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Some Results on Single Valued Neutrosophic Bi-ideals in Ordered Semigroups

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Abstract. The importance of the theory of neutrosophy is due to its connections with several fields of sciences, engineering, and technology. So, the aim of this paper is to relate neutrosophy with some algebraic structures mainly the ordered semigroups. In this regard, we define single valued neutrosophic sets in ordered semigroups. More precisely, we study single valued neutrosophic ideals of ordered semigroups and single valued neutrosophic bi-ideals of ordered semigroups.

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1. Introduction

The neutrosophic set is a generalization of intuitionistic fuzzy set which in return is a generalization of fuzzy set. Fuzzy set was introduced by Zadeh [18] in 1965 where he considered that every element in a certain set has a degree of membership (truth value) in the unit interval [0,1]. Then in 1986, Atanassov [5] introduced intuitionistic fuzzy set as a generalization of the fuzzy set where he considered that every element in the set has a degree of membership (truth value) and degree of non-membership (falsityhood value). Later in 1998, Smarandache [15] generalized these two concepts and introduced the neutrosophic sets where he considered that each element has a truth value (T), falsity value (F) and indeterminacy value (I).

Neutrosophy is considered as a connection of mathematics with philosophy where it studies the idea in a philosophical way. It is applicable in psychology, sociology, decision making, engineering, information technology, probability and statistics, etc. It proposes any idea and studies its origin, nature, scope and interactions with ideational spectra and reveals that the world is full of indeterminacy. It considers the idea studied as $< A >$, its opposite as $< Anti-A >$, its negation as $< Non-A >$ and its spectrum of neutralities as $< Neut-A >$. For details about neutrosophy, we refer to [15–17]. The new topic of neutrosophy had grabbed the attention of many algebraists and as a result, neutrosophic algebraic structures was introduced. For details about neutrosophic algebraic structures, we refer to [2,3,12–14]. Recently, a relation between neutrosophy and ordered algebraic structures was defined [1, 4].

In 2020, Al-Tahan el al. [1] defined single valued neutrosophic sets in ordered groupoids and investigated the various properties of single valued neutrosophic ideals in it. In this paper, we study the relation between neutrosophy and ordered semigroups and it is organized as follows: After a brief introduction about neutrosophy, Section 2 presents some preliminaries about some algebraic structures such as semigroups and ordered semigroups and give some concepts about neutrosophy. Section 3 presents some definitions, properties and examples about single valued neutrosophic ideals in ordered semigroups. Finally, Section 5 presents single valued neutrosophic bi-ideals in ordered semigroups and provides some related important theorems and examples.

2. **Preliminaries**

In this section, we present some definitions, concepts and examples related to (ordered) semigroups, neutrosophy, and single valued neutrosophic sets that are used throughout the paper.

2.1. *Semigroups and ordered semigroups*

Definition 2.1. [7] Let S be a non-empty set of elements and $\star$ be a binary operation defined on S. Then S is said to be semigroup if it is binary closed and the associative property holds. In other words, for every x, y and z in the set S, $(x \star y) \star z = x \star (y \star z)$.

Example 2.2. Let $\mathbb{Z}$ be the set of integers, then $(\mathbb{Z}, \cdot)$, where "$\cdot$" is the usual multiplication, is a semigroup.

Remark 2.3. (1) A semigroup is an associative groupoid.

(2) Every semigroup is a groupoid but not every groupoid is a semigroup.

(3) A semigroup with identity is called a monoid.

Definition 2.4. [11] Let $(S, \star)$ be a semigroup. Then a subset A of S is called a subsemigroup if $(A, \star)$ is a semigroup.

Remark 2.5. To prove that a non-empty subset A of a semigroup $(S, \star)$ is a subsemigroup, it suffices to show that $A \star A \subseteq A$.

Example 2.6. Let $(\mathbb{Z}, +)$ be the semigroup of integers under standard addition. Then $(\mathbb{N}, +)$, the set of non-negative integers under standard addition, is a subsemigroup of $(\mathbb{Z}, +)$.

Definition 2.7. [11] Let $(S, \star)$ be a semigroup and $A \subseteq S$ a subsemigroup of S. Then A is called a:

(1) Right ideal if $A \star S \subseteq A$,
(2) Left ideal if $S \star A \subseteq A$,
(3) Ideal if it is both right and left ideal of S,
(4) Bi-ideal if $A \star S \star A \subseteq S$.

Example 2.8. Let $(\mathbb{Z}, \cdot)$ be the semigroup of integers under standard multiplication and let $I = n\mathbb{Z} = \{nq | q \in \mathbb{Z}\}$. Then I is both right and left ideal of $(\mathbb{Z}, \cdot)$. Hence, it is an ideal of $(\mathbb{Z}, \cdot)$.

Definition 2.9. [6] Let G be a non-empty set of elements. A partial order is a binary relation "$\leq$" over a set G such that $\leq$ is reflexive, antisymmetric, and transitive.
In other words, for all $a, b, c \in G, \leq$ satisfies:

(1) $a \leq a$,
(2) If $a \leq b$ and $b \leq a$ then $a = b$,
(3) If $a \leq b$ and $b \leq c$ then $a \leq c$.

Definition 2.10. [6] A total order is a binary relation "$\leq$" over a set G such that $\leq$ is a partial order and every two elements in G are comparable.
In other words, for all $x, y \in G, x \leq y$ or $y \leq x$.

Definition 2.11. [8] Let $(S, \cdot)$ be a semigroup and "$\leq$" be a partial order on S. Then $(S, \cdot, \leq)$ is said to be an ordered semigroup if for all $z \in S$ the following condition holds

$$\text{if } x \leq y \text{ then } z \cdot x \leq z \cdot y \text{ and } x \cdot z \leq y \cdot z \text{ for all } x, y \in S.$$

Remark 2.12. Every ordered semigroup is an ordered groupoid. But the converse is not true.

