Neutrosophic Sets and Systems, Vol. 45, 2021
University of New Mexico

Impact of Complex Interval Neutrosophic Soft Set Theory in Decision making By Using Aggregate Operator

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Abstract: The main purpose of the paper is to introduce the notion of complex interval neutrosophic soft set (CIVNSS) theory, which is the generalization of the soft set, fuzzy soft set, interval-valued fuzzy soft set, interval neutrosophic soft set, etc. to describe the uncertain time-periodic phenomena in the form of an interval. After that, some important properties, and operations on CIVNSSs have been discussed. Also, we study the similarity measures on CIVNSSs. Then, an algorithm has been constructed by using the CIVNSS aggregate operator. Finally, to show the impact of CIVNSS in solving real decision-making problems, an example, which is suitable to the current theory, is chosen, which ensures the effectiveness of the proposed theory in group decision-making problem.

Keywords: Fuzzy set; soft set; complex fuzzy soft set; complex neutrosophic soft set; aggregate operator.

1. Introduction

Complex interval neutrosophic soft set (CIVNSS) is a new kind of soft set where the truth-membership function, indeterminacy-membership function, and the falsity-membership function are replaced by complex-valued functions in the form of an interval. It is the new way to handle parametric data in which time-phase plays an important role to describe the incomplete, indeterminate, inconsistent, or contradictory information systematically. The main feature in CIVNSS is the presence of phase and its membership in the form of an interval. In the group decision-making problem, researchers realized that the time period is an important factor along with the membership value so that decision-makers can make the real decision and it is more reliable and more acceptable than the other existing theories in which there is no scope of considering time-period. So, this new concept provides more scope for the decision-makers to make the real decisions with more feasibility.

In a crisp set, there are only two choices for the belongingness of an object, and, for this, we use two bits i.e., if an object belongs to a set, we assign 1, and for not belongs to we assign 0 for that particular object. There is no other option regarding the belongingness of an object. But due to the uncertainty involved in real life, we cannot restrict ourselves with only two values. This leads to the introduction of the fuzzy set theory, by Zadeh [1] in 1965. By fuzzy set theory, we represent the uncertainty with the help of a membership function. Later on, we realize that the non-membership value is also equally as important as the membership value to design the vagueness. So, Atanassov[2] introduced another mathematical tool known as the intuitionistic fuzzy set. In an intuitionistic fuzzy set, each object has membership value as well as non-membership value and they depend on each other. Researchers use the concept of fuzzy set theory in different application areas and introduce new theories and results. Later on, the fuzzy set has been extended by introducing an interval-valued intuitionistic fuzzy set [3], an interval-valued fuzzy set [4], a Pythagorean fuzzy set and its application [5], multi fuzzy set[6] ,etc.

In the fuzzy set theory, the vague concept is handled by a membership function and its nature is extremely individual. In reality, to measure the uncertainty there exist different possibilities so, setting up a membership function is a difficult task. For example, to represent the concept of 'middle-aged person' we define the membership function via a triangular form and a trapezoidal form of fuzzy membership function by setting up the age limit in different ways. So, there is a problem to choose the best criteria fit for the middle-aged person. So, there is a chance of getting different membership values for a single person. This difficulty of membership function has been removed by introducing a soft set by Russian mathematician Molodtsov[7] in 1999. Soft set theory handled uncertainty or vagueness differently by using the notion of parameterization. In soft set theory, to define an object, no need to introduce a membership function. It can be applied in different fields including game theory, social science, medical science, operation research, decision-making, pattern recognition, algebra, etc. Parameters may not be always crisp, but maybe in fuzzy words so, such types of vagueness demand several kinds of extensions of soft set theory which leads to the introduction of rough soft sets and fuzzy soft sets [8], fuzzy soft set theory, and its application [9], intuitionistic fuzzy parameterized soft set theory and its decision making [10], bipolar soft sets [11], hypersoft set[12] ,etc.

Incomplete information can be handled by the intuitionistic fuzzy set. But it cannot represent the indeterminacy involve in the data. So, there is a demand for another tool that is capable of representing incomplete, indeterminate, and uncertain information in an organized manner. This purpose is solved by introducing the neutrosophic set proposed by Smarandache [13]. The nature of indeterminacy is different as it depends on the problem so, researchers use this concept in various ways to tackle different essence of indeterminacy present in real life. Neutrosophic set is the extension of fuzzy set, intuitionistic fuzzy set, interval-valued fuzzy set, and interval-valued intuitionistic fuzzy set. For scientific implementation, we use a single-valued neutrosophic set introduced by Wang et al. [14]. The neutrosophic set has several extensions and applications among which some significant works are neutrosophic soft set [15], aggregate operators of neutrosophic hypersoft sets[16], rough neutrosophic sets[17], interval neutrosophic sets[18], interval neutrosophic tangent similarity measure based MADM strategy[19], bipolar neutrosophic sets and their application[20] , neutrosophic refined sets in medical diagnosis[21], distance-based similarity measure for refined neutrosophic sets and its application[22], an approach of TOPSIS technique for developing supplier selection under type-2 neutrosophic number[23], an integrated neutrosophic ANP and VIKOR method for supplier selection[24], neutrosophic approach for evaluation of the green supply chain management[25], group decision making model based on neutrosophic sets for heart disease diagnosis[26], bipolar neutrosophic multi-criteria decision making framework for professional selection[27], a novel intelligent medical decision support model based on soft computing IoT[28] etc.

But in all the above discussions, there is one information gap. To make it clear we consider an example. In a medical diagnosis problem, one person may have a variety of symptoms or attributes, or criteria. But in that case, we do not consider the information 'time duration of the symptom' though, it is also necessary information and should be considered together with the information's, 'belongingness level of a symptom' or 'non-belongingness level of a symptom' or 'indeterminacy level of a symptom' for proper diagnosis of a patient. To cover up such problem complex fuzzy set [29], complex fuzzy soft set [30-31], complex intuitionistic fuzzy soft set [32], complex fuzzy logic [33], interval-valued complex fuzzy soft sets [34], complex neutrosophic set [35], complex neutrosophic soft set [36],etc. are introduced.

Ramot et al. introduced complex fuzzy sets (CFSs) to ensure the accurate time-periodic representation of the fuzziness behavior of the attributes to generalize the membership structure. The problems that are intrinsic in CFSs can be handled with the help of complex intuitionistic fuzzy soft sets (CIFSSs) and complex vague soft sets(CVSSs). Selvachandran et al. generalize the CFS model by introducing the interval-valued complex fuzzy soft set(IV-CFSS). By combining the complex fuzzy sets and neutrosophic sets, Ali et al. developed complex neutrosophic sets (CNSs). In 2018, Ali et al. [37] formulate an interval complex neutrosophic set (ICNS) and apply it in decision making. To handle the parametric data, Broumi et al. [36] introduced complex neutrosophic soft sets (CNSSs).

The main objective of this paper is to introduce the notion of complex interval neutrosophic soft sets (CIVNSSs). CIVNSSs are formed by combining the interval-valued fuzzy sets (IV-FSs) and the complex neutrosophic soft sets (CNSSs). CIVNSS is the extension of CNSS. The main objective behind the modeling of CIVNSS is to provide a more general framework for time-periodic phenomena to ensure a more accurate representation of uncertainty of three-dimensional information about the problem parameters and an interval-based truth-membership, falsity-membership, and indeterminacy-membership structure. Moreover, we study some operations and distance measures on CIVNSSs. Finally, we use complex interval neutrosophic set aggregate operators to solve real-life problems in real decision-making.

The main motivation behind the introduction of complex interval neutrosophic soft set has been furnished below point wise:

- A soft set has been introduced to tackle parametric data in which the attributes associated with the parameter attain only the values 0 or 1.
- To overcome the issues which cannot be explained by a soft set, a fuzzy soft set is introduced where an attribute can take any values that belong to the unit closed interval [0, 1].
- The fuzzy soft set has been further extended by introducing an interval-valued fuzzy soft set and intuitionistic fuzzy soft set. In interval-valued fuzzy soft set, a decision-maker may take the membership value as a subset of [0,1] and in the intuitionistic fuzzy soft set, a decision-maker has a scope to assign non-membership value along with the non-membership value with the condition that their sum cannot exceed 1.
- Interval-valued fuzzy soft set and the intuitionistic fuzzy soft set has been extended further by introducing interval-valued intuitionistic fuzzy soft set where the value of an attribute can be represented by a pairwise interval in which the first interval is for membership degree and the second

interval for the non-membership degree with the condition that the sum of their supremum cannot exceed 1.

- A neutrosophic soft set has been introduced in which every attribute has three membership values and each belongs to the interval [0,1].
- Interval neutrosophic soft set has been introduced to extend the notion of the neutrosophic soft set where each membership value is a subset of [0,1].
- Sometimes time-period is an issue while solving decision making problem in real-world and such problem cannot be solved by soft set, fuzzy soft set, intuitionistic fuzzy soft set, interval-valued fuzzy soft set, neutrosophic soft set, interval-neutrosophic soft set ,etc. To eradicate such an issue, a complex interval neutrosophic soft set has been introduced in the present literature. So, complex interval neutrosophic soft set can be viewed as follows:

soft set $\subseteq$ fuzzy soft set $\subseteq$ intuitionistic fuzzy soft set / interval-valued fuzzy soft set $\subseteq$ interval-valued intuitionistic fuzzy soft set $\subseteq$ neutrosophic soft set $\subseteq$ interval neutrosophic soft set $\subseteq$ complex interval neutrosophic soft set.

The paper is organized in the following manner:
In section 2, we give a brief literature review that is relevant to the subsequent sections. In section 3, some operations on CIVNSSs have been proposed. In section 4, similarity measures on CIVNSSs have been discussed. In section 5, aggregation of CIVNSSs has been discussed. In section 6, an algorithm has been constructed by using CIVNSS aggregate operators. In section 7, an application of the proposed algorithm has been suggested. Finally, the paper is concluded in section 8.

