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Neutrosophic Boolean Rings

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Abstract: In this paper, we are going to define Neutrosophic Boolean rings and study their algebraic structure. A finite Boolean ring R satisfies the identity $a^2 = a$ for all $a \in R$, which implies the identity $a^n = a$ for each positive integer $n \geq 1$. With this as motivation, we consider a Neutrosophic Boolean ring $N(R, I)$ which fulfils the identity $(a+bI)^2 = a+bI$ for all $a+bI \in N(R, I)$ and describes several Neutrosophic rings which are Neutrosophic Boolean rings with various algebraic personalities. First, we show a necessary and sufficient condition for a Neutrosophic ring of a classical ring to be a Neutrosophic Boolean ring. Further, we achieve a couple of properties of Neutrosophic Boolean rings satisfied by utilizing the Neutrosophic self-additive inverse elements and Neutrosophic compliments.

Keywords: Boolean ring; Neutrosophic Boolean ring; Neutrosophic self-additive inverse elements; Neutrosophic compliments; Self and Mutual additive inverses.

1. Introduction

Essentially, a component a of a ring is idempotent if $a^2 = a$. A Boolean ring is a ring with unity wherein each component is idempotent. In any case, a ring with unity is by definition a ring with a recognized component 1 that goes about as a multiplicative identity and that is particular from the added substance character 0 . The impact of the last stipulation is to prohibit from thought the insignificant ring comprising of 0 alone. The expression with unit is in some cases excluded from the meaning of a Boolean ring; in that facilitate our current idea is known as a Boolean ring with unity. Every Boolean ring contains 0 and 1; the simplest Boolean ring contains nothing else. To be sure, the ring of numbers modulo 2 is a Boolean ring. This specific Boolean ring will be signified all through by a similar image as the ordinary integer 2 . However, it is exceptionally helpful. It is accordance with von Neumann's meaning of ordinal number, with sound general standards of notational economy, and in logical expressions such as two-honored with idiomatic linguistic usage. A non-trivial and common case of a Boolean ring is the set 2^X arrangement of all functions from an arbitrary non-empty set X into 2 . The components of 2^X will be called 2 -valued functions on X . The recognized components and operations in 2^X are defined point wise. This means that 0 and 1 in 2^X are the functions defined, for each x in X , by $0(x) = 0$ and $1(x) = 1$, and, if f and g are 2 -valued functions on X , then the functions $f + g$ and fg are defined by $(f+g)=f(x)+g(x)$ and $(fg)=f(x)g(x)$. These equations make sense; their right sides refer to elements of 2. The assumption $X \neq \phi$ needed to guarantee that 0 and 1 are distinct.

Next the usefulness of finite non-trivial Boolean rings has become increasingly apparent in the modern computer system theory, modern design theory, algebraic coding theory, algebraic cryptography, and electric circuit design theory. In particular, the electric circuit design of computer chips can be expressed in terms of finite Boolean rings with two components 0 and 1 as major elements. In this paper, we will consider Neutrosophic Boolean rings with three components 0, 1 and I of its significant components, and the results of these Neutrosophic Boolean rings can easily be generalized to the design of modern systems and the construction of integrated modern computer circuitry with indeterminate I.

As often occurs, the primary Neutrosophic theory research in pure and applied mathematics became indispensable in a large variety of applications in engineering and applied sciences. But Neutrosophic Boolean logic, Neutrosophic Boolean rings, and Neutrosophic Boolean algebra have become essential in the modern design of the large scale integrated circuitry found on today's modern computer chips. Additionally, sociologists and philosophical theorists have used Neutrosophic Boolean logic and their corresponding algebras to model social hierarchies; biologists, genetic engineers, and neurologists have used them to describe Neutrosophic biosystems with indeterminate I, see [1-8].

