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Solving Neutrosophic Linear Programming Problems Using Exterior Point Simplex Algorithm

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Abstract: In this manuscript, three contributions are proposed. First contribution is proposing a good evaluation between the fuzzy and neutrosophic approaches using a novel fuzzy-neutrosophic transfer. Second contribution is introducing a general framework for solving the neutrosphic linear programming problems using the advantages of the method of Abdel-Basset et al. and the advantages of Singh et al.'s method. Third contribution is proposing a new neutrosophic exterior point simplex algorithm NEPSA and its fuzzy version FEPSA. NEPSA has two paths to get optimal solutions. One path consists of basic not feasible solutions but the other path is feasible. Finally, the numerical examples and results analysis show that NEPSA more than accurate FEPSA.

Keywords: Fuzzy Linear Programming; Ranking Function; Trapezoidal Fuzzy Number; Trapezoidal Neutrosophic Number; Exterior Point Simplex Algorithm.

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1. Introduction

Fuzzy sets were introduced by Zadeh [20] to handle vague and imprecise information. But also fuzzy set does not represent vague and imprecise information efficiently, because it considers only the truthiness function. After then, Atanassov [3] introduced the concept of intuitionistic fuzzy set to handle vague and imprecise information, by considering both the truth and falsity function. But also intuitionistic fuzzy set does not simulate human decision making process. Because the proper decision is fundamentally a problem of arranging and explicate facts the concept of neutrosophic set theory was presented by Smarandache, to handle vague, imprecise and inconsistent information [9,10,11,12]. Neutrosophic set theory simulates decision-making process of humans, by considering all aspects of decision-making process. Neutrosophic set is a popularization of fuzzy and intuitionistic fuzzy sets; each element of set had a truth, indeterminacy and falsity membership function. So, neutrosophic set can assimilate inaccurate, vague and maladjusted information efficiently and effectively [18, 19].

The first EPSA was developed by Paparrizos for the assignment problem [27]. Later, Paparrizos generalized EPSA to the general LP [28]. Primal-dual versions of the algorithm are discussed in [29,30]. From the geometry of EPSA, In particular, EPSA proved to be up to ten times faster than simplex algorithm on randomly generated optimal LPs of medium size.

EPSA constructs two paths to the optimal solution. One path consists of basic but not feasible solutions; so this is an “exterior path”. The second path is feasible. It consists of line segments, the endpoints of which lie on the boundary of the feasible region. EPSA relies on the idea that making steps in directions that are linear combinations of attractive descent directions which can lead to faster practical convergence than that achievable by simplex algorithm. Although EPSA outperforms clearly the original simplex algorithm (on randomly generated dense and sparse LPs) it has two computational disadvantages. Firstly, it is difficult to construct "good moving directions". We use the term "good moving direction" loosely. A good moving direction is a direction that makes the algorithm efficient in practice. Geometrically a good moving direction is a direction that comes close to the optimal solution. In fact the two paths depend on the initial feasible segment (direction) and the initial feasible vertex. Secondly, there is no known way of moving into the interior of the feasible region. This movement will provide more flexibility in the search for computationally good directions.

Badr *et al* [8] proposed a new method to solve the fuzzy linear programming problem. It is called fuzzy exterior point simplex algorithm (FEPSA). It constructs two ways to get the optimal solution. One path consists of basic not feasible solutions. The second way is feasible.

For more details about the linear programming, the reader can refer to [13,5,4,6]. On the other hand, for more details about the fuzzy linear programming, the reader is referred to [7]. Finally, for more details about the neutrosophic linear programming, the reader may refer to [2,14,15,16,17,24,25,26,31].

The remaining parts of this research are organized as follows: In sect. 2, we introduce the basic concepts of fuzzy and neutrosophic sets and a new technique which converts the fuzzy representation to the neutrosophic representation. The fuzzy rank functions and it corresponding neutrosophic rank functions are proposed in Sec. 3. In Sec. 4, we propose Singh et al.'s modifications [32] and the proposed modification for primal neutrosophic simplex method and a new neutrosophic exterior point simplex algorithm NEPSA. In Sec. 5, we propose two numerical examples that show the importance of the proposed modification for primal neutrosophic simplex method and they show the superiority of the proposed algorithm NEPSA. Finally, we introduce the future work and conclusions in Sec. 6.

2. Preliminaries

In this section, we introduce three subsections. First one is representation of the fuzzy numbers. Second is the representation of the neutrosophic numbers. Finally, we show that how to move from fuzzy representation to neutrosophic representation. In other words, how do to convert the fuzzy numbers to neutrosophic numbers.

2.1 Fuzzy Representation

We review the fundamental notions of fuzzy set theory, initiated by Bellman and Zadeh [20].

2.1.1 Definition: A convex fuzzy set $\widetilde{A}$ on $\mathbb{R}$ is a fuzzy number if the following conditions hold:

- Its membership function is piecewise continuous.
- There exist three intervals [*a*, *b*], [*b*, *c*], [*c*, *d*] such that μ_a is increasing on [*a*, *b*], equal to 1 on [*b*, *c*], decreasing on [*c*, *d*] and equal to 0 elsewhere.

2.1.2 Definition: Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ denote the trapezoidal fuzzy number, where $(a^L - \alpha, a^U + \beta)$ is the support of ã and $[a^L, a^U]$ its core.

Remark 1*:* We denote the set of all trapezoidal fuzzy numbers by *F(*$\mathbb{R}$*).*

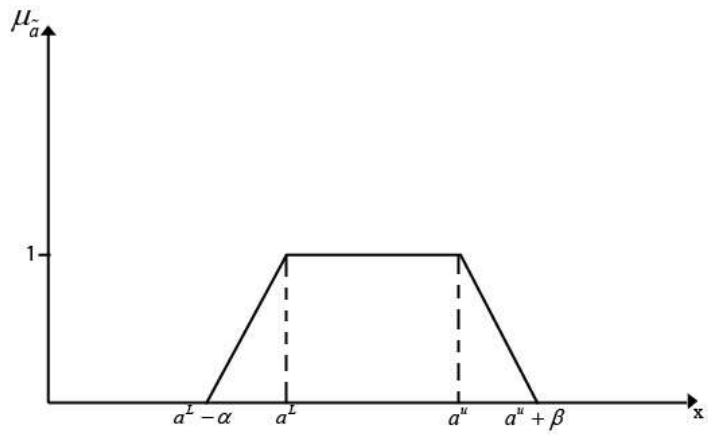

Figure 1. Truth membership function of trapezoidal fuzzy numbers

We next define arithmetic on trapezoidal fuzzy numbers. Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$be two trapezoidal fuzzy numbers. Define:

$x\tilde{a} = (xa^L, xa^U, x\alpha, x\beta):\ \ x > 0, \quad x \in \mathbb{R};$

$x\tilde{a} = (xa^U, xa^L, -x\beta, -x\alpha): x < 0, \quad x \in \mathbb{R};$

$\tilde{a}+\tilde{b}=(a^L, a^U, \alpha, \beta)+ (b^L, b^U, \gamma, \theta) = [a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta]$

We point out that the arithmetic on trapezoidal fuzzy numbers follows the Extension Principle which is discussed in [22].

2.2 Neutrosophic Representation

In this subsection, some of basic definitions in the neutrosophic set theory are introduced:

2.2.1 Definition [1]: A single-valued neutrosophic set N which is a subset of X is defined as follows: $N = \{< x, T_N(x), I_N(x), F_N(x) >: x \in X\}$where X is a universe of discourse, $T_N(x): X \to [0,1]$, $I_N(x): X \to [0,1]$ and $F_N(x): X \to [0,1]$ with $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$ for all$x \in X$, $T_N(x), I_N(x)$ and $F_N(x)$represent truth membership, indeterminacy membership and falsity membership degrees of x to N.

