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Neutrosophic Tri-Topological Space

Suman Das[1], Surapati Pramanik[2,*]

[1]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India. Email: suman.mathematics@tripurauniv.in
[2*]Department of Mathematics, Nandalal Ghosh B.T. College, Narayanpur, 743126, West Bengal, India. Email: sura_pati@yahoo.co.in

***Correspondence:** sura_pati@yahoo.co.in Tel.: (+91-9477035544)

Abstract: In this article, we present the notion of neutrosophic tri-topological space as a generalization of neutrosophic bi-topological space. Besides, we study the different types of open sets and closed sets namely neutrosophic tri-open sets, neutrosophic tri-closed sets, neutrosophic tri-semi-open sets, neutrosophic tri-pre-closed sets, etc. via neutrosophic tri-topological spaces. Further, we investigate several properties, and prove some propositions, theorems on neutrosophic tri-topological spaces.

Keywords: Tri-open set; Tri-closed set; Tri-semi-open set; Tri-pre-open set; Neutrosophic crisp tri-topology; Neutrosophic tri-topology.

1. Introduction

The concept of Neutrosophic Set (NS) was grounded by Smarandache [1] by extending the concept of Fuzzy Set 2[] and intuitionistic FS [3]. The notion of Neutrosophic Topological Space (NTS) was developed by Salama and Alblowi [4] in 2012. Afterwards, Arokiarani et al. [5] studied the neutrosophic semi-open functions and established a relation between them. Iswaraya and Bageerathi [6] introduced the notion of neutrosophic semi-closed set and neutrosophic semi-open set via NTSs. Later on, Dhavaseelan, and Jafari [7] introduced the generalized neutrosophic closed sets. Thereafter, Pushpalatha and Nandhini [8] studied the neutrosophic generalized closed sets in NTS. Shanthi et al. [9] introduced the concept of neutrosophic generalized semi closed sets in NTS. Ebenanjar et al. [10] presented the neutrosophic *b*-open sets in NTS. Maheswari et al. [11] introduced the concept of neutrosophic generalized *b*-closed sets in NTS. Afterwards, the concept of generalized neutrosophic *b*-open set via NTS was introduced by Das and Pramanik [12] in 2020. Thereafter, the concept of neutrosophic Φ-open sets and neutrosophic Φ-continuous functions was presented by Das and Pramanik [13].

The notion of neutrosophic crisp topology on neutrosophic crisp set was introduced by Salama and Alblowi [14]. Later on, the notion of neutrosophic crisp tri-topological space was introduced by Al-Hamido and Gharibah [15] in 2018.

In 1963, Kelly [16] introduced the notion of bi-topological space. Thereafter, the concept of neutrosophic bi-topological space was presented by Ozturk and Ozkan [17] in 2019. Later on, Das and Tripathy [18] introduced the pairwise neutrosophic *b*-open sets via neutrosophic bi-topological spaces. Recently, Tripathy and Das [19] studied the concept of pairwise neutrosophic *b*-continuous functions via neutrosophic bi-topological spaces.

So, we received enough motivation to do research on neutrosophic tri-topological space to extend the concept of neutrosophic bi-topological space.

In this study, we procure the notion of neutrosophic tri-topological space as a generalization of the neutrosophic bi-topological space. Besides, we introduce the different types of open sets and closed sets namely, neutrosophic tri-open sets, neutrosophic tri-closed sets, neutrosophic tri-semi-open sets, neutrosophic tri-pre-closed sets, etc. via neutrosophic tri-topological spaces. Further, we investigate several properties of these kinds of neutrosophic tri-open sets.

Research Gap: No investigation on neutrosophic tri-topological space has been reported in the recent literature.

Motivation: To reduce the research gap, we present the notion and different properties of neutrosophic tri-topological space.

The remaining part of this article is divided into the following sections:

Section-2 is on preliminaries and definitions. In this section, we give some definitions and theorems, which are relevant to this article. In section-3, we present the notion of neutrosophic tri-topology and neutrosophic tri-topological space and also we give proofs of some theorems on neutrosophic tri-topological space. In section-4, we give the concluding remarks of the work done in the present article.

Throughout this article, we use the following short terms for the clarity of the presentation.

Short Terms	
Neutrosophic Set	NS
Neutrosophic Topology	NT
Neutrosophic Topological Space	NTS
Neutrosophic Open Set	N-O-S
Neutrosophic Closed Set	N-O-S
Neutrosophic Semi-Open	NSO
Neutrosophic Pre-Open	NPO
Neutrosophic Bi-Topological Space	NBTS
Neutrosophic Tri-Topological Space	N-Tri-TS

Neutrosophic Tri-Open Set	N-tri-OS
Neutrosophic Tri-Closed Set	N-tri-CS

2. Some Relevant Definitions:

Definition 2.1.[1] A neutrosophic set L over a universe of discourse Ψ is defined as follows:

$$L=\{(n, T_L(n), I_L(n), F_L(n)): n\in\Psi\},$$

where $T_L(n)$, $I_L(n)$, $F_L(n)$ ($\in]^-0,1^+[$) are respectively denotes the truth, indeterminacy and falsity membership values of $n\in\Psi$, and so $^-0 \leq T_L(n) + I_L(n) + F_L(n) \leq 3^+$ for all $n\in\Psi$.

Definition 2.2.[1] The neutrosophic null set (0_N) and neutrosophic whole set (1_N) over a universe of discourse Ψ are defined as follows:

(*i*) $0_N=\{(n,0,0,1): n\in\Psi\}$;

(*ii*) $1_N=\{(n,1,0,0): n\in\Psi\}$.

Obviously, $0_N \subseteq 1_N$.