Example 2.13. The set defined in Example 2.2 is an ordered semigroup under the partial order "$\leq$" which is for every $x, y \in \mathbb{Z}$, $x \leq y$ if and only if $x = y$.
This order is called **trivial order**.

Definition 2.14. [8] Let $(S, \cdot, \leq)$ be an ordered semigroup and $A \subseteq S$. Then

$$(A] = \{x \in S | x \leq a \text{ for some } a \in A\}.$$

Remark 2.15. If $(S, \cdot, \leq)$ is an ordered semigroup and $A \subseteq S$ then $A \subseteq (A]$.

Definition 2.16. [8] Let $(S, \cdot, \leq)$ be an ordered semigroup and $A \subseteq S$, then A is called:

(1) Left ideal of S if $S \cdot A \subseteq A$ and $(A] \subseteq A$,
(2) Right ideal of S if $A \cdot S \subseteq A$ and $(A] \subseteq A$,
(3) Ideal if it is both right and left ideal,
(4) Bi-ideal if $A \cdot S \cdot A \subseteq A$ and $(A] \subseteq A$.

Example 2.17. Let $S = \{a, b, c, d\}$ and let $(S, \star)$ be the semigroup defined by the following table:

$\star$	a	b	c	d
a	d	d	d	d
b	d	d	d	d
c	d	d	d	d
d	d	d	d	a

And let $\leq$ be defined as:

$$\leq = \{(a, a), (b, b), (b, c), (c, c), (d, d)\}.$$

Then $(S, \star, \leq)$ is an ordered semigroup. Now, let $I = \{a, b, d\}$, then I is a right and left ideal of the ordered semigroup $(S, \star, \leq)$. Hence I is an ideal of $(S, \star)$.

2.2. *Neutrosophy and single valued neutrosophic sets*

Definition 2.18. [15] Neutrosophy is a new branch of philosophy which studies the origin, nature, scope and interactions of neutralities with ideational spectra.
It considers:

- Any idea, proposition, theory or event by $< A >$,
- Its opposite by $< Anti - A >$,
- Its negation by $< Non - A >$,
- Its of spectrum of neutralities in between them by $< Neut - A >$.

Remark 2.19. In the theory of neutrosophy, every idea $< A >$ has a truth membership value (T), false membership value (F) and indeterminacy membership value (I).

Definition 2.20. [15] Let X be a non-empty space of elements (objects). A single valued neutrosophic set(SVNS) A on X is characterized by its truth-membership function (T_A), its indeterminacy-membership function (I_A), and its falsity-membership function (F_A) where for each element $x \in X$, $0 \leq T_A(x), I_A(x), F_A(x) \leq 1$.

Remark 2.21. Let X be a non-empty space of elements (objects). A single valued neutrosophic set(SVNS) A on X is defined by $N_A(x) = (T_A(x), I_A(x), F_A(x))$ for all $x \in X$.

Definition 2.22. [1] Let X be a non empty set of elements and A and B be two single valued neutrosophic sets over X defined as follows:

$$A = \{\frac{x}{(T_A(x), I_A(x), F_A(x))} | x \in X\} \text{ and } B = \{\frac{x}{(T_B(x), I_B(x), F_B(x))} | x \in X\}$$

Then

- $A \cap B$, which is the intersection of A and B, is a single valued neutrosophic set over X defined as follows:
$$A \cap B = \{\frac{x}{(T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x))} | x \in X\}$$
where $T_{A\cap B}(x) = T_A(x) \wedge T_B(x)$, $I_{A\cap B}(x) = I_A(x) \wedge I_B(x)$ and $F_{A\cap B}(x) = F_A(x) \vee F_B(x)$ for all $x \in X$.
- $A \cup B$, which is the union of A and B, is a single valued neutrosophic set over X defined as follows:
$$A \cup B = \{\frac{x}{(T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x))} | x \in X\}$$
where $T_{A\cup B}(x) = T_A(x) \vee T_B(x)$, $I_{A\cup B}(x) = I_A(x) \vee I_B(x)$ and $F_{A\cup B}(x) = F_A(x) \wedge F_B(x)$ for all $x \in X$.

3. Single valued neutrosophic ideals in ordered semigroups

Inspired by the work in [9] done by Khan et al. related to fuzzy ideals in ordered semigroups and by the definition of single valued neutrosophic sets in ordered groupoids [1], we consider single valued neutrosophic sets in ordered semigroups. More precisely, we define single valued neutrosophic left ideals, single valued neutrosophic right ideals, study their properties, and provide some examples.

Definition 3.1. Let $(S, \cdot, \leq)$ be an ordered semigroup and A be a single valued neutrosophic set over S. Then A is a single valued neutrosophic subsemigroup of S if for all $x, y \in S$, the following conditions hold:

(1) $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y)$,
(2) $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y)$,
(3) $F_A(x \cdot y) \leq F_A(x) \vee F_A(y)$,
(4) If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$ and $F_A(x) \leq F_A(y)$.

Definition 3.2. Let $(S, \cdot, \leq)$ be an ordered semigroup and A be a single valued neutrosophic set over S. Then A is a single valued neutrosophic left ideal of S if for all $x, y \in S$, the following conditions hold:

(1) $T_A(x \cdot y) \geq T_A(y)$,
(2) $I_A(x \cdot y) \geq I_A(y)$,
(3) $F_A(x \cdot y) \leq F_A(y)$,
(4) If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$ and $F_A(x) \leq F_A(y)$.

Definition 3.3. Let $(S, \cdot, \leq)$ be an ordered semigroup and A be a single valued neutrosophic set over S. Then A is a single valued neutrosophic right ideal of S if for all $x, y \in S$, the following conditions hold:

(1) $T_A(x \cdot y) \geq T_A(x)$,
(2) $I_A(x \cdot y) \geq I_A(x)$,
(3) $F_A(x \cdot y) \leq F_A(x)$,
(4) If $x \leq y$ then $T_A(x) \geq T_A(y)$, $I_A(x) \geq I_A(y)$ and $F_A(x) \leq F_A(y)$.