2. Literature Review

2.1 Definition (Zadeh, 1965) Let X be a set of the universe. A fuzzy set on X can be defined as a set of ordered pairs of the form given by,

$A^* = \{(x, \mu_A(x)) : x \in X\}$, where μ_A denotes the membership function, and $\mu_A : X \to [0,1]$.

2.2 Definition (Molodtsov, 1999) Let X be the initial universe set and E be the set of parameters and $P(X)$ denotes the power set of X. Then the pair (F, A) is called a soft set over X, where $A \subseteq E$, and $F : A \to P(X)$.

2.3 Definition (Atanassov, 1986) Let X be a fixed set and A be a subset of X. Then an intuitionistic fuzzy set on X can be defined as a set of an ordered triplet of the form given by,

$A^* = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where μ_A and γ_A denote the membership function and non-membership function respectively such that $\mu_A, \gamma_A : X \to [0,1]$,and $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

2.4 Definition (Bustince, 2010) An interval-valued fuzzy set A^* on a universe X is a mapping such that, $A^* : X \to Int[0,1]$, where $Int[0,1]$ denotes the set of all closed subintervals of $[0,1]$ and the membership of an element $x \in X$ is defined as $\mu_X(x) = [\mu_X^l(x), \mu_X^u(x)]$.

2.5 Definition (Cagman et al., 2011) Let U be an initial universe and E be a set of parameters which are in fuzzy words. Then the pair $\left(\tilde{F}, E\right)$ is called a fuzzy soft set (FSS) over U if $\tilde{F} : E \to \tilde{P}(U)$, where $\tilde{P}(U)$ denotes the set of all fuzzy subsets over U .

2.6 Definition (Smarandache, 2005) Let U be an initial universe. A neutrosophic set $\tilde{N}$ is an object having the form $\tilde{N} = \left\{\left\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\right\rangle : x \in U\right\}$, where the functions $T, I, F : U \to]^-0,1^+[$, denote the truth, indeterminacy, and falsity membership functions, respectively and they must satisfy the condition, $^-0 \le T_{\tilde{N}}(x) + I_{\tilde{N}}(x) + F_{\tilde{N}}(x) \le 3^+$. For practical application, it is difficult to apply. So we define its special form, called single-valued neutrosophic set (SVNS).

2.7 Definition (Wang et al., 2010) Let U denotes the space of objects with generic elements $x \in U$. Then, an SVNS on U is denoted by $\hat{N}$ and it is defined as $\hat{N} = \left\{\left\langle x, T_{\hat{N}}(x), I_{\hat{N}}(x), F_{\hat{N}}(x)\right\rangle : x \in U\right\}$, where $T, I, F : U \to [0,1]$.

2.8 Definition (Ramot et al., 2002) A complex fuzzy set(CFS) C^* over a universal set U is defined by taking complex fuzzy-valued membership degree $(\mu_\varsigma(x))$ to each of the elements of U where,

$\mu_\varsigma(x) = a_\varsigma(x) e^{ib_\varsigma(x)}$, $i \equiv \sqrt{-1}$, $\forall x \in U$.

$a_{\varsigma}(x) \in [0,1]$ is called the amplitude part and $b_{\varsigma}(x) \in [0, 2\pi]$ is called the phase part in the complex fuzzy-valued membership degree $\mu_{\varsigma}(x)$ of x.

2.9 Definition (Thirunavukarasu et al., 2017) A complex fuzzy soft set (CFSS) over a universal set U is defined as an ordered pair $\left(\overset{\approx}{F}, E\right)$ where, $\overset{\approx}{F}$ is a mapping defined as, $\overset{\approx}{F} : E \to \overset{\approx}{P}(U)$; $\overset{\approx}{P}(U)$ denotes the set of all complex fuzzy subsets of the set U.

Let, $U = \{x_1, x_2, ..., x_m\}$ be the set of the universe and $E = \{e_1, e_2, ..., e_n\}$ be the set of complex fuzzy-valued parameters then, a complex fuzzy soft set $\left(\overset{\approx}{F}, E\right)$ can be defined as follows

$\left(\overset{\approx}{F}, E\right) = \left\{\left(e_j, \overset{\approx}{F}(e_j)\right) : \forall e_j \in E\right\}$,where $\overset{\approx}{F}(e_j) = \{x_1 / x_{1j}, x_2 / x_{2j}, ..., x_m / x_{sj} : \forall e_j \in E\}$

x_{sj} is a complex fuzzy evaluation of an alternative x_s over a parameter e_j as, $x_{sj} = p_{sj} e^{iu_{sj}}$, where $p_{sj} \in [0,1]$ is the amplitude part and $u_{sj} \in [0, 2\pi]$ is the periodic part; $s = 1, 2, ..., m$ and $j = 1, 2, ..., \text{n}$. So, CFSS is a combination of a soft set (SS) with CFS by taking all the parameters in the complex fuzzy sense in a soft set.

2.10 Definition (Broumi et al., 2017) Let U be an initial universe and E be a set of parameters, $A \subseteq E$, and ψ_A be a complex neutrosophic set over U for all $x \in U$. Then, a complex neutrosophic soft set (CNSS) τ_A over U is defined as a mapping $\tau_A : E \to CN(U)$, where $CN(U)$ denotes the set of complex neutrosophic sets in U and it is defined as

$\tau_A = \left\{\left(x, \psi_A(x)\right) : x \in E, \psi_A(x) \in CN(U)\right\}$,

Where $\psi_A(x) = \left(\alpha_A(x) e^{i\mu_A(x)}, \beta_A(x) e^{i\nu_A(x)}, \delta_A(x) e^{i\omega_A(x)}\right)$, $\alpha_A, \beta_A, \delta_A \in [0,1]$, and $\mu_A, \nu_A, \omega_A \in (0, 2\pi]$.

2.11 Definition Let U be an initial universe and E be a set of parameters, $A \subseteq E$, and $\overset{*}{\psi}_A$ be a complex interval neutrosophic set over U for all $x \in U$. Then, a complex interval neutrosophic soft set (CIVNSS) $\overset{*}{\tau}_A$ over U is defined as a mapping $\overset{*}{\tau} A : E \to CIVN(U)$, where $CIVN(U)$ denotes the set of complex interval neutrosophic sets in U and it is defined as

$$\overset{*}{\tau} A = \left\{ \left(x, \overset{*}{\psi}_A(x) \right) : x \in E, \overset{*}{\psi}_A(x) \in CIVN(U) \right\},$$

Where

$$\overset{*}{\psi}_A(x) = \left(\alpha_A(x) e^{i\mu_A(x)}, \beta_A(x) e^{i\nu_A(x)}, \delta_A(x) e^{i\omega_A(x)} \right) \quad , \quad \alpha_A, \beta_A, \delta_A \subseteq [0,1] \quad ,$$

$\alpha_A(x) = \left[\alpha^l{}_A(x), \alpha^u{}_A(x) \right]$, $\beta_A(x) = \left[\beta^l{}_A(x), \beta^u{}_A(x) \right]$, $\delta_A(x) = \left[\delta^l{}_A(x), \delta^u{}_A(x) \right]$ and

$\mu_A, \nu_A, \omega_A \in (0, 2\pi]$

For more clarity we consider the following example:

2.11.1 Example Let, $U = \{x_1, x_2, x_3, x_4\}$ be the set of developing countries under consideration, E be a set of parameters that signifies a country's time-dependent population indicators, and $A = \{e_1, e_2, e_3\} \subseteq E$, where the parameters stand for e_1 =birth rate, e_2 =death rate ,and e_3 =immigration rate. Then we define the CIVNSs as follows

$$\overset{*}{\psi} A(e_1) = \left\{ \begin{array}{l} \left\langle \dfrac{[0.3, 0.4] e^{i0.6\pi}, [0.5, 0.6] e^{i0.8\pi}, [0.3, 0.5] e^{i0.4\pi}}{x_1} \right\rangle, \left\langle \dfrac{[0.5, 0.8] e^{i0.4\pi}, [0.3, 0.4] e^{i\frac{\pi}{3}}, [0.25, 0.55] e^{i0.2\pi}}{x_2} \right\rangle, \\ \left\langle \dfrac{[0.2, 0.5] e^{i0.4\pi}, [0.1, 0.2] e^{i\frac{2\pi}{3}}, [0.6, 0.7] e^{i\frac{4\pi}{3}}}{x_3} \right\rangle, \left\langle \dfrac{[0.6, 0.7] e^{i\frac{5\pi}{4}}, [0.45, 0.65] e^{i0.4\pi}, [0.7, 0.8] e^{i0.5\pi}}{x_4} \right\rangle \end{array} \right\}$$

$$\overset{*}{\psi}_A(e_2)=\left\{\begin{array}{l}\left\langle\dfrac{[0.25,0.75]e^{i0.1\pi},[0.4,0.6]e^{i0.6\pi},[0.8,0.9]e^{i\frac{\pi}{3}}}{x_1}\right\rangle,\left\langle\dfrac{[0.1,0.3]e^{i0.2\pi},[0.35,0.65]e^{i\frac{2\pi}{3}},[0.6,0.7]e^{i0.5\pi}}{x_2}\right\rangle,\\ \left\langle\dfrac{[0.3,0.5]e^{i\frac{2\pi}{3}},[0.1,0.3]e^{i\frac{4\pi}{3}},[0.6,0.8]e^{i\frac{2\pi}{3}}}{x_3}\right\rangle,\left\langle\dfrac{[0.4,0.6]e^{i\frac{3\pi}{4}},[0.65,0.75]e^{i0.5\pi},[0.25,0.45]e^{i0.45\pi}}{x_4}\right\rangle\end{array}\right\}$$

$$\overset{*}{\psi}_A(e_3)=\left\{\begin{array}{l}\left\langle\dfrac{[0.25,0.35]e^{i0.3\pi},[0.4,0.6]e^{i0.2\pi},[0.3,0.5]e^{i0.1\pi}}{x_1}\right\rangle,\left\langle\dfrac{[0.45,0.65]e^{i\frac{5\pi}{6}},[0.3,0.6]e^{i\frac{3\pi}{4}},[0.65,0.85]e^{i0.4\pi}}{x_2}\right\rangle,\\ \left\langle\dfrac{[0.7,0.8]e^{i0.3\pi},[0.2,0.3]e^{i\frac{\pi}{3}},[0.8,0.9]e^{i\frac{2\pi}{3}}}{x_3}\right\rangle,\left\langle\dfrac{[0.5,0.6]e^{i\frac{3\pi}{4}},[0.35,0.65]e^{i0.3\pi},[0.6,0.7]e^{i0.6\pi}}{x_4}\right\rangle\end{array}\right\}$$