Definition 2.3.[20] Assume that Ψ be a universe of discourse. Then, a neutrosophic crisp set Q is defined by $Q=\{Q_1, Q_2, Q_3\}$, where Q_i (i=1,2,3) is a subset of Ψ such that $Q_i\cap Q_j=\phi$ (i, j= 1,2,3 and $i\neq j$)

Definition 2.4.[4] Assume that Ψ be a universe of discourse, and τ be a set of some NSs over Ψ. Then, τ is called a Neutrosophic Topology (NT) on Ψ if the following axioms hold:

(*i*) $0_N, 1_N \in \tau$;

(*ii*) $X_1, X_2 \in\tau \Rightarrow X_1\cap X_2 \in\tau$;

(*iii*) $\{X_i : i\in\Delta\} \subseteq \tau \Rightarrow \cup X_i \in\tau$.

The pair (Ψ, τ) is said to be an NTS. If $X\in\tau$, then X is called a neutrosophic-open-set (N-O-S) and its complement X^c is called a neutrosophic-closed-set (N-C-S).

Definition 2.5.[17] Assume that (Ψ, τ_1) and (Ψ, τ_2) be any two different NTSs. Then, we can call the triplet (Ψ, τ_1, τ_2) as a Neutrosophic Bi-Topological Space (NBTS).

Definition 2.6.[17] Assume that (Ψ, τ_1, τ_2) be an NBTS. Then, a neutrosophic subset X of Ψ is said to be a pairwise neutrosophic open set in (Ψ, τ_1, τ_2) if there exists an N-O-S T_1 in (Ψ, τ_1) and an N-O-S T_2 in (Ψ, τ_2) such that $X = T_1\cup T_2$.

Theorem 3.1.[18] Let (Ψ, τ_1, τ_2) be an NBTS. Then, a neutrosophic subset X of Ψ is called as

(*i*) τ_{ij} –neutrosophic-semi-open if and only if $X\subseteq N_{cl}^{i}N_{int}^{j}(X)$;

(*ii*) τ_{ij} –neutrosophic-pre-open if and only if $X\subseteq N_{int}^{j}N_{cl}^{i}(X)$;

(*iii*) τ_{ij} –neutrosophic-*b*-open if and only if $X\subseteq N_{cl}^{i}N_{int}^{j}(X) \cup N_{int}^{j}N_{cl}^{i}(X)$.

Theorem 3.2.[18] Assume that (Ψ, τ_1, τ_2) be an NBTS. Then, every τ_{ij}-neutrosophic-pre-open (τ_{ij}-neutrosophic-semi-open) set is a τ_{ij}-neutrosophic-*b*-open.

Definition 2.7.[18] Assume that (Ψ, τ_1, τ_2) be an NBTS. Then X, an NS over Ψ is said to be a

(*i*) pairwise τ_{ij}-neutrosophic-semi-open set (pairwise τ_{ij}-neutrosophic-pre-open set) in an NBTS (Ψ, τ_1, τ_2) if and only if $X=T\cup K$, where T is a τ_{ij}-neutrosophic-semi-open set (τ_{ij}-neutrosophic-pre-open set) and K is a τ_{ji}-neutrosophic-semi-open set (τ_{ji}-neutrosophic-pre-open set) in (Ψ, τ_1, τ_2).

(*ii*) pairwise τ_{ij}-neutrosophic-*b*-open set in an NBTS (Ψ, τ_1, τ_2) if $X=T\cup K$, where T is a τ_{ij}-neutrosophic-*b*-open set and K is a τ_{ji}-neutrosophic-*b*-open set in (Ψ, τ_1, τ_2).

Theorem 3.3.[18] Assume that (Ψ, τ_1, τ_2) be an NBTS. If *X* is a pairwise τ_{ij}-neutrosophic-semi open (pairwise τ_{ij}-neutrosophic-pre-open) set, then it is also a pairwise τ_{ij}-neutrosophic-*b*-open set.

Theorem 3.4.[18] In an NBTS (Ψ, τ_1, τ_2), the union of any two pairwise τ_{ij}-neutrosophic-*b*-open sets is a pairwise τ_{ij}-neutrosophic-*b*-open set.

Definition 2.8.[15] Assume that τ_1, τ_2 and τ_3 be any three neutrosophic crisp topology on a universe of discourse *W*. Then, (*W*, τ_1, τ_2, τ_3) is called a neutrosophic crisp tri-topological space.

3. Neutrosophic Tri-Topological Space:

In this section, we introduce the notion of neutrosophic tri-topological space, and formulate several results on it.

Definition 3.1. Suppose that (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) be any three different NTSs. Then, the structure (Ψ, τ_1, τ_2, τ_3) is called a neutrosophic tri-topological space (N-Tri-TS).

Example 3.1. Let Ψ={*u*, *v*, *w*} be a universe of discourse. Let X_1, X_2, Y_1, Y_2, Y_3, Z_1 and Z_2 be seven NSs over Ψ such that:

X_1={(*u*,0.9,0.3,0.7), (*v*,0.5,0.6,0.8), (*w*,0.3,0.5,0.2)};

X_2={(*u*,0.7,0.4,0.7), (*v*,0.5,0.9,0.9), (*w*,0.1,0.8,0.4)};

Y_1= {(*u*,0.9,0.3,0.7), (*v*,0.5,0.6,0.8), (*w*,0.3,0.5,0.2)};

Y_2= {(*u*,1.0,0.2,0.5), (*v*,0.9,0.5,0.8), (*w*,0.5,0.2,0.1)};

Y_3= {(*u*,1.0,0.2,0.3), (*v*,1.0,0.1,0.8), (*w*,0.9,0.1,0.1)};

Z_1= {(*u*,0.5,0.2,0.5), (*v*,0.9,0.4,0.3), (*w*,0.7,0.2,0.5)};

Z_2= {(*u*,0.4,0.5,0.6), (*v*,0.5,0.6,0.4), (*w*,0.7,0.6,0.6)};

Then, clearly τ_1={0_N, 1_N, X_1, X_2}, τ_2={0_N, 1_N, Y_1, Y_2, Y_3} and τ_3={0_N, 1_N, Z_1, Z_2} are three different NTs on Ψ. So, (Ψ, τ_1, τ_2, τ_3) is a neutrosophic tri-topological space.