Definition 3.4. Let $(S, \cdot, \leq)$ be an ordered semigroup and A be a single valued neutrosophic set over S. Then A is said to be a single valued neutrosophic ideal of S if it is both single valued neutrosophic right and left ideal of S.

Remark 3.5. Let $(S, \cdot, \leq)$ be a commutative semigroup and A be a single valued neutrosophic right (left) ideal of S. Then A is a single valued neutrosophic ideal of S.

Remark 3.6. Let $(S, \cdot, \leq)$ be a commutative semigroup and α, β, $\gamma \in [0, 1]$. Then

$$A = \{\tfrac{x}{(\alpha,\beta,\gamma)} | x \in S\}$$

is a single valued neutrosophic ideal of S and it is called the **trivial single valued neutrosophic ideal** of S.

Example 3.7. Let $S = \{1, 2, 3, 4\}$ and $(S, \cdot)$ be defined by the following table:

$\cdot$	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1

Let " $\leq$ " be the partial order on S defined as follows:

$$\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (1, 4), (2, 3), (3, 4)\}$$

Then $(S, ., \leq)$ is an ordered semigroup since "$\cdot$" is binary closed and associative, and "$\leq$" satisfies the monotone property (i.e. for all $x, y \in S, x \leq y$ and for all $z \in S$, $x \cdot z = z \cdot x = 1 \leq 1 = y \cdot z = z \cdot y$).

Let A be an SVNS on S defined by N_A as follows

$$N_A(1) = (0.9, 0.8, 0.1),\ N_A(2) = (0.7, 0.6, 0.2),\ N_A(3) = (0.6, 0.6, 0.2) \text{ and } N_A(4) = (0.5, 0.4, 0.5).$$

Then A is a single valued neutrosophic ideal of S since for all $x, y \in S$, we have

$T_A(x \cdot y) = T_A(1) = 0.9 \geq T_A(x) \vee T_A(y)$;

$I_A(x \cdot y) = I_A(1) = 0.8 \geq I_A(x) \vee I_A(y)$;

$F_A(x \cdot y) = F_A(1) = 0.1 \leq F_A(x) \wedge F_A(y)$;

Moreover, $1 \leq 2 \leq 3 \leq 4$ implies that $T_A(1) \geq T_A(2) \geq T_A(3) \geq T_A(4)$, $I_A(1) \geq I_A(2) \geq I_A(3) \geq I_A(4)$, and $F_A(1) \leq F_A(2) \leq F_A(3) \leq F_A(4)$.

Proposition 3.8. *Let $(S, \cdot, \leq)$ be an ordered semigroup with identity "e" and A be a single valued neutrosophic set over S. Then A is a single valued neutrosophic left(right) ideal of S if and only if A is the trivial single valued neutrosophic ideal of S.*

Proof. The proof is similar to the case in ordered groupoids [1]. □

Example 3.9. The only single valued neutrosophic right(left) ideal of the semigroup of non-negative integers under addition is the trivial single valued neutrosophic set.

Lemma 3.10. *Let $(S, \cdot, \leq)$ be an ordered semigroup and A_α a single valued neutrosophic left ideal, right ideal or subsemigroup of S. Then $\bigcap_\alpha A_\alpha$ which is the intersection of A_α for all α is a single valued neutrosophic left ideal, right ideal or subsemigroup of S.*

Proof. Let A_α be a single valued neutrosophic left ideal of S. Then for all $x, y \in S$, $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(y)$, $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(y)$, $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(y)$ for all α. This latter implies that

$T_{\bigcap_\alpha A_\alpha}(x \cdot y) = \inf_\alpha T_{A_\alpha}(x \cdot y) \geq \inf_\alpha T_{A_\alpha}(y) = T_{\bigcap_\alpha A_\alpha}(y)$;

$I_{\bigcap_\alpha A_\alpha}(x \cdot y) = \inf_\alpha I_{A_\alpha}(x \cdot y) \geq \inf_\alpha I_{A_\alpha}(y) = I_{\bigcap_\alpha A_\alpha}(y)$;

$F_{\bigcap_\alpha A_\alpha}(x \cdot y) = \sup_\alpha F_{A_\alpha}(x \cdot y) \leq \sup_\alpha F_{A_\alpha}(y) = F_{\bigcap_\alpha A_\alpha}(y)$.

Let $x \leq y$. Then $T_{A_\alpha}(x) \geq T_{A_\alpha}(y)$, $I_{A_\alpha}(x) \geq I_{A_\alpha}(y)$, $F_{A_\alpha}(x) \leq F_{A_\alpha}(y)$. So,

$T_{\bigcap_\alpha A_\alpha}(x) = \inf_\alpha T_{A_\alpha}(x) \geq \inf_\alpha T_{A_\alpha}(y) = T_{\bigcap_\alpha A_\alpha}(y)$;

$I_{\bigcap_\alpha A_\alpha}(x) = \inf_\alpha I_{A_\alpha}(x) \geq \inf_\alpha I_{A_\alpha}(y) = I_{\bigcap_\alpha A_\alpha}(y)$;

and $F_{\bigcap_\alpha A_\alpha}(x) = \sup_\alpha F_{A_\alpha}(x) \leq \sup_\alpha F_{A_\alpha}(y) = F_{\bigcap_\alpha A_\alpha}(y)$.

Therefore, $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic left ideal of S. Similarly, we can prove that the intersection of single valued neutrosophic right ideals or subsemigroups of S is a single valued neutrosophic right ideal of S. □

Remark 3.11. Let $(S, \cdot, \leq)$ be an ordered semigroup and A_α a single valued neutrosophic subsemigroup of S. Then $\bigcup_\alpha A_\alpha$ is not necessarily a single valued neutrosophic subsemigroup of S.

