



# The Neutrosophic Time Series-Study Its Models (Linear-Logarithmic) and test the Coefficients Significance of Its linear model

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**Abstract:** In this paper, we present the Neutrosophic time series by studying the classical time series within the framework of the Neutrosophic logic. (Logic established by the American philosopher and mathematician Florentin Smarandache presented it as a generalization of fuzzy logic, especially intuitionistic fuzzy logic). As an extension of this, A.A. Salama presented the theory of Neutrosophic crisp sets as a generalization of crisp sets theory. This study enables us to deal with all the time series values whether it is specified or not specified, we present the linear model for the Neutrosophic time series, and we test the significant of its coefficient based on Student's distribution. We present an example in which we pave the Neutrosophic time series according to the linear model, test the significant of its coefficient, and show how to deal with the unspecified values of the time series. Then we present the logarithmic model of the Neutrosophic time series. We conclude that the existence of indeterminacy in the matter we cannot ignored because it actually affects the estimated values of the time series and thus affects the prediction of the future of the series.

**Keywords:** Time Series, Neutrosophic logic, Neutrosophic Time Series, the linear model of the Neutrosophic time series, the significant of coefficients to the Neutrosophic linear model. The logarithmic model of the Neutrosophic time series.

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## 1. Introduction

Neutrosophic is a new view of Modeling , designed to effectively deal underlying doubts in the real world, as it came to replace binary logic that recognized right and wrong by introducing a third neutral case which could be interpreted as non-specific or uncertain. Founded by Florentin Smarandache, the American philosopher and mathematician, he presented it in 1999 as a generalization of fuzzy logic and as an extension of the theory of fuzzy sets presented by Lotfi A. Zadeh (1965) [6]. As an extension of this, A. A. Salama presented the theory of Neutrosophic crisp sets as a generalization of classical sets theory [5], [7], [12] and developed, inserted and formulated new concepts in the fields of mathematics, statistics, computer science and classical information systems through Neutrosophic [1-3] , [4], [5], [12]. Neutrosophic has grown significantly in recent years through its application in measurement, sets and graphs and in many scientific and practical fields [14-29].

Smarandache defined the Neutrosophic logic as a new, non-classical logic that studies the origin, nature, and field of Indeterminacy, as well as the interaction of all the different spectra that a person imagines in an Issue, so that he takes into account every idea with against it (its opposite) with the indeterminacy [11], [9].

In this paper, we highlight the application of the Neutrosophic logic to the concept of time series, so we know the concept of the Neutrosophic time series, which opens the way for dealing with time series that take precisely unspecified values such as taking range of values instead of one value (as in the classic) . We also provide the linear model for this Neutrosophic time series with an example showing how to deal with non-specified values in time series, and we test the significance of the coefficient of the model that we obtained using the Student's distribution. Finally, we present the logarithmic model of the Neutrosophic time series.

## 2. The Neutrosophic Time Series:

A time series is a set of data arranged in chronological order, the data of this series are associate to each other in the general case, and this correlation gives us reliable future forecasts. We also define it as a set of consecutive values (observations) that describe the evolution of a phenomenon over time. We say about this time series that it is Neutrosophic, if some or all of its values (its observations) are not explicitly specific, such as being a range of values instead of one value.

## 3. The Discussion:

The aim of studying time series is to monitor the changes that accompany the phenomenon during a specific period. In addition, description, analysis and classification of the phenomenon. As well as studying the reasons, that led to these changes in the phenomenon, and trying to evaluate it by accurate scientific methods. In addition, predicting what will happen to the series in the future. this is done based on the previous history of the series, by relying on statistical and mathematical laws that describe the phenomenon in the past well , and have the ability to Evaluate and estimate their future values with the least amount of errors possible. Proceeding from the importance of this goal, we must look at the data in a more comprehensive and accurate manner than they are, considering that it is the basis in predicting the future of the time series, and based on this we present the concept of the Neutrosophic time series.

The Neutrosophic time series is similar to the classic time series in terms of their types according to the unit of time that the phenomenon is measured. Where there are decennium Neutrosophic series whose values are taken every ten years such as population census (from a Neutrosophic point of view). Annual Neutrosophic time series that record their values each year such as estimating wheat production for a particular country. Quarterly Neutrosophic series such as the production of some seasonal crops. Monthly Neutrosophic series such as monthly factor production of medications. In addition, daily Neutrosophic series that record their values on a daily such as temperature, humidity, and wind speed.