Definition 3.2. Assume that (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS. Then *R*, an NS over Ψ is called a neutrosophic tri-open set (N-tri-OS) if $R\in\tau_1\cup\tau_2\cup\tau_3$. A neutrosophic set *R* is called a neutrosophic tri-closed set (N-Tri-CS) if and only if R^c is an N-Tri-OS in (Ψ, τ_1, τ_2, τ_3).

Remark 3.1. The collection of all neutrosophic tri-open sets and neutrosophic tri-closed sets in (Ψ, τ_1, τ_2, τ_3) are denoted by N-Tri-O(Ψ) and N-Tri-C(Ψ) respectively.

Theorem 3.1. Every N-O-S in (Ψ, τ_i), *i*=1,2,3 is a neutrosophic tri-open set in (Ψ, τ_1, τ_2, τ_3).

Proof. Assume that *W* be an N-O-S in (Ψ, τ_i), *i*=1,2,3. Therefore, $W\in\tau_i$, *i*=1,2,3. This implies, $W\in \bigcup_{i\in\{1,2,3\}}\tau_i$. Hence, *W* is a neutrosophic tri-open set in (Ψ, τ_1, τ_2, τ_3). Therefore, every N-O-S in (Ψ, τ_i), *i* =1,2,3 is a neutrosophic tri-open set in (Ψ, τ_1, τ_2, τ_3).

Example 3.2. Let us consider an N-Tri-TS (Ψ, τ_1, τ_2, τ_3) as shown in Example 3.1. Then, the neutrosophic open sets X_1, X_2 (in (Ψ, τ_1)), Y_1, Y_2, Y_3 (in (Ψ, τ_2)), Z_1, Z_2 (in (Ψ, τ_3)) are neutrosophic tri-open sets in (Ψ, τ_1, τ_2, τ_3).

Remark 3.2. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), the union of two neutrosophic tri-open sets may not be a neutrosophic tri-open set. This follows from the following example.

Example 3.3. Consider the N-Tri-TS (Ψ, τ_1, τ_2, τ_3) which has been shown in Example 3.1. Then, clearly Y_3 and Z_1 are two neutrosophic tri-open sets in (Ψ, τ_1, τ_2, τ_3). But their union $Y_3\cup Z_1$={(*u*,1.0,0.2,0.3),

(v,1.0,0.1,0.3), (w,0.9,0.1,0.1)} is not a neutrosophic tri-open set, because $Y_3 \cup Z_1 \notin \bigcup_{i\in\{1,2,3\}} \tau_i$. Hence, the union of two neutrosophic tri-open sets may not be a neutrosophic tri-open set.

Remark 3.3. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), the intersection of two neutrosophic tri-open sets may not be a neutrosophic tri-open set. This follows from the following example.

Example 3.4. Consider the N-Tri-TS (Ψ, τ_1, τ_2, τ_3) which is shown in Example 3.1. Then, clearly X_1 and Z_2 are two neutrosophic tri-open sets in (Ψ, τ_1, τ_2, τ_3). But their union $X_1 \cap Z_2$={(u,0.4,0.5,0.7), (v,0.5,0.6,0.8), (w,0.3,0.6,0.6)} is not a neutrosophic tri-open set, because $X_1 \cap Z_2 \notin \bigcup_{i\in\{1,2,3\}} \tau_i$. Hence, the intersection of two neutrosophic tri-open sets may not be a neutrosophic tri-open set.

Theorem 3.2. Every N-C-S in (Ψ, τ_i) (i=1,2,3) is a neutrosophic tri-closed set in (Ψ, τ_1, τ_2, τ_3).

Proof. Assume that W be an N-C-S in (Ψ, τ_i). So W^c is an N-O-S in (Ψ, τ_i) (i=1,2,3). Therefore, $W^c \in \tau_i$, i=1,2,3. This implies, $W^c \in \bigcup_{i\in\{1,2,3\}} \tau_i$. This implies, W^c is a neutrosophic tri-open set in (Ψ, τ_1, τ_2, τ_3). Therefore, W is a neutrosophic tri-closed set in (Ψ, τ_1, τ_2, τ_3). Hence, every N-C-S in (Ψ, τ_i), i=1,2,3 is a neutrosophic tri-open set in (Ψ, τ_1, τ_2, τ_3).

Example 3.5. Suppose that (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS as shown in Example 3.1. Clearly, X_1^c = {(u,0.1,0.7,0.3), (v,0.5,0.4,0.2), (w,0.7,0.5,0.8)} , X_2^c = {(u,0.3,0.6,0.3), (v,0.5,0.1,0.1), (w,0.9,0.2,0.6)} are NCSs in (Ψ, τ_1), Y_1^c = {(u,0.1,0.7,0.3), (v,0.1,0.5,0.2), (w,0.5,0.8,0.9)}, Y_2^c = {(u,0.0,0.8,0.5), (v,0.1,0.5,0.2), (w,0.5,0.8,0.9)}, Y_3^c = {(u,0.0,0.8,0.7), (v,0.0,0.9,0.2), (w,0.1,0.9,0.9)} are NCSs in (Ψ, τ_2), and Z_1^c = {(u,0.5,0.8,0.5), (v,0.1,0.6,0.7), (w,0.3,0.8,0.5)}, Z_2^c = {(u,0.6,0.5,0.4), (v,0.5,0.4,0.6), (w,0.3,0.4,0.4)} are NCSs in (Ψ, τ_3). Therefore, X_1, X_2 are NOSs in (Ψ, τ_1), Y_1, Y_2, Y_3 are NOSs in (Ψ, τ_2), and Z_1, Z_2 are NOSs in (Ψ, τ_3). By Example 3.2, X_1, X_2, Y_1, Y_2, Y_3, Z_1 and Z_2 are neutrosophic tri-open sets in (Ψ, τ_1, τ_2, τ_3). Therefore, by definition 3.2, X_1^c, X_2^c, Y_1^c, Y_2^c, Y_3^c, Z_1^c and Z_2^c are neutrosophic tri-closed sets in (Ψ, τ_1, τ_2, τ_3).