2.2.2 Definition [1]: The trapezoidal neutrosophic number $\tilde{A}$ is a neutrosophic set in R with the following truth, indeterminacy and falsity membership functions:

$$T_{\tilde{A}}(x) = \begin{cases} \frac{\alpha_{\tilde{A}}(x-a\ \ )}{a_2-a_1}: a_1 \le x \le a_2 \\ \alpha_{\tilde{A}} \quad : a_2 \le x \le a_3 \\ \alpha_{\tilde{A}}(\frac{x-a_3}{a_4-a_3}) : a_3 \le x \le a_4 \\ 0 \quad otherwise \end{cases} \quad I_{\tilde{A}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{A}}(x-a_1'))}{a_2-a_1'}: a_1' \le x \le a_2 \\ \theta_{\tilde{A}} \quad : a_2 \le x \le a_3 \\ \frac{(x-a_3+\theta_{\tilde{A}}(a_4'-x))}{a_4'-a_3}: a_3 \le x \le a_4' \\ 1 \quad otherwise \end{cases}$$

$$F_{\tilde{A}}(x)=\begin{cases}\frac{(a_2-x+\beta_{\tilde{A}}(x-a_1^{"}))}{a_2-a_1^{"}} & : a_1^{"}\le x\le a_2\\ \beta_{\tilde{A}} & : a_2\le x\le a_3\\ \frac{(x-a_3+\beta_{\tilde{A}}(a_4^{"}-x))}{a_4^{"}-a_3} & : a_3\le x\le a_4^{"}\\ 1 & otherwise\end{cases}$$

Where $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy and minimum degree of falsity, respectively, $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}} \in [0,1]$. The membership functions of trapezoidal neutrosophic number are shown in Fig. 2. It is clear that $a_1^{"} < a_1 < a_1' < a_2 < a_3 < a_4' < a_4 < a_4^{"}$.

Remark 2: Here $T_{\tilde{A}}(x)$ increases with a constant rate for [a_1, a_2] and decreases with a constant rate for [a_3, a_4]. $F_{\tilde{A}}(x)$decreases with a constant rate for [a_1'', a_2] and increases with a constant rate for [a_3, a_4'']. $I_{\tilde{A}}(x)$increases and decreases with a constant rate for [a_1', a_2] simultaneously, and it decreases and increases with a constant rate for [a_3, a_4'] simultaneously.

Remark 3: If $a_2 - a_1 = a_4 - a_3$ the trapezoidal neutrosophic number is called the symmetric trapezoidal neutrosophic number.

2.2.3 Definition [1]: Let $\tilde{A} = \ < a_1, a_2, a_3, a_4; \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} >$ and $\tilde{B} = \ < b_1, b_2, b_3, b_4; \alpha_{\tilde{B}}, \theta_{\tilde{B}}, \beta_{\tilde{B}} >$ are two trapezoidal neutrosophic numbers, then the mathematical operations are presented as follows:

$$\tilde{A}+\tilde{B} = \ < (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} >$$

$$\tilde{A}-\tilde{B} = \ < (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} >$$

$$\tilde{A}^{-1} = \ < (\tfrac{1}{a_4}, \tfrac{1}{a_3}, \tfrac{1}{a_2}, \tfrac{1}{a_1}); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} > \text{where}(\tilde{A} \neq 0)$$

$$\lambda\tilde{A} = \begin{cases} < \lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} >: & \lambda > 0\\ < \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} >: & \lambda < 0\end{cases}$$

$$\tilde{A}\tilde{B} = \begin{cases} < (a_1b_1, a_2b_2, a_3b_3, a_4b_4); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > & if\ (a_4 > 0, b_4 > 0)\\ < (a_1b_4, a_2b_3, a_3b_2, a_4b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > & if\ (a_4 < 0, b_4 > 0)\\ < (a_4b_4, a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > & if\ (a_4 < 0, b_4 < 0)\end{cases}$$

$$\frac{\tilde{A}}{\tilde{B}} = \begin{cases} < (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > & if\ (a_4 > 0, b_4 > 0)\\ < (\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > & if\ (a_4 < 0, b_4 > 0)\\ < (\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}); \alpha_{\tilde{A}} \wedge \alpha_{\tilde{A}}, \theta_{\tilde{A}} \vee \theta_{\tilde{A}}, \beta_{\tilde{A}} \vee \beta_{\tilde{A}} > & if\ (a_4 < 0, b_4 < 0)\end{cases}$$

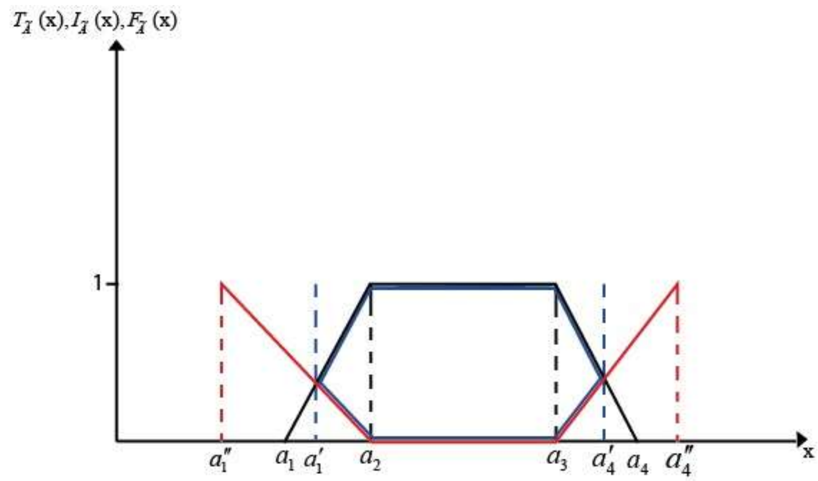

Figure 2. Truth, indeterminacy and falsity membership functions of trapezoidal neutrosophic numbers

2.3 Fuzzy-Neutrosophic Transformation

The main goal of this subsection is to explain how to convert fuzzy numbers representation into neutrosophic numbers representation. This transformation is used for simplicity and fair comparison between them. It is known that there are many rank functions for ordering the fuzzy and neutrosophic numbers. We emphasize using the same function for both fuzzy numbers and neutrosophic numbers to obtain a fair comparison between them. Here we also explain how to apply this technique.

From Figure 1 and Figure 2 we can illustrate the following relations between the two representations:

$$a_1 = a_2 - \alpha,\ a_2 = a^L,\ a_3 = a^U \text{ and } a_4 = a_3 + \beta \quad (1)$$

Assuming that the rank function is used for ordering the fuzzy numbers as follows:

$$R(\tilde{a}) = a^l + a^u + \frac{\beta-\alpha}{2} \quad (2)$$

From relations (1) we express the rank function to be used for ordering the neutrosophic numbers as follows:

$$R(\tilde{a}) = \frac{1}{2}\sum_{i=1}^{4} \tilde{a}_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \quad (3)$$

From (1), we can convert fuzzy numbers representation into neutrosophic numbers representation. On the other hand from (2) and (3), we can use the same function for both fuzzy numbers and neutrosophic numbers to obtain a fair comparison between them.

3. Rank Functions

Assuming that $T_{\tilde{A}} = 1$, $I_{\tilde{A}} = 0$, $\tilde{F}_A = 0$,then the TrNN $\tilde{a} =< a_1, a_2, a_3, a_4; T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}} >$ will be transformed into a trapezoidal fuzzy number $\tilde{a} =< a_1, a_2, a_3, a_4; 1,0,0 >$ and hence, in this case:

- The expression $R(\tilde{a}) = \frac{1}{2}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$ is equivalent to the expression $R(\tilde{a}) = \frac{1}{2}\sum_{j=1}^{4} a_i + 1$

Furthermore, it well be known that if $a_1 = a_2 = a_3 = a_4$ then the trapezoidal fuzzy number $\tilde{A} =< a_1, a_2, a_3, a_4; 1,0,0 >$ will be transformed into a real number *A = (a, a, a, a; 1, 0, 0)* and hence, in this case:

- The expression $R(\tilde{a}) = \frac{1}{2}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$ is equivalent to the expression $R(A) = 2a + 1 \neq a$

Table 1. The rank function and it corresponding neutrosophic rank function

No	Fuzzy Rank Function	Corresponding Neutrosophic Rank Function	Rank function of constraints
1	$R(\tilde{a}) = (a^l + a^u + \frac{\beta - \alpha}{2})$	$R(\tilde{a}) = \frac{1}{2}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$	$R(a) = 2a + 1$
2	$R(\tilde{a}) = (\frac{a^l + a^u}{2})$	$R(\tilde{a}) = \left(\frac{a_2 + a_3}{2}\right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$	$R(a) = a + 1$
3	$R(\tilde{a}) = (\frac{a^l + a^u}{2} + \frac{\beta - \alpha}{4})$	$R(\tilde{a}) = \frac{1}{4}\sum_{j=1}^{4} a_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$	$R(a) = a + 1$

4. Algorithms

In this section; we first present Singh *et al.*'s modifications [32] and the proposed modification about the mathematical incorrect assumptions, considered by Abdel-Basset *et al*. [1] in their proposed method to convert from neutrosophic numbers into real numbers**.** Second, we propose a new Exterior point simplex algorithm. Finally, we develop this algorithm in order to solve linear programming with neutrosophic numbers.

