ω are defined as follows:
(i) $1_{QN} = \{(q, 1, 1, 0, 0): q\in\Omega\}$;
(ii) $0_{QN} = \{(q, 0, 0, 1, 1): q\in\Omega\}$.

Example 2.2. Suppose that $\Omega=\{u, v\}$. Then, $1_{QN} = \{(u,1, 1, 0, 0), (v, 1, 1, 0, 0)\}$ and $0_{QN} = \{(u,0, 0, 1, 1), (v, 0, 0, 1, 1)\}$.

Remark 2.1. Suppose that R be a Q-NS over Ω. Then, $0_{QN} \subseteq R \subseteq 1_{QN}$.

Definition 2.3.[4] Suppose that $X = \{(q, T_X(q), C_X(q), G_X(q), F_X(q)): q\in\Omega\}$ and $Y = \{(q, T_Y(q), C_Y(q), G_Y(q), F_Y(q)): q\in\Omega\}$ be two Q-NSs over Ω. Then, $X \subseteq Y \Leftrightarrow T_X(q)\leq T_Y(q)$, $C_X(q)\leq C_Y(q)$, $G_X(q)\geq G_Y(q)$, $F_X(q)\geq F_Y(q)$, for all $q\in\Omega$.

Example 2.3. Assume that $\Omega=\{u, v\}$. Suppose that $X=\{(u,0.5,0.3,0.6,0.7), (v,0.2,0.4,0.8,0.8)\}$ and $Y=\{(u,0.3,0.3,0.8,0.8), (v,0.2,0.3,0.9,1.0)\}$ be two Q-NSs over Ω. Then, $Y\subseteq X$.

Definition 2.4.[4] Suppose that $X = \{(q, T_X(q), C_X(q), G_X(q), F_X(q)): q\in\Omega\}$ and $Y = \{(q, T_Y(q), C_Y(q), G_Y(q), F_Y(q)): q\in\Omega\}$ be two Q-NSs over Ω. Then, the union of X and Y is $X \cup Y = \{(q, \max\{T_X(q), T_Y(q)\}, \max\{C_X(q), C_Y(q)\}, \min\{G_X(q), G_X(q)\}, \min\{F_X(q), F_X(q)\}): q\in\Omega\}$.

Example 2.4. Assume that X and Y be two Q-NSs over $\Omega=\{u, v\}$ as shown in Example 2.3. Then, $X\cup Y= \{(u,0.5,0.3,0.6,0.7), (v,0.2,0.4,0.8,0.8)\}$.

Definition 2.5.[4] Suppose that X ={(q, $T_X(q)$, $C_X(q)$, $G_X(q)$, $F_X(q)$): $q\in\Omega$} and Y = {(q, $T_Y(q)$, $C_Y(q)$, $G_Y(q)$, $F_Y(q)$): $q\in\Omega$} be two Q-NSs over Ω. Then, the complement of X is X^c ={(q, $F_X(q)$, $G_X(q)$, $C_X(q)$, $T_X(q)$): $q\in\Omega$}.

Example 2.5. Suppose that Ω={u, v} and X={(u,0.8,0.8,0.5,1.0), (v,1.0,0.5,0.3,0.8)} be a Q-NS over Ω. Then, X^c={(u,1.0,0.5,0.8,0.8), (v,0.8,0.3,0.5,1.0)}.

Definition 2.6.[4] Suppose that X={(q, $T_X(q)$, $C_X(q)$, $G_X(q)$, $F_X(q)$): $q\in\Omega$} and Y = {(q, $T_Y(q)$, $C_Y(q)$, $G_Y(q)$, $F_Y(q)$): $q\in\Omega$} be two Q-NSs over Ω. Then, the intersection of X and Y is $X\cap Y$={(q, min {$T_X(q)$, $T_Y(q)$}, min {$C_X(q)$, $C_Y(q)$}, max {$G_X(q)$, $G_X(q)$}, max {$F_X(q)$, $F_X(q)$}): $q\in\Omega$}.

Example 2.6. Assume that X and Y be two Q-NSs over Ω={u, v} as shown in Example 2.3. Then, $X\cap Y$={(u,0.3,0.3, 0.8,0.8), (v,0.2,0.3,0.9, 1.0)}.

