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# A Theoretical and Analytical Approach for Fundamental Framework of Composite Mappings on Fuzzy Hypersoft Classes

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**Abstract**. Mapping is a fundamental mathematical concept that is used in many elementary areas of science and mathematics and has numerous applications. The core purpose of this study is to provide a theoretical and analytical approach for carving out a basic structure of composite mappings on the classes of Fuzzy Hypersoft (FHS) sets. It is a comprehensive study of existing concepts regarding mappings on fuzzy soft, soft and hesitant fuzzy soft classes through characterizing of composite mappings on FHS classes. Moreover, certain generalized properties of mappings on FHS classes like FHS images and FHS inverse images, are established. Some related results are verified with the help of illustrative examples.

**Keywords:** Fuzzy soft, Soft classes, Hesitant fuzzy soft classes, Composite mappings, Fuzzy hypersoft set.

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## 1. **Introduction**

In 1965, Zadeh introduced the theory of fuzzy sets [24]. It has been utilized in different decision making problem [20]- [21]- [22]. There are some theories, theory of likelihood, theory of intuitionistic fuzzy sets [2], [5], theory of vague sets [10], the theory of interval mathematics [2], [11], and theory of rough sets [13] which can be considered as scientific apparatuses for dealing with uncertainties and ambiguous. Despite that, every one of these speculations has their innate challenges as brought up in [12]. The main reason behind these troubles is potentially the inadequacy of the parametrization device of the hypothesis.

Therefore, Molodtsov [12] started the idea of a soft set (SS) as a numerical device for dealing with uncertainties which are liberated from the above challenges (We know about the SS characterized by Pawlak [14], which is an alternate idea and helpful to understand some other kind of issues).

Karaaslan [8] introduced soft class and its pertinent activities. Athar et al. [3], [4] introduced the concept of mappings on fuzzy soft classes and mappings on soft classes in 2009 and 2011 individually. They considered the properties of the soft image and soft inverse image. They also characterized the properties of fuzzy SS, fuzzy S-image, fuzzy S-inverse image of fuzzy S-sets and supported them with examples. Manash et al. [7] gave the idea of composite mappings on hesitant fuzzy soft classes in 2016 and discussed some interesting properties of this idea.

In a diversity of real-life applications, the attributes should be further sub-partitioned into attribute values for more clear understanding. Samarandache [9] fulfilled this need and developed the concept of the HSS as a generalization of the SS. He opened numerous fields in this way of thinking and generalized SS to the hyper-soft set by changing the planning F into a multi-contention function. At that point, he made the differentiation between the sorts of initial universes, crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic respectively. Thus, he also showed that a HS set can be crisp, fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic respectively. Saeed et al. [19, 25] explained some basic concepts like HS subset, HS complement, not HS set, absolute set, union, intersection, AND, OR, restricted union, extended intersection, relevant complement, restricted difference, restricted symmetric difference, HS set relation, sub relation, complement relation, HS representation in matrices form, and different operations on matrices. Saeed et al. [23] characterized mapping under a hypersoft set environment, then some of its essential properties like HS images, HS inverse images were also discussed.

The core purpose of this study is to provide a theoretical and analytical approach for carving out a basic structure of composite mappings on the classes of FHS sets. It is an comprehensive study of existing concepts regarding mappings on fuzzy soft, soft classes and hesitant fuzzy soft classes through characterizing of composite mappings on FHS classes. Moreover, certain generalized properties of mappings on FHS classes like FHS images and FHS inverse images, are established. Some related results are proved with the help of illustrative examples. The ordering of the following portion is working out as follows.

In Section 2, some pivotal regarding fuzzy set, SS, fuzzy soft class, soft class, hypersoft set, and fuzzy hypersoft set (FHSS) are re-imagined. In Section 3, composite mappings on FHS classes, FHS image, FHS inverse image, and its relevant theorems with their essential properties are considered. In the last section, some concluding remarks are described.

### 1.1. *Motivation*

In a diversity of real-life applications, the attributes should be further sub-partitioned into attribute values for more clear understanding. Samarandache [9] fulfilled this need and developed the concept of the FHSS as a generalization of the fuzzy soft set. Now, It will be a question that how do we define composite mappings for FHSS classes? FHSS set is significant? To answers these questions and getting inspiration from the above writing, it is relevant to broaden the idea of mappings for those sets managing disjoint arrangements of attributed values, i.e FHSS. In this investigation, an extension is made in existing theories with respect to mappings on fuzzy soft, soft classes and hesitant fuzzy soft classes by characterizing composite mappings on FHS classes. The striking component of composite mappings on FHS classes is that it can mirror the interrelationship between the multi-input contentions. Moreover, certain generalized properties of mappings on FHS classes like FHS images and FHS inverse images, are established. Some related results are proved with the help of illustrative examples.

## 2. **Preliminaries**

Throughout the following, let $L = F_1 \times F_2 \times F_3 \times ... \times F_n$, $M = F'_1 \times F'_2 \times F'_3 \times ... \times F'_n$, $N = F''_1 \times F''_2 \times F''_3 \times ... \times F''_n$, $O = F'''_1 \times F'''_2 \times F'''_3 \times ... \times F'''_n$, $\alpha = (\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n)$ and $\beta = (\beta_1, \beta_2, \beta_3, ..., \beta_n)$.

**Definition 2.1.** [21] The fuzzy set $X = \{(x, \xi_X(x))|x \in X\}$ such that

$$\xi_X : X \to [0, 1]$$

where $\xi_X(x)$ describes the membership percentage of $x \in X$.

**Definition 2.2.** [12] A pair $(F, A)$ is said to be soft set over $X$, where $F$ is a mapping given as

$$F : A \to P(X)$$

On the other way, a SS is the parameterized family of subsets of the universe $X$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe. For $\epsilon \in A$. $F(\epsilon)$ may be considered as the set of $\epsilon$ approximate elements of the soft set $(F, A)$.

**Definition 2.3.** [6] Let $X$ be an initial universe, indexed class of fuzzy sets $\{f_i : f_i : X \to [0, 1]), i = 1, 2, ..., n\}$ is called a fuzzy class.

**Definition 2.4.** [1] Let $X$ be a universe and E a set of attributes. Then the pair $(X, E)$ denotes the collection of all fuzzy soft sets on $X$ with attributes from E and is called a fuzzy soft class.

**Definition 2.5.** [9] Let $a_1, a_2, a_3, \cdots, a_n$ be the distinct attributes whose corresponding attribute values belongs to the sets $F_1, F_2, F_3, \cdots, F_n$ respectively, where $F_i \cap F_j = \Phi$ for $i \neq j$. A pair $(\Upsilon, L)$ is called a hypersoft set over the universal set $X$, where $\Upsilon$ is the mapping given by $\Upsilon : L \longrightarrow P(X)$.
For more definition see [15–19].