4.1. General Framework for Solving Neutrosphic Linear Programming Problems

The main objective of this section is to remove the confusion among readers regarding the contributions of Abdel-Basset et al. and the contributions of Singh et al. In this paper, we present a general framework for solving neotrosophic linear programming problems using the advantages of the method of Abdel-Basset et al. and the advantages of Singh et al.'s method.

In 2019, Abdel-Basset et al. [1] presented a simple and effective model for solving neutrosophic linear programming problems supported by a set of numerous examples and a comparison between their approaches presented and solving these examples using the fuzzy method. Consequently, Abdel-Basset et al were able to prove the effectiveness of his approach in solving neutrosophic linear programming problems. On the other hand, Singh et al, 2019 [32] introduced modifications to Abdel-Basset model. These modifications summarized in how neutrosophic numbers are converted into real numbers.

In order to illustrate the method of each of them in solving neutrosophic linear programming problems, we assume the general form of neutrosophic linear programming problems as follows:

$$max \setminus min \ \left[\tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j\right]$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ \le, \ge, = \tilde{b}_j \ , \ \ i = 1,2, \dots \dots, m; \ \ x_j \ge 0 \ , \ j = 1,2, \dots \dots, n.$$

Model (1) illustrates the method of Abdel-Basset et al in converting neutrosophic numbers into deterministic numbers (in the objective function)

$$max \setminus min \ \left[R(\tilde{z}) = \sum_{j=1}^{n} R(\tilde{c}_j) x_j\right]$$

While model (2) illustrates the method of Singh et al. in converting neutrosophic numbers into deterministic numbers (in the objective function)

$$max \setminus min \ \left[R(\tilde{z}) = R(\sum_{j=1}^{n} \tilde{c}_j x_j\right]$$

In fact, there is a complete match between the method presented by Abdel-Basset et al and the method presented by Singh et al. In the case of converting fuzzy numbers to real numbers because $R(\tilde{A}_1 \oplus \tilde{A}_2) = R(\tilde{A}_1) + R(\tilde{A}_2)$ where $\tilde{A}_1$ *and* $\tilde{A}_2$ are fuzzy numbers.

On the other hand, when converting neutrosophic numbers to real numbers, the proposed method presented by Singh et al. is more accurate than the method suggested by Abdel-Basset et al. mathematically, because $R(\tilde{A}_1 \oplus \tilde{A}_2) \neq R(\tilde{A}_1) + R(\tilde{A}_2)$ where $\tilde{A}_1 and \tilde{A}_2$ are neutrosophic numbers.

Lemma 1: Let $\tilde{A}_1$ *and* $\tilde{A}_2$ are fuzzy numbers then $\boldsymbol{R(\tilde{A}_1 \oplus \tilde{A}_2) = R(\tilde{A}_1) + R(\tilde{A}_2)}$

Proof:

Suppose that $\tilde{A}_1 = (a_1^l, a_1^u, \propto_1, \beta_1)$ and $\tilde{A}_2 = (a_2^l, a_2^u, \propto_2, \beta_2)$ are two Trapezoidal fuzzy numbers as shown in Figure 1, and the used rank function is defined as follows: $R(\tilde{A}) = \frac{a^L + a^U}{2} + \frac{\beta - \alpha}{4}$

$R(\tilde{A}_1 \oplus \tilde{A}_2)$= R $((a_1^l + a_2^l), (a_1^u + a_2^u), (\propto_1 + \propto_2), (\beta_1 + \beta_2))$ = $\frac{a_1^l + a_2^l + a_1^u + a_2^u}{2} + \frac{\beta_1 + \beta_2 - \propto_1 - \propto_2}{4}$ (4)

While,

$R(\tilde{A}_1) + R(\tilde{A}_2) = \frac{a_1^l + a_1^u}{2} + \frac{\beta_1 + \propto_1}{4} + \frac{a_2^l + a_2^u}{2} + \frac{\beta_2 + \propto_2}{4} = \frac{a_1^l + a_2^l + a_1^u + a_2^u}{2} + \frac{\beta_1 + \beta_2 - \propto_1 - \propto_2}{4}$ (5)

It is obvious from (4) and (5) that $R(\tilde{A}_1 \oplus \tilde{A}_2) = R(\tilde{A}_1) + R(\tilde{A}_2)$

Lemma 2: Let $\widetilde{\mathrm{A}}$ *and* $\widetilde{\mathrm{B}}$ are neutrosophic numbers then $\boldsymbol{R(\widetilde{\mathrm{A}} \oplus \widetilde{\mathrm{B}}) \neq R(\widetilde{\mathrm{A}}) + R(\widetilde{\mathrm{B}})}$

Proof:

Suppose that $\widetilde{\mathrm{A}} = (\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \mathrm{a}_4, \mathrm{T}_{\widetilde{\mathrm{A}}}, \mathrm{I}_{\widetilde{\mathrm{A}}}, \mathrm{F}_{\widetilde{\mathrm{A}}})$ and $\widetilde{\mathrm{B}} = (\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4, \mathrm{T}_{\widetilde{\mathrm{B}}}, \mathrm{I}_{\widetilde{\mathrm{B}}}, \mathrm{F}_{\widetilde{\mathrm{B}}})$ are two Trapezoidal neutrosophic numbers as shown in Fig. 2 and the used rank function is defined as follows:

$$\mathrm{R}(\widetilde{\mathrm{A}}) = \frac{\mathrm{a}_1 + \mathrm{a}_4 + 2\,(\mathrm{a}_2 + \mathrm{a}_3)}{2}$$

$\mathrm{R}(\widetilde{\mathrm{A}} \oplus \widetilde{\mathrm{B}}) = R\,((\mathrm{a}_1 + \mathrm{b}_1), (\mathrm{a}_2 + \mathrm{b}_2), (\mathrm{a}_3 + \mathrm{b}_3), (\mathrm{a}_4 + \mathrm{b}_4);$ min $\{\mathrm{T}_{\widetilde{\mathrm{A}}}, \mathrm{T}_{\widetilde{\mathrm{B}}}\}$, max $\{\mathrm{I}_{\widetilde{\mathrm{A}}}, \mathrm{I}_{\widetilde{\mathrm{B}}}\}$, max $\{\mathrm{F}_{\widetilde{\mathrm{A}}}, \mathrm{F}_{\widetilde{\mathrm{B}}}\}$

$= \frac{\mathrm{a}_1 + \mathrm{b}_1 + \mathrm{a}_4 + \mathrm{b}_4 + 2(\mathrm{a}_2 + \mathrm{b}_2 + \mathrm{a}_3 + \mathrm{b}_3)}{2}$ + min $\{\mathrm{T}_{\widetilde{\mathrm{A}}}, \mathrm{T}_{\widetilde{\mathrm{B}}}\}$ - max $\{\mathrm{I}_{\widetilde{\mathrm{A}}}, \mathrm{I}_{\widetilde{\mathrm{B}}}\}$ - max $\{\mathrm{F}_{\widetilde{\mathrm{A}}}, \mathrm{F}_{\widetilde{\mathrm{B}}}\}$) (6)

