Neutrosophic Sets and Systems, Vol. 45, 2021
University of New Mexico

A New Approach to Group Decision Making Problem in Medical Diagnosis using Interval Neutrosophic Soft Matrix

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Abstract: The main objective of this article is to introduce the notion of interval neutrosophic soft matrix (IVNS-Matrix), which is an extension of the neutrosophic soft matrix and reveals various types of IVNS-Matrix along with different algebraic operations on them. A new method has been proposed to solve interval neutrosophic soft set based real-life group decision making problem effectively by introducing IVNSM-Algorithm. Finally, this algorithm has been applied in medical science for disease diagnosis.

Keywords: Interval neutrosophic matrix; IVNSM-Algorithm; Choice Matrix; Decision making.

1. Introduction

Our life is surrounded by an aura of uncertainties or ambiguities or vagueness. So, when we are going to solve various real-life problems, which contain uncertainties, then we realize that such types of problems cannot be handled by traditional mathematical tools. Because in such cases we do not analyze data appropriately, as we do in case of precise and deterministic data. So, there is a problem in real decision making. The introduction of fuzzy set theory [1] by Zadeh (1965) handled the vague concept to some extent by introducing the membership function. The membership function determines the degree of belongingness of each element in a set and the membership value lies in the interval [0, 1]. Fuzzy set theory has been used extensively in different fields. In the fuzzy set theory, there is no scope of considering non-membership value as we find that the concept of non-membership value is equally as important as membership value. Practically also we used to find the dual character of an object. So, to make a balance in the characteristic of an object, an intuitionistic fuzzy set [2] was introduced by Atanassov (1986) where, for every membership function there corresponds a non-membership function and both belong to the interval [0, 1] and their sum cannot exceed one. Membership or non-membership value only assign a single real value. But sometimes the concept of uncertainty cannot be defined by a single real value. For that purpose interval-valued fuzzy set [3] was introduced(1987). Due to more complexity in the environment of uncertainty and for the dire need of an hour, many other theoretical concepts and the properties, whose base is the fuzzy set, have been introduced. Some of them are given in [4-6]. According to the nature of the problem domain, we will decide which tool is suitable for us to handle a particular problem.

Zadeh's fuzzy set theory is the most appropriate theory to deal with uncertainty with the help of the membership function. But in 1999, Molodtsov [7] observed some limitations of the fuzzy set theory. In fuzzy set theory, a concept is handled by a membership function. But we should not impose only one way to define a membership function.The nature of a membership function is extremely individual. For example, to define the attractiveness of a house it is difficult to define a membership function. If one considers the membership degree as 0.6 then everyone may understand this in his or her own manner. For instance, Mr., X may understand that the house is highly attractive. Again Mr., Y may understand that it is very highly attractive. So there is a possibility of information loss in each particular case. Molodtsov said that the reason behind these limitations is the inadequacy of the parametrization of the theory. Then, to overcome this drawback he initiated the idea of soft set theory in the parametric form which is free from the above difficulties. In soft set theory, to define an object, no need to introduce a membership function. It is the more general form to represent the concept of vagueness parametrically. As we know that, because of the amalgamation of two or more concept together, gives a better shape and it provides more flexibility to handle various uncertain problems which we face in our day to day life, that's why by embedding the idea of the soft set and other sets, some major contributions are developed in [8-13].

The concept of indeterminacy or neutrality is common in real life. For example, when there is a fight between two players there are other people surrounding them who do not support neither of the two players. In the real decision-making problem, the concept of indeterminacy is very important. There are various types of indeterminacy involved in real-world problems. To eradicate such difficulty, Smarandache(2005) introduced the notion of Neutrosophic set in[14]. With an aid of a neutrosophic set, we deal with uncertainty, incompleteness ,and indeterminacy involving a pragmatic problem. It has been used successfully in various fields such as Sociology, Economics, Logic, Artificial intelligence, Machine learning, Optimization problem, Game theory ,etc. Some extensions of the neutrosophic set have been discussed in [15-21].

Matrices play a significant role in representing, manipulating ,and modeling such a large number of data because of their compact form. In the field of computer science, mainly in Data Science, there is an abundant use of Matrix. The classical matrix theory cannot solve the problems based on uncertainty. Matrix representation of the fuzzy soft set is initiated by Yang et al. in [22] and it is successfully used in decision-making problems in [23]. Some other extensions are intuitionistic fuzzy soft matrix[24-25], interval-valued fuzzy soft matrix[26], interval-valued intuitionistic fuzzy soft matrix and its application in medical diagnosis[27], interval-valued neutrosophic matrix[28] ,etc. Cagman et al. (2010), in [29], defined soft matrices and their operations and construct a decision-making method that can be used successfully to the problems that contain uncertainty. Also, in [30-31], Wang et al. discussed single-valued neutrosophic sets and interval neutrosophic sets respectively, Deli, in [32], used interval-valued neutrosophic soft sets in decision making, Smarandache[33] proposed the extension of the soft set by introducing hypersoft set, pilthogenic hypersoft set.

Group decision-making leads to improve outcomes and it involves two or more people. It occurs when individuals collectively make a selection from the alternatives. Group decisions are subject to factors such as social influence, peer pressure ,and group dynamics. It interacts and collaborated to reach a collective decision. In the proposed study we have used the concept of the neutrosophic set because it helps to study the higher degree of uncertainty, present in various real-life problems, in an effective manner. As compared to other mathematical tools namely, fuzzy set, intuitionistic fuzzy set, interval-valued fuzzy set ,etc., the neutrosophic set is a more flexible and more general form to handle uncertain concepts. For more clarity, we know that the neutrosophic set has three components namely $T, I, F \in [0,1]$ and each component has its own importance and they are independent of each other, which makes it superior to the other existing mathematical tools of vagueness. For example, if we consider the component $I = 0$ in the neutrosophic set, then also it has the potential to tackle complete or incomplete or paraconsistent or conflicting information. We also added intervals with neutrosophic sets to address the issues where the components used in the neutrosophic sets cannot be expressed by a single real value. In interval neutrosophic set $T, I, F \in Int([0,1])$, where $Int([0,1])$ denotes the set of all subsets of $[0,1]$. Moreover, with an aid of a matrix with an interval neutrosophic set, we made our calculation speedy and it helps to store big data in a computer easily. Some recent works based on neutrosophic set discussed in [34-37].