Throughout this paper, let all classical rings and Neutrosophic rings are considered to be finite and commutative structures with unity 1 and indeterminate I. Also, the present paper deals with Neutrosophic Boolean rings with generalized algebraic properties and define corresponding Neutrosophic units and Neutrosophic compliments. Further, we consider the cardinality of the finite Neutrosophic Boolean ring $N(R,I)$ which is defined by $|N(R,I)|>2$.

Furthermore, almost all our classical notions and their corresponding results are standard and follow those from [9-10]. The other non-classical ring concepts and their terminology will be explained in detail. Let R be a finite ring. The non-empty set

$$N(R,I)=\langle R,I\rangle=\{a+bI : a,b\in R, I^2=I\}$$

is called Neutrosophic ring generated by R and I under the operations of R, where I is called Neutrosophic unit with specific properties

(i). $I \neq 0,1$,

(ii). $I^2=I$,

(iii). $I+I=2I$

(iv). I^{-1} does not exist.

(v) $aI=0$ if and only if $a=0$, and

(vi) $aI=bI$ if and only if $a=b$.

If R is a commutative ring with unity 1 then $N(R,I)$ is also a Neutrosophic commutative ring with unity 1. An element u in R is a unit (multiplicative inverse element) if there exists u^{-1} in R such that $u^{-1}u=1=uu^{-1}$. The set of units of R is denoted by $R^{\times}$. But the set of Neutrosophic units denoted by $R^{\times}I$ and defined as $R^{\times}I=\{uI : u\in R^{\times}\}$, see{14}. But, the Neutrosophic group units denoted by $N(R,I)$ and defined as $(a+bI)^2=a+bI$ $=R^{\times}\cup R^{\times}I$ where $R^{\times}\cap R^{\times}I=\phi$. For further details about finite Neutrosophic rings, the reader should refer [11-17].

In this paper, we shall adopt the definition of a modern abstract mathematical structure known as the Boolean ring introduced by famous mathematician George Boole (1815 – 1864). This ring became an essential tool for the analysis and design modern digital systems, electronic computers, dial telephones, switching systems and many kinds of electronic devices and Fuzzy systems. First, we consider some definitions and results related to finite Boolean rings. An element a of a ring R is called idempotent if $a^2 = a$. In the integral domain, the only idempotent are 0 and 1. But, there exist many rings, which contain idempotent elements of different from 0 and 1. A ring with unity is called the Boolean ring if every element of R is an idempotent element. A finite Boolean ring R is a field if and only if R is isomorphic to Z_2, where Z_2 is the ring of integers modulo 2. Also, every nontrivial Boolean ring is commutative and its characteristic is 2. These results tend to particularly easy. Most of the results in this section can be found in [18].

2. Properties of Neutrosophic Boolean Rings

In this section, we are going to define Neutrosophic Boolean rings and study their properties with different illustrations and examples.

Definition.2.1 A Neutrosophic ring $N(R,I)$ is called **Neutrosophic Boolean ring** if $(a+bI)^2 = a+bI$ for all Neutrosophic elements $a+bI$ in $N(R,I)$.

Example.2.2 The Neutrosophic Boolean ring $N(Z_2,I) = \{0,1,I,1+I\}$ is a Neutrosophic Boolean ring of integers modulo 2 because $0^2 = 0$, $1^2 = 1$, $I^2 = I$, $(1+I)^2 = 1+I$.

Now we begin a necessary and sufficient condition for Neutrosophic Boolean rings.

Theorem. 2.3 The ring R is Boolean if and only if the Neutrosophic ring $N(R,I)$ is Neutrosophic Boolean ring.

Proof: Suppose R is a finite ring with unity 1. Then by the concepts of Boolean rings, we have R is a Boolean ring if and only if $a^2 = a$, $b^2 = b$, $ab = ba$, $2ab = 0$ for every $a,b \in R$. It is clear that for any Neutrosophic element $a+bI$ in the Neutrosophic ring $N(R,I)$,

$$(a+bI)^2 = (a+bI)(a+bI) = a^2 + 2abI + b^2I = a + 0I + bI \;\; a+bI.$$

Hence, $N(R,I)$ is Neutrosophic Boolean ring. The converse part is trivial.