Then the complex interval neutrosophic soft set $\overset{*}{\tau}_A$ can be written as a collection of complex interval neutrosophic sets of the form

$$\overset{*}{\tau}_A=\left\{\overset{*}{\psi}_A(e_1),\overset{*}{\psi}_A(e_2),\overset{*}{\psi}_A(e_3)\right\}$$

2.12 Definition Let us consider the two CIVNSSs over the set of the universe U as follows:

$$\overset{*}{\tau}_A=\left\{\left(x,\overset{*}{\psi}_A(x)\right):x\in E,\overset{*}{\psi}_A(x)\in CIVN(U)\right\},$$

Where

$\overset{*}{\psi}_A(x)=\left(\alpha_A(x)e^{i\mu_A(x)},\beta_A(x)e^{i\nu_A(x)},\delta_A(x)e^{i\omega_A(x)}\right)$, $\alpha_A,\beta_A,\delta_A\subseteq[0,1]$, $\alpha_A(x)=\left[\alpha^l{}_A(x),\alpha^u{}_A(x)\right]$, $\beta_A(x)=\left[\beta^l{}_A(x),\beta^u{}_A(x)\right]$, $\delta_A(x)=\left[\delta^l{}_A(x),\delta^u{}_A(x)\right]$ and $\mu_A,\nu_A,\omega_A\in(0,2\pi]$

and $\overset{*}{\tau}_B=\left\{\left(x,\overset{*}{\psi}_B(x)\right):x\in E,\overset{*}{\psi}_B(x)\in CIVN(U)\right\},$

where

$\overset{*}{\psi}_B(x)=\left(\alpha_B(x)e^{i\mu_B(x)},\beta_B(x)e^{i\nu_B(x)},\delta_B(x)e^{i\omega_B(x)}\right)$, $\alpha_B,\beta_B,\delta_B\subseteq[0,1]$, $\alpha_B(x)=\left[\alpha^l{}_B(x),\alpha^u{}_B(x)\right]$, $\beta_B(x)=\left[\beta^l{}_B(x),\beta^u{}_B(x)\right]$, $\delta_B(x)=\left[\delta^l{}_B(x),\delta^u{}_B(x)\right]$ and $\mu_B,\nu_B,\omega_B\in(0,2\pi]$

Then we consider the following:

(i) $\overset{*}{\tau}_A$ is said to be an empty CIVNSS, denoted by $\overset{*}{\tau}_{A_\varnothing}$, if $\overset{*}{\psi}_A(x)=\varnothing$, for all $x\in U$.

(ii) $\overset{*}{\tau}_A$ is said to be an absolute CIVNSS, denoted by $\overset{*}{\tau}_{A_U}$, if $\overset{*}{\psi}_A(x)=U$, for all $x\in U$.

(iii) $\overset{*}{\tau}_A$ is said to be a normal CIVNSS, denoted by $\overset{*}{\tau}_{A_N}$, if $\alpha_A(x)=[1,1],\beta_A(x)=[1,1],\delta_A(x)=[1,1]$ and $\mu_A,\nu_A,\omega_A=2\pi,$ for all $x\in U$.

(iv) $\overset{*}{\tau}_A$ is said to be a CIVNS-subset of $\overset{*}{\tau}_B$, denoted by $\overset{*}{\tau}_A\subseteq\overset{*}{\tau}_B$, if for all $x\in U$, $\overset{*}{\psi}_A(e)\subseteq\overset{*}{\psi}_B(e)$, that is the following conditions are satisfied:

$\alpha_A(e)\subseteq\alpha_B(e),\ \beta_A(e)\subseteq\beta_B(e),\ \delta_A(x)\subseteq\delta_B(x)$

,and $\mu_A(e)\le\mu_B(e),\nu_A(e)\le\nu_B(e),\ \omega_A(e)\le\omega_B(e)$.

(v) $\overset{*}{\tau}_A$ is said to be equal to $\overset{*}{\tau}_B$, denoted by $\overset{*}{\tau}_A=\overset{*}{\tau}_B$, if for all $x\in U$, $\overset{*}{\psi}_A(e)=\overset{*}{\psi}_B(e)$, that is the following conditions are satisfied:

$\alpha_A(e)=\alpha_B(e),\ \beta_A(e)=\beta_B(e),\ \delta_A(x)=\delta_B(x)$

,and $\mu_A(e)=\mu_B(e),\nu_A(e)=\nu_B(e),\ \omega_A(e)=\omega_B(e)$

3. Operations on Complex Interval Neutrosophic Soft Sets

In this section, we discuss different sorts of set-theoretic operations on CIVNSSs.

Let $\overset{*}{\tau}_A$ and $\overset{*}{\tau}_B$ be two CIVNSSs over the common universal set U. Then we define the following operations:

3.1 Definition Complement of $\overset{*}{\tau}_A$ is denoted by $\left(\overset{*}{\tau}_A\right)^c$ and it is defined as:

$$\left(\overset{*}{\psi}_A(x)\right)^c = \left(\delta_A(x)e^{i(2\pi-\mu_A(x))}, \left(1-\beta_A(x)\right)e^{i(2\pi-\nu_A(x))}, \alpha_A(x)e^{i(2\pi-\omega_A(x))}\right)$$

It is to be noted that $\left(\left(\overset{*}{\psi}_A(x)\right)^c\right)^c = \overset{*}{\psi}_A(x)$

3.2 Definition

Let ,

$$\overset{*}{\tau}_A = \left\{\left(x, \overset{*}{\psi}_A(x)\right) : x \in E, \overset{*}{\psi}_A(x) \in CIVN(U)\right\}, \text{ and}$$

$$\overset{*}{\tau}_B = \left\{\left(x, \overset{*}{\psi}_B(x)\right) : x \in E, \overset{*}{\psi}_B(x) \in CIVN(U)\right\}$$

be two IVNSSs over the common universe U . Then, their union is denoted by $\overset{*}{\tau}_A \tilde{\bigcup} \overset{*}{\tau}_B$ and is defined as:

$$\overset{*}{\tau}_C = \overset{*}{\tau}_A \tilde{\bigcup} \overset{*}{\tau}_B = = \left\{\left(x, \overset{*}{\psi}_A(x) \tilde{\cup} \overset{*}{\psi}_B(x)\right) : x \in U\right\}, \text{ where } C = A \cup B$$

$$\overset{*}{\tau}_C(e) = \begin{cases} \left(x, \overset{*}{\psi}_A(x)\right) & if\ e \in A - B \\ \left(x, \overset{*}{\psi}_B(x)\right) & if\ e \in B - A \\ \left(x, \overset{*}{\psi}_A(x) \tilde{\cup} \overset{*}{\psi}_B(x)\right) & if\ e \in A \cap B \end{cases}$$

Where

$$\overset{*}{\psi}_A(x) \tilde{\cup} \overset{*}{\psi}_B(x) = \left\{ \begin{array}{l} \left(\left[\alpha_A^l(x) \vee \alpha_B^l(x), \alpha_A^u(x) \vee \alpha_B^u(x)\right]\right) e^{i(\mu_A(x) \cup \mu_B(x))} \\ \left(\left[\beta_A^l(x) \vee \beta_B^l(x), \beta_A^u(x) \vee \beta_B^u(x)\right]\right) e^{i(\nu_A(x) \cup \nu_B(x))} \\ \left(\left[\delta_A^l(x) \wedge \delta_B^l(x), \delta_A^u(x) \wedge \delta_B^u(x)\right]\right) e^{i(\omega_A(x) \cup \omega_B(x))} \end{array} \right\}$$

3.3Definition

Let ,

$\overset{*}{\tau}_A = \left\{ \left(x, \overset{*}{\psi}_A(x) \right) : x \in E, \overset{*}{\psi}_A(x) \in CIVN(U) \right\}$ and

$\overset{*}{\tau}_B = \left\{ \left(x, \overset{*}{\psi}_B(x) \right) : x \in E, \overset{*}{\psi}_B(x) \in CIVN(U) \right\}$

be two IVNSSs over the common universe U. Then, their intersection is denoted by $\overset{*}{\tau}_A \tilde{\bigcap} \overset{*}{\tau}_B$ and is defined as:

$$\overset{*}{\tau}_C = \overset{*}{\tau}_A \tilde{\bigcap} \overset{*}{\tau}_B = = \left\{ \left(x, \overset{*}{\psi}_A(x) \tilde{\bigcap} \overset{*}{\psi}_B(x) \right) : x \in U \right\}, \text{ where } C = A \cap B$$

$$\overset{*}{\tau}_C(e) = \begin{cases} \left(x, \overset{*}{\psi}_A(x) \right) & if\ e \in A - B \\ \left(x, \overset{*}{\psi}_B(x) \right) & if\ e \in B - A \\ \left(x, \overset{*}{\psi}_A(x) \tilde{\cap} \overset{*}{\psi}_B(x) \right) & if\ e \in A \cap B \end{cases}$$

Where

$$\overset{*}{\psi}_A(x) \tilde{\cap} \overset{*}{\psi}_B(x) = \left\{ \begin{array}{l} \left(\left[\alpha_A^l(x) \wedge \alpha_B^l(x), \alpha_A^u(x) \wedge \alpha_B^u(x) \right] \right) e^{i(\mu_A(x) \cap \mu_B(x))} \\ \left(\left[\beta_A^l(x) \wedge \beta_B^l(x), \beta_A^u(x) \wedge \beta_B^u(x) \right] \right) e^{i(\nu_A(x) \cap \nu_B(x))} \\ \left(\left[\delta_A^l(x) \vee \delta_B^l(x), \delta_A^u(x) \vee \delta_B^u(x) \right] \right) e^{i(\omega_A(x) \cap \omega_B(x))} \end{array} \right\}$$

4. Similarity measure of complex interval neutrosophic soft sets

Nowadays the concept of similarity measure has been used in almost all scientific disciplines. The Similarity measure of two objects determines the degree of closeness or the degree of sameness between them. In many different fields like pattern recognition, decision-making, disease diagnosis, etc. it has been used quite successfully.