Many conditions and attributes must be achieved by the values in order for us to call them a Neutrosophic time series:

1. The unit of time measurement, which is that all elements of the series have the same units of measurement (day - month - year ...).
2. The one place, all elements of the series must be measured in the same place and it is not permissible to take part of the values in one city and the rest in another city.

3. The unit of measurement for the series elements, where all the elements of the series must be measured in the same unit (m, kg ...).
4. The number of series values must be finite, and it may contain some indeterminate values.

**4. The Linear model of a Neutrosophic time series:**

The general form of the linear model of a Neutrosophic time series is:

$$\hat{Y}_t = a_N + b_N t$$

Where:

- $Y_t$  :The real values of the time series.
- $\hat{Y}_t$  : The estimated values.
- $a_N$  : Constant coefficient.
- $b_N$  : Regression coefficient.
- T : Time.

We compute  $a_N, b_N$  by the least squares method [10], which is:

$$\hat{b}_N = \frac{\sum(t-\bar{t})(Y_t - \bar{Y}_t)}{\sum(t-\bar{t})^2}$$

**4.1 Example:**

We have the following Neutrosophic data representing the production of a machine in a factory (by piece). We want to pave this data linearly:

T	$Y_t$	$(t - \bar{t})$	$(t - \bar{t})^2$	$(Y_t - \bar{Y}_t)$	$(t - \bar{t})(Y_t - \bar{Y}_t)$
1	[25,27]	-4.5	20.25	[-37.9 , -37.5]	[168.75 , 170.55]
2	30	-3.5	12.25	[-34.5 , -32.9]	[115.15 , 120.75]
3	45	-2.5	6.25	[-19.5 , -17.9]	[44.75 , 48.75]
4	[50,55]	-1.5	2.25	[-12.9 , -9.5]	[14.25 , 19.35]
5	[44,46]	-0.5	0.25	[-18.9 , -18.5]	[9.25 , 9.45]
6	60	0.5	0.25	[-4.5 , -2.9]	[-2.25 , -1.45]
7	80	1.5	2.25	[15.5 , 17.1]	[23.25 , 25.65]
8	[90,95]	2.5	6.25	[27.1 , 30.5]	[67.75 , 76.25]
9	100	3.5	12.25	[35.5 , 37.1]	[124.25 , 129.25]
10	[105,107]	4.5	20.25	[42.1 , 42.5]	[189.45 , 191.25]
<b>the sum</b>					[754.6 , 789.8]

Calculation of the following:  $\bar{t} = \frac{55}{10} = 5.5$

$$\bar{y} = \frac{\sum Y_t}{n} = \frac{[25,27]+[30,30]+[45,45]+[50,55]+[44,46]+[60,60]+[80,80]+[90,95]+[100,100]+[105,107]}{10} = \frac{[629,645]}{10} =$$

[ 62.9 , 64.5]

Subsequently  $\bar{Y}_t = [ 62.9 , 64.5]$

$$\hat{b}_N = \frac{\sum(t-\bar{t})(Y_t - \bar{Y}_t)}{\sum(t-\bar{t})^2} = \frac{[754.6,789.8]}{82.5} = [ 9.147 , 9.573]$$

$$\hat{a}_N = \bar{y} - b\bar{t} = [62.9, 64.5] - [9.147, 9.573](5.5) = [62.9, 64.5] - [ 50.31 , 52.65] = [11.85 , 12.59]$$