Corollary.2.4 For any finite Neutrosophic Boolean ring, the following identities hold good.

(1) $(a+bI)^n = a+bI$

(2) $(aI+bI)^n = (a+b)I$ where n **is a** positive integer .

Proof: Follows from the identities $I^2 = I$, $a^n = a$ **,** $b^n = b$,and $(a+b)^n = a+b$ for every positive integer n.

Theorem (Cauchy's theorem).2.5 Every finite abelian group has an element of prime order.

We recall that the notion $|N(R,I)|$ for the cardinality of a Neutrosophic ring $N(R,I)$. If $N(R,I)$ is a finite nontrivial Neutrosophic ring then we denote the subset of its non zero elements by $N(R,I)^*$ and the subset of Neutrosophic units by $R^{\times}I$. Note that $|R| \neq |N(R,I)| \neq \{0\}$. For finite

Neutrosophic Boolean rings, we have the following two preparatory results. Recall from[14], let $R^{\times} = \{u \in R : \exists v \in R, uv = 1 = vu\}$ be the set of group units of a ring R **.** Then

$N(R^{\times}, I) = R^{\times} \cup R^{\times} I$ and

$N(R,I)^{\times} = \{a+bI : \exists c+dI \in N(R,I), (a+bI)(c+dI)=1\}$

be the set of Neutrosophic group units and Neutrosophic ring units of the Neutrosophic ring $N(R,I)$**,** respectively. For instance,

$Z_4 = \{0,1,2,3\}$**,**

$N(Z_4, I) = \{0,1,2,3,I,2I,3I,1+I,2+I,3+I,1+2I,2+2I,3+2I,1+3I,2+3I,3+3I\}$**,**

$N(Z_4^{\times}, I) = \{1,3,I,3I\}$**,** and

$N(Z_4, I)^{\times} = \{1+2I, 3+2I\}$**.**

Theorem.2.6 For some positive integer k , the total number of elements in the finite Neutrosophic Boolean ring $N(R,I)$ is 2^{2k} .

Proof: Suppose $|N(R,I)| = n$. We shall show that $n = 2^{2k}$ for some positive integer k . Assume that $n \neq 2^{2k}$ then n has a prime factor p other than 2 . Since $N(R,I)$ is an additive group with respect to Neutrosophic addition (+), $(a+bI)+(c+dI) = (a+c)+(b+dI)$ for all Neutrosophic elements $a+bI$ and $c+dI$ in $N(R,I)$. By the Cauchy's Theorem [14] for finite abelian groups, the group $N(R,I)$ contains an element $a+bI \neq 0$ with order prime p. Therefore,

$p(a+bI) = 0 \Rightarrow (2m+1)(a+bI) = 0$ where $p = 2m+1$**,** $m > 1$

$\Rightarrow 2m(a+bI)+(a+bI) =$**,** since the characteristic of $N(R,I)$ is 2

$\Rightarrow a+bI = 0$,

which is a contradiction to the fact that $a+bI \neq 0$. This completes the proof..

Theorem.2.7 If $N(R,I)$ is a Neutrosophic Boolean ring with unity 1 then $N(R^{\times}, I) = \{1, I\}$

Proof: Since $I^2 = I$. It is evident that I is the Neutrosophic unit of the Neutrosophic Boolean ring $N(R,I)$. Therefore, $N(R^{\times}, I) = \langle R^{\times}, I \rangle = R^{\times} \cup R^{\times} I$ where $R^{\times}$ and $R^{\times} I$ are disjoint. Suppose now that $u \in R^{\times}$. Then, now multiplying the expression $u^2 = u$ by u^{-1} , we obtain $u = 1$. Thus $R^{\times}$ contains the unique element 1 if and only if R is a nontrivial Boolean ring. This implies that $R^{\times} I = \{I\}$. Hence, $N(R^{\times}, I) = R^{\times} \cup R^{\times} I = \{1\} \cup \{I\} = \{1, I\}$.