Definition 3.3. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), an NS G over Ψ is called a neutrosophic tri-semi-open set if G is a neutrosophic semi open set in at least one of three NTSs (Ψ, τ_1), (Ψ, τ_2), and (Ψ, τ_3).

Example 3.6. Consider the N-Tri-TS (Ψ, τ_1, τ_2, τ_3) which is shown in Example 3.1. Then, W={(u,1.0,0.2,0.5), (v,0.7,0.5,0.7), (w,0.9,0.8,0.3)} is a neutrosophic tri-semi-open set in (Ψ, τ_1, τ_2, τ_3), because W is a neutrosophic semi open set in (Ψ, τ_1).

Definition 3.4. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), a neutrosophic set G over Ψ is called a neutrosophic tri-pre-open set if G is a neutrosophic pre-open set in at least one of three NTSs (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3).

Example 3.7. Consider the N-Tri-TS (Ψ, τ_1, τ_2, τ_3) which is shown in Example 3.1. Then, W={(u,0.5,0.3,0.2), (v,0.6,0.3,0.2), (w,0.8,0.4,0.2)} is a neutrosophic tri-pre-open set in (Ψ, τ_1, τ_2, τ_3), because W is a neutrosophic pre- open set in (Ψ, τ_1).

Definition 3.5. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), an NS G over Ψ is called a neutrosophic tri-b-open set if G is a neutrosophic b-open set in at least one of three NTSs (Ψ, τ_1), (Ψ, τ_2), and (Ψ, τ_3).

Example 3.8. Consider the N-Tri-TS (Ψ, τ_1, τ_2, τ_3) which is shown in Example 3.1. Then, W={(u,0.9,0.3,0.6), (v,0.8,0.6,0.4), (w,0.9,0.9,0.9)} is a neutrosophic tri-b-open set in (Ψ, τ_1, τ_2, τ_3), because W is a neutrosophic b-open set in (Ψ, τ_1).

Remark 3.4. Assume that (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS. Let $\tau_{1,2,3}$= $\tau_1 \cup \tau_2 \cup \tau_3$. Then, $\tau_{1,2,3}$ may not be a neutrosophic topology on Ψ in general. This follows from the following example.

Example 3.9. Suppose that (Ψ, τ_1, τ_2, τ_3) is a neutrosophic tri-topological space, where τ_1={0_N, 1_N, {(u,0.6,0.3,0.6), (v,0.5,0.4,0.5), (w,0.8,0.5,0.8)}, {(u,0.7,0.1,0.4), (v,0.9,0.3,0.3), (w,0.9,0.1,0.5)}}, τ_2={0_N, 1_N, {(u,1.0,0.3,0.8), (v,0.8,0.4,0.7), (w,0.8,0.6,0.8)}, {(u,0.8,0.4,0.9), (v,0.5,0.5,1.0), (w,0.5,0.8,1.0)}}, τ_3={0_N, 1_N, {(u,0.5,0.2,0.5), (v,0.9,0.4,0.3), (w,0.7,0.2,0.5)}, {(u,0.4,0.5,0.6), (v,0.5,0.6,0.4), (w,0.7,0.6,0.6)}} are three different NTs on Ψ. Clearly, {(u,0.7,0.1,0.4), (v,0.9,0.3,0.3), (w,0.9,0.1,0.5)} and {(u,0.8,0.4,0.9), (v,0.5, 0.5,1.0), (w,0.5,0.8,1.0)} Ï $\tau_{1,2,3}$, but their intersection {(u,0.7,0.4,0.9), (v,0.5,0.5,1.0), (w,0.5,0.8,1.0)} Ï $\tau_{1,2,3}$. Hence, $\tau_{1,2,3}$ does not form a topology on Ψ.

Definition 3.6. Assume that (Ψ, τ_1, τ_2, τ_3) is an N-Tri-TS. Then R, an NS over Ψ is said to be a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3) if and only if there exist neutrosophic open sets R_1 in τ_1, R_2 in τ_2, and R_3 in τ_3 such that $R = R_1 \cup R_2 \cup R_3$.

Example 3.10. Consider the N-Tri-TS (Ψ, τ_1, τ_2, τ_3) as shown in Example 3.7. Then, W={(u,1.0,0.2,0.5), (v,0.9,0.4,0.3), (w,0.8,0.2,0.5)} is a neutrosophic tri-t-open set, since there exist NOSs R_1={(u,0.6,0.3,0.6), (v,0.5,0.4,0.5), (w,0.8,0.5,0.8)} in τ_1, R_2={(u,1.0,0.3,0.8), (v,0.8,0.4,0.7), (w,0.8,0.6,0.8)} in τ_2, and R_3={(u,0.5,0.2,0.5), (v,0.9,0.4,0.3), (w,0.7,0.2,0.5)} in τ_3 such that $W= R_1 \cup R_2 \cup R_3$.

Remark 3.5. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), an NS G is called a neutrosophic tri-t-closed set if and only if G^c is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3).

Theorem 3.3. Assume that (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS.

