Neutrosophic Sets and Systems, Vol. 45, 2021
University of New Mexico

Entropy Measures for Interval Neutrosophic Vague Sets and Their Application in Decision Making

Hazwani Hashim [1,*], Lazim Abdullah [2], Ashraf Al-Quran [3] and Azzah Awang [4]

[1] Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM), Campus Machang, Kelantan 18500, Malaysia; hazwanihashim@uitm.edu.my
[2] Faculty of Ocean Engineering Technology and Informatics, University Malaysia Terengganu, Kuala Nerus 21030, Terengganu, Malaysia; lazim_m@umt.edu.my
[3] Preparatory Year Deanship, King Faisal University, Hofuf 31982, Al-Ahsa, Saudi Arabia; aalquraan@kfu.edu.sa
[4] Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM), Shah Alam, Selangor 40450, Malaysia; azzah@tmsk.uitm.edu.my

* Correspondence: hazwanihashim@uitm.edu.my

Abstract: Entropy measure is an important tool in measuring uncertain information and plays a vital role in solving Multi Criteria Decision Making (MCDM). At present, various entropy measures in literature are developed to measure the degree of fuzziness. However, they could not be used to deal with interval neutrosophic vague set (INVS) environment. In this study, two kinds of entropy measures are proposed as the extension of the entropy measure of single valued neutrosophic set (SVNS). First, we construct the axiomatic definition of INVS and propose a new formula for the entropy measure of INVS. Based on this measure, we develop two multi criteria decision making methods. Subsequently, an illustrative example of investment problems under INVS is given to demonstrate the proposed entropy measures. Finally, a comparative analysis is presented to illustrate the rationality and effectiveness of the proposed entropy measures.

Keywords: decision making ; fuzzy set theory; neutrosophic set theory; interval neutrosophic vague set theory; entropy measure; uncertainty.

1. Introduction

Different types of uncertainties arise in real life problems and many complex systems such as information fusion systems, medical diagnosis, decision making, and image processing. The issue of uncertainties in decision making recently become increasingly important since the appearance of classical mathematics. Hence, entropy measure notation has been introduced for measuring fuzzy information. Fuzziness, a characteristic of incomplete information, arises from the lack of crisp distinction between the elements belonging and not belonging to a set. Shanon and Weaver [1], [2] first proposed an entropy measure known as Shanon entropy. In 1968, Zadeh [3] extended the axiom of Shanon entropy to fuzzy entropy based on the fuzzy subset with respect to the concerned probability distribution. Later, Luca and Termimi [4] presented a formal definition of fuzzy entropy and defined several axioms. In addition, Sander [5] introduced Shanon fuzzy entropy measure and proved sharpness, valuation and general additivity and all properties of the fuzzy entropy. To investigate a more comprehensive entropy, Xie and Bedrosian [6] focused on the total fuzzy entropy. To counter the disadvantages of the total fuzzy entropy, Pal and Pal [7] introduced the objective measure in hybrid entropy used to get proper defuzzification of a certain fuzzy set. Shi and Yuan [8] suggested interval entropy, interval similarity measure, interval distance measure and interval inclusion measure of fuzzy set. As for the intuitionistic fuzzy sets (IFS) and their generalization by Attanasov [9], Burillo and Bustince [10] developed an intuitionistic fuzzy entropy measure and defined an axiomatic definition. Szmidt and Kacprzyk [11] suggested a new entropy measure that is based on a geometric representation of the intuitionistic fuzzy sets (IFS). Wei et al. [12] proposed entropy measures for interval-valued intuitionistic fuzzy sets (IVIFSs) and applied them in the case study of pattern recognition. Garg [13] developed an entropy measure under IVIFSs and used the proposed measure in solving MCDM with unknown attribute weights. Meanwhile, Rashid et al. [14] constructed the concept of entropy measure distance based on IVIFSs. All the above related entropy researches were dealt with under uncertain and fuzzy information. However, fuzzy sets cannot be dealt with indeterministic and inconsistent information.

Considering this limitation, Smarandache [15] proposed a neutrosophic set (NS) which is the three components of truth, indeterminacy, and falsity degrees and that can be denoted as T, I, F respectively. NS is characterized independently and the ranges of functions T, I, F are in form of real standard and the nonstandard interval $\left]^{-}0,1^{+}\right[$ which cannot be used in real applications. Therefore, Wang et al. [16] proposed single valued neutrosophic set (SVNS) where the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree in form of real standard interval. Later, Wang [17] introduced interval neutrosophic set (INS) as an extension of SVNS whose values are interval rather than real numbers. Alkhazaleh [18], introduced a neutrosophic vague set (NV) by incorporating the features of SVNS and vague set [19]. Besides that, he also defined several operators for NV and proved related properties. NV has played a significant role in the uncertain information system. In certain NV sets, the degree of truth, falsity, and indeterminacy of a given statement cannot be strictly described in real-world contexts, but it is instead denoted by several possible interval values. To overcome this problem, Hashim et al. [20] introduced an interval neutrosophic vague set (INVS) by upgrading the membership functions in several interval membership degrees. The advantage of INVS is that it can deal with more uncertain information than NV on similar decision situations. In light of its significance, many scholars have worked to improve the concept of neutrosophic in decision making [21]–[28].

The recent rapid developments of NS and its generalization have heightened the need for measuring the fuzziness degree under NS setting. Therefore, Majumdar and Samanta [29] proposed the entropy and distance measures under SVNS. They defined the formula for entropy measure and proved related properties. Later, Wu et al. [30] suggested an entropy that overcomes the limitations in Majumdar and Samanta's entropy. They suggested a better concept of complement of SVNS where $A^{C} = \left\{ F_A(x), 1 - I_A(x), T_A(x) \middle| x \in X \right\}$ instead of $\bar{A} = \left\{ \left(1 - T_A(x), 1 - I_A(x), 1 - F_A(x)\right) \middle| x \in X \right\}$. Later, Garg [31] suggested SVNS entropy of order α. For various parameter values, the suggested entropy is more stable and scalable. In addition, Abu Qamar and Hassan [32] proposed several entropy, distance, and similarity measures for Q-Neutrosophic soft sets and applied this measure in medical diagnosis and decision making problems. The ranking method used in this example is based on the smallest entropy value. In 2020, Thao and Smarandache [33], proposed a new entropy measure based similarity measure under SVNS. They claimed that the proposed entropy by Majumdar and Samanta did not satisfy the axiomatic definition where it contradicts with the value of entropy in $[0,1]$. Hence, they proposed two entropy measures based on the intuitionistic fuzzy set entropy. Ye and Du [34] established some distances, similarity, and entropy measures and studied its relationship. They also compared the proposed entropy measure with existing entropy measures. Aydogdu and Sahin [35] defined two entropy measures for SVNS and INS. Based on this measure, they proposed a decision making problems under SVNS and INS. Quek et al. [36] proposed a new formula for the entropy measure under plithogenic sets. Very recently, Ye [37] suggested entropy measures based on trigonometric functions under simplified neutrosophic sets. In addition, he proposed a new ranking method of entropy values by considering positive and negative arguments.

Entropy measure also is a well-known approach for generating the weight in MCDM. Biswas et al. [38] used entropy proposed by [29] to measure the weight of criteria and applied it in grey relational analysis (GRA). Also, Thao and Smarandache [39] proposed complex proportional assessment (COPRAS) with weight calculated based on the proposed entropy. Besides that, Barukab et al. [40] proposed entropy measure under spherical fuzzy information to calculate unknown weights information and applied it in the technique for order performance by similarity to ideal solution (TOPSIS) method. Ye [41] proposed entropy weights-based correlation coefficients of IVIFS and used it to solve decision making problems with unknown information on criteria weights. Until now, there is no entropy measures under the INVS environment. A summary of the previous researcher's contribution is presented in Table 1.

In this paper, we introduce the INVS entropy measure that is extended from the concept of SVNS entropy in [29], [33]. This measure will resolve the limitation of the entropy proposed by Majumdar and Samanta that has been claimed as invalid by Thao and Smarandache [33]. The illustration of the limitation in [29], [33] is discussed in Subsection 3.1 and 3.2. These are the motivations that driven us to investigate a more appropriate concept of entropy measure under the INVS environment. To do so, the rest of this paper is organized as follows: Section 2 presents the definition of INVS and its relation on INVS. In Section 3, we introduce two types of entropy of INVS, propose its formula and prove related properties. The illustrative example based on the proposed entropy of INVS and comparative analysis are presented in Section 4 and the conclusion is presented in Section 5.

Table 1: Summary of contribution under neutrosophic environment

Author	Year	Set	Contributions	Gaps
Majumdar and Samanta [29]	2014	SVNS	• Introduce similarity and entropy of a neutrosophic set	• Did not satisfy the axiomatic definition for the proposed entropy measure.
Garg [31]	2016	SVNS	• Single-valued neutrosophic entropy of order α • Consider the pair of their membership functions as well as hesitation degree between them.	• Ranking methods of entropy values are not always reasonable
Wu et al. [30]	2018	SVNS	• Introduce similarity measure and cross-entropy of single-valued neutrosophic sets	• Did not consider the standard definition of a complement of SVNS
Abu Qamar and Hassan[32]	2018	Q-neutrosophic soft set	• Introduce entropy, distance, and similarity measure	• Ranking methods of entropy values are not always reasonable

Thao and Smarandache [33]	2019	SVNS	• Introduce entropy-based similarity measures of single valued neutrosophic sets • A natural extension of the concept of entropy measure of fuzzy sets and IFS	• Ranking methods of entropy values are not always reasonable
Quek et al. [36]	2020	Plithogenic sets	• Introduce Entropy Measures for Plithogenic Sets	• This entropy is limited when applied to the Plithogenic set. Due to the complexity and novelty of Plitogenic sets.
Ye [37]	2021	Simplified neutrosophic sets	• Entropy measures based on trigonometric functions	• Only considers the positive and negative arguments regarding the entropy values for different alternatives.