Theorem. 2. 8 For any finite non trivial Boolean ring R , we have $N(R,I)^{\times}$ is empty.

Proof. Suppose that R is a finite Boolean ring. Then its definition satisfies the identity $a^2 = a$ for every a in R . Now we shall show that $N(R,I)^{\times}$ is empty. If possible assume that $N(R,I)^{\times}$ is non empty, then there is a Neutrosophic element $a+bI$ in $N(R,I)^{\times}$ such that $a \neq 0$, $b \neq 0$ and $(a+bI)^2 = 1$. This implies that

$a^2 + (b^2 + 2ab)I = 1 + 0I \Rightarrow a^2 = 1, b^2 + 2ab = 0$

$\Rightarrow a = 1, b(b+2a) = 0$

$\Rightarrow a = 1, b = 0$, or, $a = 1, b = -2$, which is a contradiction to the fact that R is a finite non trivial Boolean ring and $N(R,I)$ is its corresponding Neutrosophic Boolean ring. So our assumption is not true, and hence $N(R,I)^{\times}$ is empty.

Now we can immediately prove that a special relationship between divisor of zero and simple Neutrosophic field.

Theorem. 2.9 If a Neutrosophic Boolean ring $N(R,I)$ contains no divisor of zero, then it is either $\{0\}$, or, is isomorphic to Neutrosophic field $N(Z_2,I)$.

Proof: Suppose $N(R,I)$ is a Neutrosophic Boolean ring. Then for any two Neutrosophic elements $a+bI$ and $c+dI$ in $N(R,I)$ we have the following relation

$$\begin{aligned}(a+bI)(c+dI)[(a+bI)+(c+dI)] &= (a+bI)^2(c+dI)+(a+bI)(c+dI)^2\\ &= (a+bI)(c+dI)+(a+bI)(c+dI)\\ &= 2(a+bI)(c+dI)=0,\end{aligned}$$

Since $N(R,I)$ is a Neutrosophic Boolean ring and its characteristic is 2. This implies that

$$(a+bI)(c+dI)[(a+bI)+(c+dI)]=0$$

Therefore, either $(a+bI)(c+dI)=0$, or, $(a+bI)+(c+dI)=0$. Hence, either $N(R,I)$ has a divisor of zero, or, $(a+bI)+(c+dI)=0$ for any two Neutrosophic elements $a+bI$ and $c+dI$ in $N(R,I)$. In later case, that is, $(a+bI)+(c+dI)=0$ implies that $(a+bI)=-(c+dI)=c+dI$, it follows that $a=c$ and $b=d$, and R can have only one non zero element, that is, R is isomorphic to the field Z_2, and thus $N(R,I)$ isomorphic to Neutrosophic field $N(Z_2,I)$.

For general Neutrosophic ring $N(R,I)$, the following theorem is obvious when the characteristic of $N(R,I)$ is 2, and after we shall show that a Neutrosophic ring is Neutrosophic Boolean ring when it satisfies the identity $(a+bI)^3=a+bI$.

Theorem. 2.10 Let $N(R,I)$ be a Neutrosophic commutative ring with unity and its characteristic is 2. Then the following identities are held good in $N(R,I)$.

(1) $(1+(a+bI))^2=1+(a+bI)^2$

(2) $(1+(a+bI))^4=1+(a+bI)^4$

(3) $(I+(a+bI))^2=I+(a+bI)^2$.

Theorem.2.11 Let $N(R,I)$ be a Neutrosophic commutative ring with unity and it satisfies the identity $(a+bI)^3=a+bI$ for all $a+bI$ in $N(R,I)$. Then $N(R,I)$ is a Neutrosophic Boolean ring.