(i) The neutrosophic null set (0_N) and the neutrosophic whole set (1_N) are always a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3);

(ii) Every NOS in (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) are neutrosophic tri-t-open sets in (Ψ, τ_1, τ_2, τ_3);

(iii) Every NCS in (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) are neutrosophic tri-t-closed sets in (Ψ, τ_1, τ_2, τ_3).

Proof. (i) We can write the neutrosophic null set (0_N) as $0_N = W \cup M \cup N$, where $W = 0_N$, $M = 0_N$, $N = 0_N$ are NOSs in (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) respectively. Hence, 0_N is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3).

Similarly, we can write the neutrosophic whole set (1_N) as $1_N = W \cup M \cup N$, where $W=1_N$, $M =1_N$, $N =1_N$ are NOSs in (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) respectively. Hence, 1_N is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3).

(ii) Suppose that W be an NOS in (Ψ, τ_1). Now, we can write $W = W \cup 0_N \cup 0_N$. Therefore, there exist NOSs W, 0_N, 0_N in (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) respectively such that $W = W \cup 0_N \cup 0_N$. Hence, W is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3).

Suppose that W be an NOS in (Ψ, τ_2). Now, we can write $W = 0_N \cup W \cup 0_N$. Therefore, there exist NOSs 0_N, W, 0_N in (Ψ, τ_1), (Ψ, τ_2) and (Ψ, τ_3) respectively such that $W = 0_N \cup W \cup 0_N$. Hence, W is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3).

Suppose that W be an NOS in (Ψ, τ_3). Now, we can write $W = 0_N \cup 0_N \cup W$. Therefore, there exist NOSs 0_N, 0_N, W in (Ψ, τ_1), (Ψ, τ_2), and (Ψ, τ_3) respectively such that $W = 0_N \cup 0_N \cup W$. Hence, W is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3).

(iii) Suppose that W be an NCS in (Ψ, τ_1). So W^c is an NOS in (Ψ, τ_1). By the second part of this theorem, W^c is a neutrosophic tri-t-open set in (Ψ, τ_1, τ_2, τ_3). Hence, W is a neutrosophic tri-t-closed set in (Ψ, τ_1, τ_2, τ_3).

Suppose that W be an NCS in (Ψ, τ_2). So W^c is an NOS in (Ψ, τ_2). By the second part of this theorem, W^c is a neutrosophic tri-*t*-open set in (Ψ, τ_1, τ_2, τ_3). Hence, W is a neutrosophic tri-*t*-closed set in (Ψ, τ_1, τ_2, τ_3).

Suppose that W be an NCS in (Ψ, τ_3). So W^c is an NOS in (Ψ, τ_3). By the second part of this theorem, W^c is a neutrosophic tri-*t*-open set in (Ψ, τ_1, τ_2, τ_3). Hence, W is a neutrosophic tri-*t*-closed set in (Ψ, τ_1, τ_2, τ_3).

Theorem 3.4. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), the union of two neutrosophic tri-*t*-open sets is a neutrosophic tri-*t*-open set.

Proof. Assume that X and Y are any two neutrosophic tri-*t*-open sets in an N-Tri-TS (Ψ, τ_1, τ_2, τ_3). So there exist NOSs X_1, Y_1 in (Ψ, τ_1), X_2, Y_2 in (Ψ, τ_2), and X_3, Y_3 in (Ψ, τ_3), such that $X = X_1 \cup X_2 \cup X_3$ and $Y = Y_1 \cup Y_2 \cup Y_3$. Now $X \cup Y = (X_1 \cup X_2 \cup X_3) \cup (Y_1 \cup Y_2 \cup Y_3) = (X_1 \cup Y_1) \cup (X_2 \cup Y_2) \cup (X_3 \cup Y_3)$. Since X_1, Y_1 are NOSs in (Ψ, τ_1), so $X_1 \cup Y_1$ is an NOS in (Ψ, τ_1). Since X_2, Y_2 are NOSs in (Ψ, τ_2), so $X_2 \cup Y_2$ is an NOS in (Ψ, τ_2). Since X_3, Y_3 are NOSs in (Ψ, τ_3), so $X_3 \cup Y_3$ is an NOS in (Ψ, τ_3). Therefore, $X \cup Y$ is a neutrosophic tri-*t*-open set in (Ψ, τ_1, τ_2, τ_3).

Remark 3.6. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), the intersection of any two neutrosophic tri-*t*-open sets may not be a neutrosophic tri-*t*-open set.

Definition 3.7. Assume that (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS. Then Q, an NS over Ψ is said to be a neutrosophic tri-*t*-semi-open set in (Ψ, τ_1, τ_2, τ_3) if and only if there exists a neutrosophic semi open sets Q_1 in (W, τ_1), Q_2 in (W, τ_2), and Q_3 in (W, τ_3) such that $Q = Q_1 \cup Q_2 \cup Q_3$.

Theorem 3.5. In an N-Tri-TS (Ψ, τ_1, τ_2, τ_3), every neutrosophic tri-semi-open set is a neutrosophic tri-*t*-semi-open set.

Proof. Assume that X be a neutrosophic tri-semi-open set in an N-Tri-TS (Ψ, τ_1, τ_2, τ_3). So, X must be an NSO set in at least one of the NTS (Ψ, τ_1), (Ψ, τ_2), (Ψ, τ_3). So, there will be seven cases.

Case-1: X is an NSO set in (Ψ, τ_1);

Case-2: X is an NSO set in (Ψ, τ_2);

Case-3: X is an NSO set in (Ψ, τ_3);

Case-4: X is an NSO set in (Ψ, τ_1), and (Ψ, τ_2);

Case-5: X is an NSO set in (Ψ, τ_1), and (Ψ, τ_3);

Case-6: X is an NSO set in (Ψ, τ_2), and (Ψ, τ_3);

Case-7: X is an NSO set in (Ψ, τ_1), (Ψ, τ_2), and (Ψ, τ_3).