Example 3.12. Let $(\mathbb{N}, +, \leq)$ be the ordered semigroup of natural numbers under standard addition and trivial order and let A and B be single valued neutrosophic sets on $\mathbb{N}$ defined as follows:

$$N_A(x) = \begin{cases} (0.9, 0.7, 0.2) & \text{if x is divisible by 5 ;} \\ (0.4, 0.3, 0.4) & \text{otherwise.} \end{cases}$$

$$N_B(x) = \begin{cases} (0.9, 0.7, 0.2) & \text{if x is divisible by 7 ;} \\ (0.4, 0.3, 0.4) & \text{otherwise.} \end{cases}$$

Then A and B are single valued neutrosophic subsemigroups of $\mathbb{N}$. But $A \cup B$ is not a single valued neutrosophic subsemigroup of $\mathbb{N}$ since $N_{A\cup B}(5+7) = N_{A\cup B}(12) = (0.4, 0.3, 0.4)$ so we will have that $T_{A\cup B}(5+7) = N_{A\cup B}(12) = 0.4 \ngeq N_{A\cup B}(5) \wedge N_{A\cup B}(7) = 0.9$.

Example 3.13. Let $(S, \cdot, \leq)$ be the ordered semigroup defined in Example 3.7. Let A and B be the single valued neutrosophic sets on S defined by N_A and N_B respectively as follows:

$N_A(1) = (0.9, 0.8, 0.1)$, $N_A(2) = (0.7, 0.6, 0.2)$, $N_A(3) = (0.6, 0.6, 0.2)$, $N_A(4) = (0.5, 0.4, 0.5)$;
$N_B(1) = (0.9, 0.8, 0.1)$, $N_B(2) = (0.8, 0.7, 0.2)$, $N_B(3) = (0.7, 0.6, 0.3)$, $N_B(4) = (0.6, 0.4, 0.6)$;

It is clear that A and B are single valued neutrosophic subsemigroups of S. Also $A \cup B$ and $A \cap B$, defined by $N_{A\cup B}$ and $N_{A\cap B}$ respectively as follows.

$$N_{A\cup B}(1) = (0.9, 0.8, 0.1),\ N_{A\cup B}(2) = (0.8, 0.7, 0.2),\ N_{A\cup B}(3) = (0.7, 0.6, 0.2),$$
$$N_{A\cup B}(4) = (0.6, 0.4, 0.5);$$
$$N_{A\cap B}(1) = (0.9, 0.8, 0.1),\ N_{A\cap B}(2) = (0.7, 0.6, 0.2),\ N_{A\cap B}(3) = (0.6, 0.6, 0.3),$$
$$N_{A\cap B}(4) = (0.5, 0.4, 0.6);$$

are also single valued neutrosophic subsemigoups of S.

Lemma 3.14. *Let $(S, \cdot, \leq)$ be an ordered semigroup and A_α a single valued neutrosophic ideal of S. Then $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic ideal of S.*

Proof. Let A_α be a single valued neutrosophic ideal of S. Then A_α is both, a single valued neutrosophic right and left ideal of S. So, by Lemma 3.10, $\bigcap_\alpha A_\alpha$ is both, a single valued neutrosophic right and left ideal of S. Therefore $\bigcap_\alpha A_\alpha$ is a single valued neutrsophic ideal of S. □

Lemma 3.15. *Let $(S, \cdot, \leq)$ be an ordered semigroup and A_α a single valued neutrosophic ideal of S. Then $\bigcup_\alpha A_\alpha$ is a single valued neutrosophic ideal of S.*

Proof. The proof is similar to that of ordered groupoids [1]. □

Example 3.16. Let $(S, \star, \leq)$ be an ordered semigroup where $(S, \star)$ is defined by the following table:

$\star$	1	2	3
1	1	1	1
2	1	1	3
3	1	3	1

and $\leq = \{(1,1),(1,2),(1,3),(2,2),(3,3)\}$. And let A and B be single valued neutrosophic sets over S defined by N_A and N_B respectively as follows:

$$N_A(1) = (0.9, 0.8, 0.1),\ N_A(2) = (0.6, 0.6, 0.3),\ N_A(3) = (0.4, 0.5, 0.3);$$
$$N_B(1) = (0.9, 0.8, 0.1),\ N_B(2) = (0.8, 0.5, 0.5),\ N_B(3) = (0.6, 0.5, 0.3).$$

It is clear that A and B are single valued neutrosophic ideals of S. Then $A \cap B$ and $A \cup B$ defined by $N_{A\cap B}$ and $N_{A\cup B}$ respectively as follows:

$$N_{A\cap B}(1) = (0.9, 0.8, 0.1),\ N_{A\cap B}(2) = (0.6, 0.5, 0.5),\ N_{A\cap B}(2) = (0.4, 0.5, 0.3);$$
$$N_{A\cup B}(1) = (0.9, 0.8, 0.1),\ N_{A\cup B}(2) = (0.8, 0.6, 0.3),\ N_{A\cup B}(2) = (0.6, 0.5, 0.3)$$

are also single valued neutrosophic ideals of S.

Definition 3.17. Let $(S_1, \cdot_1, \leq_1)$ and $(S_2, \cdot_2, \leq_2)$ be two ordered semigroups, then $(S_1 \times S_2, \cdot, \leq)$ is an ordered semigroup where $(x, y) \cdot (z, w) = (x \cdot_1 z, y \cdot_2 w)$ and $(x, y) \leq (z, w)$ if and only if $x \leq_1 z$ and $y \leq_2 w$.

Definition 3.18. Let $(S_1, \cdot_1, \leq_1)$ and $(S_2, \cdot_2, \leq_2)$ be two ordered semigroups, and let A and B be two single valued neutrosophic sets over $S_1 \times S_2$ defined as follows:

$$N_{A\times B}(x, y) = (T_{A\times B}(x, y), I_{A\times B}(x, y), F_{A\times B}(x, y))$$

where $T_{A\times B}(x, y) = T_A(x) \wedge T_B(y)$, $I_{A\times B}(x, y) = I_A(x) \wedge I_B(y)$ and $F_{A\times B}(x, y) = F_A(x) \vee F_B(y)$.