2. Preliminaries

In this section, we review some basic concepts related to INVS, which will be used in the rest of the paper.

Definition 2.1:[20] Let X be a universe discourse and an INVS A is written as follows:

$$A=\left\{x,\left[\bar{T}_A^L(x),\bar{T}_A^U(x)\right],\left[\bar{I}_A^L(x),\bar{I}_A^U(x)\right],\left[\bar{F}_A^L(x),\bar{F}_A^U(x)\right]>\middle|x\in X\right\} \quad (1)$$

Whose truth membership, indeterminacy membership, and falsity membership functions are defined as:

$$\bar{T}_A^L(x)=\left[T^{L-},T^{L+}\right],\ \bar{T}_A^U(x)=\left[T^{U-},T^{U+}\right],\ \bar{I}_A^L(x)=\left[I^{L-},I^{L+}\right],\ \bar{I}_A^U(x)=\left[I^{U-},I^{U+}\right] \text{ and}$$

$$\bar{F}_A^L(x)=\left[F^{L-},F^{L+}\right],\ \bar{F}_A^U(x)=\left[F^{U-},F^{U+}\right] \quad (2)$$

The symbols L and U denote the lower and upper of the intervals in which

$$T^{L+}=1-F^{L-},\ F^{L+}=1-T^{L-}$$

$$F^{U-}=1-T^{U+},\ T^{U-}=1-F^{U+}$$

and satisfying

$$0\le T^{L-}+T^{U-}+I^{L-}+I^{U-}+F^{L-}+F^{U-}\le 4 \quad (3)$$

$$0\le T^{L+}+T^{U+}+I^{L+}+I^{U+}+F^{L+}+F^{U+}\le 4 \quad (4)$$

Definition 2.2 [20] Let $A=\left\{x,\left[\bar{T}_A^L(x),\bar{T}_A^U(x)\right],\left[\bar{I}_A^L(x),\bar{I}_A^U(x)\right],\left[\bar{F}_A^L(x),\bar{F}_A^U(x)\right]>\middle| x\in X\right\}$ and $B=\left\{x,\left[\bar{T}_B^L(x),\bar{T}_B^U(x)\right],\left[\bar{I}_B^L(x),\bar{I}_B^U(x)\right],\left[\bar{F}_B^L(x),\bar{F}_B^U(x)\right]>\middle| x\in X\right\}$ be two INVSs. Then the relation of INVS is defined as follows:

i. $A=B$ if and only if $A=B$ if $T_A^L(x)=T_B^L(x), T_A^U(x)=T_B^U(x), I_A^L(x)=I_B^L(x)$

$I_A^U(x)=I_B^U(x)$, $F_A^L(x)=F_B^L(x)$ and $F_A^U(x)=F_B^U(x)$.

ii. $A\subseteq B$ if and only if $T_A^L(x)\le T_B^L(x), T_A^U(x)\le T_B^U(x), I_A^L(x)\ge I_B^L(x), I_A^U(x)\ge I_B^U(x)$,

$F_A^L(x)\ge F_B^L(x)$ and $F_A^U(x)\ge F_B^U(x)$

iii. $A\cap(\cup)B=C$

$T_C^L(x)=\left[\cap(\cup)\left(T_A^{L-},T_B^{L-}\right),\cap(\cup)\left(T_A^{L+},T_B^{L+}\right)\right]$, $T_C^U(x)=\left[\cap(\cup)\left(T_A^{U-},T_B^{U-}\right),\cap(\cup)\left(T_A^{U+},T_B^{U+}\right)\right]$,

$I_C^L(x)=\left[\cup(\cap)\left(I_A^{L-},I_B^{L-}\right),\cup(\cap)\left(I_A^{L+},I_B^{L+}\right)\right]$, $I_C^U(x)=\left[\cup(\cap)\left(I_A^{U-},I_B^{U-}\right),\cup(\cap)\left(I_A^{U+},I_B^{U+}\right)\right]$,

$F_C^L(x)=\left[\cup(\cap)\left(F_A^{L-},F_B^{L-}\right),\cup(\cap)\left(F_A^{L+},F_B^{L+}\right)\right]$, $F_C^U(x)=\left[\cup(\cap)\left(F_A^{U-},F_B^{U-}\right),\cup(\cap)\left(F_A^{U+},F_B^{U+}\right)\right]$

iv. A^C

$\left(T^L\right)^C(x)=\left[1-T_A^{L+},1-T_A^{L-}\right],\left(T^U\right)^C(x)=\left[1-T_A^{U+},1-T_A^{U-}\right]$,

$\left(I^L\right)^C(x)=\left[1-I_A^{L+},1-I_A^{L-}\right],\left(I^U\right)^C(x)=\left[1-I_A^{U+},1-I_A^{U-}\right]$,

$\left(F^L\right)^C(x)=\left[1-F_A^{L+},1-F_A^{L-}\right],\left(F^U\right)^C(x)=\left[1-F_A^{U+},1-F_A^{U-}\right]$

3. The entropy of INVS

In this section, we introduce two entropy to measure the fuzziness degree of INVS information. The entropy of INVS is defined by two formulas which are based on interval approximation and INVS entropy generalized from the existing entropy of SVNS by Majumdar and Samantha [29]. We first give the axiomatic definition of INVS entropy.

The definition is derived to satisfy several conditions need in INVS entropy, as shown below:

(i) The entropy will be null when the set is a crisp set,

(ii) The entropy will be maximum if the set is completely INVS,

(iii) The INVS entropy and its complement is equal, and

(iv) If the degree of lower and upper approximation for truth membership, indeterminacy membership and falsity membership of each element decreases, the sum will do as well, and therefore this set becomes fuzzier and consequently the entropy should increase.

In light of the conditions stated above, the axiomatic definition of INVS entropy is defined as follows:

Definition 3.1: Let INVS(X) be a set of all INVSs on (X) and $E : INVS(X) \to [0,1]$ satisfying all following properties:

(E0) (Nonenegativity) $0 \le E(A) \le 1$

(E1) (Minimality) $E(A) = 0$ if A is a crisp set i.e

$\left[T_A^{L-}(x_i), T_A^{L+}(x_i)\right] = [1,1]$, $\left[T_A^{U-}(x_i), T_A^{U+}(x_i)\right] = [1,1]$, $\left[I_A^{L-}(x_i), I_A^{L+}(x_i)\right] = [0,0]$,

$\left[I_A^{U-}(x_i), I_A^{U+}(x_i)\right] = [0,0]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [0,0]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [0,0]$ or

$\left[T_A^{L-}(x_i), T_A^{L+}(x_i)\right] = [0,0]$, $\left[T_A^{U-}(x_i), T_A^{U+}(x_i)\right] = [0,0]$, $\left[I_A^{L-}(x_i), I_A^{L+}(x_i)\right] = [0,0]$,

$\left[I_A^{U-}(x_i), I_A^{U+}(x_i)\right] = [0,0]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [1,1]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [1,1]$ for all $x_i \in X$

(E2) (Maximality) $E(A) = 1$ if $\left[T_A^{L-}(x_i), T_A^{L+}(x_i)\right] = [0.5, 0.5]$, $\left[T_A^{U-}(x_i), T_A^{U+}(x_i)\right] = [0.5, 0.5]$,

$\left[I_A^{L-}(x_i), I_A^{L+}(x_i)\right] = [0.5, 0.5]$, $\left[I_A^{U-}(x_i), I_A^{U+}(x_i)\right] = [0.5, 0.5]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [0.5, 0.5]$,

$\left[F_A^{U-}(x_i), F_A^{U+}(x_i)\right] = [0.5, 0.5]$ for all $x_i \in X$

(E3) (Symmetric) $E(A) = E(A^C)$ for all $A \in INVS(X)$,

(E4) (Resolution) $E(A) \ge E(B)$ if , i.e,

$T_A^{L-}(x_i) \ge T_B^{L-}(x_i)$, $T_A^{L+}(x_i) \ge T_B^{L+}(x_i)$, $T_A^{U-}(x_i) \ge T_B^{U-}(x_i)$, $T_A^{U+}(x_i) \ge T_B^{U+}(x_i)$;

$I_A^{L-}(x_i) \le I_B^{L-}(x_i)$, $I_A^{L+}(x_i) \le I_B^{L+}(x_i)$, $I_A^{U-}(x_i) \le I_B^{U-}(x_i)$, $I_A^{U+}(x_i) \le I_B^{U+}(x_i)$;

$F_A^{L-}(x_i) \le F_B^{L-}(x_i)$, $F_A^{L+}(x_i) \le F_B^{L+}(x_i)$, $I_A^{U-}(x_i) \le I_B^{U-}(x_i)$, $F_A^{U+}(x_i) \le F_B^{U+}(x_i)$.