Proof: Since $|N(R,I)|\geq 4$ for any ring R with $|R|>1$. Then clearly $1,I\in N(R,I)$ and the identity $(a+bI)^3=a+bI$ for all $a+bI\in N(R,I)$ implies that the characteristic of $N(R,I)$ is 2. For this reason, the following are true in $N(R,I)$.

$$a+bI=(a+bI+1)+1 \text{ and } a+bI=(a+bI+I)+I.$$

Hence, by the Theorem [2.10] and by the identity $(a+bI)^3=a+bI$, the following is holds good.

$$\begin{aligned}1+(a+bI) = (1+(a+bI))^3 &= (1+(a+bI))(1+(a+bI))^2\\ &= (1+(a+bI))(1+(a+bI)^2)\\ &= 1+(a+bI)+(a+bI)^2+(a+bI)^3\\ &= 1+(a+bI)+(a+bI)^2+(a+bI)\\ &= 1+(a+bI)^2+2(a+bI)\\ &= 1+(a+bI)^2+0 = 1+(a+bI)^2.\end{aligned}$$

Now using the additive left cancellation law of Neutrosophic rings, we obtain the identity $(a+bI)^2=a+bI$ for all $a+bI\in N(R,I)$, and hence $N(R,I)$ is a Neutrosophic Boolean ring.

Remark.2.12 The Theorem [2.11] shows that, if Neutrosophic ring with identity $(a+bI)^3 = a+bI$ is Neutrosophic Boolean ring. From this identity, we observe that the characteristic of $N(R,I)$ is 2, which is essential. Otherwise, it is evidence that the Neutrosophic ring $N(Z_6,I)$ satisfies the identity $(a+bI)^3 = a+bI$ but it is not a Neutrosophic Boolean ring because of the characteristic $N(Z_6,I)$ is 6.

Next, the following table [2.13] illustrates the main differences between Boolean rings (classical rings) and Neutrosophic Boolean rings. Consider R and $N(R,I)$ be a finite Boolean ring and its corresponding Neutrosophic Boolean ring, respectively.

Boolean rings	**Neutrosophic Boolean rings**
(i). $\lvert R\rvert = 2^k$.	(i). $\lvert N(R,I)\rvert = 2^{2k}$.
(ii). R contains two logical components 0 and 1.	(ii). $N(R,I)$ contains three logical components 0, 1 and I .
(iii). $R^{\times} = \{1\}$.	(iii). $N(R^{\times},I) = \{1, I\}$.
(iv). If R is a field then R isomorphic to Z_2.	(iv). If $N(R,I)$ is a Neutrosophic field then $N(R,I)$ is isomorphic to $N(Z_2,I)$.
(v). $1 \le \lvert R\rvert \le 2^k$.	(v). $4 \le \lvert N(R,I)\rvert \le 2^{2k}$.

Table. 2.13 Differences between Boolean rings and Neutrosophic Boolean rings.

3. Neutrosophic Complements

In this section, we have mainly obtained some properties satisfied by the Neutrosophic complements of Neutrosophic Boolean rings with unity. Note that the element a is called compliment of b in the ring R if $a+b=1$. The set of all compliments of R is denoted by $Comp(R)$, that is, $Comp(R) = \{(a,b): a+b=1\}$. Also, the two distinct elements x and y of R are called mutual additive inverses of R if $x+y=0$, and the set of all mutual additive inverses of R is denoted by $M(R)$ and $M(R) = \{(x,y): x+y=0\}$. In particular, the set $S(R) = \{(x,y): x+x=0\}$ is called the set of all self additive inverses of R. For more information about self and mutual additive inverses of R, reader refer [15]. Now begin the definition of compliments in Neutrosophic ring.

Definition.3.1 Let $N(R,I)$ be a Neutrosophic ring with unity 1 An element $a+bI$ is called **Neutrosophic compliment** of $c+dI$ in $N(R,I)$ if $(a+bI)+(c+dI)=1$. The set of all these Neutrosophic complement pairs in $N(R,I)$ is denoted by $Comp(N(R,I))$ and defined as

$$Comp(N(R,I)) = \{(a+bI, c+dI) : (a+bI)+(c+dI) = 1\}.$$

Note that if $2(a+bI)=1$ then $a+bI$ is called **Neutrosophic self-complement** and the set of all Neutrosophic self-compliments of $N(R,I)$ is denoted by $SComp(N(R,I))$. For example, the pair $(I, 1+I)$ is a Neutrosophic complement pair in $N(Z_2,I)$ because $I+(1+I)=1$.