Theorem 3.19. *Let $(S_1, \cdot_1, \leq_1)$ and $(S_2, \cdot_2, \leq_2)$ be two ordered semigroups, and let A and B be two single valued neutrosophic right (left) ideal of S_1 and S_2 respectively. Then $A \times B$ is a single valued neutrosophic right (left) ideal of $S_1 \times S_2$.*

Proof. Let A and B be single valued neutrosophic left ideal of S_1 and S_2 respectively. Then for all $x_1, y_1 \in S_1$ and $x_2, y_2 \in S_2$, we have:

(1) $T_A(x_1 \cdot y_1) \geq T_A(y_1)$ and $T_B(x_2 \cdot y_2) \geq T_B(y_2)$;
(2) $I_A(x_1 \cdot y_1) \geq I_A(y_1)$ and $I_B(x_2 \cdot y_2) \geq I_B(y_2)$;
(3) $F_A(x_1 \cdot y_1) \leq F_A(y_1)$ and $F_B(x_2 \cdot y_2) \leq F_B(y_2)$;
(4) If $(x_1, x_2) \leq (y_1, y_2)$. This latter implies that $T_A(x_1) \geq T_A(y_1)$, $T_B(x_2) \geq T_B(y_2)$, $I_A(x_1) \geq I_A(y_1)$, $I_B(x_2) \geq I_B(y_2)$, $F_A(x_1) \leq F_A(y_1)$ and $F_B(x_2) \leq F_B(y_2)$.

We get that, $T_{A\times B}((x_1, x_2) \cdot (y_1, y_2)) = T_{A\times B}(x_1 \cdot y_1, x_2 \cdot y_2) = T_A(x_1 \cdot y_1) \wedge T_B(x_2 \cdot y_2) \geq T_A(y_1) \wedge T_B(y_2) \geq T_{A\times B}(y_1, y_2)$,
$I_{A\times B}((x_1, x_2) \cdot (y_1, y_2)) = I_{A\times B}(x_1 \cdot y_1, x_2 \cdot y_2) = I_A(x_1 \cdot y_1) \wedge I_B(x_2 \cdot y_2) \geq I_A(y_1) \wedge I_B(y_2) \geq I_{A\times B}(y_1, y_2)$,
$F_{A\times B}((x_1, x_2) \cdot (y_1, y_2)) = F_{A\times B}(x_1 \cdot y_1, x_2 \cdot y_2) = F_A(x_1 \cdot y_1) \vee F_B(x_2 \cdot y_2) \leq F_A(y_1) \vee F_B(y_2) \leq F_{A\times B}(y_1, y_2)$,
and if $(x_1, x_2) \leq (y_1, y_2)$, then $x_1 \leq_1 y_1$ and $x_2 \leq_2 y_2$, so easily we can see that $T_{A\times B}(x_1, x_2) \geq T_{A\times B}(y_1, y_2)$, $I_{A\times B}(x_1, x_2) \geq I_{A\times B}(y_1, y_2)$, and $F_{A\times B}(x_1, x_2) \leq F_{A\times B}(y_1, y_2)$.
Therefore, $A \times B$ is a single valued neutrosophic left ideal of $S_1 \times S_2$.
Similarly, we can prove the case of single valued neutrosophic right ideal. □

4. Single valued neutrosophic bi-ideals in ordered semigroups

In this section, we define single valued neutrosophic bi-ideals in ordered semigroups, study some of their properties, and provide several examples. The results of this section can be considered as a generalization of fuzzy bi-ideals in ordered semigroups [10].

Definition 4.1. Let $(S, \cdot, \leq)$ be an ordered semigroup and A be a single valued neutrosophic set over S. Then A is said to be a single valued neutrosophic bi-ideal of S if it is a single valued neutrosophic subsemigroup of S and if for all $x, y, z \in S$, $N_A(x \cdot y \cdot z) \geq N_A(x) \wedge N_A(y)$(i.e. $T_A(x \cdot y \cdot z) \geq T_A(x) \wedge T_A(y)$, $I_A(x \cdot y \cdot z) \geq I_A(x) \wedge I_A(y)$ and $F_A(x \cdot y \cdot z) \leq F_A(x) \vee F_A(y)$).

Theorem 4.2. *Let $(S, \cdot, \leq)$ be an ordered semigroup and A be a single valued neutrosophic left(right) ideal over S . Then A is a single valued neutrosophic bi-ideal of S.*

Proof. Let A be a single valued neutrosophic left ideal of S, then A is a single valued neutrosophic subsemigroup and $T_A(x \cdot y) \geq T_A(y), I_A(x \cdot y) \geq I_A(y), F_A(x \cdot y) \leq F_A(y)$ and if $x \leq y, T_A(x) \geq T_A(y), I_A(x) \geq I_A(y), F_A(x) \leq F_A(y)$.
Let $x, y, z \in S$. Then $T_A(x \cdot y \cdot z) \geq T_A(y \cdot z) \geq T_A(z) \geq T_A(x) \wedge T_A(z)$;
$I_A(x \cdot y \cdot z) \geq I_A(y \cdot z) \geq I_A(z) \geq I_A(x) \wedge I_A(z)$, and $F_A(x \cdot y \cdot z) \leq F_A(y \cdot z) \leq F_A(z) \leq F_A(x) \vee F_A(z)$
Therefore, A is a single valued neutrosophic bi-ideal of S. □

Remark 4.3. Not every single valued neutrosophic bi-ideal is a single valued neutrosophic left or right ideal.

Example 4.4. Let $(S, \cdot)$ be an ordered semigroup defined by the following table:

$\cdot$	a	b	c
a	a	a	a
b	a	a	c
c	a	c	a

Let " $\leq$ " be defined as follows: $\leq = \{(a,a),(a,b),(a,c),(b,b),(c,c)\}$ and A be an SVNS on S defined by: $N_A(a) = (0.9, 0.8, 0.1)$, $N_A(b) = (0.8, 0.5, 0.4)$ and $N_A(c) = (0.7, 0.6, 0.2)$. Then A is SVN bi-ideal of S but it is neither SVN right nor left ideal of S since $T_A(b \cdot c) = T_A(c \cdot b) = T_A(c) \ngeq T_A(b)$.