Now, we define the INVS entropy based on interval approximation $\left(E_{\Delta INVS}\right)$ and entropy $\left(E_{INVS}\right)$ generalized from SVNS entropy in Subsection 3.1 and 3.2. The notations and descriptions are used in the proposed entropy measures are presented in Table 2. The proposed entropy should satisfy the axiomatic Definition 3.1.

Table 2: Some notations and descriptions

Notation	Description
X , x	Universal set, element of X
T, I, F	Truth, indeterminacy and false membership functions
$\left[\bar{T}_A^L(x), \bar{T}_A^U(x)\right]$	Truth interval valued with respect to upper bound and lower bound.
$\bar{T}_A^L(x) = \left[T^{L-}, T^{L+}\right]$	Truth lower interval valued functions with respect to the beginning of interval and end of the interval.
$\bar{T}_A^U(x) = \left[T^{U-}, T^{U+}\right]$	Truth upper interval valued functions with respect to the beginning of interval and end of the interval.
$\bar{I}_A^L(x) = \left[I^{L-}, I^{L+}\right]$	Indeterminacy lower interval valued functions with respect to the beginning of interval and end of the interval.
$\bar{I}_A^U(x) = \left[I^{U-}, I^{U+}\right]$	Indeterminacy upper interval valued functions with respect to the beginning of interval and end of the interval.
$\bar{F}_A^L(x) = \left[F^{L-}, F^{L+}\right]$	Falsity lower interval valued functions with respect to the beginning of interval and end of the interval.
$\bar{F}_A^U(x) = \left[F^{U-}, F^{U+}\right]$	Falsity lower interval valued functions with respect to the beginning of interval and end of the interval.
x_i	Element set of criteria in the universe X
$E_{\Delta INVS}$	INVS entropy based on interval approximation
E_{INVS}	INVS entropy generalized from SVNS entropy

3.1. INVS entropy based on interval approximation

In this subsection, a new concept of INVS entropy is generated from the entropy measure proposed by Thao and Smarandache [33]. This measure can deal only with SVNS set. Moreover, this entropy used the concept of natural extension of the concept of entropy measure of fuzzy sets and IFS. The SVNS entropy measure is defined as follows:

$$E_T(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{|T_A(x_i) - 0.5| + |F_A(x_i) - 0.5| + |I_A(x_i) - 0.5| + |I_{A^c}(x_i) - 0.5|}{2} \quad (5)$$

By using this measure, we presented a new concept of the INVS entropy based on interval approximation denoted as $E_{\Delta INVS}$. The interval approximation represents the average possible membership degree of truth, indeterminate, and falsity of element x . Definition of INVS entropy measure is presented as follows:

Definition 3.1.1: The entropy of the interval neutrosophic vague sets denoted as $E_{\Delta INVS}(A)$ and defined by

$$E_{\Delta INVS}(A) = 1 - \frac{1}{2n}\sum_{i=1}^{n} E_{\Delta INVS}(A)(x_i)$$

where

$$\sum_{i=1}^{n} E_{\Delta INVS}(A)(x_i) = \left|\frac{T_A^{L-}(x_i)+T_A^{L+}(x_i)+T_A^{U-}(x_i)+T_A^{U+}(x_i)}{4}-0.5\right| + \left|\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)}{4}-0.5\right| +$$
$$\left|\frac{I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)}{4}-0.5\right| + \left|\frac{I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)}{4}-0.5\right|$$

for all $i = 1, 2, \ldots, n$ (6)

Theorem 1. $E_{\Delta INVS}(A)$ as defined in Definition 3.1.1 is entropy for INVSs

Now, we show that $E_{\Delta INVS}(A)$ satisfies all properties given in Definition 3.1.

Proof:

(E1) if A is a crisp set then $\left[T_A^{L-}(x_i), T_A^{L+}(x_i)\right] = [1,1]$, $\left[T_A^{U-}(x_i), T_A^{U+}(x_i)\right] = [1,1]$, $\left[I_A^{L-}(x_i), I_A^{L+}(x_i)\right] = [0,0]$, $\left[I_A^{U-}(x_i), I_A^{U+}(x_i)\right] = [0,0]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [0,0]$, $\left[F_A^{U-}(x_i), F_A^{U+}(x_i)\right] = [0,0]$ or $\left[T_A^{L-}(x_i), T_A^{L+}(x_i)\right] = [0,0]$, $\left[T_A^{U-}(x_i), T_A^{U+}(x_i)\right] = [0,0]$, $\left[I_A^{L-}(x_i), I_A^{L+}(x_i)\right] = [0,0]$, $\left[I_A^{U-}(x_i), I_A^{U+}(x_i)\right] = [0,0]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [1,1]$, $\left[F_A^{U-}(x_i), F_A^{U+}(x_i)\right] = [1,1]$

for all $x_i \in X$ we have

$$E_{\Delta INVS}(A)(x_i) = \left|\frac{1+1+1+1}{4}-0.5\right| + \left|\frac{0+0+0+0}{4}-0.5\right| + \left|\frac{0+0+0+0}{4}-0.5\right| + \left|\frac{1+1+1+1}{4}-0.5\right| = 2$$

It implies that, $E_{\Delta INVS}(A) = 1 - \frac{1}{2(1)}(2) = 0$ or

$$E_{\Delta INVS}(A)(x_i) = \left|\frac{0+0+0+0}{4}-0.5\right| + \left|\frac{1+1+1+1}{4}-0.5\right| + \left|\frac{0+0+0+0}{4}-0.5\right| + \left|\frac{1+1+1+1}{4}-0.5\right| = 2$$

It implies that, $E_{\Delta INVS}(A) = 1 - \frac{1}{2(1)}(2) = 0$

Therefore, the INVS entropy will be null $(E_{INVS}(A) = 0)$ when the set is a crisp set.

(E2) $\left[T_A^{L-}(x_i), T_A^{L+}(x_i)\right] = [0.5, 0.5]$, $\left[T_A^{U-}(x_i), T_A^{U+}(x_i)\right] = [0.5, 0.5]$, $\left[I_A^{L-}(x_i), I_A^{L+}(x_i)\right] = [0.5, 0.5]$, $\left[I_A^{U-}(x_i), I_A^{U+}(x_i)\right] = [0.5, 0.5]$, $\left[F_A^{L-}(x_i), F_A^{L+}(x_i)\right] = [0.5, 0.5]$ and $\left[F_A^{U-}(x_i), F_A^{U+}(x_i)\right] = [0.5, 0.5]$

$$E_{\Delta INVS}(A)(x_i) = \left|\frac{0.5+0.5+0.5+0.5}{4}-0.5\right| + \left|\frac{0.5+0.5+0.5+0.5}{4}-0.5\right| + \left|\frac{0.5+0.5+0.5+0.5}{4}-0.5\right| +$$
$$\left|\frac{0.5+0.5+0.5+0.5}{4}-0.5\right| = 0$$

It implies that

$$E_{\Delta INVS}(A) = 1 - \frac{1}{2(1)}(0) = 1$$

Therefore, the entropy will be maximum $(E_{INVS}(A) = 1)$ if the set is completely INVS.

$$\textbf{(E3)}\quad E_{\Delta INVS}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{\left(1-T_A^{L+}(x_i)\right)+\left(1-T_A^{L-}(x_i)\right)+\left(1-T_A^{U+}(x_i)\right)+\left(1-T_A^{U-}(x_i)\right)}{4}-0.5\right|+$$

$$\left|\frac{\left(1-F_A^{L+}(x_i)\right)+\left(1-F_A^{L-}(x_i)\right)+\left(1-F_A^{U+}(x_i)\right)+\left(1-F_A^{U-}(x_i)\right)}{4}-0.5\right|+$$

$$\left|\frac{\left(1-I_A^{L+}(x_i)\right)+\left(1-I_A^{L-}(x_i)\right)+\left(1-I_A^{U+}(x_i)\right)+\left(1-I_A^{U-}(x_i)\right)}{4}-0.5\right|+$$

$$\left|\frac{\left(1-I_{A^c}^{L+}(x_i)\right)+\left(1-I_{A^c}^{L-}(x_i)\right)+\left(1-I_{A^c}^{U+}(x_i)\right)+\left(1-I_{A^c}^{U-}(x_i)\right)}{4}-0.5\right|$$

$$=1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{4-\left(T_A^{L+}(x_i)+T_A^{L-}(x_i)+T_A^{U+}(x_i)+T_A^{U-}(x_i)\right)}{4}-0.5\right|+\left|\frac{4-\left(F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)\right)}{4}-0.5\right|+$$

$$\left|\frac{4-\left(I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)\right)}{4}-0.5\right|+\left|\frac{4-\left(I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)\right)}{4}-0.5\right|$$

$$=1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{4-\left(T_A^{L+}(x_i)+T_A^{L-}(x_i)+T_A^{U+}(x_i)+T_A^{U-}(x_i)\right)-0.5(4)}{4}\right|+\left|\frac{4-\left(F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)\right)-0.5(4)}{4}\right|+$$

$$\left|\frac{4-\left(I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)\right)-0.5(4)}{4}\right|+\left|\frac{4-\left(I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)\right)-0.5(4)}{4}\right|$$

$$=1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{2-\left(T_A^{L+}(x_i)+T_A^{L-}(x_i)+T_A^{U+}(x_i)+T_A^{U-}(x_i)\right)}{4}\right|+\left|\frac{2-\left(F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)\right)}{4}\right|+$$

$$\left|\frac{2-\left(I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)\right)}{4}\right|+\left|\frac{2-\left(I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)\right)}{4}\right|$$

$$=1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{T_A^{L+}(x_i)+T_A^{L-}(x_i)+T_A^{U+}(x_i)+T_A^{U-}(x_i)}{4}-0.5\right|+\left|\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)}{4}-0.5\right|+$$

$$\left|\frac{I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)}{4}-0.5\right|+\left|\frac{I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)}{4}-0.5\right|=E_{\Delta INVS}(A^c)$$