**Definition 2.6.** [19] Suppose $X$ and $F(X)$ be the universal set and all fuzzy subsets of $X$ respectively. Let $a_1, a_2, a_3, \cdots, a_n$ be the distinct attributes whose corresponding attribute values belongs to the sets $F_1, F_2, F_3, ..., F_n$ respectively, where $F_i \cap F_j = \Phi$ for $i \neq j$ and $i$, $j$ $\in \{1, 2, 3, ..., n\}$. Then the FHSS is the pair $(\Sigma_L, L)$ over $X$ defined by a map $\Sigma_L : L \to F(X)$.

## 3. Main results

**Definition 3.1.** Suppose $X$ and $F(X)$ be the universal set and all fuzzy subsets of $X$ respectively, let $a_1, a_2, a_3, \cdots, a_n$ be the distinct attributes whose corresponding attribute values belongs to the sets $F_1, F_2, F_3, \cdots, F_n$ respectively, where $F_i \cap F_j = \Phi$ for $i \neq j$, let $\digamma = \{\varsigma_i : i = 1, 2, ..., n\}$ be a collection of decision makers. Indexed class of FHSS $\{\vartheta_{\varsigma_i} : \vartheta_{\varsigma_i} : L \to F(X), \varsigma_i \in \digamma\}$, is said to be fuzzy hypersoft class and it can be symbolized in such a form $\vartheta_{\digamma}$. If for any $\varsigma_i \in \digamma$, $\vartheta_{\varsigma_i} = \Phi$, the FHSS $\vartheta_{\varsigma_i} \notin \vartheta_{\digamma}$.

**Example 3.2.** Let $X = \{a =$ Holstein, $b =$ Angus, $c =$ Charolais $\}$ be the set of cow categories. Peter decide to purchase a cow for milk to get vitamin B12 and iodine of doctor instruction. They visit Bos tarus (a Europeon Cattle) to buy such cow which fulfills his requirements. Let $a_1 =$ vision and hearing, $a_2 =$ Cost, $a_3 =$ Colour, distinct attributes whose attribute values belong to the sets $F_1, F_2, F_3$. Let $F_1 = \{f_1 =$ Excellent peripheral vision, $f_2 =$ Low peripheral vision $\}$, $F_2 = \{f_3 =$High, $f_4 = Low\}$, $F_3 = \{f_5 =$ White $\}$ and let $\digamma = \{\varsigma_1, \varsigma_1, \varsigma_1\}$ be a set of decision makers. If we consider FHS sets given as

$$\vartheta_{\varsigma_1} = \left\{ \begin{array}{l} ((f_1, f_3, f_5), \{0.5/a, 0.7/b\}), ((f_1, f_4, f_5), \{0.1/c\}), \\ ((f_2, f_3, f_5), \{0.4/b, 0.2/c\}), ((f_2, f_4, f_5), \{0.02/b\}) \end{array} \right\}$$

$$\vartheta_{\varsigma_2} = \left\{ \begin{array}{l} ((f_1, f_3, f_5), \{0.05/b, 0.006/c\}), ((f_1, f_4, f_5), \{0.08/a\}), \\ ((f_2, f_3, f_5), \{0.55/a, 0.75/c\}), ((f_2, f_4, f_5), \{0.52/b\}) \end{array} \right\}$$

$$\vartheta_{\varsigma_3} = \left\{ \begin{array}{l} (f_1, f_3, f_5)\{0.008/c, 0.25/a\}), ((f_1, f_4, f_5), \{0.12/b\}), \\ ((f_2, f_3, f_5), \{0.05/b, 0.64/c\}), ((f_2, f_4, f_5), \{0.28/c\}) \end{array} \right\}$$

and

$$g_{\varsigma_1} = \left\{ \begin{array}{l} ((f_1, f_3, f_5), \{0.87/a\}), ((f_1, f_4, f_5), \{0.23/a\}), \\ ((f_2, f_3, f_5), \{0.09/c, 0.54/a\}), ((f_2, f_4, f_5), \{0.53/a\}) \end{array} \right\}$$

$$g_{\varsigma_2} = \left\{ \begin{array}{l} ((f_1, f_3, f_5), \{0.05/c\}), ((f_1, f_4, f_5), \{0.34/b\}), \\ ((f_2, f_3, f_5), \{0.32/b, 0.27/c\}), ((f_2, f_4, f_5), \{0.08/c\}) \end{array} \right\}$$

$$g_{\varsigma_3} = \left\{ \begin{array}{l} ((f_1, f_3, f_5), \{0.27/b\}), ((f_1, f_4, f_5), \{0.52/b\}), \\ ((f_2, f_3, f_5), \{0.37/a, 0.38/c\}), ((f_2, f_4, f_5), \{0.001/a\}) \end{array} \right\}$$

Then FHS classes can be written as $\{\vartheta_{\varsigma_1}, \vartheta_{\varsigma_2}, \vartheta_{\varsigma_3}\}$, $\{g_{\varsigma_1}, g_{\varsigma_2}, g_{\varsigma_3}\}$.

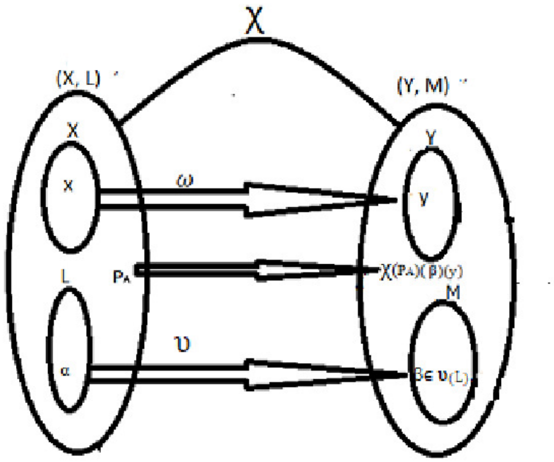


FIGURE 1. Representation of Fuzzy Hypersoft Mapping

**Definition 3.3.** Let $\overline{(X,L)}$ and $\overline{(Y,M)}$ be classes of FHS sets over $X$ and $Y$ with attributes from $L$ and $M$ respectively. Let $\omega : X \to Y$ and $\upsilon : L \to M$ be mappings. Then a FHS mappings $\chi = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(Y,M)}$ is defined as follows, for a FHSS $P_A$ in $\overline{(X,L)}$, $\chi(P_A)$ is a FHSS in $\overline{(Y,M)}$ obtained as follows, for $\beta \in \upsilon(L) \subseteq M$ and $y \in Y$, $\chi(P_A)(\beta)(y) = \cup_{\alpha \in \upsilon^{-1}(\beta) \cap A,\ s \in \omega^{-1}(y)} (\alpha) \mu_s \chi(P_A)$ is called a fuzzy hypersoft image of a FHSS $P_A$. Hence $(P_A, \chi(P_A)) \in \chi$, where $P_A \subseteq \overline{(X,L)}$, $\chi(P_A) \subseteq \overline{(Y,M)}$. For more detail see fig 1.

**Definition 3.4.** If $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ be a FHS mapping, then FHS class $\overline{(X, L)}$ is called the domain of $\chi$ and the FHS class $G_B \in \overline{(Y, M)} : G_B = \chi(H_A)$, for some $H_A \in \overline{(X, L)}$ is called the range of $\chi$. The FHS class $\overline{(Y, M)}$ is called co-domain of $\chi$.