Theorem 4.5. *Let $(S, \cdot, \leq)$ be an ordered semigroup and A_α a single valued neutrosophic bi-ideal of S. Then $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic bi-ideal of S.*

Proof. Let A_α be a single valued neutrosophic bi-ideal of S for all α. Then A_α is a single valued neutrosophic subsemigroup of S. Hence by Lemma 3.10 $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic subsemigroup of S.

Also, we have that $T_{A_\alpha}(x \cdot y \cdot z) \geq T_{A_\alpha}(x) \wedge T_{A_\alpha}(z)$, $I_{A_\alpha}(x \cdot y \cdot z) \geq I_{A_\alpha}(x) \wedge I_{A_\alpha}(z)$ and $F_{A_\alpha}(x \cdot y \cdot z) \leq F_{A_\alpha}(x) \vee F_{A_\alpha}(z)$. This latter implies that

$T_{\bigcap_\alpha A_\alpha}(x \cdot y \cdot z) = \inf_\alpha T_{A_\alpha}(x \cdot y \cdot z) \geq \inf_\alpha\{T_{A_\alpha}(x) \wedge T_{A_\alpha}(z)\} = \inf_\alpha T_{A_\alpha}(x) \wedge \inf_\alpha T_{A_\alpha}(z) = T_{\bigcap_\alpha A_\alpha}(x) \wedge T_{\bigcap_\alpha A_\alpha}(z)$;

$I_{\bigcap_\alpha A_\alpha}(x \cdot y \cdot z) = \inf_\alpha I_{A_\alpha}(x \cdot y \cdot z) \geq \inf_\alpha\{I_{A_\alpha}(x) \wedge I_{A_\alpha}(z)\} = \inf_\alpha I_{A_\alpha}(x) \wedge \inf_\alpha I_{A_\alpha}(z) = I_{\bigcap_\alpha A_\alpha}(x) \wedge I_{\bigcap_\alpha A_\alpha}(z)$;

$F_{\bigcap_\alpha A_\alpha}(x \cdot y \cdot z) = \sup_\alpha F_{A_\alpha}(x \cdot y \cdot z) \leq \sup_\alpha\{F_{A_\alpha}(x) \vee F_{A_\alpha}(z)\} = \sup_\alpha F_{A_\alpha}(x) \vee \sup_\alpha F_{A_\alpha}(z) = F_{\bigcap_\alpha A_\alpha}(x) \vee F_{\bigcap_\alpha A_\alpha}(z)$.

Therefore, $\bigcap_\alpha A_\alpha$ is a single valued neutrosophic bi-ideal of S. □

Theorem 4.6. *Let $(S_1, \cdot_1, \leq_1)$ and $(S_2, \cdot_2, \leq_2)$ be two ordered semigroups, and let A and B be two single valued neutrosophic bi-ideals of S_1 and S_2 respectively. Then $A \times B$ is a single valued neutrosophic bi-ideals of $S_1 \times S_2$.*

Proof. Let A and B be single valued neutrosophic bi-ideals of S_1 and S_2 respectively. Then for all $x_1, y_1, z_1 \in S_1$, $x_2, y_2, z_2 \in S_2$, A and B are single valued neutrosophic bi-ideals of S, $T_A(x_1 \cdot y_1 \cdot z_1) \geq T_A(x_1) \wedge T_A(z_1)$, $T_B(x_2 \cdot y_2 \cdot z_2) \geq T_B(x_2) \wedge T_B(z_2)$, $I_A(x_1 \cdot y_1 \cdot z_1) \geq I_A(x_1) \wedge I_A(z_1)$, $I_B(x_2 \cdot y_2 \cdot z_2) \geq I_B(x_2) \wedge I_B(z_2)$, $F_A(x_1 \cdot y_1 \cdot z_1) \leq F_A(x_1) \vee F_A(z_1)$ and

$F_B(x_2 \cdot y_2 \cdot z_2) \leq F_B(x_2) \vee F_B(z_2)$.

So, we get that

$T_{A\times B}((x_1,x_2).(y_1,y_2).(z_1,z_2)) = T_{A\times B}(x_1\cdot y_1\cdot z_1, x_2\cdot y_2\cdot z_2) = T_A(x_1\cdot y_1\cdot z_1)\wedge T_B(x_2\cdot y_2\cdot z_2) \geq T_A(x_1)\wedge T_A(z_1)\wedge T_B(x_2)\wedge T_B(z_2) = T_{A\times B}(x_1,x_2)\wedge T_{A\times B}(z_1,z_2)$;

$I_{A\times B}((x_1,x_2)\cdot(y_1,y_2).(z_1,z_2)) = I_{A\times B}(x_1\cdot y_1\cdot z_1, x_2\cdot y_2\cdot z_2) = I_A(x_1\cdot y_1\cdot z_1)\wedge I_B(x_2\cdot y_2\cdot z_2) \geq I_A(x_1)\wedge I_A(z_1)\wedge I_B(x_2)\wedge I_B(z_2) = I_{A\times B}(x_1,x_2)\wedge I_{A\times B}(z_1,z_2)$;

$F_{A\times B}((x_1,x_2).(y_1,y_2).(z_1,z_2)) = F_{A\times B}(x_1\cdot y_1\cdot z_1, x_2\cdot y_2\cdot z_2) = F_A(x_1\cdot y_1]\cdot z_1)\vee F_B(x_2\cdot y_2\cdot z_2) \leq F_A(x_1)\vee F_A(z_1)\vee F_B(x_2)\vee F_B(z_2) = F_{A\times B}(x_1,x_2)\vee F_{A\times B}(z_1,z_2)$.

And as A and B are single valued neutrosophic subsemigroup of S_1 and S_2 respectively. Then by Theorem 3.19, we get that $A\times B$ is a single valued neutrosophic subsemigroup of $S_1\times S_2$.