In [38], the concept of the neutrosophic soft matrix has been used in a group decision making problem. In our work, we have extended the earlier concept by introducing the notion of interval neutrosophic soft matrix and its types with concrete examples. We also introduce the choice matrix associated with interval neutrosophic soft set. An IVNSM-Algorithm has been proposed to solve real-life group decision making problem. Finally, with the help of an application, the IVNSM-algorithm has been successfully executed.

2. Preliminaries

Here we recall some basic definitions with examples that are appropriate as far as the article is concerned.

2.1 Definition[7]

Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes power set of X and $A \subseteq E$.

Then the pair (F_A, E) is called a soft set over X, where F_A is a mapping given by $F_A : E \to P(X)$.

A soft set is a parameterized family of subsets of the universe X.

2.2 Definition [14]

Let X be a universe of discourse,with a generic element in X denoted by x, the neutrosophic(NS) set A is defined as

$$A = \{\langle x : \mu_A(x), \nu_A(x), \omega_A(x)\rangle, x \in X\},$$

Where the functions $\mu, \nu, \omega : X \to]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy (neutrality), and the degree of non-membership (Falsehood) of the element $x \in X$ to the set A with the condition

$$^-0 \le \mu_A(x) + \nu_A(x) + \omega_A(x) \le 3^+$$

If t (0.4,0.1,0.5) belongs to, then it means that 40% A belongs to A, 50% does not belong to A and 10% is undecidable(not known exactly). If there is no indeterminacy involve in set A, then it reduces to an intuitionistic fuzzy set. Therefore, a neutrosophic set can be an intuitionistic fuzzy set.

2.3 Definition [30]

Let X be a universe of discourse with a generic element x. A single-valued neutrosophic set A is characterized by a truth-membership function $T_A(x)$, falsity-membership function $F_A(x)$ and the indeterminacy-membership function $I_A(x)$ and it is denoted by A = {x,< $T_A(x)$, $I_A(x)$, $F_A(x)$ >: $x \in X$ }, where $T_A(x)$, $I_A(x)$, $F_A(x)$ $\in$ [0,1] subject to the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Throughout our discussion we use the concept of single-valued neutrosophic set as it has a definite range.

2.4 Definition [31]

Let U be a space of points (objects), with a generic element u. An interval-valued neutrosophic set (IVN-set) A in U is characterized by a truth-membership function T_A , an indeterminacy-membership function I_A ,and a falsity-membership function F_A. For each point $u \in U$; T_A , I_A and $F_A \subseteq [0,1]$.

Thus, an IVN-sets over U can be represented by the set of

$$A = \left\{ \left\langle T_A(u), I_A(u), F_A(u) \right\rangle / u : u \in U \right\}$$

Here, $\left(T_A(u), I_A(u), F_A(u)\right)$ is called interval-valued neutrosophic number for all $u \in U$ and all interval-valued neutrosophic numbers over U will be denoted by $IVN(U)$.

2.4.1 Example

Let $U = \{u_1, u_2\}$ be the universe of discourse and A be an interval-valued neutrosophic set in U . Then A can be expressed as follows:

$$A = \left\{ \left\langle [0.5, 0.7], [0.5, 0.6], [0.5, 0.7] \right\rangle / u_1, \left\langle [0.4, 0.6], [0.7, 0.8], [0.3, 0.6] \right\rangle / u_2 \right\}$$

2.5 Definition [32]

Let $U = \{c_1, c_2, c_3, ..., c_m\}$ be a Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ..., e_n\}$. Let $A \subseteq E$ and (F, A) be a interval-valued neutrosophic soft set over U ,where F is a mapping given by $F{:}A \to I^U$,where I^U denotes the the collection of all interval-valued neutrosophic subsets of U. Then the interval-valued neutrosophic soft set can be expressed in a matrix form as follows:

$$\hat{A}_{m \times n} = \left[a_{ij} \right]_{m \times n} \quad \text{or} \quad \hat{A} = \left[a_{ij} \right] \; i = 1, 2, ..., m; \; j = 1, 2, ..., n$$

$$a_{ij} = \begin{cases} \left(\left\langle \left[T^l{}_j(c_i), T^u{}_j(c_i) \right], \left[I^l{}_j(c_i), I^u{}_j(c_i) \right], \left[F^l{}_j(c_i), F^u{}_j(c_i) \right] \right\rangle \right) & \text{if} \quad e_j \in A \\ \left(\left\langle [0,0], [1,1], [1,1] \right\rangle \right) & \text{if} \quad e_j \notin A \end{cases}$$

$\left[T^{l}{}_{j}(c_i),T^{u}{}_{j}(c_i)\right]$ represents the truth-membership value of c_i in the interval-valued neutrosophic set $F(e_j)$ $\left[I^{l}{}_{j}(c_i),I^{u}{}_{j}(c_i)\right]$ represents the indeterminacy-membership value of c_i in the interval-valued neutrosophic set $F(e_j)$ and $\left[F^{l}{}_{j}(c_i),F^{u}{}_{j}(c_i)\right]$ represents the falsity-membership value of c_i in the interval-valued neutrosophic set $F(e_j)$ with the condition $T^{u}{}_{j}(c_i)+I^{u}{}_{j}(c_i)+F^{u}{}_{j}(c_i)\leq 3$.

2.5.1 Example

Let $U=\{c_1,c_2\}$ be a set of cars under consideration and E is a set of parameters which is a neutrosophic word. Let $E=\{e_1=\text{value},\ e_2=\text{mileage},\ e_3=\text{safety},\ e_4=\text{performance},\ e_5=\text{looks},\ e_6=\text{sit capacity}\}$

We give an interval valued neutrosophic soft sets(ivn-soft sets) over U as follows:

$$\Upsilon_k=\left\{\begin{array}{l}\left(e_1,\left\{\langle[0.2,0.9],[0.3,0.6],[0.4,0.7]\rangle/u_1,\langle[0.1,0.7],[0.2,0.8],[0.4,0.5]\rangle/u_2\right\}\right),\\ \left(e_2,\left\{\langle[0.4,0.8],[0.3,0.7],[0.5,0.8]\rangle/u_1,\langle[0.3,0.9],[0.1,0.8],[0.4,0.7]\rangle/u_2\right\}\right),\\ \left(e_3,\left\{\langle[0.0,0.6],[0.5,0.6],[0.3,0.7]\rangle/u_1,\langle[0.5,0.7],[0.8,0.9],[0.4,0.7]\rangle/u_2\right\}\right),\\ \left(e_4,\left\{\langle[0.3,0.6],[0.5,0.8],[0.6,0.9]\rangle/u_1,\langle[0.5,0.8],[0.6,0.8],[0.1,0.8]\rangle/u_2\right\}\right),\\ \left(e_5,\left\{\langle[0.2,0.9],[0.1,0.5],[0.4,0.9]\rangle/u_1,\langle[0.6,0.8],[0.1,0.8],[0.2,0.5]\rangle/u_2\right\}\right)\end{array}\right\}$$