**Definition 3.5.** Let $\chi = (\omega, \upsilon) : \overline{(X, L)} \to \overline{(Y, M)}$ be a FHS mapping and $G_B$ a FHSS in $\overline{(Y, M)}$, where $\omega : X \to Y$, $\upsilon : L \to M$ and $B \subseteq M$. Then $\chi^{-1}(G_B)$ is a FHSS in $\overline{(X, L)}$ defined as follows, for $\alpha \in \upsilon^{-1}(B) \subseteq L$ and $x \in X$, $\chi^{-1}(G_B)(\alpha)(x) = (\upsilon(\alpha))\mu_{p(x)}\chi^{-1}(G_B)$ is called a FHS inverse image of $G_B$. For more detail see fig 2

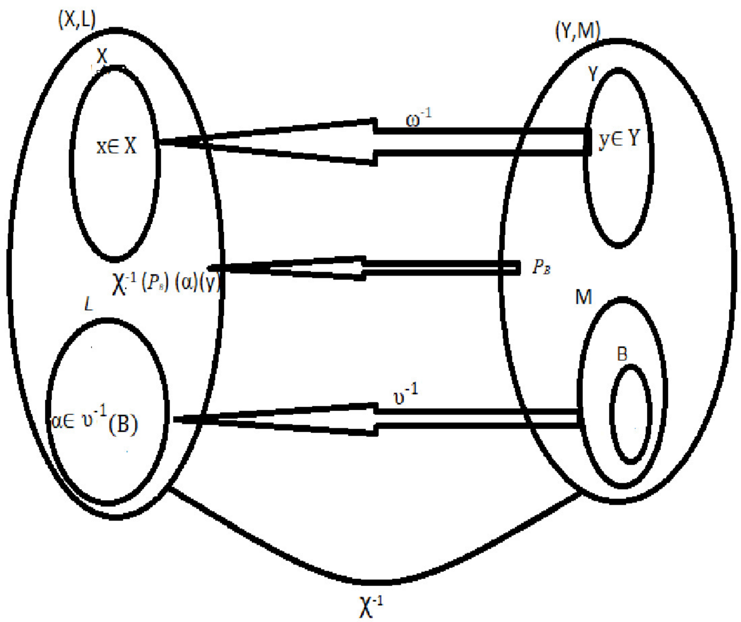


FIGURE 2. Representation of Fuzzy Hypersoft Inverse Mapping

**Example 3.6.** Let $X = \{a =$ Holstein, $b =$ Angus, $c =$ Charolais $\}$ and $Y = \{x =$ Murrah, $y =$ Siamese, $z =$ Surti $\}$ be the two universal sets of Cows and Buffalo categories respectively. Peter plans his dairy farm business and wants to understand the difference between a strategy and a tactic. For this purpose, he creates the relationship between these two types of cattle categories which is helpful regrading his dairy farm business. Let $a_1 =$ vision and hearing , $a_2 =$ Cost, $a_3 =$ Colour, and $b_1 =$ appearance, $b_2 =$ colour, $b_3 =$ price be the two types of distinct attributes whose corresponding attribute values belong to the sets $F_1, F_2, F_3$ and $F'_1, F'_2, F'_3$ respectively. Let $F_1 = \{f_1 =$ Excellent peripheral vision, $f_2 =$ Low peripheral vision $\}$, $F_2 = \{f_3 =$High $\}$, $F_3 = \{f_4 =$ White, $f_5 =$ brown $\}$. Similarly, $F'_1 = \{f'_1 =$ Good, $f'_2 =$ massive $\}$, $F'_2 = \{f'_3 =$ black $\}$, $F'_3 = \{f'_4 =$ low price $\}$ and $\overline{(X, L)}$, $\overline{(Y, M)}$ be two classes of FHS sets, where $L = F_1 \times F_2 \times F_3$ and $M = F_1' \times F_2' \times F_3'$. Let $\omega : X \to Y$, $\upsilon : F_1 \times F_2 \times F_3 \to F_1' \times F_2' \times F_3'$ be mappings as follows

$\omega(a) = y, \omega(b) = x, \omega(c) = y$ and

$\upsilon(f_1, f_3, f_4) = (f_2', f_3', f_4'), \upsilon(f_1, f_3, f_5) = (f_1', f_3', f_4'),$

$\upsilon(f_2, f_3, f_4) = (f_2', f_3', f_4'), \upsilon(f_2, f_3, f_5) = (f_1', f_3', f_4').$

Consider a FHSS $P_A$ in $\overline{(X, L)}$ as

$$P_A = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (f_1, f_3, f_4) = \{< a, \{0.5, 0.3\} >, \\ < b, \{0.9, 0.3, 0.5\} >, < c, \{0.3\} > \end{array} \right\}, \\ \left\{ \begin{array}{l} (f_1, f_3, f_5) = \{< a, \{0.3, 0.9\} >, \\ < b, \{0.5, 0.1\} >, < c, \{0.1\} >\} \end{array} \right\} \end{array} \right\}$$

Then the FHS image of $P_A$ under $\chi = (\omega, \upsilon) : \overline{(X, L)} \to \overline{(Y, M)}$ is obtained as

$\chi(P_A)(f_{1'}, f_{3'}, f_{4'})(x) = \cup_{\alpha \in \upsilon^{-1}(f_{1'}, f_{3'}, f_{4'}) \cap A, s \in \omega^{-1}(x)} (\alpha)\mu_s = \cup_{\alpha \in (f_1, f_3, f_5) \cap A, s \in b} (\alpha)\mu_s$

$= (f_1, f_3, f_5)\mu_b = \{0.5, 0.1\},$

$\chi(P_A)(f_{1'}, f_{3'}, f_{4'})(y) = \{0.3, 0.9\},$

$\chi(P_A)(f_{1'}, f_{3'}, f_{4'})(z) = \{0\}$

$\chi(P_A)(f_{2'}, f_{3'}, f_{4'})(x) = \{0.9, 0.3, 0.5\}$

$\chi(P_A)(f_{2'}, f_{3'}, f_{4'})(y) = \{0.3, 0.5\}$

$\chi(P_A)(f_{2'}, f_{3'}, f_{4'})(z) = \{0\}$

$$\chi(P_A) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (f'_1, f'_3, f'_4) = \{< x, \{0.5, 0.1\} >, \\ < y, \{0.3, 0.9\} >, < z, \{0\} > \end{array} \right\}, \\ \left\{ \begin{array}{l} (f'_2, f'_3, f'_4) = \{< x, \{0.9, 0.3, 0.5\} >, \\ < y, \{0.3, 0.5\} >, < z, \{0\} >\} \end{array} \right\} \end{array} \right\}$$

Again consider a FHSS $P_B'$ in $\overline{(Y, M)}$ as

$$P_B' = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (f'_1, f'_3, f'_4) = \{< x, \{0.3, 0.4\} >, \\ < y, \{0.4, 0.5, 0.1\} >, < z, \{0.9, 0.3\} > \end{array} \right\}, \\ \left\{ \begin{array}{l} (f'_2, f'_3, f'_4) = \{< x, \{0.4, 0.5\} >, \\ < y, \{0.9, 0.3\} >, < z, \{0.5, 0.4\} >\} \end{array} \right\} \end{array} \right\}$$