On the other hand,

$\mathrm{R}(\widetilde{\mathrm{A}}) + \mathrm{R}(\widetilde{\mathrm{B}}) = \frac{\mathrm{a}_1 + \mathrm{a}_4 + 2\,(\mathrm{a}_2 + \mathrm{a}_3)}{2} + (\mathrm{T}_{\widetilde{\mathrm{A}}} - \mathrm{I}_{\widetilde{\mathrm{A}}} - \mathrm{F}_{\widetilde{\mathrm{A}}}) + \frac{\mathrm{b}_1 + \mathrm{b}_4 + 2\,(\mathrm{b}_2 + \mathrm{b}_3)}{2} + (\mathrm{T}_{\widetilde{\mathrm{B}}} - \mathrm{I}_{\widetilde{\mathrm{B}}} - \mathrm{F}_{\widetilde{\mathrm{B}}})$

$= \frac{\mathrm{a}_1 + \mathrm{b}_1 + \mathrm{a}_4 + \mathrm{b}_4 + 2(\mathrm{a}_2 + \mathrm{b}_2 + \mathrm{a}_3 + \mathrm{b}_3)}{2}$ + min $\{\mathrm{T}_{\widetilde{\mathrm{A}}}, \mathrm{T}_{\widetilde{\mathrm{B}}}\}$ - max $\{\mathrm{I}_{\widetilde{\mathrm{A}}}, \mathrm{I}_{\widetilde{\mathrm{B}}}\}$ - max $\{\mathrm{F}_{\widetilde{\mathrm{A}}}, \mathrm{F}_{\widetilde{\mathrm{B}}}\}$) (7)

It is obvious from (6) and (7) that $\mathrm{R}(\widetilde{\mathrm{A}} \oplus \widetilde{\mathrm{B}}) \neq \mathrm{R}(\widetilde{\mathrm{A}}) + \mathrm{R}(\widetilde{\mathrm{B}})$

Remark 4:

Other considerations were not discussed by Singh et al. such as:

1. Abdel-Basset et al. [1] used the rank function for the maximization problems of NLP, and used another rank function for the minimization problems, which means that he used the two rank functions in his proposed model.

2. Abdel-Basset et al [1], compared his proposed model with other models, using different rank functions, thus the comparison is unfair.

Section 2.3 addressed these considerations by finding a relationship between the representation of fuzzy numbers and the representation of neutrosophic numbers.

Now, we can introduce a general framework for solving the linear programming problems using neutrosophic numbers as follows:

Step 1: neutrosophic or uncertain information is generally processed by transforming into an accurate or crisp number by using the same ranking function for maximization and minimization problem for both fuzzy numbers and neutrosophic numbers to obtain a fair comparison between them using the method suggested by Singh et al. [32].

All parameters are represented by trapezoidal neutrosophic numbers, except variables are exemplified only by real values.

$$max \setminus min\left[\sum_{j=1}^{n} = \tilde{c}_j x_j\right]$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq, \geq, = \tilde{b}_j$$
$$i = 1,2,\ldots\ldots, m; \; x_j \geq 0, \; j = 1,2,\ldots\ldots, n \qquad (8)$$

The Equation (8) can be transformed into Exact crisp linear programming problem

$$Max\,/Min\,[\sum_{j=1}^{n} R(\tilde{c}_j x_j) - \sum_{j=1}^{n} T_{\tilde{c}_j} x_j + \sum_{j=1}^{n} I_{\tilde{c}_j} x_j + \sum_{j=1}^{n} F_{\tilde{c}_j} x_j + \min_{1\leq j\leq n}\left\{T_{\tilde{c}_j} x_j\right\} - \max_{1\leq j\leq n}\left\{I_{\tilde{c}_j} x_j\right\} - \max_{1\leq j\leq n}\left\{F_{\tilde{c}_j} x_j\right\}]$$

s.t.

$$(\sum_{j=1}^{n} R(\tilde{a}_{ij}) x_j) + 1 \leq, \geq, = R(\tilde{b}_j), \quad i = 1,2,\ldots\ldots, m;$$
$$x_j \geq 0, \; j = 1,2,\ldots\ldots, n.(2)$$

This transformation can happen at the beginning of the decision process, or in the middle or final stage.

Step 2: Let $\tilde{A} = (a_1, a_2, a_3, a_4, T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}})$ be a trapezoidal neutrosophic number, where a_1, a_2, a_3, a_4; are lower bound, first, second median value and upper bound for trapezoidal neutrosophic number, respectively. Also $T_{\tilde{A}}, I_{\tilde{A}}, F_{\tilde{A}}$ are the truth, indeterminacy and falsity degree of trapezoidal neutrosophic number. Ranking function for this trapezoidal neutrosophic number is as follows:

$$R(\tilde{A}) = \frac{a_1 + a_4 + 2\,(a_2 + a_3)}{2} + \text{Confirmation degree}$$

Mathematically, this function can be written as follows:

$$R(\tilde{A}) = \frac{a_1 + a_4 + 2\,(a_2 + a_3)}{2} + (T_{\tilde{A}} - I_{\tilde{A}} - F_{\tilde{A}})$$

Step 3: Solve the crisp model using the standard method and obtain the optimal solution of problem

Table 2. Singh et al.'s modifications.

no	NLPP- (Type)	NLPP- (Form)	Exact Crisp LPP
1	The coefficients of the objective function are represented by trapezoidal neutrosophic numbers	$max\backslash\min\ \ z = [\sum_{j=1}^{n} \tilde{c}_j x_j]$ s.t $\sum_{j=1}^{n} a_{ij}x_j \ \le , \ge , = b_j$, $i = 1,2, \dots \dots , m;$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$	$Max\ /Min\ z = [\sum_{j=1}^{n} R(\tilde{c}_j x_j) - \sum_{j=1}^{n} T_{\tilde{c}_j} x_j + \sum_{j=1}^{n} I_{\tilde{c}_j} x_j + \sum_{j=1}^{n} F_{\tilde{c}_j} x_j + \min_{1\le j\le n}\{T_{\tilde{c}_j} x_j\} - \max_{1\le j\le n}\{I_{\tilde{c}_j} x_j\} - \max_{1\le j\le n}\{F_{\tilde{c}_j} x_j\}]$ $s.t.\ \ \sum_{j=1}^{n} a_{ij}x_j \ \le , \ge , = b_j$, $i = 1,2, \dots \dots , m;$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$
2	The coefficients of constraints variables and right hand side are represented by trapezoidal neutrosophic numbers	$\max\backslash\ \min\ z = [\sum_{j=1}^{n} c_j x_j]$ s.t. $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ \le , \ge , = \tilde{b}_j$, $i = 1,2, \dots \dots , m;$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$	Max / min $z = \sum_{j=1}^{n} c_j x_j$ $s.t.\ \ [\sum_{j=1}^{n} R(\tilde{a}_{ij} x_j) - \sum_{j=1}^{n} T_{\tilde{a}_{ij}} x_j + \sum_{j=1}^{n} I_{\tilde{a}_{ij}} x_j + \sum_{j=1}^{n} F_{\tilde{a}_{ij}} x_j + \min_{1\le j\le n}\{T_{\tilde{a}_{ij}} x_j\} - \max_{1\le j\le n}\{I_{\tilde{a}_{ij}} x_j\} - \max_{1\le j\le n}\{F_{\tilde{a}_{ij}} x_j\}] \le , \ge , = R(\tilde{b}_i)$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$
3	All parameters are represented by trapezoidal neutrosophic numbers, except variables are exemplified only by real values	$max\ \backslash\ \min z = [\sum_{j=1}^{n} \tilde{c}_j x_j]$ s.t. $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ \le , \ge , = \tilde{b}_j$, $i = 1,2, \dots \dots , m;$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$	$Max\ /Min\ z = [\sum_{j=1}^{n} R(\tilde{c}_j x_j) - \sum_{j=1}^{n} T_{\tilde{c}_j} x_j + \sum_{j=1}^{n} I_{\tilde{c}_j} x_j + \sum_{j=1}^{n} F_{\tilde{c}_j} x_j + \min_{1\le j\le n}\{T_{\tilde{c}_j} x_j\} - \max_{1\le j\le n}\{I_{\tilde{c}_j} x_j\} - \max_{1\le j\le n}\{F_{\tilde{c}_j} x_j\}]$ s.t. $(\sum_{j=1}^{n} R(\tilde{a}_{ij}) x_j) + 1 \ \le , \ge , = R(\tilde{b}_j)$, $i = 1,2, \dots \dots , m;$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$
4	The coefficients of objective function and constraints variables are represented by real numbers and right hand side are represented by trapezoidal neutrosophic numbers	$\max\backslash\ \min z = [\sum_{j=1}^{n} c_j x_j]$ s.t. $\sum_{j=1}^{n} a_{ij} x_j \ \le , \ge , = \tilde{b}_j$, $i = 1,2, \dots \dots , m;$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$	Max / min $\sum_{j=1}^{n} c_j x_j$ $s.t.$ $R\left[\sum_{j=1}^{n} (a_{ij} x_j)\right] \le, \ge, = R(\tilde{b}_i)$ $x_j \ge 0$, $j = 1,2, \dots \dots , n.$

Remark 5: If $R(a) = a + 1$ and the coefficients of the objective function & constraints variables are real, then the fuzzy linear programming problem is equivalent to the neutrosophic linear programming problem.