The tabular representation of ivn-soft set Υ_k is as follows:

U	c_1	c_2
e_1	$\langle[0.2,0.9],[0.3,0.6],[0.4,0.7]\rangle$	$\langle[0.1,0.7],[0.2,0.8],[0.4,0.5]\rangle$
e_2	$\langle[0.1,0.7],[0.2,0.8],[0.4,0.5]\rangle$	$\langle[0.3,0.9],[0.1,0.8],[0.4,0.7]\rangle$
e_3	$\langle[0.0,0.6],[0.5,0.6],[0.3,0.7]\rangle$	$\langle[0.5,0.7],[0.8,0.9],[0.4,0.7]\rangle$
e_4	$\langle[0.3,0.6],[0.5,0.8],[0.6,0.9]\rangle$	$\langle[0.5,0.8],[0.6,0.8],[0.1,0.8]\rangle$
e_5	$\langle[0.2,0.9],[0.1,0.5],[0.4,0.9]\rangle$	$\langle[0.6,0.8],[0.1,0.8],[0.2,0.5]\rangle$

Table 1: Tabular representation of the ivn-soft set Υ_k

2.6 Definition [29]

Let (F_A,E) be a soft set over U. Then a subset of $U\times E$ is uniquely defined by

$R_A=\{(u,e): e\in A, u\in F_A(e)\}$, which is called a relation form of (F_A,E). Now the characteristic function of R_A is written by,

$$\chi_{R_A} : U \times E \to \{0,1\},\ \chi_{R_A} = \begin{cases} 1, (u,e) \in R_A \\ 0, (u,e) \notin R_A \end{cases}$$

Let $U = \{u_1, u_2, ..., u_m\}$, $E = \{e_1, e_2, ..., e_n\}$, then R_A can be presented by a table as in the following form

	e_1	e_2	…………..	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	…………….	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	……………	$\chi_{R_A}(u_2,e_n)$
………	……………	……………	……………	…………
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	…………….	$\chi_{R_A}(u_m,e_n)$

Table 2 Tabular representation of R_A of the soft set (F_A, E)

If $a_{ij} = \chi_{R_A}(u_i,e_j)$, we can define a matrix

$$\left[a_{ij}\right]_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

Which is called a soft matrix of the order $m \times n$ corresponding to the soft set (F_A, E) over U. A soft set (F_A, E) is uniquely characterized by the matrix $\left[a_{ij}\right]_{m\times n}$. Therefore we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

3. Some Notions of Interval Neutrosophic Soft Matrix Theory

3.1 Definition

Let $(F_A^{\neg}, E)$ be an interval neutrosophic soft set (here soft set in the sense of Cagman and Enginglu) over U, where $F_A^{\neg} : E \to IVN^U$, here IVN^U denotes the set of all interval neutrosophic sets over U, then a subset of $U \times E$ is uniquely defined by

$$R_A^{\neg} = \{(u,e) : e \in A, u \in F_A^{\neg}\}$$

The relation $R_A^{\neg}$ is described by the truth-membership function $T_A : U \times E \to \subseteq [0,1]$, indeterminacy-membership function $I_A : U \times E \to \subseteq [0,1]$, and the falsity-membership function $F_A : U \times E \to \subseteq [0,1]$.

Suppose $U = \{u_1, u_2, ..., u_m\}$ and $E = \{\mathrm{e}_1, \mathrm{e}_2, ..., \mathrm{e}_m\}$, then the relation set $R_A^{\neg}$ can be expressed in the following matrix form

	e_1	e_2	....	e_m
u_1	$\left\langle \left[T_{11}^l, T_{11}^u\right], \left[I_{11}^l, I_{11}^u\right], \left[F_{11}^l, F_{11}^u\right] \right\rangle$	$\left\langle \left[T_{12}^l, T_{12}^u\right], \left[I_{12}^l, I_{12}^u\right], \left[F_{12}^l, F_{12}^u\right] \right\rangle$	....	$\left\langle \left[T_{1m}^l, T_{1m}^u\right], \left[I_{1m}^l, I_{1m}^u\right], \left[F_{1m}^l, F_{1m}^u\right] \right\rangle$
u_2	$\left\langle \left[T_{21}^l, T_{21}^u\right], \left[I_{21}^l, I_{21}^u\right], \left[F_{21}^l, F_{21}^u\right] \right\rangle$	$\left\langle \left[T_{22}^l, T_{22}^u\right], \left[I_{22}^l, I_{22}^u\right], \left[F_{22}^l, F_{22}^u\right] \right\rangle$	....	$\left\langle \left[T_{2m}^l, T_{2m}^u\right], \left[I_{2m}^l, I_{2m}^u\right], \left[F_{2m}^l, F_{2m}^u\right] \right\rangle$
…… ..	....	....	....	....
u_m	$\left\langle \left[T_{m1}^l, T_{m1}^u\right], \left[I_{m1}^l, I_{m1}^u\right], \left[F_{m1}^l, F_{m1}^u\right] \right\rangle$	$\left\langle \left[T_{m2}^l, T_{m2}^u\right], \left[I_{m2}^l, I_{m2}^u\right], \left[F_{m2}^l, F_{m2}^u\right] \right\rangle$	....	$\left\langle \left[T_{mm}^l, T_{mm}^u\right], \left[I_{mm}^l, I_{mm}^u\right], \left[F_{mm}^l, F_{mm}^u\right] \right\rangle$

Table3 Tabular representation of $R_A^{\neg}$ of IVN^U

The above matrix representation is useful for computer storage of such a big expression in concise form so that we can retrieve it very easily and it is handy for doing different algebraic operations under certain condition.

3.2 Definition

An interval neutrosophic soft matrix is said to be a null or void interval neutrosophic soft matrix if all the entries of $R_A^{\neg}$ are $\left\langle [0,0], [0,0], [1,1] \right\rangle$ and it is denoted by $\Phi^{\neg}$.

3.3 Definition

An interval neutrosophic soft matrix is said to be a complete interval neutrosophic soft matrix if all the entries of $R_A^{\neg}$ are $\left\langle [1,1], [0,0], [0,0] \right\rangle$ and it is denoted by $\square^{\neg}$.