Therefore,

$\chi^{-1}(P'_B)(f_1, f_3, f_4)(a) = \upsilon((f_1, f_3, f_4))\mu_{\omega(a)}$

$= (f'_2, f'_3, f'_4)\mu_{\omega(a)} = (f'_2, f'_3, f'_4)\mu_y = \{0.9, 0.3\}$

$\chi^{-1}(P'_B)(f_1, f_3, f_4)(b) = \{0.4, 0.5\}$

$\chi^{-1}(P'_B)(f_1, f_3, f_4)(c) = \{0.9, 0.3\}$

$\chi^{-1}(P'_B)(f_1, f_3, f_5)(a) = \{0.4, 0.5, 0.1\}$

$\chi^{-1}(P'_B)(f_1, f_3, f_5)(b) = \{0.3, 0.4\}$

$\chi^{-1}(P'_B)(f_1, f_3, f_5)(c) = \{0.4, 0.5, 0.1\}$

$\chi^{-1}(P'_B)(f_2, f_3, f_4)(a) = \{0.9, 0.3\}$

$\chi^{-1}(P'_B)(f_2, f_3, f_4)(b) = \{0.4, 0.5\}$

$\chi^{-1}(P'_B)(f_2, f_3, f_4)(c) = \{0.9, 0.3\}$

$\chi^{-1}(P'_B)(f_2, f_3, f_5)(a) = \{0.4, 0.5, 0.1\}$
$\chi^{-1}(P'_B)(f_2, f_3, f_5)(b) = \{0.3, 0.4\}$
$\chi^{-1}(P'_B)(f_2, f_3, f_5)(c) = \{0.4, 0.5, 0.1\}$
Similarly,

$$\chi^{-1}(P'_B) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \{(f_1, f_3, f_4) = \{< a, \{0.9, 0.3\} >, \\ < b, \{0.4, 0.5\} >, < c, \{0.9, 0.3\} >\} \end{array} \right\}, \\ \left\{ \begin{array}{l} \{(f_1, f_3, f_5) = \{< a, \{0.4, 0.5, 0.1\} >, \\ < b, \{0.3, 0.4\} >, < c, \{0.4, 0.5, 0.1\} >\} \end{array} \right\}, \\ \left\{ \begin{array}{l} \{(f_2, f_3, f_4) = \{< a, \{0.9, 0.3\} >, \\ < b, \{0.4, 0.5\} >, < c, \{0.9, 0.3\} >\} \end{array} \right\}, \\ \left\{ \begin{array}{l} \{(f_2, f_3, f_5) = \{< a, \{0.4, 0.5, 0.1\} >, \\ < b, \{0.3, 0.4\} >, < c, \{0.4, 0.5, 0.1\} >\} \end{array} \right\} \end{array} \right\}.$$

In case of $\chi : \overline{(X, L)} \to \overline{(Y, M)}$, Peter should purchase those buffalo having attribute values ($f_1$ = Excellent peripheral vision, $f_3$ = High, $f_4$ = White) or can purchase those cow having this attributes values ($f'_1$ = Good, $f'_3$ = black, $f'_4$ = low price ) because both of the categories are interrelated and fulfills the requirements individually. For more clarity see 3.

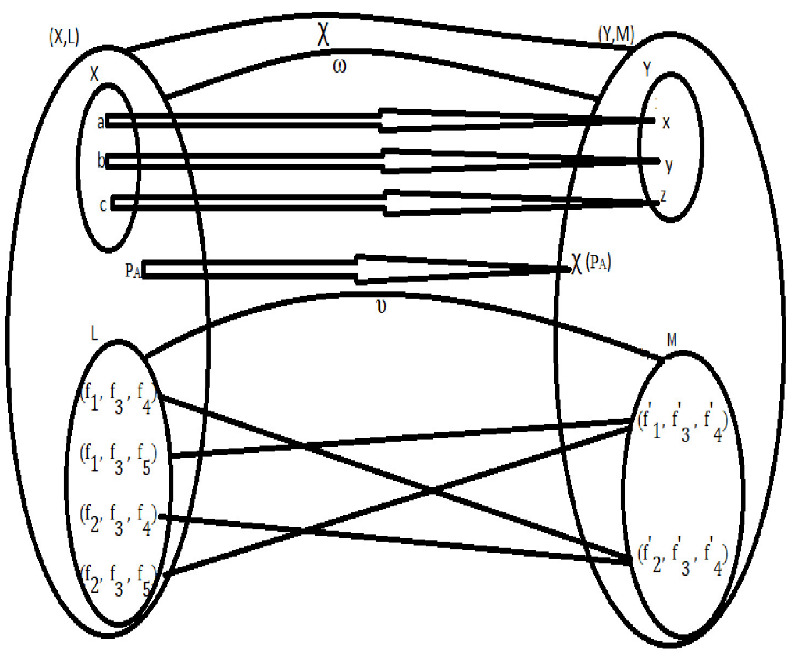


FIGURE 3. Representation of Fuzzy Hypersoft Mapping

Similarly, for inverse FHSS see fig 4.

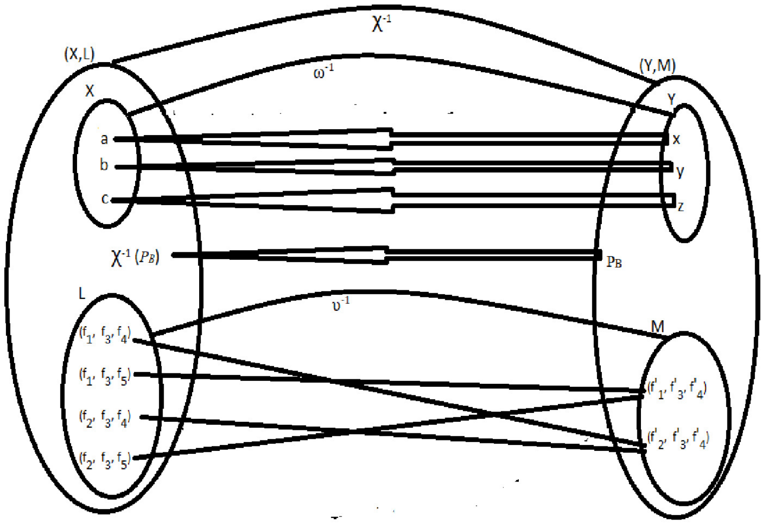


FIGURE 4. Representation of Fuzzy Hypersoft Inverse Mapping

**Definition 3.7.** Let $\chi = (\omega, \upsilon)$ be a FHS mapping of a FHS class $\overline{(X,L)}$ into a FHS class $\overline{(Y,M)}$. Then

(1) $\chi$ is said to be a one-one (or injection) FHS mapping if for both $\omega : X \to Y$ and $\upsilon : L \to M$ are one-one.

(2) $\chi$ is said to be a onto (or surjection) FHS mapping if for both $\omega : X \to Y$ and $\upsilon : L \to M$ are onto.