Therefore, INVS entropy and its complement is equal; $E_{\Delta INVS}(A)=E_{\Delta INVS}(A^c)$ for all $A\in INVS(X)$

(E4) we have $E_{INVS}(A)\geq E_{INVS}(B)$ if $T_A^{L-}(x_i)\geq T_B^{L-}(x_i)$, $T_A^{L+}(x_i)\geq T_B^{L+}(x_i)$, $T_A^{U-}(x_i)\geq T_B^{U-}(x_i)$, $T_A^{U+}(x_i)\geq T_B^{U+}(x_i)$, $I_A^{L-}(x_i)\leq I_B^{L-}(x_i)$, $I_A^{L+}(x_i)\leq I_B^{L+}(x_i)$, $I_A^{U-}(x_i)\leq I_B^{U-}(x_i)$, $I_A^{U+}(x_i)\leq I_B^{U+}(x_i)$; $F_A^{L-}(x_i)\leq F_B^{L-}(x_i)$, $F_A^{L+}(x_i)\leq F_B^{L+}(x_i)$, $I_A^{U-}(x_i)\leq I_B^{U-}(x_i)$, $F_A^{U+}(x_i)\leq F_B^{U+}(x_i)$ for $x_i\in X$;

Then we obtain the following relation:

a) $$\left|\frac{T_A^{L-}(x_i)+T_A^{L+}(x_i)+T_A^{U-}(x_i)+T_A^{U+}(x_i)}{4}-0.5\right| \geq \left|\frac{T_B^{L-}(x_i)+T_B^{L+}(x_i)+T_B^{U-}(x_i)+T_B^{U+}(x_i)}{4}-0.5\right|$$

b) $$\left|\frac{I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)}{4}-0.5\right| \leq \left|\frac{I_B^{L-}(x_i)+I_B^{L+}(x_i)+I_B^{U-}(x_i)+I_B^{U+}(x_i)}{4}-0.5\right|$$

c) $$\left|\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)}{4}-0.5\right| \leq \left|\frac{F_B^{L-}(x_i)+F_B^{L+}(x_i)+F_B^{U-}(x_i)+F_B^{U+}(x_i)}{4}-0.5\right|$$

d) $$\left|\frac{I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)}{4}-0.5\right| \leq \left|\frac{I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)}{4}-0.5\right|$$

Combining a), b), c), and d) we obtain

$$E_{\Delta INVS}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{T_A^{L-}(x_i)+T_A^{L+}(x_i)+T_A^{U-}(x_i)+T_A^{U+}(x_i)}{4}-0.5\right|+\left|\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{U-}(x_i)+F_A^{U+}(x_i)}{4}-0.5\right|+$$
$$\left|\frac{I_A^{L-}(x_i)+I_A^{L+}(x_i)+I_A^{U-}(x_i)+I_A^{U+}(x_i)}{4}-0.5\right|+\left|\frac{I_{A^c}^{L-}(x_i)+I_{A^c}^{L+}(x_i)+I_{A^c}^{U-}(x_i)+I_{A^c}^{U+}(x_i)}{4}-0.5\right|\geq$$
$$1-\frac{1}{2n}\sum_{i=1}^{n}\left|\frac{T_B^{L-}(x_i)+T_B^{L+}(x_i)+T_B^{U-}(x_i)+T_B^{U+}(x_i)}{4}-0.5\right|+\left|\frac{F_B^{L-}(x_i)+F_B^{L+}(x_i)+F_B^{U-}(x_i)+F_B^{U+}(x_i)}{4}-0.5\right|+$$
$$\left|\frac{I_B^{L-}(x_i)+I_B^{L+}(x_i)+I_B^{U-}(x_i)+I_B^{U+}(x_i)}{4}-0.5\right|+\left|\frac{I_{B^c}^{L-}(x_i)+I_{B^c}^{L+}(x_i)+I_{B^c}^{U-}(x_i)+I_{B^c}^{U+}(x_i)}{4}-0.5\right|$$

That is $E_{\Delta INVS}(A)\geq E_{\Delta INVS}(B)$. Thus, the property (E4) is satisfied.

The proof is completed.

Apart from INVS based on interval approximation, we also have the INVS entropy based on SVNS entropy. This definition is given as follows.

3.2. *INVS entropy generalized from SVNS entropy*

In this subsection, we developed another approach to measure the degree of fuzziness of an INVS. It is generalized from SVNS entropy proposed by Majumdar and Samanta [29]. The SVNS entropy is defined as follows:

$$E_{MM}(A)=1-\frac{1}{n}\sum_{i=1}^{n}\left[T_A(x_i)+F_A(x_i)\right]\left|I_A(x_i)-I_{A^C}(x_i)\right| \quad (7)$$

The SVNS entropy meets the principle of entropy measure. But, Thao and Smarandache [29] claimed in some conditions such as when $A=\{(x,0.8,0.0.7)\}$ on $X=\{x\}$ then we substitute in the SVNS entropy $E_{MM}(A)=1-\frac{1}{1}(0.8+0.7)|0-1|=-0.5\notin[0,1]$.

Therefore, to overcome this limitation we improved to

$E_{MM}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}\left[T_A(x_i)+F_A(x_i)\right]\left|I_A(x_i)-I_{A^c}(x_i)\right|$. So we have,

$E_{MM}(A)=1-\frac{1}{2}(0.8+0.7)\left|0-1\right|=0.75\in[0,1]$.

For convenient and suitability of the INVS entropy, Equation 6 is simplified using the complement definition $I_{A^c}=1-I(x)$ as follows:

$$\begin{aligned}E_{MM}(A)&=1-\frac{1}{2n}\sum_{i=1}^{n}\left[T_A(x_i)+F_A(x_i)\right]\left|\ I_A(x_i)-\left(1-I_A(x_i)\right)\right|\\&=1-\frac{1}{2n}\sum_{i=1}^{n}\left[T_A(x_i)+F_A(x_i)\right]\left|\ 2\ I_A(x_i)-1\ \right|\end{aligned}\tag{8}$$

Based on the SVNS entropy we generalized the entropy formula under the INVS environment. The INVS entropy based on SVNS entropy denoted as $E_{INVS}(A)$ is defined as follows:

Definition 3.2.1: An entropy $E_{INVS}(A)$ on interval neutrosophic vague sets is a function $E:INVS(X)\to[0,1]$ satisfying given condition in Definition 3.1. Then

$$E_{INVS}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}E_{INVS}(A)(x_i)$$

where

$$\begin{aligned}\sum_{i=1}^{n}E_{INVS}(A)(x_i)=\sum_{i=1}^{n}&\left\{\left(\left|\frac{T_A^{L-}(x_i)+T_A^{L+}(x_i)}{2}-\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)}{2}\right|\cdot\left|I_A^{L-}(x_i)+I_A^{L+}(x_i)-1\right|\right)\right.\\&\left.+\left(\left|\frac{T_A^{U-}(x_i)+T_A^{U+}(x_i)}{2}-\frac{F_A^{U-}(x_i)+F_A^{U+}(x_i)}{2}\right|\cdot\left|I_A^{U-}(x_i)+I_A^{U+}(x_i)-1\right|\right)\right\}\end{aligned}$$

for all $i=1,2,\ldots,n$ (9)

Theorem 2. $E_{INVS}(A)$ as specified in Definition 3.2.1 is entropy for INVS

To show that $E_{INVS}(A)$ is a valid measure, we must show that it satisfies the axioms mentioned in Definition 3.1.

Proof

(E1) if A is a crisp set then $\left[T_A^{L-}(x_i),T_A^{L+}(x_i)\right]=[1,1]$, $\left[T_A^{U-}(x_i),T_A^{U+}(x_i)\right]=[1,1]$, $\left[I_A^{L-}(x_i),I_A^{L+}(x_i)\right]=[0,0]$, $\left[I_A^{U-}(x_i),I_A^{U+}(x_i)\right]=[0,0]$, $\left[F_A^{L-}(x_i),F_A^{L+}(x_i)\right]=[0,0]$, $\left[F_A^{U-}(x_i),F_A^{U+}(x_i)\right]=[0,0]$ or $\left[T_A^{L-}(x_i),T_A^{L+}(x_i)\right]=[0,0]$, $\left[T_A^{U-}(x_i),T_A^{U+}(x_i)\right]=[0,0]$, $\left[I_A^{L-}(x_i),I_A^{L+}(x_i)\right]=[0,0]$, $\left[I_A^{U-}(x_i),I_A^{U+}(x_i)\right]=[0,0]$, $\left[F_A^{L-}(x_i),F_A^{L+}(x_i)\right]=[1,1]$, $\left[F_A^{U-}(x_i),F_A^{U+}(x_i)\right]=[1,1]$ for all $x_i\in X$ we have

$$E_{INVS}(A)(x_i)=\left(\left|\frac{1+1}{2}-\frac{0+0}{2}\right|\cdot\left|0+0-1\right|\right)+\left(\left|\frac{1+1}{2}-\frac{0+0}{2}\right|\cdot\left|0+0-1\right|\right)=2$$

It implies that, $E_{INVS}(A)=1-\frac{1}{2(1)}\left(2\right)=0$ or

$$E_{INVS}(A)(x_i)=\left(\left|\frac{0+0}{2}-\frac{1+1}{2}\right|\cdot\left|0+0-1\right|\right)+\left(\left|\frac{0+0}{2}-\frac{1+1}{2}\right|\cdot\left|0+0-1\right|\right)=2$$

It implies that, $E_{INVS}(A)=1-\frac{1}{2(1)}\left(2\right)=0$

Therefore, the INVS entropy will be null $(E_{INVS}(A)=0)$ when the set is a crisp set.