□

Example 4.7. Let $(S,\star)$ be the semigroup defined by the following table:

$\star$	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

and let " $\leq$ " be defined as follows: $\leq = \{(0,0),(0,1),(2,2),(2,1),(0,2)\}$. Then $(S,\star,\leq)$ is an ordered semigroup. Let A be an SVNS on S defined by N_A as follows:

$$N_A(0) = (0.9,0.3,0.1),\ N_A(1) = (0.9,0.2,0.2) \text{ and } N_A(2) = (0.9,0.2,0.2).$$

Then A is a single valued neutrosophic ideal of S since

$T_A(0\star 0) = T_A(0) = 0.9 \geq T_A(0)\vee T_A(0) = 0.9$;

$T_A(0\star 1) = T_A(0) = 0.9 \geq T_A(0)\vee T_A(1) = 0.9$;

$T_A(0\star 2) = T_A(0) = 0.9 \geq T_A(0)\vee T_A(2) = 0.9$;

$T_A(1\star 0) = T_A(0) = 0.9 \geq T_A(1)\vee T_A(0) = 0.9$;

$T_A(1\star 1) = T_A(1) = 0.9 \geq T_A(1)\vee T_A(1) = 0.9$;

$T_A(1\star 2) = T_A(2) = 0.9 \geq T_A(1)\vee T_A(2) = 0.9$;

$T_A(2\star 0) = T_A(0) = 0.9 \geq T_A(2)\vee T_A(0) = 0.9$;

$T_A(2\star 1) = T_A(1) = 0.9 \geq T_A(2)\vee T_A(1) = 0.9$;

$T_A(2\star 2) = T_A(2) = 0.9 \geq T_A(2)\vee T_A(2) = 0.9$;

$I_A(0\star 0) = T_A(0) = 0.3 \geq I_A(0)\vee I_A(0) = 0.3$;

$I_A(0\star 1) = I_A(0) = 0.3 \geq I_A(0)\vee I_A(1) = 0.3$;

$I_A(0\star 2) = I_A(0) = 0.3 \geq I_A(0)\vee T_A(2) = 0.3$;

$I_A(1\star 0) = I_A(0) = 0.3 \geq I_A(1)\vee I_A(0) = 0.3$;

$I_A(1\star 1) = I_A(1) = 0.2 \geq I_A(1)\vee I_A(1) = 0.2$;

$I_A(1 \star 2) = I_A(2) = 0.2 \geq I_A(1) \vee I_A(2) = 0.2;$
$I_A(2 \star 0) = I_A(0) = 0.3 \geq I_A(2) \vee I_A(0) = 0.3;$
$I_A(2 \star 1) = I_A(1) = 0.2 \geq I_A(2) \vee I_A(1) = 0.2;$
$I_A(2 \star 2) = I_A(2) = 0.2 \geq I_A(2) \vee I_A(2) = 0.2;$
$F_A(0 \star 0) = F_A(0) = 0.1 \leq F_A(0) \wedge F_A(0) = 0.1;$
$F_A(0 \star 1) = F_A(0) = 0.1 \leq F_A(0) \wedge F_A(1) = 0.1;$
$F_A(0 \star 2) = F_A(0) = 0.1 \leq F_A(0) \wedge F_A(2) = 0.1;$
$F_A(1 \star 0) = F_A(0) = 0.1 \leq F_A(1) \wedge F_A(0) = 0.1;$
$F_A(1 \star 1) = F_A(1) = 0.2 \leq F_A(1) \wedge F_A(1) = 0.2;$
$F_A(1 \star 2) = F_A(2) = 0.2 \leq F_A(1) \wedge F_A(2) = 0.2;$
$F_A(2 \star 0) = F_A(0) = 0.1 \leq T_A(2) \wedge F_A(0) = 0.2;$
$F_A(2 \star 1) = F_A(1) = 0.2 \leq F_A(2) \wedge F_A(1) = 0.2;$
$F_A(2 \star 2) = F_A(2) = 0.2 \leq F_A(2) \wedge F_A(2) = 0.2.$

Moreover, $0 \leq 1 \leq 2$ implies that $T_A(0) \geq T_A(1) \geq T_A(2)$, $I_A(0) \geq I_A(1) \geq I_A(2)$ and $F_A(0) \leq F_A(1) \leq F_A(2)$.
Therefore, A is a single valued neutrosophic ideal of S.

Example 4.8. Let $(S, \star, \leq)$ be the semigroup defined in Example 4.7 and B a single valued neutrosophic set over S defined by N_B as follows

$$N_B(0) = (0.9, 0.2, 0.1),\ N_B(1) = (0.8, 0.1, 0.3) \text{ and } N_B(2) = (0.7, 0.1, 0.4).$$

Then B is a single valued neutrosophic left ideal of S since
$T_B(0 \star 0) = T_B(0) = 0.9 \geq T_B(0) = 0.9;$
$T_B(0 \star 1) = T_B(0) = 0.9 \geq T_B(1) = 0.8;$
$T_B(0 \star 2) = T_B(0) = 0.9 \geq T_B(2) = 0.7;$
$T_B(1 \star 0) = T_B(0) = 0.9 \geq T_B(0) = 0.9;$
$T_B(1 \star 1) = T_B(1) = 0.8 \geq T_B(1) = 0.8;$
$T_B(1 \star 2) = T_B(2) = 0.7 \geq T_B(2) = 0.7;$
$T_B(2 \star 0) = T_B(0) = 0.9 \geq T_B(0) = 0.9;$
$T_B(2 \star 1) = T_B(1) = 0.8 \geq T_B(1) = 0.8;$
$T_B(2 \star 2) = T_B(2) = 0.7 \geq T_B(2) = 0.7;$
$I_B(0 \star 0) = T_B(0) = 0.2 \geq I_B(0) = 0.2;$
$I_B(0 \star 1) = I_B(0) = 0.2 \geq I_B(1) = 0.1;$
$I_B(0 \star 2) = I_B(0) = 0.2 \geq I_B(2) = 0.1;$
$I_B(1 \star 0) = I_B(0) = 0.2 \geq I_B(0) = 0.2;$
$I_B(1 \star 1) = I_B(1) = 0.1 \geq I_B(1) = 0.1;$
$I_B(1 \star 2) = I_B(2) = 0.1 \geq I_B(2) = 0.1;$