If $\chi$ is both one-one and onto then $\chi$ is called a one-one onto (or bijective) correspondence of FHS mapping.

**Theorem 3.8.** *Let $\chi = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(X,M)}$ and $\phi = (m, n) : \overline{(X,L)} \to \overline{(X,M)}$ are two FHS mappings. Then $\chi$ and $\phi$ are equal if and only if $\omega = m$ and $\upsilon = n$.*

*Proof.* Obvious.

**Theorem 3.9.** *Two FHS mappings $\chi$ and $\phi$ of a FHS class $\overline{(X,L)}$ into a FHS class $\overline{(Y,M)}$ are equal if and only if $\chi(P_A) = \phi(P_A)$, for all $P_A \in \overline{(X,L)}$.*

*Proof.* Let $\chi = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(X,M)}$ and $\phi = (m, n) : \overline{(X,L)} \to \overline{(X,M)}$ are two FHS mappings. Since $\omega$ and $\upsilon$ are equal, this implies $\omega = m$ and $\upsilon = n$, let $\beta \in \upsilon(L) \subseteq M$ and $y \in Y$, $\chi(P_A)(\beta)(y) = \cup_{\alpha \in \upsilon^{-1}(\beta) \cap A,\, s \in \omega^{-1}(y)} (\alpha)\mu_s = \cup_{\alpha \in n^{-1}(\beta) \cap A,\, s \in m^{-1}(y)} (\alpha)\mu_s$. Hence $\chi(P_A) = \phi(P_A)$.
Conversely,
let $\chi(P_A) = \phi(P_A)$, for all $P_A \in \overline{(X,L)}$, let $(P, Q) \in \chi$, where $P \in \overline{(X,L)}$ and $Q \in \overline{(Y,M)}$.

Therefore $Q = \chi(P) = \phi(P)$, this gives $(P, Q) \in \phi$. Therefore $\chi \subseteq \phi$. Similarly, it can be proved that $\phi \subseteq \chi$. Hence $\phi = \chi$.

**Definition 3.10.** If $\chi = (\omega, \upsilon) : \overline{(X, L)} \to \overline{(Y, M)}$ and $\phi = (m, n) : \overline{(Y, M)} \to \overline{(Z, N)}$ are two FHS mappings, then their composite $\phi o \chi$ is a FHS mapping of $\overline{(X, L)}$ into $\overline{(Z, N)}$ such that for every $P_A \in \overline{(X, L)}$, $(\phi o \chi)(P_A) = \phi(\chi(P_A))$. We defined as for $\beta \in n(M) \subseteq N$ and $y \in Z$, $\phi(\chi(P_A))(\beta)(y) = \cup_{\alpha \in n^{-1}(\beta) \cap \chi(A),\ s \in m^{-1}(y)}(\alpha)\mu_s$. For more detail see fig 5.

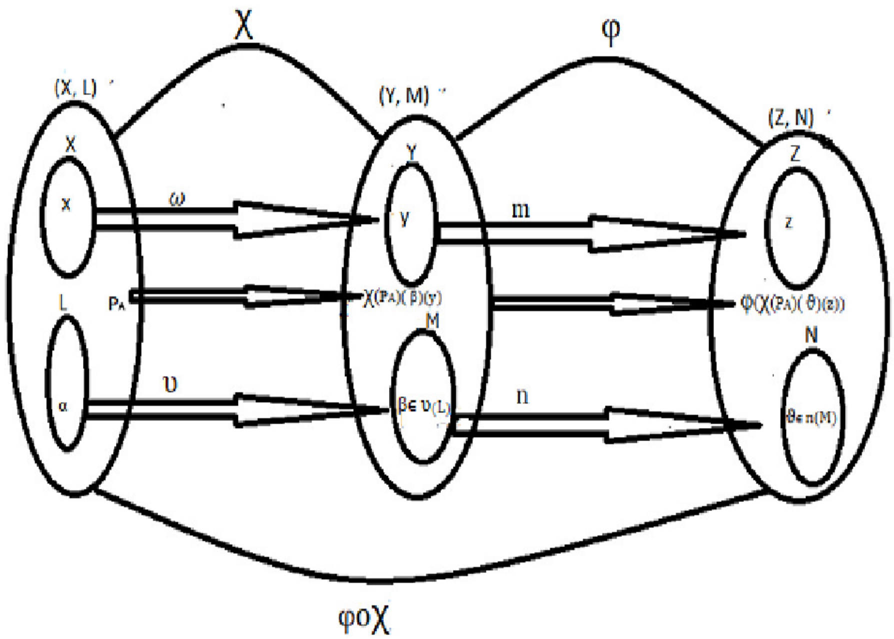


FIGURE 5. Representation of Composite Fuzzy Hypersoft Mapping

**Example 3.11.** From 3.6, consider the FHS mapping $\phi = (m, n) : \overline{(Y, M)} \to \overline{(Z, N)}$ in such a way $m(x) = h_2, m(y) = h_3, m(z) = h_2$,
and

$$n(f_1', f_3', f_4') = (f_1'', f_2'', f_3''),$$
$$n(f_2', f_3', f_4') = (f_1'', f_2'', f_4'')$$

where $Z = \{h_1, h_2, h_3\}$, $N = F_1'' \times F_2'' \times F_3'' = \{(f_1'', f_2'', f_3''), (f_1'', f_2'', f_4'')\}$. Therefore

$$\phi(\chi(P_A))(f_1'', f_2'', f_3'')(h_1) = \cup_{\alpha \in n^{-1}(f_1'', f_2'', f_3'') \cap A,\ s \in m^{-1}(h_1)}(\alpha)\mu_s$$

$$= \cup_{\alpha \in (f_1', f_3', f_4') \cap \chi(A),\ s \in \Phi}(\alpha)\mu_s = (f_1', f_3', f_4')\mu_\Phi = \{0\},$$

$$\phi(\chi(P_A))(f_1'', f_2'', f_3'')(h_2) = \{0.3, 0.4, 0.9\},$$

$$\phi(\chi(P_A))(f''_1, f''_2, f''_3)(h_3) = \{0.4, 0.5, 0.1\},$$

So,

$$\phi(\chi(P_A))(f''_1, f''_2, f''_4)(h_1) = \{0\}$$
$$\phi(\chi(P_A))(f''_1, f''_2, f''_4)(h_2) = \{0.4, 0.5\},$$
$$\phi(\chi(P_A))(f''_1, f''_2, f''_4)(h_3) = \{0.9, 0.3\}$$

$$(\phi o \chi)(P_A) = \phi(\chi(P_A) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (f''_1, f''_2, f''_4) = \{< h_1, \{0\} >, \\ < h_2, \{0.3, 0.4, 0.9\} >, < h_3, \{0.4, 0.5, 0.1\} > \end{array} \right\} \\ \left\{ \begin{array}{l} (f''_1, f''_2, f''_4) = \{< h_1, \{0\} >, \\ < h_2, \{0.4, 0.5\} >, < h_3, \{0.9, 0.3\} > \end{array} \right\} \end{array} \right\}$$