(E2) $\left[T_A^{L-}(x_i),T_A^{L+}(x_i)\right]=\left[0.5,0.5\right]$, $\left[T_A^{U-}(x_i),T_A^{U+}(x_i)\right]=\left[0.5,0.5\right]$, $\left[I_A^{L-}(x_i),I_A^{L+}(x_i)\right]=\left[0.5,0.5\right]$, $\left[I_A^{U-}(x_i),I_A^{U+}(x_i)\right]=\left[0.5,0.5\right]$, $\left[F_A^{L-}(x_i),F_A^{L+}(x_i)\right]=\left[0.5,0.5\right]$ and $\left[F_A^{U-}(x_i),F_A^{U+}(x_i)\right]=\left[0.5,0.5\right]$ for all $x_i\in X$

$$E_{INVS}(A)(x_i)=\left(\left|\frac{0.5+0.5}{2}-\frac{0.5+0.5}{2}\right|\cdot\left|0.5+0.5-1\right|\right)+\left(\left|\frac{0.5+0.5}{2}-\frac{0.5+0.5}{2}\right|\cdot\left|0.5+0.5-1\right|\right)=0$$

It implies that, $E_{INVS}(A)=1-\frac{1}{2(1)}\left(0\right)=1$

Therefore, the entropy will be maximum $(E_{INVS}(A)=1)$ if the set is completely INVS.

(E3)

$$E_{INVS}(A)=1-\frac{1}{2n}\left\{\left(\left|\frac{\left(1-T_A^{L+}(x_i)\right)+\left(1-T_A^{L-}(x_i)\right)}{2}-\frac{\left(1-F_A^{L+}(x_i)\right)+\left(1-F_A^{L-}(x_i)\right)}{2}\right|\cdot\left|\left(1-I_A^{L+}(x_i)\right)+\left(1-I_A^{L-}(x_i)\right)-1\right|\right)\right.$$

$$\left.+\left(\left|\frac{\left(1-T_A^{U+}(x_i)\right)+\left(1-T_A^{U-}(x_i)\right)}{2}-\frac{\left(1-F_A^{U+}(x_i)\right)+\left(1-F_A^{U-}(x_i)\right)}{2}\right|\cdot\left|\left(1-I_A^{U+}(x_i)\right)+\left(1-I_A^{U-}(x_i)\right)-1\right|\right)\right\}$$

$$=1-\frac{1}{2n}\left\{\left(\left|\frac{2-2-T_A^{L+}(x_i)-T_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{L-}(x_i)}{2}\right|\cdot\left|2-I_A^{L+}(x_i)-I_A^{L-}(x_i)-1\right|\right)+\right.$$

$$\left.\left(\left|\frac{2-2-T_A^{U+}(x_i)-T_A^{U-}(x_i)+F_A^{U+}(x_i)+F_A^{U-}(x_i)}{2}\right|\cdot\left|2-I_A^{U+}(x_i)-I_A^{U-}(x_i)\right|-1\right)\right\}$$

$$=1-\frac{1}{2n}\left\{\left(\left|\frac{-T_A^{L+}(x_i)-T_A^{L-}(x_i)+F_A^{L+}(x_i)+F_A^{L-}(x_i)}{2}\right|\cdot\left|1-I_A^{L+}(x_i)-I_A^{L-}(x_i)\right|\right)+\right.$$

$$\left.\left(\left|\frac{-T_A^{U+}(x_i)-T_A^{U-}(x_i)+F_A^{U+}(x_i)+F_A^{U-}(x_i)}{2}\right|\cdot\left|1-I_A^{U+}(x_i)-I_A^{U-}(x_i)\right|\right)\right\}$$

$$=1-\frac{1}{2n}\left\{\left(\left|\frac{T_A^{L+}(x_i)+T_A^{L-}(x_i)-F_A^{L+}(x_i)-F_A^{L-}(x_i)}{2}\right|\cdot\left|I_A^{L+}(x_i)+I_A^{L-}(x_i)-1\right|\right)+\right.$$

$$\left.\left(\left|\frac{T_A^{U+}(x_i)+T_A^{U-}(x_i)-F_A^{U+}(x_i)-F_A^{U-}(x_i)}{2}\right|\cdot\left|I_A^{U+}(x_i)+I_A^{U-}(x_i)-1\right|\right)\right\}$$

$$=1-\frac{1}{2n}\left\{\left(\left|\frac{T_A^{L+}(x_i)+T_A^{L-}(x_i)-\left(F_A^{L+}(x_i)+F_A^{L-}(x_i)\right)}{2}\right|\cdot\left|I_A^{L+}(x_i)+I_A^{L-}(x_i)-1\right|\right)+\right.$$

$$\left.\left(\left|\frac{\left(T_A^{U+}(x_i)+T_A^{U-}(x_i)\right)-\left(F_A^{U+}(x_i)+F_A^{U-}(x_i)\right)}{2}\right|\cdot\left|I_A^{U+}(x_i)+I_A^{U-}(x_i)-1\right|\right)\right\}=E_{INVS}(A^c)$$

Therefore, INVS entropy and its complement is equal; $E_{INVS}(A)=E_{INVS}(A^c)$ for all $A\in INVS(X)$

(**E4**) we have $E_{INVS}(A)\ge E_{INVS}(B)$ if $T_A^{L-}(x_i)\ge T_B^{L-}(x_i)$, $T_A^{L+}(x_i)\ge T_B^{L+}(x_i)$, $T_A^{U-}(x_i)\ge T_B^{U-}(x_i)$, $T_A^{U+}(x_i)\ge T_B^{U+}(x_i)$, $I_A^{L-}(x_i)\le I_B^{L-}(x_i)$, $I_A^{L+}(x_i)\le I_B^{L+}(x_i)$, $I_A^{U-}(x_i)\le I_B^{U-}(x_i)$, $I_A^{U+}(x_i)\le I_B^{U+}(x_i)$; $F_A^{L-}(x_i)\le F_B^{L-}(x_i)$, $F_A^{L+}(x_i)\le F_B^{L+}(x_i)$, $I_A^{U-}(x_i)\le I_B^{U-}(x_i)$, $F_A^{U+}(x_i)\le F_B^{U+}(x_i)$ for $x_i\in X$;

Then we obtain the following relation:

a) $$\left|\frac{T_A^{L-}(x_i)+T_A^{L+}(x_i)}{2}-\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)}{2}\right|\ge\left|\frac{T_B^{L-}(x_i)+T_B^{L+}(x_i)}{2}-\frac{F_B^{L-}(x_i)+F_B^{L+}(x_i)}{2}\right|,$$

$$\left|\frac{T_A^{U-}(x_i)+T_A^{U+}(x_i)}{2}-\frac{F_A^{U-}(x_i)+F_A^{U+}(x_i)}{2}\right|\ge\left|\frac{T_B^{U-}(x_i)+T_B^{U+}(x_i)}{2}-\frac{F_B^{U-}(x_i)+F_B^{U+}(x_i)}{2}\right|$$

b) $$\left|I_A^{L-}(x_i)+I_A^{L+}(x_i)-1\right|\le\left|I_B^{L-}(x_i)+I_B^{L+}(x_i)-1\right|$$

$$\left|I_A^{U-}(x_i)+I_A^{U+}(x_i)-1\right|\le\left|I_B^{U-}(x_i)+I_B^{U+}(x_i)-1\right|$$

Combining a) and b)

$$1-\frac{1}{2n}\sum_{i=1}^{n}E_{INVS}\left\{\left(\left|\frac{T_A^{L-}(x_i)+T_A^{L+}(x_i)}{2}-\frac{F_A^{L-}(x_i)+F_A^{L+}(x_i)}{2}\right|\cdot\left|I_A^{L-}(x_i)+I_A^{L+}(x_i)-1\right|\right)\right.$$

$$\left.+\left(\left|\frac{T_A^{U-}(x_i)+T_A^{U+}(x_i)}{2}-\frac{F_A^{U-}(x_i)+F_A^{U+}(x_i)}{2}\right|\cdot\left|I_A^{U-}(x_i)+I_A^{U+}(x_i)-1\right|\right)\right\}\ge$$

$$1-\frac{1}{2n}\sum_{i=1}^{n}\left\{\left(\left|\frac{T_B^{L-}(x_i)+T_B^{L+}(x_i)}{2}-\frac{F_B^{L-}(x_i)+F_B^{L+}(x_i)}{2}\right|\cdot\left|I_A^{L-}(x_i)+I_A^{L+}(x_i)-1\right|\right)+\right.$$

$$\left.\left(\left|\frac{T_B^{U-}(x_i)+T_B^{U+}(x_i)}{2}-\frac{F_B^{U-}(x_i)+F_B^{U+}(x_i)}{2}\right|\cdot\left|I_B^{U-}(x_i)+I_B^{U+}(x_i)-1\right|\right)\right\}$$

That is $E_{INVS}(A) \geq E_{INVS}(B)$. Thus, the property (E4) is satisfied.