Now considering $U=\{x_1,x_2,x_3,.....,x_m\}$ be the set of the universe and $E=\{e_1,e_2,e_3,.....,e_n\}$ be the set of parameters where they are used in complex interval neutrosophic sense. Then, a mapping

$\xi:\chi_{CIVNSS}(U)\times\chi_{CIVNSS}(U)\to([0,1],[0,1],[0,1])$, where $\chi_{CIVNSS}(U)$ denotes the set of all complex interval neutrosophic soft set over the universe U, is said to be a similarity measure if it satisfies the following conditions:

For all $\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{Q}},E\right),\left(\tilde{\tilde{R}},E\right)\in\chi_{CIVNSS}(U)$

(i) $\overset{*}{S}\left(\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{Q}},E\right)\right)\in([0,1],[0,1],[0,1])$

(ii) $\overset{*}{S}\left(\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{Q}},E\right)\right)=\overset{*}{S}\left(\left(\tilde{\tilde{Q}},E\right),\left(\tilde{\tilde{P}},E\right)\right)$

(iii) $\overset{*}{S}\left(\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{Q}},E\right)\right)=(1,1,1)\Leftrightarrow\left(\tilde{\tilde{P}},E\right)=\left(\tilde{\tilde{Q}},E\right)$

(iv) If $\left(\tilde{\tilde{P}},E\right)\subseteq\left(\tilde{\tilde{Q}},E\right)\subseteq\left(\tilde{\tilde{R}},E\right)$ then, $\overset{*}{S}\left(\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{Q}},E\right)\right)\geq\overset{*}{S}\left(\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{R}},E\right)\right)$

, and $\overset{*}{S}\left(\left(\tilde{\tilde{Q}},E\right),\left(\tilde{\tilde{R}},E\right)\right)\geq\overset{*}{S}\left(\left(\tilde{\tilde{P}},E\right),\left(\tilde{\tilde{R}},E\right)\right)$

4.1 Ratio Similarity measure of two complex interval neutrosophic soft sets

Let $\left(\tilde{\tilde{F}},E\right)$ and $\left(\tilde{\tilde{G}},E\right)$ be two complex interval neutrosophic soft sets over U as follows:

$$\left(\tilde{\tilde{F}},E\right)=\left\{\left(e_j,\tilde{\tilde{F}}(e_j)\right):\forall e_j\in E\right\}=\left\{e_j,\left(x_s/\left(p^F_{s_j}e^{i\mu^F_{s_j}},q^F_{s_j}e^{i\nu^F_{s_j}},r^F_{s_j}e^{i\delta^F_{s_j}}\right)\right):\forall e_j\in E,x_s\in U\right\}\text{ and}$$

$$\left(\tilde{\tilde{G}},E\right)=\left\{\left(e_j,\tilde{\tilde{G}}(e_j)\right):\forall e_j\in E\right\}=\left\{e_j,\left(x_s/\left(p^G_{s_j}e^{i\mu^G_{s_j}},q^G_{s_j}e^{i\nu^G_{s_j}},r^G_{s_j}e^{i\delta^G_{s_j}}\right)\right):\forall e_j\in E,x_s\in U\right\}$$

Where $p_{s_j}^{F}, q_{s_j}^{F}, r_{s_j}^{F} \subseteq [0,1]$ are the amplitude parts of the truth-membership, indeterminacy-membership ,and falsity-membership values respectively and $\mu_{s_j}^{F}, \nu_{s_j}^{F}, \delta_{s_j}^{F} \in [0, 2\pi]$ are the respective phase parts of the evaluation of an alternative x_s concerning for to the parameter e_j over the CIVNSS $\left(\overset{\approx}{F}, E\right)$. Similarly, we can write for $\left(\overset{\approx}{G}, E\right)$.

Since in every evaluation there exist two decision information to each membership value, one is amplitude part another one is phase part. So, to measure the similarity degree between the two CIVNSSs $\left(\overset{\approx}{F}, E\right)$ and $\left(\overset{\approx}{G}, E\right)$, we have measured one similarity for the amplitude part and phase part individually and then added them for deriving the total similarity.

4.2 Definition

The ratio similarity between $\left(\overset{\approx}{F}, E\right)$ and $\left(\overset{\approx}{G}, E\right)$ is denoted by $\overset{\approx}{S}_R\left(\left(\overset{\approx}{F}, E\right), \left(\overset{\approx}{G}, E\right)\right)$ and is defined by the following equation:

$$\overset{\approx}{S}_R\left(\left(\overset{\approx}{F}, E\right), \left(\overset{\approx}{G}, E\right)\right) = \frac{\sum_{j=1}^{n} w_j \overset{\approx}{S}_R\left(\overset{\approx}{F}\left(e_j\right), \overset{\approx}{G}\left(e_j\right)\right)}{\sum_{j=1}^{n} w_j}$$

Where

$$\tilde{\tilde{S}}_R\left(\tilde{\tilde{F}}\left(e_j\right),\tilde{\tilde{G}}\left(e_j\right)\right) =$$

$$\frac{\sum_{j=1}^{n}\max\left(\min\left(p_{s_j}^{l^F},p_{s_j}^{l^G}\right),\min\left(p_{s_j}^{u^F},p_{s_j}^{u^G}\right)\right)+\max\left(\min\left(q_{s_j}^{l^F},q_{s_j}^{l^G}\right),\min\left(q_{s_j}^{u^F},q_{s_j}^{u^G}\right)\right)+\min\left(\max\left(r_{s_j}^{l^F},r_{s_j}^{l^G}\right),\max\left(r_{s_j}^{u^F},r_{s_j}^{u^G}\right)\right)}{\sum_{j=1}^{n}\left(\min\left(p_{s_j}^{l^F},p_{s_j}^{l^G}\right)+\min\left(p_{s_j}^{u^F},p_{s_j}^{u^G}\right)+\min\left(q_{s_j}^{l^F},q_{s_j}^{l^G}\right)+\min\left(q_{s_j}^{u^F},q_{s_j}^{u^G}\right)+\max\left(r_{s_j}^{l^F},r_{s_j}^{l^G}\right)+\max\left(r_{s_j}^{u^F},r_{s_j}^{u^G}\right)\right)}$$

,and $w=\{w_1,w_2,........,w_n\}$ are the weights of the parameters and each $w_j\in[0,1]$.

If , $\sum_{j=1}^{n}w_j=1$, then the above equation takes the form as,

$$\tilde{\tilde{S}}_R\left(\left(\tilde{\tilde{F}},E\right),\left(\tilde{\tilde{G}},E\right)\right)=\sum_{j=1}^{n}w_j\,\tilde{\tilde{S}}_R\left(\tilde{\tilde{F}}\left(e_j\right),\tilde{\tilde{G}}\left(e_j\right)\right)$$

5. Aggregation of complex interval neutrosophic soft sets

Let, $U=\{u_1,u_2,.....,u_m\}$ be the set of alternatives and $E=\{e_1,e_2,.....,e_n\}$ be the set of parameters which are in complex interval neutrosophic sense. Consider K-CIVNSSs $\left(\tilde{\tilde{F}}^1,E\right),\left(\tilde{\tilde{F}}^2,E\right),......,\left(\tilde{\tilde{F}}^k,E\right)$ from $\tau_{CIVN}(U)$ (set of all CIVNSSs over U. Then the mapping

$\tilde{\tilde{B}}:\tau_{CIVN}(U)\times\tau_{CIVN}(U)\times..........\times\tau_{CIVN}(U)\rightarrow\tau_{CIVN}(U)$ satisfies the following properties:

(a) $\tilde{\tilde{B}}\left(\left(\tilde{\tilde{F}}^1,E\right)_{\underset{1}{\prec}},\left(\tilde{\tilde{F}}^2,E\right)_{\underset{1}{\prec}},........,\left(\tilde{\tilde{F}}^k,E\right)_{\underset{1}{\prec}}\right)=\left(\tilde{\tilde{F}},E\right)_{\underset{1}{\prec}}$, where $\left(\tilde{\tilde{F}},E\right)_{\underset{1}{\prec}}$ is the absolute complex interval neutrosophic soft set over U.

(b) $\tilde{\tilde{B}}\left(\left(\tilde{\tilde{F}}^1,E\right)_{\underset{0}{\prec}},\left(\tilde{\tilde{F}}^2,E\right)_{\underset{0}{\prec}},........,\left(\tilde{\tilde{F}}^k,E\right)_{\underset{0}{\prec}}\right)=\left(\tilde{\tilde{F}},E\right)_{\underset{0}{\prec}}$, where $\left(\tilde{\tilde{F}},E\right)_{\underset{0}{\prec}}$ is the null complex interval neutrosophic soft set over U.

(c) If, for all $i = 1, 2, 3, ..., k$, $\left(\overset{\approx}{F^i}, E\right) \subseteq \left(\overset{\approx}{G^i}, E\right)$ then,

$\tilde{\tilde{B}}\left(\left(\overset{\approx}{F^1}, E\right), \left(\overset{\approx}{F^2}, E\right), ..., \left(\overset{\approx}{F^k}, E\right)\right) \leq \tilde{\tilde{B}}\left(\left(\overset{\approx}{G^1}, E\right), \left(\overset{\approx}{G^2}, E\right), ..., \left(\overset{\approx}{G^k}, E\right)\right)$, where

$\left(\overset{\approx}{G^1}, E\right), \left(\overset{\approx}{G^2}, E\right), ..., \left(\overset{\approx}{G^k}, E\right)$ be another k-CIVNSSs over U .