- NLPP: neutrosophic linear programming problem.

4.2 A novel neutrosophic Exterior Point Simplex Algorithm (NEPSA)

Badr *et al* [8] proposed a fuzzy exterior point simplex algorithm (FEPSA) for solving the linear programming problems with fuzzy numbers. In this section, we propose a new algorithm which solves linear programming with neutrosophic numbers (Neutrosophic exterior point simplex algorithm NEPSA).

Neutrosophic Exterior Point Simplex Algorithm (NEPSA)

Step0: (Initialization)

- Transfer fuzzy numbers into neutrosophic numbers (see section 3)
- Apply the general framework (see section 4)
- Start with a feasible basic point and construct the corresponding tableau exterior simplex.

Step1: (Test of termination)

Find the set $J_- = \{j: \tilde{a}_{0j} \underset{R}{<} \tilde{0}\}$. If $J_- = \Phi$, STOP. The problem is optimal.

Otherwise, calculate $\tilde{a}_{00} = \sum_{j\in J_-} \tilde{a}_{ij}$ and $a_{i0} = \sum_{j\in J_-} a_{ij}$ where $i = 1, 2, \ldots, m$

Step2: (Choice of entering variable)

Find the set $I_+ = \{i: a_{i0} > 0\}$. If $I_+ = \Phi$, STOP. The problem is unbounded.

Otherwise, determine the index of entering variable r from the relation :

$$\frac{b_r}{a_{r0}} = min\left\{\frac{b_j}{a_{r0}} : i \in I_+\right\}$$

Step3: (Choice of leaving variable)

Put $J_+ = \{j: \tilde{a}_{0j} > \tilde{0}\}$ and calculate

$$\theta_1 = \frac{-\tilde{a}_{0k}}{a_{rk}} = min\left\{\frac{-\tilde{a}_{0j}}{a_{rj}} : j \in J_-, a_{rj} > 0\right\}$$

$$\theta_2 = \frac{-\tilde{a}_{0l}}{a_{rl}} = min\left\{\frac{-\tilde{a}_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\}$$

Find the index of the leaving variable s, if $\theta_1 \le \theta_2$ put $s = k$ otherwise $s = l$.

Step4: (Pivoting)

Form the next tableau by the pivoting variable a_{rs} and go to Step1

5. Numerical Examples and Results Analysis

In this section, two benchmark examples (P1 and P2) are proposed to compare between the proposed algorithm NEPSA and its fuzzy version FEPSA.

Table 3. Special fuzzy linear programming from different references

Problem No.	Problem object function and constrained	Reference
P1	$Max\ \tilde{z} = (2,4,2,6)x_1 + (2,6,1,3)x_2 + (1,3,1,3)x_3$ s.t $x_1 + x_2 + 2x_3 \leq 2$ $2x_1 + 3x_2 + 4x_3 \leq 3$ $6x_1 + 6x_2 + 2x_3 \leq 8$ $x_1, x_2, x_3 \geq 0$	[8]
P2	$Max\ \tilde{z} = (13,15,2,2)x_1 + (12,14,3,3)x_2 + (15,17,2,2)x_3$ s.t. $12x_1 + 13x_2 + 12x_3 \leq (475,505,6,6)$ $14x_1 + 13x_3 \leq (460,480,8,8)$ $12x_1 + 15x_2 \leq (465,495,5,5)$ $x_1, x_2, x_3 \geq 0$	[21]

5.1 Example 1 (P1) [8] :

Consider the following linear programming problem

$$Max\ \tilde{z} = (2,4,2,6)x_1 + (2,6,1,3)x_2 + (1,3,1,3)x_3$$
s.t
$$x_1 + x_2 + 2x_3 \leq 2$$
$$2x_1 + 3x_2 + 4x_3 \leq 3$$
$$6x_1 + 6x_2 + 2x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0$$

First: We will convert the fuzzy numbers into neutrosophic numbers

Then, using the following rank function:

$$R(\check{a}) = \frac{1}{2}\sum_{i=1}^{4} \tilde{a}_i + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$$
$$R(a) = 2a + 1$$

$$Max\ z = R[(0,2,4,10)]x_1 + R[(1,2,6,9)]x_2 + R[(0,1,3,6)]x_3$$
s.t.
$$x_1 + x_2 + 2x_3 \leq 2$$
$$2x_1 + 3x_2 + 4x_3 \leq 3$$
$$6x_1 + 6x_2 + 2x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0$$

Putting the last formula into the standard form, we have:

$$Max\ z = 9x_1 + 10x_2 + 6x_3$$
s.t.
$$x_1 + x_2 + 2x_3 + x_4 = 2$$
$$2x_1 + 3x_2 + 4x_3 + x_5 = 3$$
$$6x_1 + 6x_2 + 2x_3 + x_6 = 8$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Step (0): we construct the initial tableau of exterior simplex:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		-9	-10	-6	0	0	0	2
x_4	4	1	1	2	1	0	0	2
x_5	9	2	3	4	0	1	0	3
x_6	14	6	6	2	0	0	1	8

Step (1): $J_- = \{j: a_{0j} < 0\} = \{1, 2, 3\} \neq \emptyset$ the algorithm does not stop.

Step (2): $I_+ = \{i: a_{i0} > 0\} = \{1, 2, 3\} \neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}}\right\} = min\left\{\frac{2}{4}, \frac{3}{9}, \frac{8}{14}\right\} = \frac{3}{9} \Rightarrow r = 2$$

Then, the leaving variable is x_5

Step (3): $J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \Phi$

$$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{0l}}{a_{21}}, \frac{-a_{02}}{a_{22}}, \frac{-a_{03}}{a_{23}}\right\} = min\left\{\frac{9}{2}, \frac{10}{3}, \frac{6}{4}\right\} = \frac{6}{4} \Rightarrow k = 3$$

Then, the entering variable is x_3

$\theta_2 \underset{R}{=} \frac{-a_{0l}}{a_{rl}} \underset{R}{=} min\left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\Phi\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 3$, the pivot element is a_{23}

Step (4): the next tableau by pivot element:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		-6	$\frac{-11}{2}$	0	0	$\frac{3}{2}$	0	$\frac{13}{2}$
x_4	$\frac{-1}{2}$	0	$\frac{-1}{2}$	0	1	$\frac{-1}{2}$	0	$\frac{1}{2}$
x_3	$\frac{5}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	0	$\frac{1}{4}$	0	$\frac{3}{4}$
x_6	$\frac{19}{2}$	5	$\frac{9}{2}$	0	0	$\frac{-1}{2}$	1	$\frac{13}{2}$

Step (1): $J_- = \{j: a_{0j} < 0\} = \{1, 2\} \neq \emptyset$ the algorithm does not stop.

Step (2): $I_+ = \{i: a_{i0} > 0\} = \{2, 3\} \neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_2}{a_{20}}, \frac{b_3}{a_{30}}\right\} = min\left\{\frac{3}{5}, \frac{13}{19}\right\} = \frac{3}{5} \Rightarrow r = 2$$

Then, the leaving variable is x_3

Step (3): $J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \{5\}$

$$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{01}}{a_{21}}, \frac{-a_{02}}{a_{22}}\right\} = min\left\{12, \frac{22}{3}\right\} = \frac{22}{3} \Rightarrow k = 2$$

Then, the entering variable is x_2

$\theta_2 \underset{R}{=} \frac{-a_{0l}}{a_{rl}} \underset{R}{=} min\left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\Phi\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 2$, the pivot element is a_{22}

Step (4): the next tableau by pivot element:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		$\frac{-7}{3}$	0	$\frac{22}{3}$	0	$\frac{10}{3}$	0	12
x_4	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	1	$\frac{-1}{3}$	0	1
x_2	$\frac{2}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	0	$\frac{1}{3}$	0	1
x_6	2	2	0	-6	0	-2	1	2

Step (1): $J_- = \{j: a_{0j} < 0\} = \{1\} \neq \emptyset$ the algorithm does not stop.