3. Quadripartitioned Neutrosophic Topology:

Definition 3.1. Let Ω be a fixed set. A collection ℑ of some Q-NSs over Ω is called a Q-NT on Ω, if the following conditions holds:

(i) 1_{QN}, 0_{QN} ∈ ℑ;

(ii) $M_1\cap M_2$ ∈ℑ whenever M_1, M_2 ∈ ℑ;

(iii) $\cup M_i$ ∈ℑ, whenever {M_i: $i\in\Delta$} ⊆ ℑ.

Then, (Ω,ℑ) is called a Q-NTS. Every element of ℑ are called a quadripartitioned neutrosophic open set (Q-NOS). If $M\in$ℑ, then M^c is called a quadripartitioned neutrosophic closed set (Q-NCS).

Remark 3.1. In every Q-NTS, 0_{QN} and 1_{QN} are both Q-NOS and Q-NCS.

Example 3.1. Let Ω = {u, v}. Assume that M = {(u,0.9,0.5,0.7,1.0), (v,0.3,0.1,0.5,0.7): u, v ∈ Ω} and N = {(u,0.9,0.7,0.1,0.9), (v,0.4,0.6,0.1,0.2): u, v ∈Ω} be two Q-NSs over Ω. Then, ℑ ={0_{QN}, 1_{QN}, M, N} forms a Q-NT on Ω.

The quadripartitioned-neutrosophic interior and quadripartitioned-neutrosophic closure of a Q-NS in a Q-NTS are defined as follows:

Definition 3.2. Let us consider a quadripartitioned neutrosophic subset X of a Q-NTS (Ω,ℑ). Then, the quadripartitioned-neutrosophic closure (Q-N_{cl}) of X is the intersection of all Q-NCSs containing X and the quadripartitioned-neutrosophic interior (Q-N_{int}) of X is the union of all Q-NOSs contained in X, i.e.

$Q\text{-}N_{cl}(X) = \cap\{Z : X \subseteq Z$ and Z is a Q-NCS in (Ω,ℑ)};

$Q\text{-}N_{int}(X) = \cup\{Y : Y \subseteq X$ and Y is a Q-NOS in (Ω,ℑ)}.

Remark 3.2. It is clear that $Q\text{-}N_{cl}(X)$ is the smallest Q-NCS in (Ω,ℑ) that contains X and $Q\text{-}N_{int}(X)$ is the largest Q-NOS in (Ω,ℑ) which is contained in X.

Theorem 3.1. If T and R be any two quadripartitioned neutrosophic subsets of a Q-NTS (Ω,ℑ), then

(i) $Q\text{-}N_{int}(T) \subseteq T \subseteq Q\text{-}N_{cl}(T)$;

(ii) $T \subseteq R \Rightarrow Q\text{-}N_{cl}(T) \subseteq Q\text{-}N_{cl}(R)$;

(iii) $T \subseteq R \Rightarrow Q\text{-}N_{int}(T) \subseteq Q\text{-}N_{int}(R)$;

(iv) T is an N^*-OS iff $Q\text{-}N_{int}(T) = T$;

(v) T is an N^*-CS iff $Q\text{-}N_{cl}(T) = T$.

Proof. (i) From the previous definition, we have $Q\text{-}N_{int}(T) = \cup\{R$: R is a Q-NOS in (Ω,ℑ) and $R \subseteq T\}$.

Since, each $R \subseteq T$, so $\cup\{R$: R is a Q-NOS in $(\Omega,\Im)$ and $R \subseteq T\} \subseteq T$, i.e. $Q\text{-}N_{int}(T) \subseteq T$.

Again, $Q\text{-}N_{cl}(T) = \cap\{Z$: Z is a Q-NCS in $(\Omega,\Im)$ and $T \subseteq Z\}$. Since, each $Z \supseteq T$, so $\cap\{Z$: Z is a Q-NCS in $(\Omega,\Im)$ and $T \subseteq Z\} \supseteq T$, i.e. $Q\text{-}N_{cl}(T) \supseteq T$.

Therefore, $Q\text{-}N_{int}(T) \subseteq T \subseteq Q\text{-}N_{cl}(T)$.

(ii) Assume that T and R be any two quadripartitioned neutrosophic subsets of a Q-NTS $(\Omega,\Im)$ such that $T \subseteq R$.