**Theorem 3.12.** *Let $\chi = (\omega, \upsilon) : \overline{(X, L)} \to \overline{(Y, M)}$ and $\phi = (m, n) : \overline{(Y, M)} \to \overline{(Z, N)}$ are two FHS mappings.Then*

(1) *if $\chi$ and $\phi$ are one-one then so is $\phi o \chi$.*
(2) *if $\phi$ and $\chi$ are onto then so is $\chi o \phi$.*
(3) *if $\chi$ and $\phi$ are both bijections then so is $\phi o \chi$.*

**Example 3.13.** Fom 3.6, 3.11, let $\omega(a) = z, \omega(b) = x, \omega(c) = y$,
$\upsilon(f_1, f_3, f_4) = (f'_2, f'_3, f'_5)$,
$\upsilon(f_1, f_3, f_5) = (f'_1, f'_3, f'_4)$
and
$m(x) = h_2, m(y) = h_3, m(z) = h_1$

$$\begin{aligned} n(f'_1, f'_3, f'_4) &= (f''_1, f''_2, f''_3), \\ n(f'_2, f'_3, f'_4) &= (f''_1, f''_2, f''_4). \end{aligned}$$

Also we consider a FHSS$P_A$ in $\overline{(X, L)}$ as

$$P_A = \left\{ \begin{array}{l} \{(f_1, f_3, f_4) = \left\{ \begin{array}{l} < a, \{0.5, 0.3\} >, \\ < b, \{0.9, 0.3, 0.5\} >, \\ < c, \{0.3\} > \end{array} \right\}, \\ \{(f_1, f_3, f_5) = \left\{ \begin{array}{l} < a, \{0.3, 0.9\} >, \\ < b, \{0.5, 0.1\} >, \\ < c, \{0.1\} >\} \end{array} \right\} \end{array} \right\}$$

Then the FHS image of $P_A$ under $\chi = (\omega, \upsilon) : \overline{(X, L)} \to \overline{(Y, M)}$ is obtained as

$$\chi(P_A)(f'_1, f'_3, f'_4)(x) = \cup_{\alpha \in \upsilon^{-1}(f'_1, f'_3, f'_4) \cap A,\ s \in \omega^{-1}(x)} (\alpha)\mu_s$$
$$= \cup_{\alpha \in (f_1, f_3, f_5) \cap A,\ s \in b} (\alpha)\mu_s = (f_1, f_3, f_5)\mu_b = \{0.5, 0.1\},$$
$$\chi(P_A)(f'_1, f'_3, f'_4)(y) = \{0.1\},$$
$$\chi(P_A)(f'_1, f'_3, f'_4)(z) = \{0.3, 0.9\},$$

$$\chi(P_A)(f_2', f_3', f_4')(x) = \{0.9, 0.3, 0.5\},$$

$$\chi(P_A)(f_2', f_3', f_4')(y) = \{0.5, 0.3\},$$

$$\chi(P_A)(f_2', f_3', f_4')(z) = \{0.3\}$$

$$\chi(P_A) = \left\{ \begin{array}{l} \left\{ (f'_1, f'_3, f'_4) = \left\{ \begin{array}{l} < x, \{0.5, 0.1\} >, \\ < y, \{0.1\} >, \\ < z, \{0.3, 0.9\} > \end{array} \right\} \right\}, \\ \left\{ (f'_2, f'_3, f'_5) = \left\{ \begin{array}{l} < x, \{0.9, 0.3, 0.5\} >, \\ < y, \{0.3, 0.5\} >, \\ < z, \{0.3\} > \end{array} \right\} \right\} \end{array} \right\}$$

Again,

$$\phi(\chi(P_A))(f_1'', f_2'', f_3'')(h_1) = \cup_{\alpha \in n^{-1}(f_1'', f_2'', f_3'') \cap A,\ s \in m^{-1}(h_1)}(\alpha)\mu_s$$

$$= \cup_{\alpha \in (f_1', f_3', f_4') \cap \chi(A),\ s \in z}(\alpha)\mu_s = (f_1', f_3', f_4')\mu_z = \{0.9, 0.3\},$$

$$\phi(\chi(P_A))(f_1'', f_2'', f_3'')(h_2) = \{0.5, 0.1\},$$

$$\phi(\chi(P_A))(f_1'', f_2'', f_3'')(h_3) = \{0.1\},$$

$$\phi(\chi(P_A))(f_1'', f_2'', f_4'')(h_1) = \{0.3\},$$

$$\phi(\chi(P_A))(f_1'', f_2'', f_4'')(h_2) = \{0.9, 0.3, 0.5\},$$

$$\phi(\chi(P_A))(f_1'', f_2'', f_4'')(h_3) = \{0.3, 0.5\},$$

$$(\phi o \chi)(P_A) = \phi(\chi(P_A) = \left\{ \begin{array}{l} \left\{ \begin{array}{l} (f''_1, f''_2, f''_4) = \{< h_1, \{0.9, 0.3\} >, \\ < h_2, \{0.5, 0.1\} >, < h_3, \{0.1\} > \end{array} \right\} \\ \left\{ \begin{array}{l} (f''_1, f''_2, f''_4) = \{< h_1, \{0.3\} >, \\ < h_2, \{0.9, 0.3, 0.5\} >, < h_3, \{0.3, 0.5\} > \end{array} \right\} \end{array} \right\}$$

Therefore, from above example we see that, if $\chi$ and $\phi$ are one-one then so is $\phi o \chi$. Similarly, for onto as well as bijections.

**Theorem 3.14.** *Let us consider three FHS mappings* $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ , $\phi : \overline{(Y, M)} \to \overline{(Z, N)}$ *and* $\varphi : \overline{(Z, N)} \to \overline{(A, O)}$ . *Then* $\varphi o(\phi o \chi) = (\varphi o \phi) o \chi$.

*Proof.* Let $P_A \in \overline{(X, L)}$, now from definition we have, $[\varphi o(\phi o \chi)](P_A) = (\varphi o \phi) o \chi(P_A)] = \varphi[\phi(\chi(P_A))]$, also $[(\varphi o \phi) o \chi](P_A) = (\varphi o \phi)(\chi(P_A)) = \varphi[\phi(\chi(P_A))]$. Hence $\varphi o(\phi o \chi) = (\varphi o \phi) o \chi$.

**Theorem 3.15.** *A FHS mapping* $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ *is said to be many one FHS mapping if two (or more than two) FHS sets in* $\overline{(X, L)}$ *have the same FHS image in* $\overline{(Y, M)}$.