Step (2): $I_+ = \{i: a_{i0} > 0\} = \{1,2,3\} \neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}}\right\} = min\left\{3, \frac{3}{2}, 1\right\} = 1 \Rightarrow r = 3$$

Then, the leaving variable is x_6

Step (3): $J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \{3,5\}$

$$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{01}}{a_{31}}\right\} = min\left\{\frac{7}{6}\right\} \Rightarrow k = 1$$

Then, the entering variable is x_1

$\theta_2 \underset{R}{=} \frac{-a_{0l}}{a_{rl}} \underset{R}{=} min\left\{\frac{-a_{0j}}{a_{rj}}: j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\Phi\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 1$, the pivot element is a_{31}

Step (4): the next tableau by pivot element:

	x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z	0	0	$\frac{1}{3}$	0	1	$\frac{7}{6}$	$\frac{43}{3}$
x_4	0	0	$\frac{5}{3}$	1	0	$\frac{-1}{6}$	$\frac{2}{3}$
x_2	0	1	$\frac{10}{3}$	0	1	$\frac{-1}{3}$	$\frac{1}{3}$
x_1	1	0	-3	0	-1	$\frac{1}{2}$	1

Step (1): $J_-: \left\{j: a_{0j} \underset{R}{<} 0\right\} = \Phi$, the algorithm stops .

The solution is: $z = \frac{43}{3}, x_1 = 1, x_2 = \frac{1}{3}, x_3 = 0$

Table 4. A comparison between fuzzy EPSA & Neutrosophic EPSA

	FEPSA[7]	NEPSA
$Iteration\ no.$	3	3
Z	11	14.33
x_1	1	1
x_2	$\frac{1}{3}$	$\frac{1}{3}$
x_3	0	0

In Table 4, we make a comparison between FEPSA and NEPSA. It is clear that the neutrosophic approach NEPSA is more accurate than the fuzzy approach FEPSA according to the value of objective function. The value of objective function of NEPSA is 14.33 while FEPSA has 11 where the type of this problem is maximization. From Table 4, we deduce that NEPSA is more accurate than FEPSA.

5.1 Case study (P_2) [21]:

A company produces three products P1, P2 and P3. These products are processed on three different machines M1, M2 and M3. The time required to manufacture one unit of each product and the daily capacity of the machines are given below:

	Time per unit(minutes)			
Machines	p_1	p_2	p_3	**Machine Capacity (min/day)**
M1	12	13	12	490
M2	14	-	13	470
M3	12	15	-	480

Note that the time availability can vary from day to day due to break down of machines, overtime work etc. Finally the profit for each product can also vary due to variations in price. At the same time the company wants to keep the profit somewhat close to 14 for P1, 13 for P2 and 16 for P3. The company wants to determine the range of each product to be produced per day to maximize its profit. It is assumed that all the amounts produced are consumed in the market.

Since the profit from each product and the time availability on each machine are uncertain, the number of units to be produced on each product will also be uncertain. So we will model the problem as a fuzzy linear programming problem. We use symmetric trapezoidal fuzzy numbers for each uncertain value. Profit for P1 which is close to 14 is modelled as [13, 15, 2, 2]. Similarly the other parameters are also modelled as symmetric trapezoidal fuzzy numbers taking into account the nature of the problem and other requirements. So we formulate the given fuzzy linear programming problem as:

$$Max\ \tilde{z} = (13,15,2,2)x_1 + (12,14,3,3)x_2 + (15,17,2,2)x_3$$
$$s.t.$$
$$12x_1 + 13x_2 + 12x_3 \le (475,505,6,6)$$
$$14x_1 + \qquad 13x_3 \le (460,480,8,8)$$
$$12x_1 + 15x_2 \le (465,495,5,5)$$
$$x_1, x_2, x_3 \ge 0$$

5.1.1 Solving case study using fuzzy exterior point simplex method

Putting the formula into the standard form, we have:

$$Max\ \tilde{z} = (13,15,2,2)x_1 + (12,14,3,3)x_2 + (15,17,2,2)x_3$$
$$\text{s.t.}$$
$$12x_1 + 13x_2 + 12x_3 + x_4 = (475,505,6,6)$$
$$14x_1 + 13x_3 + x_5 = (460,480,8,8)$$
$$12x_1 + 15x_2 + x_6 = (465,495,5,5)$$
$$x_1, x_2, x_3 \ge 0$$

Step (0): we construct the initial tableau of fuzzy exterior simplex:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		$-(13,15,2,2)$	$-(12,14,3,3)$	$-(15,17,2,2)$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
x_4	37	12	13	12	1	0	0	(475,505,6,6)
x_5	27	14	0	13	0	1	0	(460,480,8,8)
x_6	27	12	15	0	0	0	1	(465,495,5,5)

Step (1): $J- = \{j: a_{0j} \underset{R}{<} 0\} = \{1, 2, 3\} \neq \emptyset$ the algorithm does not stop.

Step (2): *$I_+ = \{i: a_{i0} > 0\} = \{1, 2,3\}$* $\neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}}\right\} = min\left\{\frac{R(475,505,6,6)}{37}, \frac{R(460,480,8,8)}{27}, \frac{R(465,495,5,5)}{27}\right\}$$
$$= \frac{490}{37} \Rightarrow r = 1$$

Then, the leaving variable is x_4

Step (3): *$J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \Phi$*

$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{0l}}{a_{11}}, \frac{-a_{02}}{a_{12}}, \frac{-a_{03}}{a_{13}}\right\} = min\left\{\frac{R(13,15,2,2)}{12}, \frac{R(12,14,3,3)}{13}, \frac{R(15,17,2,2)}{12}\right\} = \frac{13}{13} = 1 \Rightarrow k = 2$

Then, the entering variable is x_2

$\theta_2 = \frac{-a_{0l}}{a_{rl}} = min\left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\Phi\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 2$, the pivot element is a_{12}

Step (4): the next tableau by pivot element:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		$(\frac{-25}{13}, \frac{-27}{13}, \frac{10}{13}, \frac{10}{13})$	0	$(\frac{-51}{13}, \frac{-53}{13}, \frac{10}{13}, \frac{10}{13})$	$(\frac{12}{13}, \frac{14}{13}, \frac{3}{13}, \frac{3}{13})$	$\tilde{0}$	$\tilde{0}$	(475,505,6,6)
x_2	$\frac{24}{13}$	$\frac{12}{13}$	1	$\frac{12}{13}$	$\frac{1}{13}$	0	0	$(\frac{475}{13}, \frac{505}{13}, \frac{6}{13}, \frac{6}{13})$
x_5	27	14	0	13	0	1	0	(460,480,8,8)
x_6	$\frac{-204}{13}$	$\frac{-24}{13}$	0	$\frac{-180}{13}$	$\frac{15}{13}$	0	1	$(\frac{-1080}{13}, \frac{-1140}{13}, \frac{-25}{13}, \frac{-25}{13})$

Step (1): *$J- = \{j: a_{0j} \underset{R}{<} 0\} = \{1,3\}$* $\neq \emptyset$ the algorithm does not stop.