3.4 Definition

Let $\mathrm{X}^{\neg}$ be an interval neutrosophic soft matrix. Then the transpose of $\mathrm{X}^{\neg}$ is obtained by interchanging its rows and columns and it is denoted by $\left(\mathrm{X}^{\neg}\right)^t$.

3.5 Definition

Let $\mathrm{A}^{\neg}$ and $\mathrm{B}^{\neg}$ be two interval neutrosophic soft matrices of the same order. Then $\mathrm{A}^{\neg}$ is said to be a interval neutrosophic soft sub matrix of $\mathrm{B}^{\neg}$ if for every elements of $\mathrm{A}^{\neg}$ their corresponds another element in $\mathrm{B}^{\neg}$ such that $T_A^l \le T_B^l$, $T_A^u \le T_B^u$; $I_A^l \ge I_B^l$, $I_A^u \ge I_B^u$ and $F_A^l \ge F_B^l$, $F_A^u \ge F_B^u$ and it is denoted by

$\mathrm{A}^{\neg} \subseteq \mathrm{B}^{\neg}$.

3.6 Definition

Let $\mathrm{A}^{\neg}$ and $\mathrm{B}^{\neg}$ be two interval neutrosophic soft matrices of the same order. Then their sum is denoted by $\mathrm{A}^{\neg} \oplus \mathrm{B}^{\neg}$ and it is defined as

$$\mathrm{A}^{\neg} \oplus \mathrm{B}^{\neg} = \left\langle \left[\max\left(T_A^l, T_B^l\right), \max\left(T_A^u, T_B^u\right)\right], \left[\frac{I_A^l + I_B^l}{2}, \frac{I_A^u + I_B^u}{2}\right], \left[\min\left(F_A^l, F_B^l\right), \min\left(F_A^u, F_B^u\right)\right] \right\rangle$$

We can extend it for more than two matrices.

3.7 Definition

Let $\mathrm{A}^{\neg}$ and $\mathrm{B}^{\neg}$ be two interval neutrosophic soft matrices of the same order. Then their sum is denoted by $\mathrm{A}^{\neg} \ominus \mathrm{B}^{\neg}$ and it is defined as

$$\mathrm{A}^{\neg} \ominus \mathrm{B}^{\neg} = \left\langle \left[\min\left(T_A^l, T_B^l\right), \min\left(T_A^u, T_B^u\right)\right], \left[\frac{I_A^l - I_B^l}{2}, \frac{I_A^u - I_B^u}{2}\right], \left[\max\left(F_A^l, F_B^l\right), \max\left(F_A^u, F_B^u\right)\right] \right\rangle$$

3.8 Definition

Let $\mathrm{A}^{\neg}$ and $\mathrm{B}^{\neg}$ be two interval neutrosophic soft matrices of the same order. Then the product of $\mathrm{A}^{\neg}$ and $\mathrm{B}^{\neg}$ exist if the number of column in $\mathrm{A}^{\neg}$ is equal to the number of rows in $\mathrm{B}^{\neg}$ and it is denoted by $\mathrm{A}^{\neg} \otimes \mathrm{B}^{\neg}$ and it is defined as

$$\mathrm{A}^{\neg} \otimes \mathrm{B}^{\neg}$$

$$= \left\langle \left[\max\left(\min\left(T_{A_j}^l, T_{B_j}^l\right)\right), \max\left(\min\left(T_{A_j}^u, T_{B_j}^u\right)\right)\right], \left[\max\left(\min\left(I_{A_j}^l, I_{B_j}^l\right)\right), \min\left(\max\left(I_{A_j}^u, I_{B_j}^u\right)\right)\right], \left[\begin{matrix} \min\left(\max\left(F_{A_j}^l, F_{B_j}^l\right)\right), \\ \min\left(\max\left(F_{A_j}^u, F_{B_j}^u\right)\right) \end{matrix}\right] \right\rangle$$

3.9 Definition

Let $\mathrm{X}^{\neg}$ be an interval neutrosophic soft matrix. Then the complement of $\mathrm{X}^{\neg}$ is obtained by interchanging the truth-membership and falsity-membership intervals, without altering the indeterminacy-membership interval of all the elements of $\mathrm{X}^{\neg}$ and it is denoted by $\left(\mathrm{X}^{\neg}\right)^c$.

3.10 Definition

A square interval neutrosophic soft matrix $\mathrm{X}^{\neg}$ is said to be a diagonal interval neutrosophic soft matrix if all of its non-diagonal elements are $\langle[0,0],[0,0],[1,1]\rangle$.

3.11 Definition

Let $X^{\neg}$ be a square interval neutrosophic soft matrix of order $m \times n$, where m=n. Then the Trace of $X^{\neg}$ is denoted by $tr(X^{\neg})$ and it is defined as

$$tr(X^{\neg})$$

$$=\left(\left[\max(T_j^{\,l}(c_i)),\max(T_j^{\,u}(c_i))\right]\left[\max(I_j^{\,l}(c_i)),\max(I_j^{\,u}(c_i))\right]\left[\min(F_j^{\,l}(c_i)),\min(F_j^{\,u}(c_i))\right]\right).$$

3.12 Definition

Let X^* be a choice matrix corresponding to a square interval neutrosophic soft matrix $X^{\neg}$. The elements x_{ij}^* of X^* are defined as follows

$$x_{ij}^* = \begin{cases} \langle[1,1][0,0][0,0]\rangle, \text{ where both the entries are the entries of the choice parameters of the decision-makers} \\ \langle[0,0][0,0][1,1]\rangle, \text{ where atleast one of the parameters be not under choice} \end{cases}$$

Choice matrices depend upon the number of decision makers. To get a clear idea about choice matrices, we consider the following algorithm and apply this algorithm in example 4.1

4. **Construction of IVNSM-algorithm for decision making and its application**

For solving a real decision making problem we consider the following steps:

Step 1: Input the interval neutrosophic soft set over the set of attributes and construct interval neutrosophic soft matrix.

Step 2: Compute the product interval neutrosophic soft matrices by multiplying the given interval neutrosophic soft matrix with the combined choice matrices as per the rule of multiplication of interval neutrosophic soft matrices.

Step 3: Calculate the sum of all the product interval neutrosophic soft matrices as per the rule of matrix addition of interval neutrosophic soft matrices.

Step 4: Construct the lower-value matrix and the upper-value matrix corresponding to the resultant matrix.

Step 5: Compute the value matrices corresponding to the lower-value matrix and the upper-value matrix.