The proof is completed.

The proposed entropy is further embedded into a MCDM.

4. MCDM problem based on proposed entropy

In this section, the proposed entropy measures are applied in MCDM problems. Measuring uncertainty is important in decision making problems, the DMs will obtain significant preference and priority to avoid losing out in the selection process. Based on the proposed entropy measures, when the entropy value is smaller, then DMs can provide more valuable knowledge from this alternative. As a result, the alternative with the lowest entropy value should be considered as a priority.

Consider the set of different alternatives denoted as $A = \{A_1, A_2, \ldots, A_m\}$ and set of criteria is denoted by $C = \{C_1, C_2, \ldots, C_n\}$ in INVS environment and the algorithm to evaluate the best alternative is presented as follows:

Step 1: Construction of decision making matrix

Organize each A_i alternatives under criteria C_j according to the DM's preferences in the form of INVS environment as follows

$D = (x_{ij})_{m\times n}$ where $x_{ij} = \left\{\left[T_{ij}^{L-}, T_{ij}^{L+}\right], \left[T_{ij}^{U-}, T_{ij}^{U+}\right]\right\}, \left\{\left[I_{ij}^{L-}, I_{ij}^{L+}\right], \left[I_{ij}^{U-}, I_{ij}^{U+}\right]\right\}, \left\{\left[F_{ij}^{L-}, F_{ij}^{L+}\right], \left[F_{ij}^{U-}, F_{ij}^{U+}\right]\right\}$

$0 \leq T^{L-} + T^{U-} + I^{L-} + I^{U-} + F^{L-} + F^{U-} \leq 4$ and $0 \leq T^{L+} + T^{U+} + I^{L+} + I^{U+} + F^{L+} + F^{U+} \leq 4$.

$\left[T_{ij}^{L-}, T_{ij}^{L+}\right]$ represents the degree that alternative A_i relatively satisfies the criteria C_j, $\left[T_{ij}^{U-}, T_{ij}^{U+}\right]$ represents the degree that alternative A_i absolutely satisfies the criteria C_j, $\left[I_{ij}^{L-}, I_{ij}^{L+}\right]$ represents the degree relatively indeterminant the criteria C_j, $\left[I_{ij}^{U-}, I_{ij}^{U+}\right]$ represents the degree absolutely indeterminant the criteria C_j, $\left[F_{ij}^{L-}, F_{ij}^{L+}\right]$ represents the degree that alternative A_i relatively doesn't satisfies the criteria C_j and $\left[F_{ij}^{U-}, F_{ij}^{U+}\right]$ represents the degree that alternative A_i absolutely doesn't satisfies the criteria C_j. Therefore, a decision matrix D is arranged as follows:

$$D = (x_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

Where x_{ij} is $x_{ij} = \left\{\left[T_{ij}^{L-}, T_{ij}^{L+}\right], \left[T_{ij}^{U-}, T_{ij}^{U+}\right]\right\}, \left\{\left[I_{ij}^{L-}, I_{ij}^{L+}\right], \left[I_{ij}^{U-}, I_{ij}^{U+}\right]\right\}, \left\{\left[F_{ij}^{L-}, F_{ij}^{L+}\right], \left[F_{ij}^{U-}, F_{ij}^{U+}\right]\right\}$

Step 2: Transform the INVS decision matrix $D=\left(x_{ij}\right)_{m\times n}$ into the normalized INVS decision matrix denoted as $\tilde{D}=\left(\tilde{x}_{ij}\right)_{m\times n}$ where

$$\tilde{x}_{ij}=\begin{cases} x_{ij}^{c};\, j\in B \\ x_{ij};\, j\in C \end{cases} \tag{10}$$

$$x_{ij}^{c}=\left\{\left[1-T_{ij}^{L+},1-T_{ij}^{L-}\right],\left[1-T_{ij}^{U+},1-T_{ij}^{U-}\right]\right\},\left\{\left[1-I_{ij}^{L+},1-I_{ij}^{L-}\right],\left[1-I_{ij}^{U+},1-I_{ij}^{U-}\right]\right\},\left\{\left[1-F_{ij}^{L+},1-F_{ij}^{L-}\right],\left[1-F_{ij}^{U+},1-F_{ij}^{U-}\right]\right\}$$

is complement of $x_{ij}=\left\{\left[T_{ij}^{L-},T_{ij}^{L+}\right],\left[T_{ij}^{U-},T_{ij}^{U+}\right]\right\},\left\{\left[I_{ij}^{L-},I_{ij}^{L+}\right],\left[I_{ij}^{U-},I_{ij}^{U+}\right]\right\},\left\{\left[F_{ij}^{L-},F_{ij}^{L+}\right],\left[F_{ij}^{U-},F_{ij}^{U+}\right]\right\}$.

Step 3: Calculate the entropy

$$E_{\Delta INVS}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}E_{\Delta INVS}(A)(x_i) \tag{11}$$

or

$$E_{INVS}(A)=1-\frac{1}{2n}\sum_{i=1}^{n}E_{INVS}(A)(x_i) \tag{12}$$

Step 4: Select the best option A_i for $i=1,2,\ldots,m$ based on smallest entropy.

4.1. Illustrative example

In this section, the proposed entropy measure is applied to the case study of investment decisions adapted from [31]. There is a DM with four potential investment options namely A_1 is a food company, A_2 is a transport company, A_3 is an electronic company, and A_4 is a tire company. The DM makes a choice based on three criteria, C_1 is growth analysis, C_2 is risk analysis, C_3 is an environment impact analysis. We begin by using the INVS entropy based on interval approximation $\left(E_{\Delta INVS}\right)$.

Method 1: Using INVS entropy based on interval approximation $\left(E_{\Delta INVS}\right)$.

Step 1: The linguistic evaluation consists of $A=\left\{A_1,A_2,A_3,A_4\right\}$ with respect to criteria $C=\left\{C_1,C_2,C_3\right\}$ are obtained from the expert evaluation. The INVS decision matrix denoted as $D=\left(x_{ij}\right)_{m\times n}$ is represented as follows:

	C_1	C_2	C_3
A_1	$[0.5,0.7],[0.6,0.8]$	$[0.5,0.6],[0.4,0.9]$	$[0.7,0.8],[0.5,0.8]$
	$[0.2,0.3],[0.3,0.4]$	$[0.1,0.2],[0.3,0.4]$	$[0.1,0.2],[0.3,0.4]$
	$[0.3,0.5],[0.2,0.4]$	$[0.4,0.5],[0.1,0.6]$	$[0.2,0.3],[0.2,0.5]$
A_2	$[0.4,0.7],[0.6,0.9]$	$[0.3,0.6],[0.1,0.5]$	$[0.6,0.7],[0.5,0.9]$
	$[0.2,0.3],[0.4,0.5]$	$[0.2,0.4],[0.4,0.5]$	$[0.3,0.4],[0.4,0.5]$
	$[0.3,0.6],[0.1,0.4]$	$[0.4,0.7],[0.5,0.9]$	$[0.3,0.4],[0.1,0.5]$
A_3	$[0.4,0.9],[0.7,0.9]$	$[0.5,0.7],[0.5,0.9]$	$[0.5,0.6],[0.4,0.7]$
	$[0.3,0.4],[0.4,0.5]$	$[0.1,0.2],[0.2,0.3]$	$[0.1,0.2],[0.2,0.3]$
	$[0.1,0.6],[0.1,0.3]$	$[0.3,0.5],[0.1,0.5]$	$[0.4,0.5],[0.3,0.6]$
A_4	$[0.6,0.8],[0.5,0.9]$	$[0.2,0.5],[0.1,0.4]$	$[0.4,0.8],[0.5,0.9]$
	$[0.1,0.2],[0.3,0.5]$	$[0.2,0.3],[0.3,0.4]$	$[0.3,0.4],[0.4,0.5]$
	$[0.2,0.4],[0.1,0.5]$	$[0.5,0.6],[0.6,0.9]$	$[0.2,0.6],[0.1,0.5]$

Step 2: since the criteria C_1 is the benefit criteria and C_2, C_3 are cost criteria, so the INVS decision matrix is transformed into the normalized INVS decision matrix using Equation 10.