In case-1, we can express, $X = X \cup 0_N \cup 0_N$, that is X is the union of NSO sets X (in (W, τ_1)), 0_N (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in (Ψ, τ_1, τ_2, τ_3).

In case-2, we can express, $X = 0_N \cup X \cup 0_N$, that is X is the union of NSO sets 0_N (in (W, τ_1)), X (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in (Ψ, τ_1, τ_2, τ_3).

In case-3, we can express, $X = 0_N \cup 0_N \cup X$, that is X is the union of NSO sets 0_N (in (W, τ_1)), 0_N (in (W, τ_2)), and X (in (W,τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in ((Ψ, τ_1, τ_2, τ_3).

In case-4, we can express, $X = X \cup X \cup 0_N$, that is X is the union of NSO sets X (in (W, τ_1)), X (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in (Ψ, τ_1, τ_2, τ_3).

In case-5, we can express, $X = X \cup 0_N \cup X$, that is X is the union of NSO sets X (in (W, τ_1)), 0_N (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in (Ψ, τ_1, τ_2, τ_3).

In case-6, we can express, $X = 0_N \cup X \cup X$, that is X is the union of NSO sets 0_N (in (W, τ_1)), X (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

In case-7, we can express, $X = X \cup X \cup X$, that is X is the union of NSO sets X (in (W, τ_1)), X (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-semi-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

Hence, every neutrosophic tri-semi-open set is a neutrosophic tri-*t*-semi-open set.

Definition 3.8. Assume that $(\Psi, \tau_1, \tau_2, \tau_3)$ be an N-Tri-TS. Then Q, an NS over Ψ is said to be a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$ iff there exist a neutrosophic-pre-open sets Q_1 in τ_1, Q_2 in τ_2, and Q_3 in τ_3 such that $Q = Q_1 \cup Q_2 \cup Q_3$.

Theorem 3.6. In an N-Tri-TS $(\Psi, \tau_1, \tau_2, \tau_3)$, every neutrosophic tri-pre-open set is a neutrosophic tri-*t*-pre-open set.

Proof. Assume that X is a neutrosophic tri-pre-open set in an N-Tri-TS $(\Psi, \tau_1, \tau_2, \tau_3)$. So, X must be an NPO set in at least one of the NTS (Ψ, τ_1), (Ψ, τ_2), (Ψ, τ_3). So, there will be seven cases.

Case-1: X is an NPO set in (Ψ, τ_1);

Case-2: X is an NPO set in (Ψ, τ_2);

Case-3: X is an NPO set in (Ψ, τ_3);

Case-4: X is an NPO set in (Ψ, τ_1), and (Ψ, τ_2);

Case-5: X is an NPO set in (Ψ, τ_1), and (Ψ, τ_3);

Case-6: X is an NPO set in (Ψ, τ_2), and (Ψ, τ_3);

Case-7: X is an NPO set in (Ψ, τ_1), (Ψ, τ_2), and (Ψ, τ_3).

In case-1, we can express, $X = X \cup 0_N \cup 0_N$, that is X is the union of NPO sets X (in (W, τ_1)), 0_N (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

In case-2, we can express, $X = 0_N \cup X \cup 0_N$, that is X is the union of NPO sets 0_N (in (W, τ_1)), X (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

In case-3, we can express, $X = 0_N \cup 0_N \cup X$, that is X is the union of NPO sets 0_N (in (W, τ_1)), 0_N (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

In case-4, we can express, $X = X \cup X \cup 0_N$, that is X is the union of NPO sets X (in (W, τ_1)), X (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi,\tau_1,\tau_2,\tau_3)$.

In case-5, we can express, $X = X \cup 0_N \cup X$, that is X is the union of NPO sets X (in (W, τ_1)), 0_N (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

In case-6, we can express, $X = 0_N \cup X \cup X$, that is X is the union of NPO sets 0_N (in (W, τ_1)), X (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

In case-7, we can express, $X = X \cup X \cup X$, that is X is the union of NPO sets X (in (W, τ_1)), X (in (W, τ_2)), and X (in (W, τ_3)) . Therefore, X is a neutrosophic tri-*t*-pre-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

Hence, every neutrosophic tri-pre-open set is a neutrosophic tri-*t*-pre-open set.

Definition 3.9. Assume that $(\Psi, \tau_1, \tau_2, \tau_3)$ is an N-Tri-TS. Then Q, an NS over Ψ is said to be a neutrosophic tri-*t*-*b*-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$ if and only if there exist three neutrosophic-*b*-open sets, namely Q_1 in τ_1, Q_2 in τ_2, and Q_3 in τ_3 such that $Q = Q_1 \cup Q_2 \cup Q_3$.

Theorem 3.7. In an N-Tri-TS $(\Psi, \tau_1, \tau_2, \tau_3)$, every neutrosophic tri-*b*-open set is a neutrosophic tri-*t*-*b*-open set.

Proof. Assume that X be a neutrosophic tri-b-open set in an N-Tri-TS (Ψ, τ_1, τ_2, τ_3). So, X must be a neutrosophic b-open set in at least one of the NTS (Ψ, τ_1), (Ψ, τ_2), (Ψ, τ_3). So there will be seven cases.

Case-1: X is a neutrosophic b-open set in (Ψ, τ_1);

Case-2: X is a neutrosophic b-open set in (Ψ, τ_2);

Case-3: X is a neutrosophic b-open set in (Ψ, τ_3);

Case-4: X is a neutrosophic b-open set in (Ψ, τ_1), and (Ψ, τ_2);

Case-5: X is a neutrosophic b-open set in (Ψ, τ_1), and (Ψ, τ_3);

Case-6: X is a neutrosophic b-open set in (Ψ, τ_2), and (Ψ, τ_3);

Case-7: X is a neutrosophic b-open set in (Ψ, τ_1), (Ψ, τ_2), and (Ψ, τ_3).