Step 6: Find the row sum of the value matrices.

Step 7: By adding the corresponding elements of the row sum of the value matrices, we obtain the weight of each object. Among these, the highest weight becomes the optimal choice object. If more than one object having the highest weight then any one of them may be chosen as the optimal choice object.

To understand IVNSM-algorithm properly, we have the following example.

4.1 Example

Suppose Mr., Debnath wants to buy a house and for that purpose, he appointed three brokers to inspect and report the house. According to their report, he will choose the house which fulfills the optimality criteria i.e the best option he affords. But the problem is that each broker has its own point of view and the owner has come to one decision. Keeping it in mind we consider the following problem:

Let $U=\{h_1,h_2,h_3,h_4\}$ be the set of four houses under consideration and E= {good location, cheap, green surrounding, costly}= $\{e_1,e_2,e_3,e_4\}$ be the set of parameters. The set of brokers is denoted by B= {Mrs., Rama, Mr., Advik, Mrs., Shewly}.

Now, let us construct the interval neutrosophic soft set $(F_A^{\neg},E)$ which describes the attractiveness of the houses and it is given by

$$\left(F_A^{\neg},E\right)=\begin{Bmatrix} \textit{good location of houses}=\{h_1/\langle[0.4,0.7][0.5,0.8][0.5,0.7]\rangle,h_2/\langle[0.5,0.6][0.3,0.5][0.4,0.6]\rangle,h_3/\langle[0.2,0.5][0.4,0.6][0.6,0.7]\rangle,h_4/\langle[0.3,0.5][0.4,0.6][0.5,0.8]\rangle\} \\ \textit{cheap house}=\{h_1/\langle[0.3,0.6][0.7,0.9][0.6,0.8]\rangle,h_2/\langle[0.1,0.3][0.3,0.6][0.5,0.7]\rangle,h_3/\langle[0.3,0.4][0.5,0.7][0.8,0.9]\rangle,h_4/\langle[0.4,0.5][0.2,0.4][0.3,0.6]\rangle\} \\ \textit{houses at green surroundings}=\{h_1/\langle[0.6,0.9][0.1,0.2][0.6,0.7]\rangle,h_2/\langle[0.3,0.4][0.2,0.4][0.5,0.6]\rangle,h_3/\langle[0.3,0.4][0.6,0.7][0.3,0.5]\rangle,h_4/\langle[0.4,0.5][0.3,0.5][0.7,0.8]\rangle\} \\ \textit{costly house}=\{h_1/\langle[0.2,0.5][0.5,0.7][0.4,0.6]\rangle,h_2/\langle[0.3,0.6][0.4,0.6][0.5,0.7]\rangle,h_3/\langle[0.1,0.3][0.3,0.5][0.7,0.8]\rangle,h_4/\langle[0.4,0.5][0.4,0.6][0.6,0.8]\rangle\} \end{Bmatrix}$$

The matrix representation of the set $(F_A^{\neg},E)$ is given by

$$M=\begin{bmatrix} \langle[0.4,0.7][0.5,0.8][0.5,0.7]\rangle & \langle[0.3,0.6][0.7,0.9][0.6,0.8]\rangle & \langle[0.6,0.9][0.1,0.2][0.6,0.7]\rangle & \langle[0.2,0.5][0.5,0.7][0.4,0.6]\rangle \\ \langle[0.5,0.6][0.3,0.5][0.4,0.6]\rangle & \langle[0.1,0.3][0.3,0.6][0.5,0.7]\rangle & \langle[0.3,0.4][0.2,0.4][0.5,0.6]\rangle & \langle[0.3,0.6][0.4,0.6][0.5,0.7]\rangle \\ \langle[0.2,0.5][0.4,0.6][0.6,0.7]\rangle & \langle[0.3,0.4][0.5,0.7][0.8,0.9]\rangle & \langle[0.3,0.4][0.6,0.7][0.3,0.5]\rangle & \langle[0.1,0.3][0.3,0.5][0.7,0.8]\rangle \\ \langle[0.3,0.5][0.4,0.6][0.5,0.8]\rangle & \langle[0.4,0.5][0.2,0.4][0.3,0.6]\rangle & \langle[0.4,0.5][0.3,0.5][0.7,0.8]\rangle & \langle[0.4,0.5][0.4,0.6][0.6,0.8]\rangle \end{bmatrix}$$

Suppose the choice parameter sets of Mrs., Rama, Mr., Advik ,and Mrs., Shewly are respectively $X=\{e_2,e_3,e_4\}$, $Y=\{e_1,e_3,e_4\}$ and $Z=\{e_1,e_2,e_4\}$ and all these are the subsets of E.

By the definition of the choice matrix, we consider the separate choice matrices of Mrs., Rama, Mr., Advik and Mrs. Shewly are given by,

$$\left(x^{*}_{ij}\right)_X=\begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

$$\left(x^*_{ij}\right)_Y = \begin{pmatrix} \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

and $$\left(x^*_{ij}\right)_Z = \begin{pmatrix} \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

Now combined choice matrix of Mrs. Rama and Mr. Advik , i.e the matrix in which they come to one decision

$$\left(x^*_{ij}\right)_{X\wedge Y} = \begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

Similarly, we can find another two combined choice matrices given by

$$\left(x^*_{ij}\right)_{Y\wedge Z} = \begin{pmatrix} \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

and $$\left(x^*_{ij}\right)_{Z\wedge X} = \begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

Next, we consider another three choice matrices which predict that when any of the two brokers are agree with their decision then the third broker also agrees. They can be presented in the following form

$$\left(x^*_{ij}\right)_{(X\wedge Y,Z)} = \begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

$$\left(x_{ij}^{*}\right)_{(Y\wedge Z,\,X)} = \begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

and

$$\left(x_{ij}^{*}\right)_{(Z\wedge X,\,Y)} = \begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