	C_1	C_2	C_3
A_1	$[0.3,0.5],[0.2,0.4]$	$[0.5,0.6],[0.4,0.9]$	$[0.7,0.8],[0.5,0.8]$
	$[0.7,0.8],[0.6,0.7]$	$[0.1,0.2],[0.3,0.4]$	$[0.1,0.2],[0.3,0.4]$
	$[0.5,0.7],[0.6,0.8]$	$[0.4,0.5],[0.1,0.6]$	$[0.2,0.3],[0.2,0.5]$
A_2	$[0.3,0.6],[0.1,0.4]$	$[0.3,0.6],[0.1,0.5]$	$[0.6,0.7],[0.5,0.9]$
	$[0.7,0.8],[0.5,0.6]$	$[0.2,0.4],[0.4,0.5]$	$[0.3,0.4],[0.4,0.5]$
	$[0.4,0.7],[0.6,0.9]$	$[0.4,0.7],[0.5,0.9]$	$[0.3,0.4],[0.1,0.5]$
A_3	$[0.1,0.6],[0.1,0.3]$	$[0.5,0.7],[0.5,0.9]$	$[0.5,0.6],[0.4,0.7]$
	$[0.6,0.7],[0.5,0.6]$	$[0.1,0.2],[0.2,0.3]$	$[0.1,0.2],[0.2,0.3]$
	$[0.4,0.9],[0.7,0.9]$	$[0.3,0.5],[0.1,0.5]$	$[0.4,0.5],[0.3,0.6]$
A_4	$[0.2,0.4],[0.1,0.5]$	$[0.2,0.5],[0.1,0.4]$	$[0.4,0.8],[0.5,0.9]$
	$[0.8,0.9],[0.5,0.7]$	$[0.2,0.3],[0.3,0.4]$	$[0.3,0.4],[0.4,0.5]$
	$[0.6,0.8],[0.5,0.9]$	$[0.5,0.6],[0.6,0.9]$	$[0.2,0.6],[0.1,0.5]$

Step 3: Calculate the aggregated entropy measure for all the alternative $A=\{A_1,A_2,A_3,A_4\}$

By equation 11, we calculate for $i=1, j=1$, then

$$=\left|\frac{0.3+0.5+0.2+0.4}{4}-0.5\right|+\left|\frac{0.5+0.7+0.6+0.8}{4}-0.5\right|+\left|\frac{0.7+0.8+0.6+0.7}{4}-0.5\right|$$

$$+\left|\frac{0.2+0.3+0.3+0.4}{4}-0.5\right|=0.7$$

Therefore;

$A_1: \quad \sum_{i=1}^{3} E_{\Delta INVS}(A) = 0.7 + 0.7 + 0.9 = 2.3$ and $E_{\Delta INVS}(A) = 1 - \frac{1}{6}(2.3) = 0.6167$ and the rest of entropy measure for A_2, A_3, A_4 are calculated similarly and presented as follows:

$A_2 = 0.7250, A_3 = 0.6250, A_4 = 0.65$

Step 4: Based on the smallest entropy values, we conclude that the ranking of given alternatives is as follows:

$A_1 \prec A_3 \prec A_4 \prec A_2$

Since A_1 is the less uncertainty information**.** Therefore, the best option to invest is in a food company.

Method 2: Using INVS entropy generalized based on SVNS entropy (E_{INVS}).

Step 1: Similar to Step 1 of Method 1.
Step 2: Similar to Step 2 of Method 1.

Step 3: Calculate the aggregated entropy measure for all the alternative $A = \{A_1, A_2, A_3, A_4\}$

By equation 12, we calculate for $i = 1, j = 1$, then

$= \left\{ \left(\left| \frac{0.3+0.5}{2} - \frac{0.5+0.7}{2} \right| \cdot |0.7+0.8-1| \right) + \left(\left| \frac{0.2+0.4}{2} - \frac{0.6+0.8}{2} \right| \cdot |0.6+0.7-1| \right) \right\} = 0.22$. Hence,

$A_1: \quad \sum_{i=1}^{3} E_{INVS}(A) = 0.22 + 0.16 + 0.44 = 0.82$ and $E_{INVS}(A) = 1 - \frac{1}{6}(0.82) = 0.8633$ and the rest of entropy measure for A_2, A_3, A_4 are calculated similarly and presented as follows:

$A_2 = 0.9463, A_3 = 0.8983, A_4 = 0.8817$

Step 4: Based on the smallest entropy values, we conclude that the ranking of given alternatives is as follows:

Since A_1 is the less uncertainty information, we conclude that the ranking of given alternatives is as follows:

$A_1 \prec A_3 \prec A_4 \prec A_2$

Since the smallest entropy value is A_1. Therefore, the best option to invest is in a food company.

According to the findings, investing in a food company is the best option based on the proposed entropy. Both entropy measure produce different fuzziness degree but the ranking of alternatives is similar which could assist DMs to choose the best alternative. The ranking of four alternatives using proposed entropy measures is presented in Figure 1.

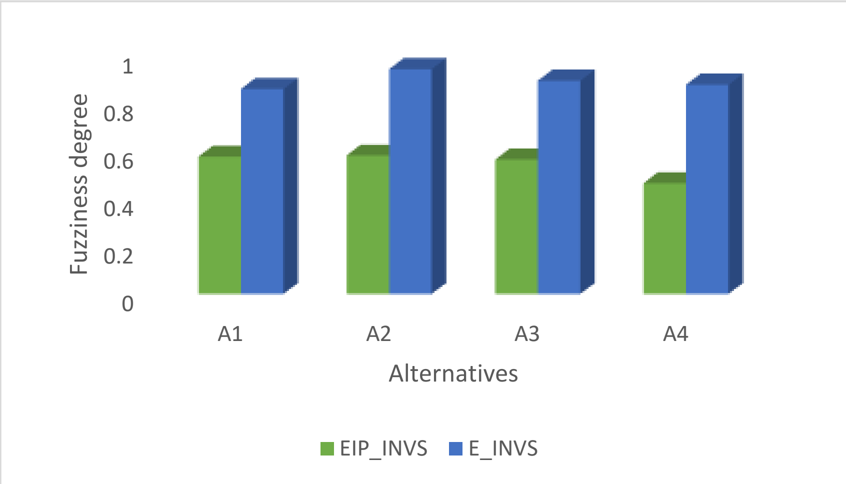

Figure 1: Ranking of the four alternatives using $(E_{\Delta INVS})$ and (E_{INVS})

4.2. *Comparison analysis and discussion*

Based on the illustrative example in [31] and the computational procedures in section 4, the proposed entropy measures are compared with two existing entropy measures under the INVS environment. The proposed entropy measures are denoted as $E_{\Delta INVS}$ and E_{INVS} are based on interval approximation and existing SVNS entropy, respectively. The existing entropy measures by Majumdar and Samanta E_{MM} [29], and entropy measures by Ali E_{δ} [35] are incorporated in this comparative analysis. Table 1 represents the ranking results based on entropy values.

Table 3: The comparison with other entropy measures

Entropy measure	Aggregated entropy measure $A_i\,(i=1,2,3,4)$	Ranking
Proposed entropy $E_{\Delta INVS}$	$A_1=0.6167$, $A_2=0.6250$ $A_3=0.7250$, $A_4=0.65$	$A_1 \prec A_3 \prec A_4 \prec A_2$
Proposed entropy E_{INVS}	$A_1=0.863$, $A_2=0.9463$ $A_3=0.8983$, $A_4=0.8817$	$A_1 \prec A_4 \prec A_3 \prec A_2$
Entropy E_{MM} [33]	$A_1=0.7267$, $A_2=0.8967$ $A_3=0.7967$, $A_4=0.7633$	$A_1 \prec A_4 \prec A_3 \prec A_2$
Entropy E_{δ} [35]	$A_1=0.4474$, $A_2=0.5690$ $A_3=0.4567$, $A_4=0.4860$	$A_1 \prec A_3 \prec A_4 \prec A_2$

The following conclusions are drawn from a comparison of different entropy measures:

- The ranking result of our proposed entropy is almost consistent with the existing entropy in the literature. The smallest entropy value is A_1 meanwhile the largest entropy value is A_2
- The entropy measures show a similar ranking of alternatives with different fuzziness degree.
- The proposed entropy measures are reliable in measuring the degree of fuzziness in terms of the INVS data set.
- The fuzziness degree in the proposed entropy measures may assist DMs to choose the most significant alternative based on the lowest fuzziness degree.
- The new entropy measures resolve the arguments claimed by Thao and Smarandache [33] towards the entropy measures proposed by Majumdar and Samanta.
- The suggested entropy measure can address the same decision making problem as existing entropy measures.
- The proposed entropy measures can take into account the incompleteness and vagueness environments and may assist to better understand the degree of fuzziness in terms of the INVS data set.

5. Conclusions

In this paper, we have presented two entropy measures of INVS and some desirable properties corresponding to these entropies including nonnegativity, minimality, maximality, symmetric and resolution have been proved. Based on the extension of SVNS entropy, we define the concept of INVS entropy by including some improvements. Specifically, the improved entropy measures resolve the arguments claimed in [33]. In addition, this entropy measures can measure the degree of fuzziness in terms of INVS environment. Then, the proposed entropies are applied in a MCDM problem, in which the alternatives on criteria are represented in the INVS environment. Subsequently, an illustrative example was presented to illustrate the application of the proposed MCDM. Finally, a comparative analysis with other entropy measures is presented.

The advantages of proposed entropy are in form of INVS where truth, indeterminacy, and falsity are defined by several membership degree and also complement the NV and INS in representing uncertain, indeterminate, and inconsistent information. The result shows that the proposed entropy measures are reliable in measuring the degree of fuzziness. Measuring uncertainty information is important in decision making, the least value of fuzziness degree will assist DMs to make effective decisions to prevent loss. The suggested entropy measures may be used to assess uncertainty information in other decision making problems such as selection of renewable energy, waste water treatment, and supplier selection. The limitation of the study is the idea to generalize entropy measure may be utilized for the interval concept only.