$I_B(2 \star 0) = I_B(0) = 0.2 \geq I_B(0) = 0.2;$
$I_B(2 \star 1) = I_B(1) = 0.1 \geq I_B(1) = 0.1;$
$I_B(2 \star 2) = I_B(2) = 0.1 \geq I_B(2) = 0.1;$
$F_B(0 \star 0) = F_B(0) = 0.1 \leq F_B(0) = 0.1;$
$F_B(0 \star 1) = F_B(0) = 0.1 \leq F_B(1) = 0.3;$
$F_B(0 \star 2) = F_B(0) = 0.1 \leq F_B(2) = 0.4;$
$F_B(1 \star 0) = F_B(0) = 0.1 \leq F_B(0) = 0.1;$
$F_B(1 \star 1) = F_B(1) = 0.3 \leq F_B(1) = 0.3;$
$F_B(1 \star 2) = F_B(2) = 0.4 \leq F_B(2) = 0.4;$
$F_B(2 \star 0) = F_B(0) = 0.1 \leq F_B(0) = 0.1;$
$F_B(2 \star 1) = F_B(1) = 0.3 \leq F_B(1) = 0.3;$
$F_B(2 \star 2) = F_B(2) = 0.4 \leq F_B(2) = 0.4.$

Moreover, $0 \leq 1 \leq 2$ implies that $T_B(0) \geq T_B(1) \geq T_B(2)$, $I_B(0) \geq I_B(1) \geq I_B(2)$ and $F_B(0) \leq F_B(1) \leq F_B(2)$.
Moreover, since B is an SVN left ideal of S, it follows by Theorem 4.2 that B is a single valued neutrosophic bi-ideal of S.

Example 4.9. Let $M_2(\mathbb{N})$ be the set of 2×2 matrices (i.e. $M_2(\mathbb{N}) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \mathbb{N}\}$).
And let A be an SVNS on $M_2(\mathbb{N})$ defined by N_A as follows

$$N_A(X) = \begin{cases} (0.8, 0.4, 0.2) & \text{if } X \in I; \\ (0.6, 0.3, 0.5) & \text{if } X \notin I. \end{cases}$$

where $I = \{\begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}; k \in \mathbb{N}\}$.
Then A is neither a single valued neutrosophic left ideal nor single valued neutrosophic right ideal of $M_2(\mathbb{N})$. Moreover, it is a single valued neutrosophic bi-ideal of $M_2(\mathbb{N})$.

Proof. First we show that A is neither a single valued neutrosophic right ideal nor a single valued neutrosophic left ideal of $M_2(\mathbb{N})$.
Let $X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in I$ and $Y = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \notin I$
So, we have, $X.Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} . \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \notin I.$
Then $N_A(X.Y) = (0.6, 0.3, 0.5)$. But $T_A(X.Y) = 0.6 \ngeq T_A(X) = 0.8$. So, A is not an SVN right ideal of $M_2(\mathbb{N})$.
Also we have, $Y.X = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix} \notin I.$
Then $N_A(Y.X) = (0.6, 0.3, 0.5)$. But $T_A(Y.X) = 0.6 \ngeq T_A(X) = 0.8$. So, A is not an SVN left

ideal of $M_2(\mathbb{N})$.

Next, we show that A is a single valued neutrosophic subsemigroup of $M_2(\mathbb{N})$.
Let $X, Y \in M_2(\mathbb{N})$. We consider the following cases.

- **Case** $X, Y \in I$, then we have $N_A(X) = N_A(Y) = (0.8, 0.4, 0.2)$. Then $X.Y \in I$, then $N_A(X.Y) = (0.8, 0.4, 0.2)$.
  So, easily we can see that, $T_A(X.Y) = 0.8 \geq T_A(X) \wedge T_A(Y) = 0.8$, $I_A(X.Y) = 0.4 \geq I_A(X) \wedge I_A(Y) = 0.4$, $F_A(X.Y) = 0.2 \leq F_A(X) \vee F_A(Y) = 0.2$, and if $X \leq Y$, then $T_A(X) = 0.8 \geq T_A(Y) = 0.8$, $I_A(X) = 0.4 \geq I_A(Y) = 0.4$ and $F_A(X) = 0.2 \leq F_A(Y) = 0.2$.
- **Case** $X, Y \notin I$, then we have $N_A(X) = N_A(Y) = (0.6, 0.3, 0.5)$. So, easily we can see that, $T_A(X.Y) \geq 0.6 = T_A(X) \wedge T_A(Y)$, $I_A(X.Y) \geq 0.3 = I_A(X) \wedge I_A(Y)$, $F_A(X.Y) \leq 0.5 = F_A(X) \vee F_A(Y)$.
- **Case** $X \in I$, $Y \notin I$, then we have $N_A(X) = (0.8, 0.4, 0.2)$ and $N_A(Y) = (0.6, 0.3, 0.5)$. So, easily we can see that, $T_A(X.Y) \geq 0.6 = T_A(X) \wedge T_A(Y)$, $I_A(X.Y) \geq 0.3 = I_A(X) \wedge I_A(Y)$, $F_A(X.Y) \leq 0.5 = F_A(X) \vee F_A(Y)$.

Therefore, A is an SVN subsemigroup of $M_2(\mathbb{N})$.
Simple computations show that A is an SVN bi-ideal of $M_2(\mathbb{N})$. □

5. Conclusion

This paper dealt with single valued neutrosophic sets in ordered semigroups where several concepts about single valued neutrosophic ideals and single valued neutrosophic bi-ideals were defined and studied with several examples. The results in this paper are generalization of fuzzy ideals (bi-ideals) in ordered semigroups.

For future research, it will be interesting to discuss single valued neutrosophic sets in other ordered algebraic structures.

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