**Example 3.16.** From example 3.6, consider the FHSS $Q_A \in \overline{(X,L)}$,

$$Q_A = \left\{ \begin{array}{l} \left\{ (f_1, f_3, f_4) = \left\{ \begin{array}{l} < a, \{0.5, 0.1\} >, \\ < b, \{0.9, 0.3, 0.5\} >, \\ < c, \{0.3\} > \end{array} \right\} \right\}, \\ \left\{ (f_1, f_3, f_5) = \left\{ \begin{array}{l} < a, \{0.3, 0.9\} >, \\ < b, \{0.5, 0.1\} >, \\ < c, \{0.1\} > \end{array} \right\} \right\} \end{array} \right\}$$

Then the FHS image of $P_A$ under $\chi = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(Y,M)}$ can be written as

$$\chi(Q_A) = \left\{ \begin{array}{l} \left\{ (f'_1, f'_3, f'_4) = \left\{ \begin{array}{l} \{< x, \{0.5, 0.1\} >, \\ < y, \{0.3, 0.9\} >, \\ < z, \{0\} >\} \end{array} \right\} \right\}, \\ \left\{ (f'_2, f'_3, f'_5) = \left\{ \begin{array}{l} < x, \{0.9, 0.3, 0.5\} >, \\ < y, \{0.3, 0.5\} >, \\ < z, \{0\} > \end{array} \right\} \right\} \end{array} \right\}$$

Therefore $\chi(P_A) = \chi(Q_A)$. Hence $\chi$ is many one FHS mapping.

**Definition 3.17.** Let $i = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(X,L)}$ be a FHS mapping, where $\omega : X \to X$ and $\upsilon : L \to L$. Then $\chi$ is said to be a FHS identity mapping if both $\omega$ and $\upsilon$ are identity mappings.

**Remark 3.18.** $i = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(X,L)}$ be a FHS identity mapping, then $i(P_A) = P_A$, where $P_A \in \overline{(X,L)}$.

**Definition 3.19.** Let $\chi = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(Y,M)}$ be a FHS mapping and let $i = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(X,L)}$ and $j = (r,t) : \overline{(Y,M)} \to \overline{(Y,M)}$ are FHS identity mappings then $\chi o i = \chi$ and $j o \chi = \chi$.

**Example 3.20.** Consider the following example, we consider $P_A$ from example 3.6 and consider the FHS mappings $i = (\omega, \upsilon) : \overline{(X,L)} \to \overline{(X,L)}$, where $\omega : X \to X$ and $\upsilon : L \to L$, such that $\omega(a) = a, \omega(b) = b, \omega(c) = c$, and

$$\upsilon(f_1, f_3, f_4) = (f_1, f_3, f_4), \upsilon(f_1, f_3, f_5) = (f_1, f_3, f_5)$$

$$\upsilon(f_2, f_3, f_4) = (f_2, f_3, f_4), \upsilon(f_2, f_3, f_5) = (f_2, f_3, f_4)$$

Therefore,

$$i(P_A)(f_1, f_3, f_4)(a) = \cup_{\alpha \in \upsilon^{-1}(f_1, f_3, f_4) \cap A, s \in \omega^{-1}(a)} (\alpha)\mu_s$$

$$\cup_{\alpha \in (f_1, f_3, f_4), s \in \{a\}} (\alpha)\mu_s = (f_1, f_3, f_4)\mu_a = \{0.5, 0.1\}$$

$$i(P_A)(f_1, f_3, f_4)(b) = \{0.9, 0.3, 0.5\},$$

$$i(P_A)(f_1, f_3, f_4)(c) = \{0.3\},$$

So,

$$i(P_A)(f_1, f_3, f_5)(a) = \{0.3, 0.9\},$$

$$i(P_A)(f_1, f_3, f_5)(b) = \{0.5, 0.1\},$$

$$i(P_A)(f_1, f_3, f_5)(c) = \{0.1\}$$

we get,

$$i(P_A) = \left\{ \begin{array}{l} \left\{ (f_1, f_3, f_4) = \left\{ \begin{array}{l} < a, \{0.5, 0.3\} >, \\ < b, \{0.9, 0.3, 0.5\} >, \\ < c, \{0.3\} > \end{array} \right\} \right\}, \\ \left\{ (f_1, f_3, f_5) = \left\{ \begin{array}{l} < a, \{0.3, 0.9\} >, \\ < b, \{0.5, 0.1\} >, \\ < c, \{0.1\} > \end{array} \right\} \right\} \end{array} \right\}$$

Hence $i(P_A) = P_A \Rightarrow \chi(i(P_A)) = \chi(P_A) \Rightarrow (\chi oi)(P_A) = \chi(P_A) \Rightarrow \chi oi = \chi$.
Similarly, we get $\chi(P_A) \in \overline{(X, L)}$ and $j(\chi(P_A)) = (\chi(P_A)) \Rightarrow (jo\chi)(P_A) = \chi(P_A)$.
Hence $jo\chi = \chi$.

**Definition 3.21.** A one-one onto FHS mapping $\chi = (\omega, \upsilon) : \overline{(X, L)} \rightarrow \overline{(Y, M)}$ is called FHS invertable mapping. Its FHS inverse mapping is denoted by $\chi^{-1} = (\omega^{-1}, \upsilon^{-1}) : \overline{(Y, M)} \rightarrow \overline{(X, L)}$.

**Remark 3.22.** In a FHS invertable mapping $\chi : \overline{(X, L)} \rightarrow \overline{(Y, M)}$, for $P_A \in \overline{(X, L)}, P_B \in \overline{(Y, M)}, \chi^{-1}(P_B) = P_A$, whenever $\chi(P_A) = P_B$.

**Example 3.23.** We consider $\chi(L_A) = P_B$ (see 3.13). Therefore,

$$\chi^{-1}(P_B)(f_1, f_3, f_4)(a) = (\upsilon(f_1, f_3, f_4)\mu_{\omega(a)} = (f_1', f_3', f_4')\mu_z = \{0.3, 0.9\}$$

$$\chi^{-1}(P_B)(f_1, f_3, f_4)(b) = \{0.5, 0.1\},$$

$$\chi^{-1}(P_B)(f_1, f_3, f_4)(c) = \{0.1\},$$

So,

$$\chi^{-1}(P_B)(f_1, f_3, f_5)(a) = \{0.3\},$$

$$\chi^{-1}(P_B)(f_1, f_3, f_5)(b) = \{0.9, 0.3, 0.5\},$$

$$\chi^{-1}(P_B)(f_1, f_3, f_5)(c) = \{0.3, 0.5\}$$

we get,

$$\chi^{-1}(P_B) = \left\{ \begin{array}{l} \left\{ (f_1, f_3, f_4) = \left\{ \begin{array}{l} < a, \{0.9, 0.3\} >, \\ < b, \{0.5, 0.1\} >, \\ < c, \{0.1\} > \end{array} \right\} \right\}, \\ \left\{ (f_1, f_3, f_5) = \left\{ \begin{array}{l} < a, \{0.3\} >, \\ < b, \{0.9, 0.3, 0.5\} >, \\ < c, \{0.3, 0.5, \} > \end{array} \right\} \right\} \end{array} \right\}$$

Hence, $\chi^{-1}(P_B) = L_A$.