In case-1, we can express, $X = X\cup 0_N \cup 0_N$, that is X is the union of neutrosophic b-open sets X (in (W, τ_1)), 0_N (in (W, τ_2)), and 0_N (in (W, τ_3)) . Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

In case-2, we can express, $X = 0_N\cup X\cup 0_N$, that is X is the union of neutrosophic b-open sets 0_N (in (W, τ_1)), X (in (W, τ_2)), and 0_N (in (W, τ_3)). Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

In case-3, we can express, $X = 0_N\cup 0_N\cup X$, that is X is the union of neutrosophic b-open sets 0_N (in (W, τ_1)), 0_N (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

In case-4, we can express, $X = X\cup X\cup 0_N$, that is X is the union of neutrosophic b-open sets X (in (W, τ_1)), X (in (W, τ_2)), and 0_N (in (W, τ_3)) . Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

In case-5, we can express, $X = X\cup 0_N\cup X$, that is X is the union of neutrosophic b-open sets X (in (W, τ_1)), 0_N (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

In case-6, we can express, $X = 0_N\cup X\cup X$, that is X is the union of neutrosophic b-open sets 0_N (in (W, τ_1)), X (in (W, τ_2)), and X (in (W, τ_3)). Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

In case-7, we can express, $X = X\cup X \cup X$, that is X is the union of neutrosophic b-open sets X (in (W, τ_1)), X (in (W, τ_2)), and X (in (W, τ_3)) . Therefore, X is a neutrosophic tri-t-b-open set in (Ψ, τ_1, τ_2, τ_3).

Hence, every neutrosophic tri-b-open set is a neutrosophic tri-t-b-open set.

Definition 3.10. Assume that (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS. Let X be an NS over Ψ. The neutrosophic tri-t-interior ($N\text{-}tri\text{-}t_{int}$) and neutrosophic tri-$t$-closure ($N\text{-}tri\text{-}t_{cl}$) of X is defined as follows:

$N\text{-}tri\text{-}t_{int}(X)= \cup\{Y: Y$ is a neutrosophic tri-t-open set and $Y\subseteq X\}$;

$N\text{-}tri\text{-}t_{cl}(X)= \cap\{Y: Y$ is a neutrosophic tri-t-closed set and $X\subseteq Y\}$.

It is clearly observed that $N\text{-}tri\text{-}t_{int}(X)$ is the largest neutrosophic tri-t-open set which is contained in X and $N\text{-}tri\text{-}t_{cl}(X)$ is the smallest neutrosophic tri-t-closed set which contains X.

Theorem 3.8. Let (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS. Let X and Y be two neutrosophic sets over Ψ. Then,

(*i*) $N\text{-}tri\text{-}t_{int}(X) \subseteq X$;

(*ii*) $X \subseteq Y \Rightarrow N\text{-}tri\text{-}t_{int}(X) \subseteq N\text{-}tri\text{-}t_{int}(Y)$;

(*iii*) If X is a neutrosophic tri-t-open set, then $N\text{-}tri\text{-}t_{int}(X)=X$;

(*iv*) $N\text{-}tri\text{-}t_{int}(0_N)=0_N$, and $N\text{-}tri\text{-}t_{int}(1_N)=1_N$.

Proof.

(i) From the definition 3.10, we see that $N\text{-}tri\text{-}t_{int}(X) = \cup\{B: B$ is a neutrosophic tri-t-open set and $B\subseteq X\}$. Since $B\subseteq X$, so $\cup\{B: B$ is a neutrosophic tri-t-open set and $B\subseteq X\} \subseteq X$. Therefore, $N\text{-}tri\text{-}t_{int}(X) \subseteq X$.

(ii) Suppose that X and Y are two neutrosophic sets over Ψ such that $X \subseteq Y$. Then,

$N\text{-}tri\text{-}t_{int}(X)$

$= \cup\{B$: B is a neutrosophic tri-t-open set and $B \subseteq X\}$;

$\subseteq \cup\{B$: B is a neutrosophic tri-t-open set and $B \subseteq Y\}$ [since $X \subseteq Y$]

$= N\text{-}tri\text{-}t_{int}(Y)$

$\Rightarrow N\text{-}tri\text{-}t_{int}(X) \subseteq N\text{-}tri\text{-}t_{int}(Y)$.

Therefore, $X \subseteq Y \Rightarrow N\text{-}tri\text{-}t_{int}(X) \subseteq N\text{-}tri\text{-}t_{int}(Y)$.

(iii) Assume that X be a neutrosophic tri-t-open set in an N-Tri-TS $(\Psi, \tau_1, \tau_2, \tau_3)$. Now, $N\text{-}tri\text{-}t_{int}(X) = \cup\{B$: B is a neutrosophic tri-t-open set and $B \subseteq X\}$. Since X is a neutrosophic tri-t-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$, so X is the largest neutrosophic tri-t-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$, which is contained in X. Hence $\cup\{B$: B is a neutrosophic tri-t-open set and $B \subseteq X\} = X$. Therefore, $N\text{-}tri\text{-}t_{int}(X) = X$.

(iv) We know that 0_N, and 1_N are neutrosophic tri-t-open sets in $(\Psi, \tau_1, \tau_2, \tau_3)$, so by the third part of this theorem, we have $N\text{-}tri\text{-}t_{int}(0_N) = 0_N$, $N\text{-}tri\text{-}t_{int}(1_N) = 1_N$.