By the definition 3.8, we find the following products :

$$M \otimes \left(x_{ij}^{*}\right)_{(X\wedge Y,\,Z)} =$$

$$\begin{bmatrix} \langle[0.4,0.7][0.5,0.8][0.5,0.7]\rangle & \langle[0.3,0.6][0.7,0.9][0.6,0.8]\rangle & \langle[0.6,0.9][0.1,0.2][0.6,0.7]\rangle & \langle[0.2,0.5][0.5,0.7][0.4,0.6]\rangle \\ \langle[0.5,0.6][0.3,0.5][0.4,0.6]\rangle & \langle[0.1,0.3][0.3,0.6][0.5,0.7]\rangle & \langle[0.3,0.4][0.2,0.4][0.5,0.6]\rangle & \langle[0.3,0.6][0.4,0.6][0.5,0.7]\rangle \\ \langle[0.2,0.5][0.4,0.6][0.6,0.7]\rangle & \langle[0.3,0.4][0.5,0.7][0.8,0.9]\rangle & \langle[0.3,0.4][0.6,0.7][0.3,0.5]\rangle & \langle[0.1,0.3][0.3,0.5][0.7,0.8]\rangle \\ \langle[0.3,0.5][0.4,0.6][0.5,0.8]\rangle & \langle[0.4,0.5][0.2,0.4][0.3,0.6]\rangle & \langle[0.4,0.5][0.3,0.5][0.7,0.8]\rangle & \langle[0.4,0.5][0.4,0.6][0.6,0.8]\rangle \end{bmatrix}$$

$$\otimes$$

$$\begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

$$= \begin{bmatrix} \langle[0.6,0.9][0.0,0.2][0.4,0.6]\rangle & \langle[0.6,0.9][0.0,0.2][0.4,0.6]\rangle & \langle[0.0,0.0][0.0,0.2][1,1]\rangle & \langle[0.6,0.9][0.0,0.2][0.4,0.6]\rangle \\ \langle[0.3,0.6][0.0,0.4][0.5,0.6]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.6]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.6]\rangle \\ \langle[0.3,0.4][0.0,0.5][0.3,0.5]\rangle & \langle[0.3,0.4][0.0,0.5][0.3,0.5]\rangle & \langle[0.0,0.0][0.0,0.5][1,1]\rangle & \langle[0.3,0.4][0.0,0.5][0.8,0.9]\rangle \\ \langle[0.4,0.5][0.0,0.4][0.6,0.8]\rangle & \langle[0.4,0.5][0.0,0.4][0.6,0.8]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.4,0.5][0.0,0.4][0.6,0.8]\rangle \end{bmatrix}$$

$$M \otimes \left(x_{ij}^{*}\right)_{(Y \wedge Z,\, X)} =$$

$$\begin{bmatrix} \langle[0.4,0.7][0.5,0.8][0.5,0.7]\rangle & \langle[0.3,0.6][0.7,0.9][0.6,0.8]\rangle & \langle[0.6,0.9][0.1,0.2][0.6,0.7]\rangle & \langle[0.2,0.5][0.5,0.7][0.4,0.6]\rangle \\ \langle[0.5,0.6][0.3,0.5][0.4,0.6]\rangle & \langle[0.1,0.3][0.3,0.6][0.5,0.7]\rangle & \langle[0.3,0.4][0.2,0.4][0.5,0.6]\rangle & \langle[0.3,0.6][0.4,0.6][0.5,0.7]\rangle \\ \langle[0.2,0.5][0.4,0.6][0.6,0.7]\rangle & \langle[0.3,0.4][0.5,0.7][0.8,0.9]\rangle & \langle[0.3,0.4][0.6,0.7][0.3,0.5]\rangle & \langle[0.1,0.3][0.3,0.5][0.7,0.8]\rangle \\ \langle[0.3,0.5][0.4,0.6][0.5,0.8]\rangle & \langle[0.4,0.5][0.2,0.4][0.3,0.6]\rangle & \langle[0.4,0.5][0.3,0.5][0.7,0.8]\rangle & \langle[0.4,0.5][0.4,0.6][0.6,0.8]\rangle \end{bmatrix}$$

$$\otimes$$

$$\begin{pmatrix} \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\ \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \end{pmatrix}$$

$$= \begin{bmatrix} \langle[0.0,0.0][0.0,0.2][1,1]\rangle & \langle[0.4,0.7][0.0,0.2][0.4,0.6]\rangle & \langle[0.4,0.7][0.0,0.2][0.4,0.6]\rangle & \langle[0.4,0.7][0.0,0.2][0.4,0.6]\rangle \\ \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.5,0.6][0.0,0.4][0.4,0.6]\rangle & \langle[0.5,0.6][0.0,0.4][0.4,0.6]\rangle & \langle[0.5,0.6][0.0,0.4][0.4,0.6]\rangle \\ \langle[0.0,0.0][0.0,0.5][1,1]\rangle & \langle[0.2,0.5][0.0,0.5][0.6,0.7]\rangle & \langle[0.2,0.5][0.0,0.5][0.6,0.7]\rangle & \langle[0.2,0.5][0.0,0.5][0.6,0.7]\rangle \\ \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.4,0.5][0.0,0.4][0.5,0.8]\rangle & \langle[0.4,0.5][0.0,0.4][0.5,0.8]\rangle & \langle[0.4,0.5][0.0,0.4][0.5,0.8]\rangle \end{bmatrix}$$

$$M\otimes\left(x_{ij}^{*}\right)_{(Z\wedge X,Y)}=$$

$$\begin{bmatrix}
\langle[0.4,0.7][0.5,0.8][0.5,0.7]\rangle & \langle[0.3,0.6][0.7,0.9][0.6,0.8]\rangle & \langle[0.6,0.9][0.1,0.2][0.6,0.7]\rangle & \langle[0.2,0.5][0.5,0.7][0.4,0.6]\rangle \\
\langle[0.5,0.6][0.3,0.5][0.4,0.6]\rangle & \langle[0.1,0.3][0.3,0.6][0.5,0.7]\rangle & \langle[0.3,0.4][0.2,0.4][0.5,0.6]\rangle & \langle[0.3,0.6][0.4,0.6][0.5,0.7]\rangle \\
\langle[0.2,0.5][0.4,0.6][0.6,0.7]\rangle & \langle[0.3,0.4][0.5,0.7][0.8,0.9]\rangle & \langle[0.3,0.4][0.6,0.7][0.3,0.5]\rangle & \langle[0.1,0.3][0.3,0.5][0.7,0.8]\rangle \\
\langle[0.3,0.5][0.4,0.6][0.5,0.8]\rangle & \langle[0.4,0.5][0.2,0.4][0.3,0.6]\rangle & \langle[0.4,0.5][0.3,0.5][0.7,0.8]\rangle & \langle[0.4,0.5][0.4,0.6][0.6,0.8]\rangle
\end{bmatrix}$$

$$\otimes$$

$$\begin{pmatrix}
\langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\
\langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle \\
\langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[0,0][0,0][1,1]\rangle \\
\langle[1,1][0,0][0,0]\rangle & \langle[0,0][0,0][1,1]\rangle & \langle[1,1][0,0][0,0]\rangle & \langle[1,1][0,0][0,0]\rangle
\end{pmatrix}$$