Now, $Q\text{-}N_{cl}(T) = \cap\{Z$: Z is a Q-NCS in $(\Omega,\Im)$ and $T \subseteq Z\}$

$\subseteq\cap\{Z$: Z is a Q-NCS in $(\Omega,\Im)$ and $R \subseteq Z\}$ [since $T \subseteq R$]

$= Q\text{-}N_{cl}(R)$

$\Rightarrow Q\text{-}N_{cl}(T) \subseteq Q\text{-}N_{cl}(R)$.

Therefore, $T \subseteq R \Rightarrow Q\text{-}N_{cl}(T) \subseteq Q\text{-}N_{cl}(R)$.

(iii) Assume that T and R be any two quadripartitioned neutrosophic subsets of a Q-NTS $(\Omega,\Im)$ such that $T \subseteq R$.

Now, $Q\text{-}N_{int}(T) = \cup\{Z$: Z is a Q-NOS in $(\Omega,\Im)$ and $Z \subseteq T\}$

$\subseteq\cup\{Z$: Z is a Q-NOS in $(\Omega,\Im)$ and $Z \subseteq R\}$ [since $T \subseteq R$]

$= Q\text{-}N_{int}(R)$

$\Rightarrow Q\text{-}N_{int}(T) \subseteq Q\text{-}N_{int}(R)$.

Therefore, $T \subseteq R \Rightarrow Q\text{-}N_{int}(T) \subseteq Q\text{-}N_{int}(R)$.

(iv) Assume that T be a Q-NOS in a Q-NTS $(\Omega,\Im)$. Now, $Q\text{-}N_{int}(T) = \cup\{Z$: Z is a Q-NOS in $(\Omega,\Im)$ and $Z \subseteq T\}$. Since, T is a Q-NOS in $(\Omega,\Im)$, so T is the largest Q-NOS, which is contained in T. Therefore, $\cup\{Z$: Z is a Q-NOS in $(\Omega,\Im)$ and $Z \subseteq T\} = T$. This implies, $Q\text{-}N_{int}(T) = T$.

(v) Assume that T be a Q-NCS in a Q-NTS $(\Omega,\Im)$. Now, $Q\text{-}N_{cl}(T) = \cap\{Z : Z$ is a Q-NCS in $(\Omega,\Im)$ and $T \subseteq Z\}$. Since, T is a Q-NCS in $(\Omega,\Im)$, so T is the smallest Q-NCS, which contains T. Therefore, $\cap\{Z$: Z is a Q-NCS in $(\Omega,\Im)$ and $T \subseteq Z\} = T$. This implies, $Q\text{-}N_{cl}(T) = T$.

Theorem 3.2. Let E be a quadripartitioned neutrosophic subset of a Q-NTS $(\Omega,\Im)$. Then,

(i) $(Q\text{-}N_{int}(E))^c = Q\text{-}N_{cl}(E^c)$;

(ii) $(Q\text{-}N_{cl}(E))^c = Q\text{-}N_{int}(E^c)$.

Proof. (i) Suppose that $(\Omega,\Im)$ be a Q-NTS and $E=\{(w, T_E(w), C_E(w), G_E(w), F_E(w))$: $w \in \Omega\}$ be a quadripartitioned neutrosophic subset of Ω. Now, $Q\text{-}N_{int}(E) = \cup\{Z_i$: $i \in \Delta$ and Z_i is a Q-NOS in $(\Omega,\Im)$ such that $Z_i \subseteq E\} = \{(w, \vee T_{Z_i}(w), \vee C_{Z_i}(w), \wedge G_{Z_i}(w), \wedge F_{Z_i}(w))$: $w \in \Omega\}$, where for all $i \in \Delta$ and Z_i is a Q-NOS in $(\Omega,\Im)$ such that $Z_i \subseteq E$. This implies, $(Q\text{-}N_{int}(E))^c = \{(w, \wedge T_{Z_i}(w), \wedge C_{Z_i}(w), \vee G_{Z_i}(w), \vee F_{Z_i}(w))$: $w \in \Omega\}$. Since, $\wedge T_{Z_i}(w) \leq T_E(w)$, $\wedge C_{Z_i}(w) \leq C_E(w)$, $\vee G_{Z_i}(w) \geq G_E(w)$, $\vee F_{Z_i}(w) \geq F_E(w)$, for each $i \in \Delta$ and $w \in \Omega$, so $Q\text{-}N_{cl}(E^c) = \{(w, \wedge T_{Z_i}(w), \wedge C_{Z_i}(w), \vee G_{Z_i}(w), \vee F_{Z_i}(w))$: $w \in \Omega\}\} = \cap\{Z_i$: $i \in \Delta$ and Z_i is a Q-NCS in $(\Omega,\Im)$ such that $E^c \subseteq Z_i\}$. Therefore, $(Q\text{-}N_{int}(E))^c = Q\text{-}N_{cl}(E^c)$.