(d) The aggregate operator satisfies the inequality

$\left(\overset{\approx}{F^-}, E\right) \leq \tilde{\tilde{B}}\left(\left(\overset{\approx}{F^1}, E\right), \left(\overset{\approx}{F^2}, E\right), ..., \left(\overset{\approx}{F^k}, E\right)\right) \leq \left(\overset{\approx}{F^+}, E\right)$, where $\left(\overset{\approx}{F^-}, E\right)$ is the worst(min-min-max-valued for amplitude part and min-valued for phase part) CIVNSS and $\left(\overset{\approx}{F^+}, E\right)$ is the best(max-max-min-valued for amplitude part and max-valued for phase part)CIVNSS over k-CIVNSSs.

Now we consider the following tables for better understanding:

Table1 for absolute CIVNSS $\left(\overset{\approx}{F}, E\right)_{1^{\prec}}$

	e_1	e_2	……………..	e_n
x_1	$([1,1][1,1][1,1])e^{i2\pi}$	$([1,1][1,1][1,1])e^{i2\pi}$	………………	$([1,1][1,1][1,1])e^{i2\pi}$
x_2	$([1,1][1,1][1,1])e^{i2\pi}$	$([1,1][1,1][1,1])e^{i2\pi}$	……………	$([1,1][1,1][1,1])e^{i2\pi}$
………….	………….	…………………	……………..	
x_m	$([1,1][1,1][1,1])e^{i2\pi}$	$([1,1][1,1][1,1])e^{i2\pi}$	…………………	$([1,1][1,1][1,1])e^{i2\pi}$

Table1. Absolute CIVNSS $\left(\overset{\approx}{F}, E\right)_{1^{\prec}}$

Table2 for null CIVNSS $\left(\overset{\approx}{F},E\right)_{0^{\prec}}$

	e_1	e_2	…………….	e_n
x_1	$([0,0][0,0][0,0])e^{i0\pi}$	$([0,0][0,0][0,0])e^{i0\pi}$	………………	$([0,0][0,0][0,0])e^{i0\pi}$
x_2	$([0,0][0,0][0,0])e^{i0\pi}$	$([0,0][0,0][0,0])e^{i0\pi}$	……………	$([0,0][0,0][0,0])e^{i0\pi}$
…………….	………….	…………………	…………….	
x_m	$([0,0][0,0][0,0])e^{i0\pi}$	$([0,0][0,0][0,0])e^{i0\pi}$	……………… …	$([0,0][0,0][0,0])e^{i0\pi}$

Table2. Null CIVNSS $\left(\overset{\approx}{F},E\right)_{1^{\prec}}$

Table 3 for K-CIVNSSs

	e_1	e_2	…… …	e_n
x_1	$\left(\begin{array}{l}\left[p_{11}^l,p_{11}^u\right]e^{i\alpha_{11}^1},\left[q_{11}^l,q_{11}^u\right]e^{i\beta_{11}^1},\\ \left[r_{11}^l,r_{11}^u\right]e^{i\delta_{11}^1}\end{array}\right)$	$\left(\begin{array}{l}\left[p_{12}^l,p_{12}^u\right]e^{i\alpha_{12}^1},\left[q_{12}^l,q_{12}^u\right]e^{i\beta_{12}^1},\\ \left[r_{12}^l,r_{12}^u\right]e^{i\delta_{12}^1}\end{array}\right)$	…… …	$\left(\begin{array}{l}\left[p_{1n}^l,p_{1n}^u\right]e^{i\alpha_{1n}^1},\left[q_{1n}^l,q_{1n}^u\right]e^{i\beta_{1n}^1},\\ \left[r_{1n}^l,r_{1n}^u\right]e^{i\delta_{1n}^1}\end{array}\right)$
x_2	$\left(\begin{array}{l}\left[p_{21}^l,p_{21}^u\right]e^{i\alpha_{21}^2},\left[q_{21}^l,q_{21}^u\right]e^{i\beta_{21}^2},\\ \left[r_{21}^l,r_{21}^u\right]e^{i\delta_{21}^2}\end{array}\right)$	$\left(\begin{array}{l}\left[p_{22}^l,p_{22}^u\right]e^{i\alpha_{22}^2},\left[q_{22}^l,q_{22}^u\right]e^{i\beta_{22}^2},\\ \left[r_{22}^l,r_{22}^u\right]e^{i\delta_{22}^2}\end{array}\right)$	…… …	$\left(\begin{array}{l}\left[p_{2n}^l,p_{2n}^u\right]e^{i\alpha_{2n}^2},\left[q_{2n}^l,q_{2n}^u\right]e^{i\beta_{2n}^2},\\ \left[r_{2n}^l,r_{2n}^u\right]e^{i\delta_{2n}^2}\end{array}\right)$
…….	………….	…………………	…… …	……………

x_m	$\left(\begin{array}{l}\left[p_{m1}^{l}, p_{m1}^{u}\right]e^{i\alpha_{m1}^{k}}, \left[q_{m1}^{l}, q_{m1}^{u}\right]e^{i\beta_{m1}^{k}}, \\ \left[r_{m1}^{l}, r_{m1}^{u}\right]e^{i\delta_{m1}^{k}}\end{array}\right)$	$\left(\begin{array}{l}\left[p_{m2}^{l}, p_{m2}^{u}\right]e^{i\alpha_{m2}^{k}}, \left[q_{m2}^{l}, q_{m2}^{u}\right]e^{i\beta_{m2}^{k}}, \\ \left[r_{m2}^{l}, r_{m2}^{u}\right]e^{i\delta_{m2}^{k}}\end{array}\right)$	….	$\left(\begin{array}{l}\left[p_{mn}^{l}, p_{mn}^{u}\right]e^{i\alpha_{mn}^{k}}, \left[q_{mn}^{l}, q_{mn}^{u}\right]e^{i\beta_{mn}^{k}}, \\ \left[r_{mn}^{l}, r_{mn}^{u}\right]e^{i\delta_{mn}^{k}}\end{array}\right)$

Table3. K- CIVNSSs

Table4 for the best CIVNSS $\left(\overset{\approx}{F^{+}}, E\right)$

	e_1	...	e_n
x_1	$\left\langle\begin{array}{l}\left[\max(p_{11}^{l1}, p_{11}^{l2}, ..., p_{11}^{lk}), \max(p_{11}^{u1}, p_{11}^{u2}, ..., p_{11}^{uk})\right]e^{i\max(\alpha_{11}^{1}, \alpha_{11}^{2}, ..., \alpha_{11}^{k})}, \\ \left[\max(q_{11}^{l1}, q_{11}^{l2}, ..., q_{11}^{lk}), \max(q_{11}^{u1}, q_{11}^{u2}, ..., q_{11}^{uk})\right]e^{i\max(\beta_{11}^{1}, \beta_{11}^{2}, ..., \beta_{11}^{k})}, \\ \left[\min(r_{11}^{l1}, r_{11}^{l2}, ..., r_{11}^{lk}), \min(r_{11}^{u1}, r_{11}^{u2}, ..., r_{11}^{uk})\right]e^{i\max(\delta_{11}^{1}, \delta_{11}^{2}, ..., \delta_{11}^{k})}\end{array}\right\rangle$	….	$\left\langle\begin{array}{l}\left[\max(p_{1n}^{l1}, p_{1n}^{l2}, ..., p_{1n}^{lk}), \max(p_{1n}^{u1}, p_{1n}^{u2}, ..., p_{1n}^{uk})\right]e^{i\max(\alpha_{1n}^{1}, \alpha_{1n}^{2}, ..., \alpha_{1n}^{k})}, \\ \left[\max(q_{1n}^{l1}, q_{1n}^{l2}, ..., q_{1n}^{lk}), \max(q_{1n}^{u1}, q_{1n}^{u2}, ..., q_{1n}^{uk})\right]e^{i\max(\beta_{1n}^{1}, \beta_{1n}^{2}, ..., \beta_{1n}^{k})}, \\ \left[\min(r_{1n}^{l1}, r_{1n}^{l2}, ..., r_{1n}^{lk}), \min(r_{1n}^{u1}, r_{1n}^{u2}, ..., r_{1n}^{uk})\right]e^{i\max(\delta_{1n}^{1}, \delta_{1n}^{2}, ..., \delta_{1n}^{k})}\end{array}\right\rangle$
x_2	$\left\langle\begin{array}{l}\left[\max(p_{21}^{l1}, p_{21}^{l2}, ..., p_{21}^{lk}), \max(p_{21}^{u1}, p_{21}^{u2}, ..., p_{21}^{uk})\right]e^{i\max(\alpha_{21}^{1}, \alpha_{21}^{2}, ..., \alpha_{21}^{k})}, \\ \left[\max(q_{21}^{l1}, q_{21}^{l2}, ..., q_{21}^{lk}), \max(q_{21}^{u1}, q_{21}^{u2}, ..., q_{21}^{uk})\right]e^{i\max(\beta_{21}^{1}, \beta_{21}^{2}, ..., \beta_{21}^{k})}, \\ \left[\min(r_{21}^{l1}, r_{21}^{l2}, ..., r_{21}^{lk}), \min(r_{21}^{u1}, r_{21}^{u2}, ..., r_{21}^{uk})\right]e^{i\max(\delta_{21}^{1}, \delta_{21}^{2}, ..., \delta_{21}^{k})}\end{array}\right\rangle$	….	$\left\langle\begin{array}{l}\left[\max(p_{2n}^{l1}, p_{2n}^{l2}, ..., p_{2n}^{lk}), \max(p_{2n}^{u1}, p_{2n}^{u2}, ..., p_{2n}^{uk})\right]e^{i\max(\alpha_{2n}^{1}, \alpha_{2n}^{2}, ..., \alpha_{2n}^{k})}, \\ \left[\max(q_{2n}^{l1}, q_{2n}^{l2}, ..., q_{2n}^{lk}), \max(q_{2n}^{u1}, q_{2n}^{u2}, ..., q_{2n}^{uk})\right]e^{i\max(\beta_{2n}^{1}, \beta_{2n}^{2}, ..., \beta_{2n}^{k})}, \\ \left[\min(r_{2n}^{l1}, r_{2n}^{l2}, ..., r_{2n}^{lk}), \min(r_{2n}^{u1}, r_{2n}^{u2}, ..., r_{2n}^{uk})\right]e^{i\max(\delta_{2n}^{1}, \delta_{2n}^{2}, ..., \delta_{2n}^{k})}\end{array}\right\rangle$
...		...	
x_m	$\left\langle\begin{array}{l}\left[\max(p_{m1}^{l1}, p_{m1}^{l2}, ..., p_{m1}^{lk}), \max(p_{m1}^{u1}, p_{m1}^{u2}, ..., p_{m1}^{uk})\right]e^{i\max(\alpha_{m1}^{1}, \alpha_{m1}^{2}, ..., \alpha_{m1}^{k})}, \\ \left[\max(q_{m1}^{l1}, q_{m1}^{l2}, ..., q_{m1}^{lk}), \max(q_{m1}^{u1}, q_{m1}^{u2}, ..., q_{m1}^{uk})\right]e^{i\max(\beta_{m1}^{1}, \beta_{m1}^{2}, ..., \beta_{m1}^{k})}, \\ \left[\min(r_{m1}^{l1}, r_{m1}^{l2}, ..., r_{m1}^{lk}), \min(r_{m1}^{u1}, r_{m1}^{u2}, ..., r_{m1}^{uk})\right]e^{i\max(\delta_{m1}^{1}, \delta_{m1}^{2}, ..., \delta_{m1}^{k})}\end{array}\right\rangle$	…. .	$\left\langle\begin{array}{l}\left[\max(p_{mn}^{l1}, p_{mn}^{l2}, ..., p_{mn}^{lk}), \max(p_{mn}^{u1}, p_{mn}^{u2}, ..., p_{mn}^{uk})\right]e^{i\max(\alpha_{mn}^{1}, \alpha_{mn}^{2}, ..., \alpha_{mn}^{k})}, \\ \left[\max(q_{mn}^{l1}, q_{mn}^{l2}, ..., q_{mn}^{lk}), \max(q_{mn}^{u1}, q_{mn}^{u2}, ..., q_{mn}^{uk})\right]e^{i\max(\beta_{mn}^{1}, \beta_{mn}^{2}, ..., \beta_{mn}^{k})}, \\ \left[\min(r_{mn}^{l1}, r_{mn}^{l2}, ..., r_{mn}^{lk}), \min(r_{mn}^{u1}, r_{mn}^{u2}, ..., r_{mn}^{uk})\right]e^{i\max(\delta_{mn}^{1}, \delta_{mn}^{2}, ..., \delta_{mn}^{k})}\end{array}\right\rangle$