In the future, the proposed entropy measures can be extended further based on exponential entropy, symmetric entropy, and trigonometric functions. Under the INVS environment, we shall propose other information measures such as similarity and cross-entropy. Besides that, the proposed entropy measures may be used to measure the weight of criteria and DMs in MCDM.

Funding: This research received no extra funding

Conflicts of Interest: The authors declare no conflict of interest

References

1. Shannon, C.E. A Mathematical Theory of Communication. *Bell Syst. Tech. J*. **1948**, 27, 379–423.

2. Shannon, C.E., & Weaver, W. The Mathematical Theory of Communications. **1949**. The University of Illinois Press: Urbana, IL, USA.

3. Zadeh, L.A. Probability Measures of Fuzzy Events. *J. Math. Anal. Appl.,*. **1968**, 23(2), 421-427.

4. De Luca, A. & Termini, S. A Definition of a Nonprobabilistic Entropy in the Setting of Fuzzy Sets Theory. *Inf. Control,*. **1972** ,20(4), 301–312.

5. Sander, W. On Measure of Fuzziness. *Fuzzy Sets Syst*. **1989**, 29, 49–55.

6. Xie, W.X.&Bedrosian, S.D. An Information Measure for Fuzzy Sets. *IEEE Trans. Syst. Man Cybern*.,. **1984**, SMC-14(1), 151–156, 1984.

7. Pal, N.R. & Pal, S.K. Higher Order Fuzzy Entropy and Hybrid Entropy of a Set. *Inf. Sci. (Ny).,*. **1992** ,61(3), 211–231.

8. Shi, Y., & Yuan, X. (2015). Interval Entropy of Fuzzy Sets and The Application to Fuzzy Multiple Attribute Decision Making. *Mathematical Problems in Engineering*. **2015**, 2015, 1-21.

9. Atanassov, K.T. Intuitionistic Fuzzy Sets. *Fuzzy sets syst*. **1986**, 20(1), 87–96.

10. Burillo, P. & Bustince, H. Entropy on Intuitionistic Fuzzy Sets and on Interval-Valued Fuzzy Sets. *Fuzzy Sets Syst*. **1996**, 78(3), 305–316.

11. Szmidt, E. & Kacprzyk, J. Entropy for Intuitionistic Fuzzy Sets. *Fuzzy Sets Syst*. **2001**, 118(3), 467–477.

12. Wei, C.P., Wang, P. & Zhang, Y.Z. Entropy, Similarity Measure of Interval-Valued Intuitionistic Fuzzy Sets and Their Applications. *Inf. Sci. (Ny).* **2011**, 181(19), 4273–4286.

13. Garg, H. Generalized Intuitionitsic Fuzzy Entropy-Based Approach for Solving Multi-Attribute Decision Making. *Proc. Natl. Acad. Sci. India Sect. A Phys. Sci*. **2017**, 89,129-139.

14. Rashid, T., Faizi, S., & Zafar,S. Distance Based Entropy Measure of Interval-Valued Intuitionistic Fuzzy Sets and Its Application in Multicriteria Decision Making. *Adv. Fuzzy Syst*. **2018,** 2018,1-10.

15. Smarandache, F. Neutrosophy: Neutrosophic Probability, Set, and Logic. *RehobothAmerican Res. Press*. **1998**, 1–141.

16. Wang, H., Smarandache, F., Zhang, Y. & Sunderraman, R. Single Valued Neutrosophic Set. *Multispace and Multistructure*. **2010**. 4, 410–413.

17. Wang, H., Smarandache, F., Zhang Y. Q., & Sunderrman, R. Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. *in Hexis, Phoenix*. **2005**.

18. Alkhazaleh, S., Neutrosophic Vague Set Theory. *Crit. Rev*. **2015**, X (July), 29-39.

19. Gau, W. L. & Buehrer, D. J. Vague Sets. IEEE Trans. Syst. Man. *Cybern*. **1993**, 23 (2), 610–614.

20. Hashim, H., Abdullah, L., & Al-Quran. A. Interval Neutrosophic Vague Sets. *Neutrosophic Sets and System*. **2019**, 25, 66-75.

21. Al-Quran, & Hassan, A. Neutrosophic Vague Soft Set and Its applications. *Malaysian J. Math. Sci*. **2017**, 11(2), 141–163.

22. Al-Quran, A., & Hassan, N. Neutrosophic Vague Soft Expert Set Theory. *Artic. J. Intell. Fuzzy Syst*. **2016**, 30(6), 3691–3702.

23. Awang, A., Ali, M., & Abdullah, L. Hesitant Bipolar-Valued Neutrosophic Set: Formulation, Theory and Application. *IEEE Access*. **2019**, 7, 176099–176114.

24. Liu, P. & Shi, L. Some Neutrosophic Uncertain Linguistic Number Heronian Mean Operators and Their Application to Multi-Attribute Group Decision Making. *Neural Comput. Appl*. **2017**, 28(5), 1079–1093.

25. Awang, A., Aizam, N.A.H., Ab Ghani, A.T., Othman, M., & Abdullah, L. A Normalized Weighted Bonferroni Mean Aggregation Operator Considering Shapley Fuzzy Measure Under Interval-valued Neutrosophic Environment for Decision-Making,. *Int. J. Fuzzy Syst*. **2020** , 22(1), 321–336.

26. Tian, C., Peng, J. J., Zhang, Z. Q., Goh, M., & Wang, J. Q. A Multi-Criteria Decision-Making Method Based on Single-Valued Neutrosophic Partitioned Heronian Mean Operator. *Mathematics*. **2020**, 8(7), 1189.

27. Yang, H., Wang, X. & Qin, K. New Similarity and Entropy Measures of Interval Neutrosophic Sets with Applications in Multi-Attribute Decision-Making. *Symmetry*. **2019**, 11(3), 370.

28. Al-Quran, A. & Hassan, N. Fuzzy Parameterized Single Valued Neutrosophic Soft Expert Set Theory and Its Application in Decision Making. *Int. J. Applied Decision Sciences*. **2016**. 9, 212-227.

29. Majumdar, P. & Samanta, S.K. On Similarity and Entropy of Neutrosophic Sets. *J. Intell. Fuzzy Syst*. **2014**, 26(3), 1245–1252.

30. Wu, H., Yuan, Y., Wei, L. & Pei, L. On Entropy, Similarity Measure and Cross-Entropy of Single-Valued Neutrosophic Sets and Their Application in Multi-Attribute Decision Making. *Soft Comput*. **2018**, 22(22), 7367–7376.

31. Garg, N. H and Garg, H. On Single-Valued Neutrosophic Entropy of Order α. *Neutrosophic Set and System*. **2016**, 14, 21–28.

32. Abu Qamar, M. & Hassan, N. Entropy, Measures of Distance and Similarity of Q-neutrosophic Soft Sets and Some Applications. *Entropy*. **2018,** 20(9), 672.

33. Thao, N.X., & Smarandache, F. Apply New Entropy Based Similarity Measures of Single Valued Neutrosophic Sets to Select Supplier Material," *J. Intell. Fuzzy Syst*. **2020**, 39(1), 1005–1019.

34. Ye, J., & Du, S. Some distances, similarity and entropy measures for interval-valued neutrosophic sets and their relationship. *International Journal of Machine Learning and Cybernetics*. **2019**, 10(2), 347-355.

35. Aydoğdu. A., & Şahin, R. New Entropy Measures Based on Neutrosophic Set and Their Applications to Multi-Criteria Decision Making. *J. Nat. Appl. Sci*. **2019**, 23(1), 40–45.

36. Quek, S. G., Selvachandran, G., Smarandache, F., Vimala, J., Le, S. H., Bui, Q. T., & Gerogiannis, V.C. Entropy Measures for Plithogenic Sets and Applications in Multi-Attribute Decision Making. *Mathematics*. **2020**, 8(6), 965.

37. Ye., Entropy Measures of Simplified Neutrosophic Sets and Their Decision-Making Approach with Positive and Negative Arguments. *J. Manag. Anal*. 2021, 1–15.

38. Biswas, P. Pramanik, S. & Giri, B.C. Entropy Based Grey Relational Analysis Method for Multi-Attribute Decision Making Under Single Valued Neutrosophic Assessments," *Neutrosophic Sets Syst.* 2014 , 2, 105–113.

39. Thao, N. X., & Smarandache, F. A New Fuzzy Entropy on Pythagorean Fuzzy Sets. *J. Intell. Fuzzy Syst*. **2019,** 37(1), 1065–1074.

40. Barukab, O., Abdullah, S., Ashraf, S., Arif, M. & Khan, S.A. A New Approach to Fuzzy TOPSIS Method Based on Entropy Measure Under Spherical Fuzzy Information. *Entropy*. **2019**. 21(12), 1231.

41. Ye, J. Multicriteria fuzzy decision-making method using entropy weights-based correlation coefficients of interval-valued intuitionistic fuzzy sets. *Applied Mathematical Modelling*. **2010**, 34(12), 3864-3870.

Received: June 8, 2021. Accepted: August 26, 2021