$$=\begin{bmatrix}
\langle[0.3,0.6][0.0,0.2][0.4,0.6]\rangle & \langle[0.0,0.0][0.0,0.2][1,1]\rangle & \langle[0.3,0.6][0.0,0.2][0.4,0.6]\rangle & \langle[0.3,0.6][0.0,0.2][0.4,0.6]\rangle \\
\langle[0.3,0.6][0.0,0.4][0.5,0.7]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.7]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.7]\rangle \\
\langle[0.3,0.4][0.0,0.5][0.7,0.8]\rangle & \langle[0.0,0.0][0.0,0.5][1,1]\rangle & \langle[0.3,0.4][0.0,0.5][0.7,0.8]\rangle & \langle[0.3,0.4][0.0,0.5][0.7,0.8]\rangle \\
\langle[0.4,0.5][0.0,0.4][0.3,0.6]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.4,0.5][0.0,0.4][0.3,0.6]\rangle & \langle[0.4,0.5][0.0,0.4][0.3,0.6]\rangle
\end{bmatrix}$$

By the definition of 3.6, we take the sum of all the product matrices and obtain

$$\left(M\otimes\left(x_{ij}^{*}\right)_{(X\wedge Y,Z)}\right)\oplus\left(M\otimes\left(x_{ij}^{*}\right)_{(Y\wedge Z,X)}\right)\oplus\left(M\otimes\left(x_{ij}^{*}\right)_{(Z\wedge X,Y)}\right)$$

$$=$$

$$\begin{bmatrix}
\langle[0.6,0.9][0.0,0.2][0.4,0.6]\rangle & \langle[0.6,0.9][0.0,0.2][0.4,0.6]\rangle & \langle[0.0,0.0][0.0,0.2][1,1]\rangle & \langle[0.6,0.9][0.0,0.2][0.4,0.6]\rangle \\
\langle[0.3,0.6][0.0,0.4][0.5,0.6]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.6]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.6]\rangle \\
\langle[0.3,0.4][0.0,0.5][0.3,0.5]\rangle & \langle[0.3,0.4][0.0,0.5][0.3,0.5]\rangle & \langle[0.0,0.0][0.0,0.5][1,1]\rangle & \langle[0.3,0.4][0.0,0.5][0.8,0.9]\rangle \\
\langle[0.4,0.5][0.0,0.4][0.6,0.8]\rangle & \langle[0.4,0.5][0.0,0.4][0.6,0.8]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.4,0.5][0.0,0.4][0.6,0.8]\rangle
\end{bmatrix}$$

$$\oplus$$

$$\begin{bmatrix} \langle[0.0,0.0][0.0,0.2][1,1]\rangle & \langle[0.4,0.7][0.0,0.2][0.4,0.6]\rangle & \langle[0.4,0.7][0.0,0.2][0.4,0.6]\rangle & \langle[0.4,0.7][0.0,0.2][0.4,0.6]\rangle \\ \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.5,0.6][0.0,0.4][0.4,0.6]\rangle & \langle[0.5,0.6][0.0,0.4][0.4,0.6]\rangle & \langle[0.5,0.6][0.0,0.4][0.4,0.6]\rangle \\ \langle[0.0,0.0][0.0,0.5][1,1]\rangle & \langle[0.2,0.5][0.0,0.5][0.6,0.7]\rangle & \langle[0.2,0.5][0.0,0.5][0.6,0.7]\rangle & \langle[0.2,0.5][0.0,0.5][0.6,0.7]\rangle \\ \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.4,0.5][0.0,0.4][0.5,0.8]\rangle & \langle[0.4,0.5][0.0,0.4][0.5,0.8]\rangle & \langle[0.4,0.5][0.0,0.4][0.5,0.8]\rangle \end{bmatrix}$$

$$\oplus$$

$$\begin{bmatrix} \langle[0.3,0.6][0.0,0.2][0.4,0.6]\rangle & \langle[0.0,0.0][0.0,0.2][1,1]\rangle & \langle[0.3,0.6][0.0,0.2][0.4,0.6]\rangle & \langle[0.3,0.6][0.0,0.2][0.4,0.6]\rangle \\ \langle[0.3,0.6][0.0,0.4][0.5,0.7]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.7]\rangle & \langle[0.3,0.6][0.0,0.4][0.5,0.7]\rangle \\ \langle[0.3,0.4][0.0,0.5][0.7,0.8]\rangle & \langle[0.0,0.0][0.0,0.5][1,1]\rangle & \langle[0.3,0.4][0.0,0.5][0.7,0.8]\rangle & \langle[0.3,0.4][0.0,0.5][0.7,0.8]\rangle \\ \langle[0.4,0.5][0.0,0.4][0.3,0.6]\rangle & \langle[0.0,0.0][0.0,0.4][1,1]\rangle & \langle[0.4,0.5][0.0,0.4][0.3,0.6]\rangle & \langle[0.4,0.5][0.0,0.4][0.3,0.6]\rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle[0.6,0.9][0.0,0.3][0.4,0.6]\rangle & [0.6,0.9][0.0,0.3][0.4,0.6] & [0.4,0.7][0.0,0.3][0.4,0.6] & [0.6,0.9][0.0,0.3][0.4,0.6] \\ [0.3,0.6][0.0,0.6][0.5,0.6] & [0.5,0.6][0.0,0.6][0.4,0.6] & [0.5,0.6][0.0,0.6][0.4,0.6] & [0.5,0.6][0.0,0.6][0.4,0.6] \\ [0.3,0.4][0.0,0.75][0.3,0.5] & [0.3,0.5][0.0,0.75][0.3,0.5] & [0.3,0.5][0.0,0.75][0.6,0.7] & [0.3,0.5][0.0,0.75][0.6,0.7] \\ [0.4,0.5][0.0,0.6][0.3,0.6] & [0.4,0.5][0.0,0.6][0.5,0.8] & [0.4,0.5][0.0,0.6][0.3,0.6] & [0.4,0.5][0.0,0.6][0.3,0.6] \end{bmatrix}$$