Table4. Best CIVNSS $\left(\overset{\approx}{F^{+}}, E\right)$

Table5 for the worst CIVNSS $\left(\overset{\approx}{F^{-}}, E\right)$

	e_1	...	e_n
x_1	$\left\langle \begin{matrix} \left[\min(p_{11}^{l1}, p_{11}^{l2}, ..., p_{11}^{lk}), \min(p_{11}^{u1}, p_{11}^{u2}, ..., p_{11}^{uk})\right] e^{i\min(\alpha_{11}^{1}, \alpha_{11}^{2}, ..., \alpha_{11}^{k})}, \\ \left[\min(q_{11}^{l1}, q_{11}^{l2}, ..., q_{11}^{lk}), \min(q_{11}^{u1}, q_{11}^{u2}, ..., q_{11}^{uk})\right] e^{i\min(\beta_{11}^{1}, \beta_{11}^{2}, ..., \beta_{11}^{k})}, \\ \left[\max(r_{11}^{l1}, r_{11}^{l2}, ..., r_{11}^{lk}), \max(r_{11}^{u1}, r_{11}^{u2}, ..., r_{11}^{uk})\right] e^{i\min(\delta_{11}^{1}, \delta_{11}^{2}, ..., \delta_{11}^{k})} \end{matrix} \right\rangle$	....	$\left\langle \begin{matrix} \left[\min(p_{1n}^{l1}, p_{1n}^{l2}, ..., p_{1n}^{lk}), \min(p_{1n}^{u1}, p_{1n}^{u2}, ..., p_{1n}^{uk})\right] e^{i\min(\alpha_{1n}^{1}, \alpha_{1n}^{2}, ..., \alpha_{1n}^{k})}, \\ \left[\min(q_{1n}^{l1}, q_{1n}^{l2}, ..., q_{1n}^{lk}), \min(q_{1n}^{u1}, q_{1n}^{u2}, ..., q_{1n}^{uk})\right] e^{i\min(\beta_{1n}^{1}, \beta_{1n}^{2}, ..., \beta_{1n}^{k})}, \\ \left[\max(r_{1n}^{l1}, r_{1n}^{l2}, ..., r_{1n}^{lk}), \max(r_{1n}^{u1}, r_{1n}^{u2}, ..., r_{1n}^{uk})\right] e^{i\min(\delta_{1n}^{1}, \delta_{1n}^{2}, ..., \delta_{1n}^{k})} \end{matrix} \right\rangle$
x_2	$\left\langle \begin{matrix} \left[\min(p_{21}^{l1}, p_{21}^{l2}, ..., p_{21}^{lk}), \min(p_{21}^{u1}, p_{21}^{u2}, ..., p_{21}^{uk})\right] e^{i\min(\alpha_{21}^{1}, \alpha_{21}^{2}, ..., \alpha_{21}^{k})}, \\ \left[\min(q_{21}^{l1}, q_{21}^{l2}, ..., q_{21}^{lk}), \min(q_{21}^{u1}, q_{21}^{u2}, ..., q_{21}^{uk})\right] e^{i\min(\beta_{21}^{1}, \beta_{21}^{2}, ..., \beta_{21}^{k})}, \\ \left[\max(r_{21}^{l1}, r_{21}^{l2}, ..., r_{21}^{lk}), \max(r_{21}^{u1}, r_{21}^{u2}, ..., r_{21}^{uk})\right] e^{i\min(\delta_{21}^{1}, \delta_{21}^{2}, ..., \delta_{21}^{k})} \end{matrix} \right\rangle$	....	$\left\langle \begin{matrix} \left[\min(p_{2n}^{l1}, p_{2n}^{l2}, ..., p_{2n}^{lk}), \min(p_{2n}^{u1}, p_{2n}^{u2}, ..., p_{2n}^{uk})\right] e^{i\min(\alpha_{2n}^{1}, \alpha_{2n}^{2}, ..., \alpha_{2n}^{k})}, \\ \left[\min(q_{2n}^{l1}, q_{2n}^{l2}, ..., q_{2n}^{lk}), \min(q_{2n}^{u1}, q_{2n}^{u2}, ..., q_{2n}^{uk})\right] e^{i\min(\beta_{2n}^{1}, \beta_{2n}^{2}, ..., \beta_{2n}^{k})}, \\ \left[\max(r_{2n}^{l1}, r_{2n}^{l2}, ..., r_{2n}^{lk}), \max(r_{2n}^{u1}, r_{2n}^{u2}, ..., r_{2n}^{uk})\right] e^{i\min(\delta_{2n}^{1}, \delta_{2n}^{2}, ..., \delta_{2n}^{k})} \end{matrix} \right\rangle$
...		...	
x_m	$\left\langle \begin{matrix} \left[\min(p_{m1}^{l1}, p_{m1}^{l2}, ..., p_{m1}^{lk}), \min(p_{m1}^{u1}, p_{m1}^{u2}, ..., p_{m1}^{uk})\right] e^{i\min(\alpha_{m1}^{1}, \alpha_{m1}^{2}, ..., \alpha_{m1}^{k})}, \\ \left[\min(q_{m1}^{l1}, q_{m1}^{l2}, ..., q_{m1}^{lk}), \min(q_{m1}^{u1}, q_{m1}^{u2}, ..., q_{m1}^{uk})\right] e^{i\min(\beta_{m1}^{1}, \beta_{m1}^{2}, ..., \beta_{m1}^{k})}, \\ \left[\max(r_{m1}^{l1}, r_{m1}^{l2}, ..., r_{m1}^{lk}), \max(r_{m1}^{u1}, r_{m1}^{u2}, ..., r_{m1}^{uk})\right] e^{i\min(\delta_{m1}^{1}, \delta_{m1}^{2}, ..., \delta_{m1}^{k})} \end{matrix} \right\rangle$	.....	$\left\langle \begin{matrix} \left[\min(p_{mn}^{l1}, p_{mn}^{l2}, ..., p_{mn}^{lk}), \min(p_{mn}^{u1}, p_{mn}^{u2}, ..., p_{mn}^{uk})\right] e^{i\min(\alpha_{mn}^{1}, \alpha_{mn}^{2}, ..., \alpha_{mn}^{k})}, \\ \left[\min(q_{mn}^{l1}, q_{mn}^{l2}, ..., q_{mn}^{lk}), \min(q_{mn}^{u1}, q_{mn}^{u2}, ..., q_{mn}^{uk})\right] e^{i\min(\beta_{mn}^{1}, \beta_{mn}^{2}, ..., \beta_{mn}^{k})}, \\ \left[\max(r_{mn}^{l1}, r_{mn}^{l2}, ..., r_{mn}^{lk}), \max(r_{mn}^{u1}, r_{mn}^{u2}, ..., r_{mn}^{uk})\right] e^{i\min(\delta_{mn}^{1}, \delta_{mn}^{2}, ..., \delta_{mn}^{k})} \end{matrix} \right\rangle$

Table5. Worst CIVNSS $\left(\overset{\approx}{F}^{-}, E\right)$

5.1 Complex interval neutrosophic soft geometric mean aggregation operator

Aggregation of some CIVNSSs produces a CIVNSS. We have introduced the geometric mean aggregation of CIVNSSs.