Theorem. 3.2 Let R be a finite ring with unity 1. Then the pair $(a+bI, c+dI)$ is a Neutrosophic complement pair in $N(R,I)$ if and only if (a,c) is a complimenting pair and (b,d) mutual additive inverse pair in R.

Proof: Suppose $a+bI$ and $c+dI$ be two elements in $N(R,I)$. By the definition of Neutrosophic complement pair, the pair $(a+bI, c+dI)$ is a Neutrosophic complement pair in $N(R,I)$ if and only if $(a+bI)+(c+dI)=1$ if and only if $(a+c)+(b+d)I = 1+0I$ if and only if $a+c=1$ and $b+d=0$.

Corollary. 3.3 If $n>1$ be a positive integer then the total number of Neutrosophic Complement pairs in $N(Z_n, I)$ is $n/2$ if n is even and is $(n-1)/2$ if n is odd.

Proof: It is obvious from the well-known formula that

$$|Comp(N(Z_n, I))| = n/2 \text{ if } n \text{ is even and } (n-1)/2 \text{ if } n \text{ is odd.}$$

Example.3.4 Since the ring of integers modulo 4 is $Z_4 = \{1,2,3,4\}$. The set complement and Neutrosophic complement pairs of the ring Z_4 is

$$Comp(Z_4) = \{(0,1),(2,3)\} \text{ and } Comp(N(Z_4, I)) = \{(2I, 1+2I),(2+2I, 3+2I)\},$$

respectively.

Theorem.3.5 The following conditions on the Neutrosophic ring $N(R, I)$ with unity 1 are equivalent.

(i). $N(R, I)$ is a Neutrosophic Boolean ring.

(ii). The complement of the Neutrosophic idempotent element is Neutrosophic idempotent

(iii). The Neutrosophic complements are Neutrosophic zero divisors.

Proof: (i)⇒(ii). First suppose $N(R, I)$ is a Neutrosophic Boolean ring with unity 1. Let $c+dI$ be the Neutrosophic complement of Neutrosophic idempotent $a+bI$ in $N(R, I)$. Then

$$(c+dI)^2 = (c+dI)(c+dI) = (1-(a+bI))(1-(a+bI))$$
$$= 1-(a+bI)-(a+bI)+(a+bI)^2 = 1-(a+bI) = c+dI.$$

This proves (ii).

(ii) ⇒ (iii). From (ii) we have $(a+bI)+(c+dI)=1$.

Therefore,

$$(a+bI)(c+dI) = (a+bI)(1-(a+bI)) = (a+bI)-(a+bI)^2 = (a+bI)-(a+bI) = 0.$$

This completes (iii).

(iii) ⇒ (i). From (iii) we have

$$(a+bI)+(c+dI)=1 \Rightarrow (a+bI)(c+dI)=0$$

It is clear that the Neutrosophic elements $a+bI$ and $c+dI$ are both Neutrosophic Complements to each other, and this forces that the identity $(1-(a+bI))^2 = 1-(a+bI)$. Hence $N(R, I)$ is a Neutrosophic Boolean ring.

4. Conclusions

In this paper, we address our self a twofold aim: first to review the theory of classical Boolean rings, as we understand it recent, and to construct certain Neutrosophic Boolean rings. Next, we have introduced Neutrosophic complement elements and mainly obtained some properties satisfied by the Neutrosophic complement elements of Neutrosophic Boolean rings. This study understands the new structure basis in Neutrosophic hypothesis which builds up another idea for the comparison of classical Boolean ring and Neutrosophic Boolean ring structures dependent on the use of the indeterminacy idea and the structural information.

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