Step (2): *$I_+ = \{i: a_{i0} > 0\} = \{1,2\}$* $\neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}\right\} = min\left\{\frac{R(\frac{475}{13}, \frac{505}{13}, \frac{6}{13}, \frac{6}{13})}{\frac{24}{13}}, \frac{R(460,480,8,8)}{27}\right\}$$
$$= min\{\frac{245}{12}, \frac{470}{27}\} = \frac{470}{27} \Rightarrow r = 2$$

Then, the leaving variable is x_5

Step (3): *$J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \{4,5\}$*

$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{03}}{a_{23}}\right\} = min\{\frac{R(\frac{389}{182}, \frac{391}{182}, \frac{-5}{91}, \frac{-5}{91})}{\frac{13}{14}}\} = \frac{30}{13} \Rightarrow k = 3$

Then, the entering variable is x_3

$\theta_2 = \frac{-a_{0l}}{a_{rl}} = min\left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\Phi\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 3$, the pivot element is a_{23}

Step (4): the next tableau by pivot element:

	x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z	$(\frac{389}{169},\frac{391}{169},\frac{-10}{169},\frac{-10}{169})$	0	0	$(\frac{12}{13},\frac{14}{13},\frac{3}{13},\frac{3}{13})$	$(\frac{51}{169},\frac{53}{169},\frac{-10}{169},\frac{-10}{169})$	$\tilde{0}$	$(\frac{8015}{13},\frac{8485}{13},\frac{110}{13},\frac{110}{13})$
x_2	$\frac{-12}{169}$	1	0	$\frac{1}{14}$	$\frac{-12}{169}$	0	$(\frac{655}{169},\frac{805}{169},\frac{-18}{169},\frac{-18}{169})$
x_3	$\frac{14}{13}$	0	1	0	$\frac{1}{13}$	0	$(\frac{460}{13},\frac{480}{13},\frac{8}{13},\frac{8}{13})$
x_6	$\frac{2208}{169}$	0	0	$\frac{-15}{13}$	$\frac{180}{169}$	1	$(\frac{481320}{1183},\frac{501060}{1183},\frac{1115}{169},\frac{1115}{169})$

The solution is: $z = 634.6$, $x_1 = 0, x_2 = \frac{731}{169}, x_3 = \frac{471}{13}$

5.1.2 Solving case study using neutrosophic exterior point simplex method

First: We will convert the fuzzy numbers into neutrosophic numbers

Then, using the following rank function:

$$R(\breve{a}) = \frac{a_2 + a_3}{2} + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$$

$$R(a) = a + 1$$

$Max\ \tilde{z} = R[(13,15,2,2)]x_1 + R[(12,14,3,3)]x_2 + R[(15,17,2,2)]x_3$

s.t.

$$12x_1 + 13x_2 + 12x_3 \le R[(475,505,6,6)]$$
$$14x_1 + \quad 13x_3 \le R[(460,480,8,8)]$$
$$12x_1 + 15x_2 \le R[(465,495,5,5)]$$
$$x_1, x_2, x_3 \ge 0$$

Putting the last formula into the standard form, we have:

$Max\ z = 15x_1 + 14x_2 + 17x_3 - 2$

s.t.

$$12x_1 + 13x_2 + 12x_3 + x_4 = 491$$
$$14x_1 + 13x_3 + x_5 = 471$$
$$12x_1 + 15x_2 + x_6 = 481$$
$$x_1, x_2, x_3, x_4 \ge 0$$

Step (0): we construct the initial tableau of exterior simplex:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		−15	−14	−17	0	0	0	2
x_4	37	12	13	12	1	0	0	491
x_5	27	14	0	13	0	1	0	471
x_6	27	12	15	0	0	0	1	481

Step (1): $J_- = \{j: a_{0j} < 0\} = \{1, 2, 3\} \neq \emptyset$ the algorithm does not stop.

Step (2): $I_+ = \{i: a_{i0} > 0\} = \{1, 2, 3\} \neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}, \frac{b_3}{a_{30}}\right\} = min\left\{\frac{491}{37}, \frac{471}{27}, \frac{481}{27}\right\} = \frac{491}{37} \Rightarrow r = 1$$

Then, the leaving variable is x_4

Step (3): $J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \Phi$

$$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{0l}}{a_{11}}, \frac{-a_{02}}{a_{12}}, \frac{-a_{03}}{a_{13}}\right\} = min\left\{\frac{15}{12}, \frac{14}{13}, \frac{17}{12}\right\} = \frac{14}{13} \Rightarrow k = 2$$

Then, the entering variable is x_2

$\theta_2 = \frac{-a_{0l}}{a_{rl}} = mi \left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\Phi\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 2$, the pivot element is a_{12}

Step (4): the next tableau by pivot element:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		$\frac{-27}{13}$	0	$\frac{-53}{13}$	$\frac{14}{13}$	0	0	$\frac{6900}{13}$
x_2	$\frac{24}{13}$	$\frac{12}{13}$	1	$\frac{12}{13}$	$\frac{1}{13}$	0	0	$\frac{491}{13}$
x_5	27	14	0	13	0	1	0	471
x_6	$\frac{-204}{13}$	$\frac{-24}{13}$	0	$\frac{-180}{13}$	$\frac{-15}{13}$	0	1	$\frac{-1112}{13}$

Step (1): $J_- = \{j: a_{0j} < 0\} = \{1,3\} \neq \emptyset$ the algorithm does not stop.

Step (2): $I_+ = \{i: a_{i0} > 0\} = \{1,2\} \neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}\right\} = min\left\{\frac{491}{24}, \frac{471}{27}\right\} = \frac{471}{27} \Rightarrow r = 2$$

Then, the leaving variable is x_5

Step (3): $J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \{4\}$

$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{01}}{a_{21}}, \frac{-a_{03}}{a_{23}}\right\} = min\left\{\frac{27}{182}, \frac{53}{169}\right\} = \frac{27}{182} \Rightarrow k = 1$

Then, the entering variable is x_2

$\theta_2 \underset{R}{=} \frac{-a_{0l}}{a_{rl}} \underset{R}{=} min\left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\emptyset\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 1$, the pivot element is a_{21}

Step (4): the next tableau by pivot element:

		x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z		0	0	$\frac{-391}{182}$	$\frac{14}{13}$	$\frac{27}{182}$	0	$\frac{8409}{14}$
x_2	$\frac{6}{91}$	0	1	$\frac{6}{91}$	$\frac{1}{13}$	$\frac{-6}{91}$	0	$\frac{47}{7}$
x_1	$\frac{13}{14}$	1	0	$\frac{13}{14}$	0	$\frac{1}{14}$	0	$\frac{471}{14}$
x_6	$\frac{-1104}{91}$	0	0	$\frac{-1104}{91}$	$\frac{-15}{13}$	$\frac{12}{91}$	1	$\frac{-164}{7}$

Step (1): $J_- = \{j: a_{0j} < 0\} = \{3\} \neq \emptyset$ the algorithm does not stop.

Step (2): $I_+ = \{i: a_{i0} > 0\} = \{1,2\} \neq \Phi$ the problem is not unbounded

$$\frac{br}{a_{r0}} = min\left\{\frac{b_i}{a_{i0}}, i \in I_+\right\} = min\left\{\frac{b_1}{a_{10}}, \frac{b_2}{a_{20}}\right\} = min\left\{\frac{611}{6}, \frac{471}{13}\right\} = \frac{471}{13} \Rightarrow r = 2$$

Then, the leaving variable is x_1

Step (3): $J_+ = \{j: a_{0j} \underset{R}{>} 0\} = \{4,5\}$

$\theta_1 = \frac{-a_{0k}}{a_{rk}} = min\left\{\frac{-a_{0j}}{a_{rj}} = j \in J-, a_{rj} > 0\right\} = min\left\{\frac{-a_{03}}{a_{23}}\right\} = min\left\{\frac{391}{169}\right\} = 2.3 \Rightarrow k = 3$

Then, the entering variable is x_3

$\theta_2 \underset{R}{=} \frac{-a_{0l}}{a_{rl}} \underset{R}{=} min\left\{\frac{-a_{0j}}{a_{rj}} : j \in J_+, a_{rj} < 0\right\} \Rightarrow \theta_2 = min\{\emptyset\} = \infty \Rightarrow \theta_1 < \theta_2 \Rightarrow s = k = 3$, the pivot element is a_{23}

Step (4): the next tableau by pivot element:

	x_1	x_2	x_3	x_4	x_5	x_6	$R.H.S$
z	$\frac{391}{169}$	0	0	$\frac{14}{13}$	$\frac{53}{169}$	0	$\frac{114663}{169}$
x_2	$\frac{-12}{169}$	1	0	$\frac{1}{13}$	$\frac{-12}{169}$	0	$\frac{731}{169}$
x_3	$\frac{14}{13}$	0	1	0	$\frac{1}{13}$	0	$\frac{471}{13}$
x_6	$\frac{2208}{169}$	0	0	$\frac{-15}{13}$	$\frac{180}{169}$	1	$\frac{70324}{169}$

Step (1): $J_{-}: \left\{ j: a_{0j} \underset{R}{<} 0 \right\} = \Phi$, the algorithm stops .