Let $\left(\overset{\approx}{F}^{1}, E\right), \left(\overset{\approx}{F}^{2}, E\right), ..., \left(\overset{\approx}{F}^{k}, E\right)$ be k-CIVNSSs over a universe U and $w = \{w_1, w_2, ..., w_k\}$ be k real numbers such that $w_i \in [0,1]$, and $\sum_{i=1}^{k} w_i = 1$. Then, the complex interval neutrosophic soft geometric aggregation of k-CIVNSSs is denoted by,

$\overset{\approx}{\tilde{B}}_{GM}\left(w_1, w_2, ..., w_k\right)\left(\left(\overset{\approx}{F}^{1}, E\right), \left(\overset{\approx}{F}^{2}, E\right), ..., \left(\overset{\approx}{F}^{k}, E\right)\right)$, and is defined as follows:

$$\tilde{\tilde{B}}_{GM}\left(w_1, w_2, \ldots, w_k\right)\left(\left(\tilde{\tilde{F}}^1, E\right), \left(\tilde{\tilde{F}}^2, E\right), \ldots\ldots, \left(\tilde{\tilde{F}}^k, E\right)\right) = \left(\tilde{\tilde{F}}, E\right)$$

$=$

$$\left\{\left(e_j, \left(\frac{x_s}{\left(\left(\prod_{i=1}^{k}\left(p_{s_j}^{ui} - p_{s_j}^{li}\right)e^{i\alpha_{s_j}^{i}}\right)\left(\prod_{i=1}^{k}\left(q_{s_j}^{ui} - q_{s_j}^{li}\right)e^{i\beta_{s_j}^{i}}\right)\left(\prod_{i=1}^{k}\left(r_{s_j}^{ui} - r_{s_j}^{li}\right)e^{i\delta_{s_j}^{i}}\right)\right)^{w_i}}\right)\right) : \forall e_j \in E, x_s \in U\right\}$$

We have the following properties:

(a) $\tilde{\tilde{B}}_{GM}\left(w_1, w_2, \ldots, w_k\right)\left(\left(\tilde{\tilde{F}}^1, E\right)_{1^{\prec}}, \left(\tilde{\tilde{F}}^2, E\right)_{1^{\prec}}, \ldots\ldots, \left(\tilde{\tilde{F}}^k, E\right)_{1^{\prec}}\right) = \left(\tilde{\tilde{F}}, E\right)_{1^{\prec}}$, where $\left(\tilde{\tilde{F}}, E\right)_{1^{\prec}}$ denotes the absolute complex interval neutrosophic soft set over the universe U.

(b) $\tilde{\tilde{B}}_{GM}\left(w_1, w_2, \ldots, w_k\right)\left(\left(\tilde{\tilde{F}}^1, E\right)_{0^{\prec}}, \left(\tilde{\tilde{F}}^2, E\right)_{0^{\prec}}, \ldots\ldots, \left(\tilde{\tilde{F}}^k, E\right)_{0^{\prec}}\right) = \left(\tilde{\tilde{F}}, E\right)_{0^{\prec}}$, where $\left(\tilde{\tilde{F}}, E\right)_{0^{\prec}}$ denotes the null complex interval neutrosophic soft set over the universe U.

(c) If, for all $i = 1, 2, \ldots, k$, $\left(\tilde{\tilde{F}}^i, E\right) \le \left(\tilde{\tilde{G}}^i, E\right)$ then

$$\tilde{\tilde{B}}GM\left(w_1, w_2, \ldots, w_k\right)\left(\left(\tilde{\tilde{F}}^1, E\right), \left(\tilde{\tilde{F}}^2, E\right), \ldots\ldots, \left(\tilde{\tilde{F}}^k, E\right)\right) \le \tilde{\tilde{B}}GM\left(w_1, w_2, \ldots, w_k\right)\left(\left(\tilde{\tilde{G}}^1, E\right), \left(\tilde{\tilde{G}}^2, E\right), \ldots\ldots, \left(\tilde{\tilde{G}}^k, E\right)\right);$$

where $\left(\tilde{\tilde{G}}^1, E\right), \left(\tilde{\tilde{G}}^2, E\right), \ldots\ldots, \left(\tilde{\tilde{G}}^k, E\right)$ be another set of K-CIVNSSs over U.

(d) $\left(\tilde{\tilde{F}}^-, E\right) \le \tilde{\tilde{B}}GM\left(w_1, w_2, \ldots, w_k\right)\left(\left(\tilde{\tilde{F}}^1, E\right), \left(\tilde{\tilde{F}}^2, E\right), \ldots\ldots, \left(\tilde{\tilde{F}}^k, E\right)\right) \le \left(\tilde{\tilde{F}}^+, E\right)$.

6. Construction of an algorithm by using complex interval neutrosophic soft sets aggregate operator

In this section, a step-wise method is described by using complex interval neutrosophic soft sets aggregate operators and it is useful and effective to deal with real-life decision-making problems.

Step1: Input a set of m alternatives $U=\{u_1,u_2,.....,u_m\}$ that have been defined by k-experts $D=\{d_1,d_2,.....,d_k\}$ concerning for to n complex interval neutrosophic parameters $E=\{e_1,e_2,.....,e_n\}$.

Step2: Opinions of K-experts have been described by K-CIVNSSs $\left(\overset{\approx}{F^1},E\right),\left(\overset{\approx}{F^2},E\right),......,\left(\overset{\approx}{F^k},E\right)$ defined as

$$\left(\overset{\approx}{F^l},E\right)=\left\{\left(e_j,\overset{\approx}{F}\left(e_j\right)\right):\forall e_j\in E\right\}=\left\{e_j,\left(x_s/\left(p^{lF}_{s_j}e^{il\mu^F_{s_j}},q^{lF}_{s_j}e^{il\nu^F_{s_j}},r^{lF}_{s_j}e^{il\delta^F_{s_j}}\right)\right):\forall e_j\in E,x_s\in U\right\}$$

Where $p^{lF}_{s_j},q^{lF}_{s_j},r^{lF}_{s_j}\subseteq[0,1]$ are the amplitude parts of the truth-membership, indeterminacy-membership ,and falsity-membership values respectively and $\mu^{lF}_{s_j},\nu^{lF}_{s_j},\delta^{lF}_{s_j}\in[0,2\pi]$ are the respective phase parts of the evaluation of an alternative x_s concerning for to the parameter e_j over the CIVNSS $\left(\overset{\approx}{F},E\right)$.

Step3: Construct the best $\left(\overset{\approx}{F^+},E\right)$ and the worst $\left(\overset{\approx}{F^-},E\right)$ CIVNSS over K-CIVNSSs.

Step4: Evaluate the approximate index $P_l\left(d_l\right)$ of an expert d_l is given by,

$$P_l\left(d_l\right)=\frac{\hat{S}\left(\left(\overset{\approx}{F^+},E\right),\left(\overset{\approx}{F^l},E\right)\right)}{\hat{S}\left(\left(\overset{\approx}{F^+},E\right),\left(\overset{\approx}{F^l},E\right)\right)+\hat{S}\left(\left(\overset{\approx}{F^-},E\right),\left(\overset{\approx}{F^l},E\right)\right)}$$, where $\hat{S}$ indicates similarity measure.

Step5: Measure the nearness index $C_l\left(d_l\right)$ of an expert d_l is given by,

$$C_l\left(d_l\right)=\sum_{l\neq l',l'=1}^{k}\frac{\hat{S}\left(\left(\overset{\approx}{F^l},E\right),\left(\overset{\approx}{F^{l'}},E\right)\right)}{l+1}$$

Step6: Derive the preference rate $\varpi\left(d_l\right)$ of an expert d_l as,

$\varpi(d_l) = \frac{\left(P_l(d_l) \otimes C_l(d_l)\right)}{\sum_{l=1}^{k}\left(P_l(d_l) \otimes C_l(d_l)\right)}$, where $\otimes$ denotes the linear product.

Step7: Construct the collective CIVNSS $\left(\overset{\approx}{F}, E\right)$ from K-CIVNSSs which is derived by using complex interval neutrosophic soft geometric mean aggregation operators as follows:

$$\overset{\approx}{B}_{GM}(w_1, w_2, \ldots\ldots, w_k)\left(\left(\overset{\approx}{F^1}, E\right), \left(\overset{\approx}{F^2}, E\right), \ldots\ldots\ldots, \left(\overset{\approx}{F^k}, E\right)\right)$$

Step8: Determined the combined weight of a parameter e_j as follows:

$$W_j = \frac{\lambda\left(w_j^*\right) \otimes (1-\lambda)\left(w_j^\#\right)}{\sum_{j=1}^{n}\left(\lambda\left(w_j^*\right) \otimes (1-\lambda)\left(w_j^\#\right)\right)}, \text{ where } w_j^* = \frac{1}{k}\left(w_j^1 \otimes w_j^2 \otimes \ldots\ldots\ldots \otimes w_j^k\right);$$

$$w_j^\# = \frac{1}{k(k-1)} \times \frac{\sum_{S'=1, S \neq S'}^{k}\left\{\frac{1}{2}\left(\left(\left(p_{s_j}^u - p_{s_j}^l\right)\mu_{s_j}\right)^2 + \left(\left(q_{s_j}^u - q_{s_j}^l\right)\nu_{s_j}\right)^2 + \left(\left(r_{s_j}^u - r_{s_j}^l\right)\delta_{s_j}\right)^2\right)\right\}^{\frac{1}{2}}}{\sum_{j=1}^{k}\sum_{S'=1, S \neq S'}^{k}\left\{\frac{1}{2}\left(\left(\left(p_{s_j}^u - p_{s_j}^l\right)\mu_{s_j}\right)^2 + \left(\left(q_{s_j}^u - q_{s_j}^l\right)\nu_{s_j}\right)^2 + \left(\left(r_{s_j}^u - r_{s_j}^l\right)\delta_{s_j}\right)^2\right)\right\}^{\frac{1}{2}}}$$

,and λ is the influence parameter such that $\lambda \in [0,1]$.

Since, experts came from different environments along with different specialization, judgment powers, and knowledge, so they may impose different weights on the associated parameters. If $w^l = \left\{w_1^l, w_2^l, w_3^l, \ldots\ldots, w_n^l\right\}$ be the associated weights of the parameters given by an expert d_l such that $\sum_{j=1}^{n} w_j^l = 1$,and $w_j^l \in [0,1]$.