**Theorem 3.24.** *Let $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ be a FHS invertable mapping. Therefore its FHS inverse mapping is unique.*

*Proof.* Let $\chi^{-1}$ and $\phi^{-1}$ are two FHS inverse mappings of $\chi$. Therefore, $\chi^{-1}(P_B) = P_A$, whenever $\chi(P_A) = P_B, P_A \in \overline{(X, L)}$, $P_B \in \overline{(Y, M)}$, and $\phi^{-1}(P_B) = H_A$, whenever $\phi(H_A) = P_B, H_A \in \overline{(X, L)}, P_B \in \overline{(Y, M)}$. Thus $\chi(P_A) = \phi(H_A)$. Since $\chi$ is one-one, therefore $P_A = H_A$. Hence $\chi^{-1}(P_B) = \phi^{-1}(P_B)$ i.e $\chi^{-1} = \phi^{-1}$.

3.1. *Theorem*

Let $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ and $\phi : \overline{(Y, M)} \to \overline{(Z, N)}$ are two one-one onto FHS mappings. If $\chi^{-1} : \overline{(Y, M)} \to \overline{(X, L)}$ and $\phi^{-1} : \overline{(Z, N)} \to \overline{(Y, M)}$ are FHS inverse mappings of $\chi$ and $\phi$, respectively, then the inverse of the mapping $\phi o \chi : \overline{(X, L)} \to \overline{(Z, N)}$ is the FHS mapping $\chi^{-1} o \phi^{-1} : \overline{(Z, N)} \to \overline{(X, L)}$. For more detail see fig 6

*Proof.* Obvious.

**Theorem 3.25.** *A FHS mapping $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ is invertable if and only if there exists a FHS inverse mapping $\chi^{-1} : \overline{(Y, M)} \to \overline{(X, L)}$ such that $\chi^{-1} o \chi = i_{\overline{(X,L)}}$ and $\chi o \chi^{-1} = i_{\overline{(Y,M)}}$, where $i_{\overline{(X,L)}}$ and $i_{\overline{(Y,M)}}$ is FHS identity mapping on $\overline{(X, L)}$ and $\overline{(Y, M)}$, respectively.*

*Proof.* Let $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ be a FHS invertable mapping. Therfore, by definition we have $\chi^{-1}(P_B) = P_A$, whenever $\chi(P_A) = P_B, P_A \in \overline{(X, L)}, P_B \in \overline{(Y, M)}$. Since $(\chi^{-1} o \chi)(P_A) = \chi^{-1}(\chi(P_A)) = \chi^{-1}(P_B) = P_A$. Therefore, $\chi^{-1} o \chi = i_{\overline{(X,L)}}$. Similarly, we prove that $\chi o \chi^{-1} = i_{\overline{(Y,M)}}$.

**Theorem 3.26.** *If $\chi : \overline{(X, L)} \to \overline{(Y, M)}$ and $\phi : \overline{(Y, M)} \to \overline{(Z, N)}$ are two one-one onto FHS mapping then $(\phi o \chi)^{-1} = \chi^{-1} o \phi^{-1}$.*

*Proof.* Since $\chi$ and $\phi$ are one-one onto FHS mapping, then there exists $\chi^{-1} : \overline{(Y, M)} \to \overline{(X, L)}$ and $\phi^{-1} : \overline{(Z, N)} \to \overline{(Y, M)}$ such that $\chi^{-1}(P_B) = P_A$, whenever $\chi(P_A) = P_B, P_A \in \overline{(X, L)}, P_B \in \overline{(Y, M)}$, and $\phi^{-1}(H_C) = P_B$, whenever $\phi(P_B) = H_C, H_C \in \overline{(Z, N)}, P_B \in \overline{(Y, M)}$. Therefore, $(\phi o \chi)(P_A) = \phi[\chi(P_A)] = \phi(P_B) = H_C$. As $\phi o \chi$ is one-one onto, therefore

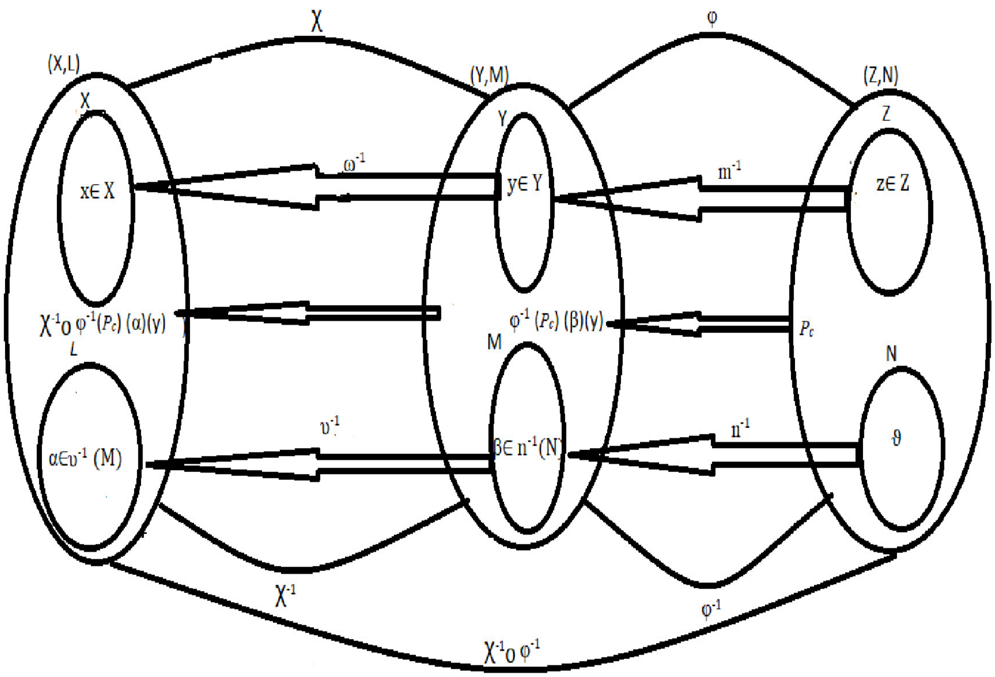


FIGURE 6. Representation of Composite Fuzzy Hypersoft Inverse Mapping

$(\phi o \chi)^{-1}$ exists such that $(\phi o \chi)(P_A) = H_C \Rightarrow (\phi o \chi)^{-1}(H_C) = P_A$. Also $(\phi^{-1} o \phi^{-1})(H_C) = \chi^{-1}[\phi^{-1}(H_C)] = \chi^{-1}(P_B) = P_A$. Hence $(\phi o \chi)^{-1}(H_C) = (\chi^{-1} o \phi^{-1})(H_C) \Rightarrow (\phi o \chi)^{-1} = \chi^{-1} o \phi^{-1}$.

## 4. **Conclusion**

A basic structure of composite mapping of FHS classes is established along with generalization of certain properties and results. It is very helpful for solving problems involving uncertainty and vagueness. In the future, we will expand our exploration in the domain of Neutrosophic Hypersoft Set, Plithogenic Crisp Hypersoft Set, Plithogenic Fuzzy Hypersoft Set, Plithogenic Intuitionistic Fuzzy Hypersoft Set, Plithogenic Neutrosophic Hypersoft Set, complex Intuitionistic Fuzzy Hypersoft Set, complex Neutrosophic Hypersoft Set, and their hybrid structures. We will apply them in medical imaging issues, pattern recognition, recommender frameworks, social, the monetary framework estimated thinking, image processing, and game theory.

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