The solution is: $z = 678.4$, $x_1 = 0, x_2 = \frac{731}{169}, x_3 = \frac{471}{13}$

Table 5. A comparison between Fuzzy EPSA & Neutrosophic EPSA

	FEPSA	**NEPSA**
$\boldsymbol{Iter.no.}$	3	3
Z	634.6	678.4
$\mathbf{x_1}$	0	0
$\mathbf{x_2}$	$\frac{730}{169}$	$\frac{731}{169}$
$\mathbf{x_3}$	$\frac{470}{13}$	$\frac{471}{13}$

In Table 5, we make a comparison between FEPSA and NEPSA. It is clear that the neutrosophic approach NEPSA is more accurate than the fuzzy approach FEPSA according to the value of objective function. The value of objective function of NEPSA is 678.4 while FEPSA has 634.6 where the type of this problem is maximization. From Table 5, we deduce that NEPSA is more accurate than FEPSA.

6. Conclusion

Three contributions were proposed. First contribution was proposing a good evaluation between the fuzzy and neutrosophic approaches using a novel fuzzy-neutrosophic transfer. Second contribution was introducing a general framework for solving the neutrosphic linear programming problems using the advantages of the method of Abdel-Basset et al. and the advantages of Singh et al.'s method. Third contribution was proposing a new neutrosophic exterior point simplex algorithm NEPSA and its fuzzy version FEPSA. NEPSA has two paths to get optimal solutions. One path consists of basic not feasible solutions but the other path is feasible. Finally, the numerical examples and results analysis showed that NEPSA more than accurate FEPSA.

References

1. Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. (2019). A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, *31*(5), 1595-1605.
2. Abdel-Basset, M. & Smarandache, F. Neutrosophic Sets in Decision Analysis and Operations Research. Publisher: IGI Global Dissmenator knowledge (2019).

3. Atanassov, K. T. (1999). Intuitionistic fuzzy sets. In *Intuitionistic fuzzy sets* (pp. 1-137). Physica, Heidelberg.
4. Badr, E., Almotairi, S., Eirokh, A., Abdel-Hay, A., & Almutairi, B. (2020). An Integer Linear Programming Model for Solving Radio Mean Labeling Problem. *IEEE Access*, *8*, 162343-162349.
5. Badr, E., & Almotairi, S. (2020). On a dual direct cosine simplex type algorithm and its computational behavior. *Mathematical Problems in Engineering*, *2020*.
6. Badr, E.; Paparrizos.; K., Samaras.; N., Baloukas, T. Some computational results on the efficiency of an exterior point algorithm, Proceedings of the 18th National Conference of Hellenic Operartional Research Society (HELORS), 2006.
7. Badr, E. M. (2014, May). On Fuzzy Primal Exterior Point Simplex Algorithm for Solving Linear Programming Problems with Fuzzy Numbers. In *The International Conference on Mathematics and Engineering Physics* (Vol. 7, No. International Conference on Mathematics and Engineering Physics (ICMEP-7), pp. 1-13). Military Technical College.
8. Badr, E. M. (2014, May). On Fuzzy Primal Exterior Point Simplex Algorithm for Solving Linear Programming Problems with Fuzzy Numbers. In *The International Conference on Mathematics and Engineering Physics* (Vol. 7, No. International Conference on Mathematics and Engineering Physics (ICMEP-7), pp. 1-13). Military Technical College.
9. Broumi, S., Bakali, A., Talea, M., Smarandache, F., & Vladareanu, L. (2016, December). Shortest path problem under triangular fuzzy neutrosophic information. In *2016 10th International Conference on Software, Knowledge, Information Management & Applications (SKIMA)* (pp. 169-174). IEEE.
10. Broumi, S., Bakali, A., Talea, M., & Smarandache, F. (2016). *Isolated single valued neutrosophic graphs*. Infinite Study
11. Broumi, S., Smarandache, F., Talea, M., & Bakali, A. (2016, July). Single valued neutrosophic graphs: degree, order and size. In *2016 IEEE international conference on fuzzy systems (FUZZ-IEEE)* (pp. 2444-2451). IEEE.
12. Broumi, S., Talea, M., Smarandache, F., & Bakali, A. (2016, December). Decision-making method based on the interval valued neutrosophic graph. In *2016 Future Technologies Conference (FTC)* (pp. 44-50). IEEE.
13. Dantzig, G. (2016). *Linear programming and extensions*. Princeton university press.
14. Das, S., & Pramanik, S. (2020). Generalized neutrosophic b-open sets in neutrosophic topological space. *Neutrosophic Sets and Systems*, *35*(1), 30.
15. Das, S., & Pramanik, S. (2020). Neutrosophic Simply Soft Open Set in Neutrosophic Soft Topological Space. *Neutrosophic Sets and Systems*, *38*(1), 16.
16. Das, S., Das, R., & Tripathy, B. C. (2020). *Multi-criteria group decision making model using single-valued neutrosophic set*. Infinite Study.
17. Das, S., & Pramanik, S. (2020). *Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions* (Vol. 38). Infinite Study.
18. Deli, I., & Şubaş, Y. (2017). A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *International Journal of Machine Learning and Cybernetics*, *8*(4), 1309-1322.
19. Deli, I., & Şubaş, Y. (2017). Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision making problems. *Journal of Intelligent & Fuzzy Systems*, *32*(1), 291-301.
20. Zadeh, L. A. (1996). Fuzzy sets. In *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh* (pp. 394-432).
21. Ganesan, K., & Veeramani, P. (2006). Fuzzy linear programs with trapezoidal fuzzy numbers. *Annals of Operations Research*, *143*(1), 305-315.
22. Ganesan, K., & Veeramani, P. (2006). Fuzzy linear programs with trapezoidal fuzzy numbers. *Annals of Operations Research*, *143*(1), 305-315.
23. Lai, Y. J., & Hwang, C. L. (1992). Fuzzy mathematical programming. In *Fuzzy mathematical programming* (pp. 74-186). Springer, Berlin, Heidelberg.
24. Maiti, I., Mandal, T., & Pramanik, S. (2019). Neutrosophic goal programming strategy for multi-level multi-objective linear programming problem. *Journal of ambient intelligence and humanized computing*, 1-12.

25. Maiti, I., Mandal, T., & Pramanik, S. (2020). A goal programming strategy for bi-level decentralized multi-objective linear programming problem with neutrosophic numbers. *International Journal of Applied Management Science*. *11*(8), 3175-3186.
26. Maiti, I., Pramanik, S., Mandal, T., & Das, S. K. (2020). Solving Multi-objective linear fractional programming problem based on Stanojevic's normalization technique under fuzzy environment. *International journal of operation research*, *10*.
27. Paparrizos, K. (1991). An infeasible (exterior point) simplex algorithm for assignment problems. *Mathematical Programming*, *51*(1), 45-54.
28. Paparrizos, K. (1993). An exterior point simplex algorithm for (general) linear programming problems. *Annals of Operations Research*, *46*(2), 497-508.
29. Paparrizos, K. (1996). A new primal and dual pivoting rule for the simplex algorithm. *Proceedings of SYMOPIS*, *96*, 1.
30. Paparrizos, K., Samaras, N., & Sifaleras, A. (2015). Exterior point simplex-type algorithms for linear and network optimization problems. *Annals of Operations Research*, *229*(1), 607-633.
31. Pramanik, S., & Dey, P. P. (2019). *Multi-level linear programming problem with neutrosophic numbers: A goal programming strategy*. Infinite Study.
32. Singh, A., Kumar, A., & Appadoo, S. S. (2019). A novel method for solving the fully neutrosophic linear programming problems: Suggested modifications. *Journal of Intelligent & Fuzzy Systems*, *37*(1), 885-895.

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