(ii) Suppose that $(\Omega,\Im)$ be a Q-NTS and $E=\{(w, T_E(w), C_E(w), G_E(w), F_E(w))$: $w \in \Omega\}$ be a quadripartitioned neutrosophic subset of Ω. Now, $Q\text{-}N_{cl}(E) = \cap\{Z_i$: $i \in \Delta$ and Z_i is a Q-NCS in $(\Omega,\Im)$ such that $Z_i \supseteq E\} = \{(w, \wedge T_{Z_i}(w), \wedge C_{Z_i}(w), \vee G_{Z_i}(w), \vee F_{Z_i}(w))$: $w \in \Omega\}$, where for all $i \in \Delta$ and Z_i is a Q-NCS in $(\Omega,\Im)$ such that $Z_i \supseteq E$. This implies, $(Q\text{-}N_{cl}(E))^c = \{(w, \vee T_{Z_i}(w), \vee C_{Z_i}(w), \wedge G_{Z_i}(w), \wedge F_{Z_i}(w))$: $w \in \Omega\}$. Since, $\vee T_{Z_i}(w) \geq T_E(w)$, $\vee C_{Z_i}(w) \geq C_E(w)$, $\wedge G_{Z_i}(w) \leq G_E(w)$, $\wedge F_{Z_i}(w) \leq F_E(w)$, for each $i \in \Delta$ and $w \in \Omega$, so $Q\text{-}N_{int}(E^c) =$

$\{(w,\vee T_{Z_i}(w),\vee C_{Z_i}(w),\wedge G_{Z_i}(w),\wedge F_{Z_i}(w))\colon w\in\Omega\}=\cup\{Z_i\colon i\in\Delta$ and Z_i is a Q-NOS in $(\Omega,\Im)$ such that $Z_i\subseteq E^c\}$. Therefore, $(Q\text{-}N_{cl}(E))^c = Q\text{-}N_{int}(E^c)$.

Definition 3.3. Suppose that $(\Omega,\Im)$ be a Q-NTS. Then, a Q-NS W over Ω is called a

(i) quadripartitioned neutrosophic semi-open (Q-NSO) set iff $W \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$;

(ii) quadripartitioned neutrosophic pre-open (Q-NPO) set iff $W \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))$.

The complement of Q-NSO sets and Q-NPO sets are called Q-NSC sets and Q-NPC sets respectively.

Theorem 3.3. Suppose that $(\Omega,\Im)$ be a Q-NTS. If W and M are two Q-NSO sets, then $W\cup M$ is also a Q-NSO set.

Proof. Suppose that $(\Omega,\Im)$ be a Q-NTS. Let W, M be two Q-NSO sets in $(\Omega,\Im)$. Therefore,

$$W\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W)) \quad (1)$$

and $$M\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(M)) \quad (2)$$

From (1) and (2), we have

$W\cup M\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W))\cup Q\text{-}N_{cl}(Q\text{-}N_{int}(M)) = Q\text{-}N_{cl}(Q\text{-}N_{int}(W) \cup Q\text{-}N_{int}(Y)) \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W \cup M))$.

This implies, $W\cup M \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W \cup M))$. Therefore, $W\cup M$ is a Q-NSO set in $(\Omega,\Im)$.

Theorem 3.4. Suppose that $(\Omega,\Im)$ be a Q-NTS. If W is a Q-NOS, then W is also a Q-NSO set.

Proof. Suppose that $(\Omega,\Im)$ be a Q-NTS and W be a Q-NOS. Therefore, $W=Q\text{-}N_{int}(W)$. It is known that $W\subseteq Q\text{-}N_{cl}(W)$. This implies, $W\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$. Therefore, W is a Q-NSO set.

Theorem 3.5. Suppose that $(\Omega,\Im)$ be a Q-NTS. If W and M are two Q-NPO sets, then $W\cup M$ is also a Q-NPO set.