Let

$$S= \begin{bmatrix} \langle[0.6,0.9][0.0,0.3][0.4,0.6]\rangle & [0.6,0.9][0.0,0.3][0.4,0.6] & [0.4,0.7][0.0,0.3][0.4,0.6] & [0.6,0.9][0.0,0.3][0.4,0.6] \\ [0.3,0.6][0.0,0.6][0.5,0.6] & [0.5,0.6][0.0,0.6][0.4,0.6] & [0.5,0.6][0.0,0.6][0.4,0.6] & [0.5,0.6][0.0,0.6][0.4,0.6] \\ [0.3,0.4][0.0,0.75][0.3,0.5] & [0.3,0.5][0.0,0.75][0.3,0.5] & [0.3,0.5][0.0,0.75][0.6,0.7] & [0.3,0.5][0.0,0.75][0.6,0.7] \\ [0.4,0.5][0.0,0.6][0.3,0.6] & [0.4,0.5][0.0,0.6][0.5,0.8] & [0.4,0.5][0.0,0.6][0.3,0.6] & [0.4,0.5][0.0,0.6][0.3,0.6] \end{bmatrix}$$

Taking all the lower limits and all the upper limits separately of each entry of S we construct another two matrices, given by

$$S^{l} = \begin{bmatrix} \langle 0.6,0.0,0.4\rangle & \langle 0.6,0.0,0.4\rangle & \langle 0.4,0.0,0.4\rangle & \langle 0.6,0.0,0.4\rangle \\ \langle 0.3,0.0,0.5\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.5,0.0,0.4\rangle \\ \langle 0.3,0.0,0.3\rangle & \langle 0.3,0.0,0.3\rangle & \langle 0.3,0.0,0.6\rangle & \langle 0.3,0.0,0.6\rangle \\ \langle 0.4,0.0,0.3\rangle & \langle 0.4,0.0,0.5\rangle & \langle 0.4,0.0,0.3\rangle & \langle 0.4,0.0,0.3\rangle \end{bmatrix}$$

and $$S^{u} = \begin{bmatrix} \langle 0.9,0.3,0.6\rangle & \langle 0.9,0.3,0.6\rangle & \langle 0.7,0.3,0.6\rangle & \langle 0.9,0.3,0.6\rangle \\ \langle 0.6,0.6,0.6\rangle & \langle 0.6,0.6,0.6\rangle & \langle 0.6,0.6,0.6\rangle & \langle 0.6,0.6,0.6\rangle \\ \langle 0.4,0.75,0.5\rangle & \langle 0.5,0.75,0.5\rangle & \langle 0.5,0.75,0.7\rangle & \langle 0.5,0.75,0.7\rangle \\ \langle 0.5,0.6,0.6\rangle & \langle 0.5,0.6,0.8\rangle & \langle 0.5,0.6,0.6\rangle & \langle 0.5,0.6,0.6\rangle \end{bmatrix}$$

Compute the value matrices corresponding to S^{l} and S^{u} are

$$V(S^{l}) = \begin{bmatrix} 0.2 & 0.2 & 0.0 & 0.2 \\ -0.2 & 0.1 & 0.1 & 0.1 \\ 0.0 & 0.0 & -0.3 & -0.3 \\ 0.1 & -0.1 & 0.1 & 0.1 \end{bmatrix}$$

$$V(S^u) = \begin{bmatrix} 0.6 & 0.6 & 0.4 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.65 & 0.75 & 0.55 & 0.55 \\ 0.5 & 0.3 & 0.5 & 0.5 \end{bmatrix}$$

$$\sum_{Row} V(S^l) = \begin{bmatrix} 0.6 \\ 0.1 \\ -0.6 \\ 0.2 \end{bmatrix} \text{ and } \sum_{Row} V(S^u) = \begin{bmatrix} 2.2 \\ 2.4 \\ 2.5 \\ 1.8 \end{bmatrix}$$

$$W(h_1) = 0.6 + 2.2 = 2.8$$

$$W(h_2) = 0.1 + 2.4 = 2.5$$

$$W(h_3) = -0.6 + 2.5 = 1.9$$

$$W(h_4) = 0.2 + 1.8 = 2.0$$

Among all the values above, h_1 has the highest weight. So, Mr., Debnath will prefer to buy the house h_1 .

5. Application in medical science

In medical science, one symptom is related to various diseases. For instance, the symptom fever is related to different diseases including typhoid, peptic ulcer, food poisoning ,etc. So, proper disease diagnosis is a very difficult task. For medical diagnosis problems, we consider the following example.

Suppose Mr. X is suffering from a fever having the symptoms of body pain , breathing difficulty, headache ,and cough and the possible diseases, as per experts advice, relating to the proposed symptoms are viral fever,dengue,food poisoning ,and diptheria. But all experts should come to a common decision so that it helps to diagnose the patient. For this, we consider the set of four diseases $U =$ {viral fever, dengue, food poisoning, diphtheria}, as a universal set and some related symptoms of these four diseases $E =$ {body pain, breathing difficulty, headache, cough, }, as a set of parameters and a set of doctors *D*= { d_1, d_2, d_3 },called a set of decision makers or experts or doctors. In reference to example 4.2, if we compare the set of houses as a set of diseases , set of parameters as a set of symptoms , set of decision makers as a set of doctors and then by using this information we construct interval neutrosophic soft set frame work, by considering the same data set proposed in section 4.1. Then, with the help of IVNSM-algoritm, proposed in section 4, we come to the conclusion that Mr. X is suffering from viral fever. The main advantage of using IVNSM-algoritm is that it helps the doctors to diagnose the correct disese and so it prevents the wrong treatment of the patient and it saves time as well. In case no such conclusion can be drawn with the given information then we need to reassess all the symptoms with the help of expert and then repeat all the steps proposed in IVNSM-algorithm.

6. Conclusions

In this paper, we have studied the various concept of interval neutrosophic soft set, which is an extension of neutrosophic soft set. We also introduce the choice matrices, which are associated with the interval neutrosophic soft sets. An algorithm, called IVNSM-algorithm, has been introduced for real decision making problem with the help of a group of decision makers. Finally, it has been discussed, how we use the IVNSM-algorithm in medical diagnosis problem.

7. Future Scope

In our work , we do not consider the information 'time duration' of the symptom or attribute though it has its own significance together with the belongingness or non-belongingness or indeterminacy level of a symptom for proper diagnosis of a patient. Which may be cover up by introducing complex interval neutrosophic soft set. Also, instead of introducing a soft set, we introduce a hypersoft set, pilthogenic hypersoft set, introduced by Smarandache(2018) in [33], to extend the concept used in this paper.

Acknowledgments: Author is thankful to the reviewers for their valuable suggestions.

Conflicts of Interest: Author has no conflict of interest regarding the publication of the article.

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Received: May 22, 2021. Accepted: August 13, 2021