Step9: Select the best alternative by determining the upper-alternative $\left(\tilde{X}\right)$ and the lower-alternative $\left(\underset{\sim}{X}\right)$ as follows:

$$\tilde{X}=\left\{e_j,\left(\begin{array}{l}\left(\left(p_{1j}^u-p_{1j}^l\right)e^{i\mu_{1j}}\otimes\left(q_{1j}^u-q_{1j}^l\right)e^{i\nu_{1j}}\otimes\left(r_{1j}^u-r_{1j}^l\right)e^{i\delta_{1j}}\right)\cup\left(\left(p_{2j}^u-p_{2j}^l\right)e^{i\mu_{2j}}\otimes\left(q_{2j}^u-q_{2j}^l\right)e^{i\nu_{2j}}\otimes\left(r_{2j}^u-r_{2j}^l\right)e^{i\delta_{2j}}\right)\ldots\ldots\\ \cup\left(\left(p_{mj}^u-p_{mj}^l\right)e^{i\mu_{mj}}\otimes\left(q_{mj}^u-q_{mj}^l\right)e^{i\nu_{mj}}\otimes\left(r_{mj}^u-r_{mj}^l\right)e^{i\delta_{mj}}\right)\end{array}\right):\forall e_j\in E\right\}$$

$$=\left\{\left(e_j,\tilde{P}e^{i\tilde{\mu}_j},\tilde{Q}e^{i\tilde{\nu}_j},\tilde{R}e^{i\tilde{\delta}_j}\right):\forall e_j\in E\right\}$$

$$\underset{\sim}{X}=\left\{e_j,\left(\begin{array}{l}\left(\left(p_{1j}^u-p_{1j}^l\right)e^{i\mu_{1j}}\otimes\left(q_{1j}^u-q_{1j}^l\right)e^{i\nu_{1j}}\otimes\left(r_{1j}^u-r_{1j}^l\right)e^{i\delta_{1j}}\right)\cap\left(\left(p_{2j}^u-p_{2j}^l\right)e^{i\mu_{2j}}\otimes\left(q_{2j}^u-q_{2j}^l\right)e^{i\nu_{2j}}\otimes\left(r_{2j}^u-r_{2j}^l\right)e^{i\delta_{2j}}\right)\ldots\ldots\\ \cap\left(\left(p_{mj}^u-p_{mj}^l\right)e^{i\mu_{mj}}\otimes\left(q_{mj}^u-q_{mj}^l\right)e^{i\nu_{mj}}\otimes\left(r_{mj}^u-r_{mj}^l\right)e^{i\delta_{mj}}\right)\end{array}\right):\forall e_j\in E\right\}$$

$$=\left\{\left(e_j,\underset{\sim}{P}e^{i\underset{\sim}{\mu}_j},\underset{\sim}{Q}e^{i\underset{\sim}{\nu}_j},\underset{\sim}{R}e^{i\underset{\sim}{\delta}_j}\right):\forall e_j\in E\right\}$$

Step10: Determine the separation level of $\left(\tilde{X}\right)$ and $\left(\underset{\sim}{X}\right)$ as follows:

$$D\left(\frac{x_s}{\tilde{X}}\right)=\left(\frac{1}{2n}\left\{\begin{array}{l}\left(w_1^2\otimes\left(\left(\frac{p_{s_1}^l+p_{s_1}^u}{2}-\tilde{p}_1\right)\left(\mu_{s_1}-\tilde{\mu}_{s_1}\right)\right)^2\otimes\left(\left(\frac{q_{s_1}^l+q_{s_1}^u}{2}-\tilde{q}_1\right)\left(\nu_{s_1}-\tilde{\nu}_{s_1}\right)\right)^2\otimes\left(\left(\frac{r_{s_1}^l+r_{s_1}^u}{2}-\tilde{r}_1\right)\left(\delta_{s_1}-\tilde{\delta}_{s_1}\right)\right)^2\right)+\\ \left(w_2^2\otimes\left(\left(\frac{p_{s_2}^l+p_{s_2}^u}{2}-\tilde{p}_2\right)\left(\mu_{s_2}-\tilde{\mu}_{s_2}\right)\right)^2\otimes\left(\left(\frac{q_{s_2}^l+q_{s_2}^u}{2}-\tilde{q}_2\right)\left(\nu_{s_2}-\tilde{\nu}_{s_2}\right)\right)^2\otimes\left(\left(\frac{r_{s_2}^l+r_{s_2}^u}{2}-\tilde{r}_2\right)\left(\delta_{s_2}-\tilde{\delta}_{s_2}\right)\right)^2\right)+\\ \ldots\ldots\ldots\ldots\ldots+\left(w_n^2\otimes\left(\left(\frac{p_{s_n}^l+p_{s_n}^u}{2}-\tilde{p}_n\right)\left(\mu_{s_n}-\tilde{\mu}_{s_n}\right)\right)^2\otimes\left(\left(\frac{q_{s_n}^l+q_{s_n}^u}{2}-\tilde{q}_n\right)\left(\nu_{s_n}-\tilde{\nu}_{s_n}\right)\right)^2\otimes\left(\left(\frac{r_{s_n}^l+r_{s_n}^u}{2}-\tilde{r}_n\right)\left(\delta_{s_n}-\tilde{\delta}_{s_n}\right)\right)^2\right)\end{array}\right\}\right)^{\frac{1}{2}}$$

$$D\left(\frac{x_s}{\underset{\sim}{X}}\right)=\left(\frac{1}{2n}\left\{\begin{array}{l}\left(w_1^2\otimes\left(\left(\frac{p_{s_1}^l+p_{s_1}^u}{2}-\underset{\sim}{p}_1\right)\left(\mu_{s_1}-\underset{\sim}{\mu}_{s_1}\right)\right)^2\otimes\left(\left(\frac{q_{s_1}^l+q_{s_1}^u}{2}-\underset{\sim}{q}_1\right)\left(\nu_{s_1}-\underset{\sim}{\nu}_{s_1}\right)\right)^2\otimes\left(\left(\frac{r_{s_1}^l+r_{s_1}^u}{2}-\underset{\sim}{r}_1\right)\left(\delta_{s_1}-\underset{\sim}{\delta}_{s_1}\right)\right)^2\right)+\\ \left(w_2^2\otimes\left(\left(\frac{p_{s_2}^l+p_{s_2}^u}{2}-\underset{\sim}{p}_2\right)\left(\mu_{s_2}-\underset{\sim}{\mu}_{s_2}\right)\right)^2\otimes\left(\left(\frac{q_{s_2}^l+q_{s_2}^u}{2}-\underset{\sim}{q}_2\right)\left(\nu_{s_2}-\underset{\sim}{\nu}_{s_2}\right)\right)^2\otimes\left(\left(\frac{r_{s_2}^l+r_{s_2}^u}{2}-\underset{\sim}{r}_2\right)\left(\delta_{s_2}-\underset{\sim}{\delta}_{s_2}\right)\right)^2\right)+\\ \ldots\ldots\ldots\ldots\ldots+\left(w_n^2\otimes\left(\left(\frac{p_{s_n}^l+p_{s_n}^u}{2}-\underset{\sim}{p}_n\right)\left(\mu_{s_n}-\underset{\sim}{\mu}_{s_n}\right)\right)^2\otimes\left(\left(\frac{q_{s_n}^l+q_{s_n}^u}{2}-\underset{\sim}{q}_n\right)\left(\nu_{s_n}-\underset{\sim}{\nu}_{s_n}\right)\right)^2\otimes\left(\left(\frac{r_{s_n}^l+r_{s_n}^u}{2}-\underset{\sim}{r}_n\right)\left(\delta_{s_n}-\underset{\sim}{\delta}_{s_n}\right)\right)^2\right)\end{array}\right\}\right)^{\frac{1}{2}}$$

Step11: Obtain the ranking index of an alternative x_s by using the formula

$$\tilde{R}(x_s) = \frac{D\left(\frac{x_s}{\tilde{X}}\right)}{D\left(\frac{x_s}{\tilde{X}}\right) + D\left(\frac{x_s}{\underset{\sim}{X}}\right)}; s = 1, 2, ..., m$$

The alternative having a maximum ranking index $\tilde{R}$ will be selected as the best or optimal alternative for this multi-expert decision-making. If more than one alternative has the same maximum ranking index, then, we will select any one of them as an optimal solution.

7. An application on a financial problem

A trader wants to set up a car manufacturing company and the set of alternatives $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ represent a set of six cars among which he or she has to choose any one of the alternatives which fulfilled all the pre-assigned criteria. Selection of any one of the alternatives influenced by the set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$. Here, the parameters stand for land, labor, capital, entrepreneurship, and raw material cost respectively. Now, a set of four experts denoted by $D = \{d_1, d_2, d_3, d_4\}$ have been assigned for monitoring the parameters to reach a common decision about which a car manufacturing company is more likely to choose which have these parametric characters. Here, the belongingness level of a parameter has been taken through the amplitude part (interval form due to the more complexity involved in the problem which has neutrosophic nature) and the time duration of a parameter has been taken through the phase part. All the data has been collected by the decision-makers on 20 consecutive days. To express this data in the interval $[0, 2\pi]$, 2π has been taken here instead of 20 days.

About of on this idea and by using the algorithm discussed in section6, the trader will be able to choose the best alternatives and it is possible when all the decision-makers come to a common solution and it is possible due to the aggregate operators used in the said algorithm. The calculation part is left for the readers as an exercise.

8. Conclusions

In this article, we first give the basic definition of complex interval neutrosophic soft sets and some basic operations on them. We then discuss similarity measures on complex interval neutrosophic soft sets and their aggregation. An algorithm has been introduced by using complex interval neutrosophic soft sets aggregate operators. To apply the algorithm to the decision-making problem we give an application that shows the algorithm can be successfully applied in financial problems. In the future, there is a scope to extend the notion of complex interval neutrosophic soft set by introducing hypersoft set introduced by Smarandache [12] in 2018. Also, the comlex interval neutrosophic soft set may be applied comprehensively in different fields such as engineering, medical science, finance, game theory, computer science, decision-making,etc.

Acknowledgments: Author is very much thankful to the anonymous referees for their valuable suggestions which help to improve the standard of the paper.

Conflicts of Interest: There is no conflict of interest regarding the publication of the paper.

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Received: Month Day, Year. Accepted: Month Day, Year