Proof. Suppose that $(\Omega,\Im)$ be a Q-NTS. Let W, M be two Q-NPO sets in $(\Omega,\Im)$. Therefore,

$$W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \quad (3)$$

and $$M\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \quad (4)$$

From (3) and (4), we have,

$W\cup M \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))\cup Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)\cup Q\text{-}N_{cl}(M)) = Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M))$.

This implies, $W\cup M\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W \cup M))$. Therefore, $W\cup M$ is a Q-NPO set in $(\Omega,\Im)$.

Theorem 3.6. Assume that $(\Omega,\Im)$ be a Q-NTS. If W is a Q-NOS, then W is also a Q-NPO set.

Proof. Suppose that $(\Omega,\Im)$ be a Q-NTS and W be a Q-NOS in $(\Omega,\Im)$. So $W=Q\text{-}N_{int}(W)$. It is known that $W\subseteq Q\text{-}N_{cl}(W)$. This implies, $Q\text{-}N_{int}(W) \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))$ i.e. $W =Q\text{-}N_{int}(W) \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))$. Hence, $W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))$. Therefore, W is a Q-NPO set.

Definition 3.4. Let us assume that $(\Omega,\Im)$ be a Q-NTS. Then W, a Q-NS over Ω is called a quadripartitioned neutrosophic α-open (Q-Nα-O) set if and only if $W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(Q\text{-}N_{int}(W)))$.

Remark 3.3. (i) The complement of a Q-Nα-O set is called a quadripartitioned neutrosophic α-closed (Q-Nα-C) set.

(ii) In a Q-NTS $(\Omega,\Im)$, every Q-NOS is a Q-Nα-O set.

(iii) In a Q-NTS $(\Omega,\Im)$, every Q-NCS is a Q-Nα-C set.

Theorem 3.7. In a Q-NTS $(\Omega,\Im)$,

(i) every Q-Nα-O set is a Q-NPO set;

(ii) every Q-Nα-O set is a Q-NSO set.

Proof. (i) Suppose that W be a Q-Nα-O set in a Q-NTS $(\Omega,\Im)$. Therefore, $W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(Q\text{-}N_{int}(W)))$. It is known that, $Q\text{-}N_{int}(W)\subseteq W$. This implies, $Q\text{-}N_{cl}(Q\text{-}N_{int}(W))\subseteq Q\text{-}N_{cl}(W)$. Therefore,

$Q\text{-}N_{int}(Q\text{-}N_{cl}(Q\text{-}N_{int}(W)))\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)$. This implies, $W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)$. Therefore, W is a Q-NPO set. Hence, every Q-Nα-O set is a Q-NPO set.

(ii) Suppose that W be a Q-Nα-O set in a Q-NTS $(\Omega,\Im)$. So, $W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(Q\text{-}N_{int}(W)))$. It is known that, $Q\text{-}N_{int}(Q\text{-}N_{cl}(Q\text{-}N_{int}(W)))\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$. This implies, $W\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$. Therefore, W is a Q-NSO set. Hence, every Q-Nα-O set is a Q-NSO set.

Definition 3.5. Suppose that $(\Omega,\Im)$ be a Q-NTS. Then W, a Q-NS over Ω is called a quadripartitioned neutrosophic b-open (Q-N-b-O) set if and only if $W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))\cup Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$. A quadripartitioned neutrosophic set W is called a quadripartitioned neutrosophic b-closed (Q-N-b-C) set if and only if W^c is a Q-N-b-O set.

Remark 3.4. A Q-NS W over Ω is called a Q-N-b-C set if and only if $Q\text{-}N_{int}(Q\text{-}N_{cl}(W))\cap Q\text{-}N_{cl}(Q\text{-}N_{int}(W))\subseteq W$.

Theorem 3.8. Suppose that $(\Omega,\Im)$ be a Q-NTS. If W, M be two Q-N-b-O sets in $(\Omega,\Im)$, then $W\cup M$ is also a Q-N-b-O set.