Theorem 3.9. Let $(\Psi, \tau_1, \tau_2, \tau_3)$ be an N-Tri-TS. Let X and Y be two neutrosophic sets over Ψ. Then,

(*i*) $X \subseteq N\text{-}tri\text{-}t_{cl}(X)$;

(*ii*) $X \subseteq Y \Rightarrow N\text{-}tri\text{-}t_{cl}(X) \subseteq N\text{-}tri\text{-}t_{cl}(Y)$;

(*iii*) X is a neutrosophic tri-t-closed set iff $N\text{-}tri\text{-}t_{cl}(X) = X$;

(*iv*) $N\text{-}tri\text{-}t_{cl}(0_N)=0_N$, and $N\text{-}tri\text{-}t_{cl}(1_N)=1_N$;

Proof. (i) From the definition 3.10, we see that $N\text{-}tri\text{-}t_{cl}(X)= \cap\{B$: B is a neutrosophic tri-t-closed set and $X \subseteq B\}$. Since each $X \subseteq B$, so $X \subseteq \cap\{B$: B is a neutrosophic tri-t-closed set and $X \subseteq B\}$. Therefore, $X \subseteq N\text{-}tri\text{-}t_{cl}(X)$.

(ii) Suppose that X and Y are two neutrosophic sets over Ψ such that $X \subseteq Y$. Then,

$N\text{-}tri\text{-}t_{cl}(X)$

$= \cap\{B$: B is a neutrosophic tri-t-closed set and $X \subseteq B\}$.

$\subseteq \cap\{B$: B is a neutrosophic tri-t-closed set and $Y \subseteq B\}$ [since $X \subseteq Y$]

$= N\text{-}tri\text{-}t_{cl}(Y)$.

Therefore, $X \subseteq Y \Rightarrow N\text{-}tri\text{-}t_{cl}(X) \subseteq N\text{-}tri\text{-}t_{cl}(Y)$.

(iii) Assume that X be a neutrosophic tri-t-closed set in $(\Psi, \tau_1, \tau_2, \tau_3)$. Now, $N\text{-}tri\text{-}t_{cl}(X) = \cap\{B$: B is a neutrosophic tri-t-closed set and $X \subseteq B\}$. Since X is a neutrosophic tri-t-closed set in $(\Psi, \tau_1, \tau_2, \tau_3)$, so X is the smallest neutrosophic tri-t-closed set in $(\Psi, \tau_1, \tau_2, \tau_3)$, which contains X. Therefore, $\cap\{B$: B is a neutrosophic tri-t-closed set and $X \subseteq B\} = X$. Therefore, $N\text{-}tri\text{-}t_{cl}(X) = X$.

(iv) It is known that, 0_N and 1_N are neutrosophic tri-t-closed sets in $(\Psi, \tau_1, \tau_2, \tau_3)$. So, by the third part of this theorem, we have $N\text{-}tri\text{-}t_{cl}(0_N) = 0_N$, $N\text{-}tri\text{-}t_{cl}(1_N) = 1_N$.

Theorem 3.11. Let $(\Psi, \tau_1, \tau_2, \tau_3)$ be an N-Tri-TS. Let X be an NS over Ψ. Then, $\tau_i\text{-}N_{int}(X) = N\text{-}tri\text{-}t_{int}(X)$.

Proof. Assume that X be a neutrosophic subset of an N-Tri-TS $(\Psi, \tau_1, \tau_2, \tau_3)$. Now, $\tau_i\text{-}N_{int}(X) = \cup\{Y$: Y is an NOS in (Ψ, τ_i) and $Y \subseteq X\}$. Since Y is an NOS in (Ψ, τ_i), so by second part of Theorem 3.3., Y is a neutrosophic tri-t-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$.

Therefore, $\tau_i\text{-}N_{int}(X)$

$= \cup\{Y$: Y is an NOS in (Ψ, τ_i) and $Y \subseteq X\}$

$= \cup\{Y$: Y is a neutrosophic tri-t-open set in $(\Psi, \tau_1, \tau_2, \tau_3)$, and $Y \subseteq X\}$

$= N\text{-}tri\text{-}t_{int}(X)$.

Hence, in an N-Tri-TS $(\Psi, \tau_1, \tau_2, \tau_3)$, $\tau_i\text{-}N_{int}(X) = N\text{-}tri\text{-}t_{int}(X)$ for any neutrosophic set X.

Theorem 3.12. Let (Ψ, τ_1, τ_2, τ_3) be an N-Tri-TS. Let X be an NS over Ψ. Then, τ_i-$N_{cl}(X) \subseteq$ N-tri-$t_{cl}(X)$.

Proof. Assume that X be a neutrosophic subset of an N-Tri-TS (Ψ, τ_1, τ_2, τ_3). Now, τ_i-$N_{cl}(X) = \cap\{Y$: Y is an NCS in (Ψ, τ_i) and $X \subseteq Y\}$. Since Y is an NCS in (Ψ, τ_i), so by third part of Theorem 3.3., Y is a neutrosophic tri-t-closed set in (Ψ, τ_1, τ_2, τ_3).

Therefore, τ_i-$N_{cl}(X)$

$= \cap\{Y$: Y is an NCS in (Ψ, τ_i) and $X \subseteq Y\}$

$= \cap\{Y$: Y is a neutrosophic tri-t-closed set in (Ψ, τ_1, τ_2, τ_3) and $X \subseteq Y\}$

= N-tri-$t_{cl}(X)$.

Hence, τ_i-$N_{cl}(X)$ = N-tri-$t_{cl}(X)$, for any neutrosophic subset X of (Ψ, τ_1, τ_2, τ_3).

4. Conclusions

In this study, we introduce the notion neutrosophic tri-topological spaces. Also, we establish some of their basic properties. By defining neutrosophic tri-topology and neutrosophic tri-topological space, we present well described examples and proofs of some theorems on neutrosophic tri-topological spaces.

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