Proof. Suppose that $(\Omega,\Im)$ be a Q-NTS. Let W, M be two Q-N-b-O sets in $(\Omega,\Im)$. So

$$W\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \cup Q\text{-}N_{cl}(Q\text{-}N_{int}(W)), \tag{5}$$

and $$M\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \cup Q\text{-}N_{cl}(Q\text{-}N_{int}(M)). \tag{6}$$

We know that, $W\subseteq W\cup M$ and $M\subseteq W\cup M$. This implies,

$$Q\text{-}N_{cl}(Q\text{-}N_{int}(W))\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cup M)), \tag{7}$$

$$Q\text{-}N_{int}(Q\text{-}N_{cl}(W))\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M)), \tag{8}$$

$$Q\text{-}N_{cl}(Q\text{-}N_{int}(M))\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cup M)), \tag{9}$$

$$Q\text{-}N_{int}(Q\text{-}N_{cl}(M))\subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M)). \tag{10}$$

From Eq. (5) and Eq. (6), we have,

$$\begin{aligned} W\cup M &\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \cup Q\text{-}N_{cl}(Q\text{-}N_{int}(M)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \\ &\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cup M)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M)) \cup Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cup M)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M)) \\ &\qquad [\text{ by eqs } (7), (8), (9), (10)] \\ &= Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cup M)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M)). \end{aligned}$$

This implies, $W\cup M\subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cup M)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cup M))$. Hence, $W\cup M$ is a Q-N-b-O set.

Theorem 3.9. Suppose that $(\Omega,\Im)$ be a Q-NTS. If W, M be two Q-N-b-C sets in $(\Omega,\Im)$, then $W\cap M$ is also a Q-N-b-C set.

Proof. Suppose that $(\Omega,\Im)$ be a Q-NTS. Let W, M be two Q-N-b-C sets in $(\Omega,\Im)$. So,

$$Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(W)) \subseteq W \tag{11}$$

and $$Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(M)) \subseteq M \tag{12}$$

We know that, $W\cap M\subseteq W$ and $W\cap M\subseteq M$. This implies,

$$Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cap M)) \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W)) \tag{13}$$

$$Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cap M)) \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \tag{14}$$

$$Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cap M)) \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(M)) \tag{15}$$

$$Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cap M)) \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \tag{16}$$

From Eq. (11) and Eq. (12), we have

$$\begin{aligned} W\cap M &\supseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(W)) \cap Q\text{-}N_{int}(Q\text{-}N_{cl}(M)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(M)) \\ &\supseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cap M)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cap M)) \cap Q\text{-}N_{int}(Q\text{-}N_{cl}(W\cap M)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(W\cap M)) \end{aligned}$$

[by using eqs. (13), (14), (15) & (16)]

$= Q\text{-}N_{int}(Q\text{-}N_{cl}(W \cap M)) \cap Q\text{-}N_{cl}(Q\text{-}N_{int}(W \cap M))$.

This implies, $W \cap M \supseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W \cap M)) \cap Q\text{-}N_{int}(Q\text{-}N_{cl}(W \cap M))$. Hence, $W \cap M$ is a Q-N-*b*-C set in $(\Omega,\Im)$.

Theorem 3.10. In a Q-NTS $(\Omega,\Im)$, every Q-NPO set is a Q-N-*b*-O set.

Proof. Let W be a Q-NPO set in a Q-NTS $(\Omega,\Im)$. So, $W \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W))$. This implies, $W \subseteq Q\text{-}N_{int}(Q\text{-}N_{cl}(W)) \cup Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$. Therefore, W is a Q-N-*b*-O set. Hence, every Q-NPO set is a Q-N-*b*-O set.

Theorem 3.11. In a Q-NTS $(\Omega,\Im)$, every Q-NSO set is a Q-N-*b*-O set.

Proof. Let W be a Q-NSO set in a Q-NTS $(\Omega,\Im)$. Therefore, $W \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W))$. This implies, $W \subseteq Q\text{-}N_{cl}(Q\text{-}N_{int}(W)) \cup Q\text{-}N_{int}(Q\text{-}N_{cl}(W))$. Therefore, W is a Q-N-*b*-O set. Hence, every Q-NSO set is a Q-N-*b*-O set.

4. Conclusion: In this article, we introduce topology on Q-NSs. We study different types of open sets like Q-NPO set, Q-NSO set, Q-Nα-O set, and Q-N-*b*-O set, etc. By defining Q-NPO set, Q-NSO set, Q-N-*b*-O set, and Q-Nα-O set, we formulate some theorems, remarks on quadripartitioned neutrosophic topological space. Further, few illustrative examples are given. In the future, based on these notions and various open sets on Q-NTSs, many new investigations (compectness, para-compectness, conectedness, separation axioms) can be easily done. Also, the quadripartitioned neutrosophic topological operators can be used in the area of multi criteria decision making problems.

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