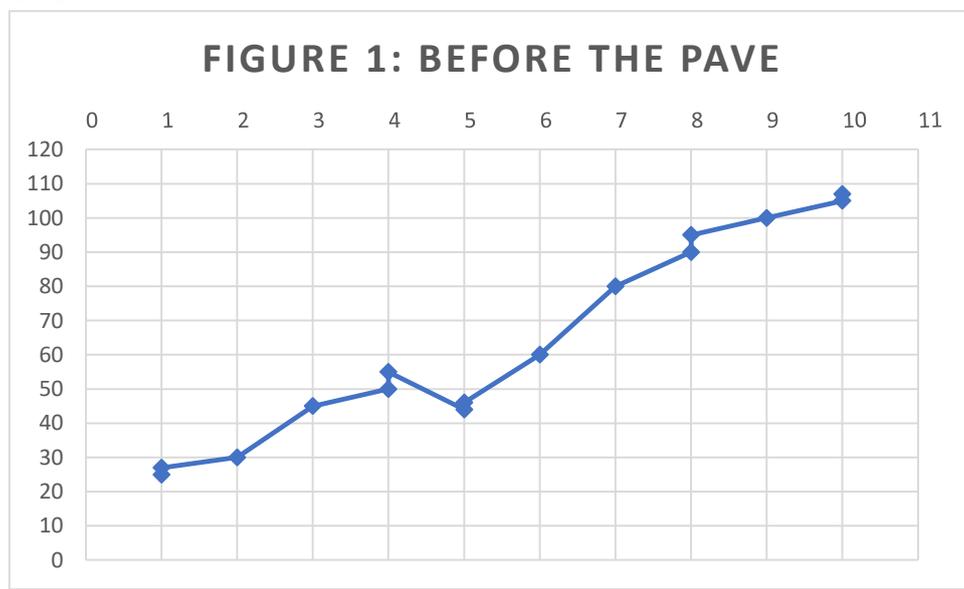
$$\hat{a}_N = [ 11.85 , 12.59]$$

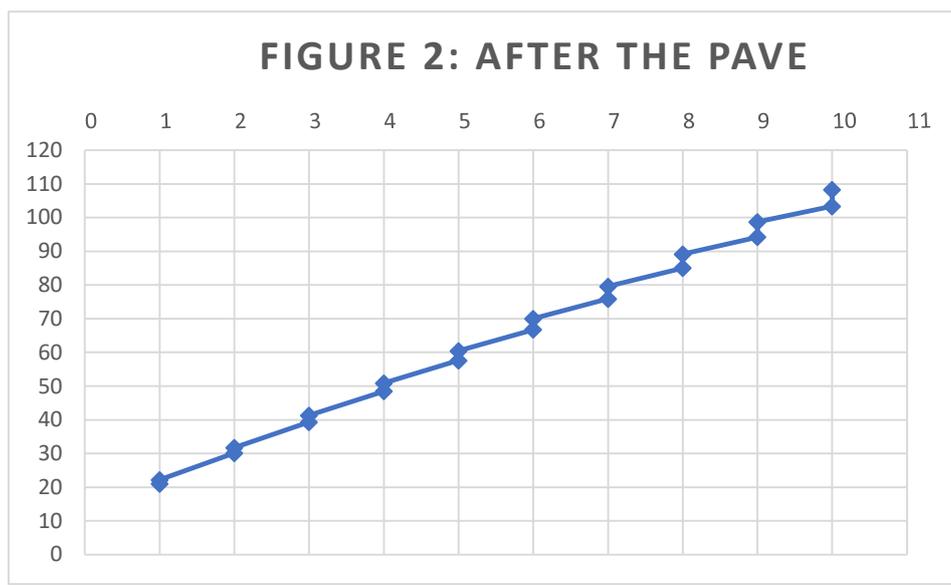
Subsequently  $\hat{Y}_t = a_N + b_N t = [11.85 , 12.59] + [9.147 , 9.573] t$

Now we calculate the estimated values for  $\hat{Y}_t$ :

T	$\hat{Y}_t$
1	[21 , 22.16]
2	[30.15 , 31.73]
3	[39.3 , 41.3]
4	[48.45 , 50.87]
5	[57.6 , 60.44]
6	[66.75 , 70.01]
7	[75.9 , 79.58]
8	[85.05 , 89.15]
9	[94.2 , 98.72]
10	[103.35 , 108.29]

4.1.1 The graph:





**We note that:**

1. From figures (1) and (2) it is clear that the model represents the data well, and this appears from the approximation of the graphic line of the estimated values from the graphic line of the real values.
2. We did not merge the diagram of series before and after the pave into a single diagram, so that the Neutrosophic values appear clearly, and no confusion occurs.
3. We note in both cases (before and after the pave of the series) that the Neutrosophic values of the series did not affect its general direction neither by increasing nor decreasing (only the time series moves slightly within the range), without affecting the properties of the series and the characteristics of its general direction. It gives us an accurate view of the series and thus more accurate and objective prediction of the future of the series.

**5. Coefficient's Significance test of the Neutrosophic linear model:**

We say about the regression coefficient (the coefficient of the model) that it is significant if its value is effective and its effect cannot be ignored. In addition, we say that the regression coefficient is not significant if its value can be neglected and its effect neglected.

We perform the test on the coefficient 'b' as in the classic method, but here we deal with Neutrosophic data. (The constant coefficient 'a' mostly has no statistical significance).

The model coefficient test is subjected to a student test with (n-2) degrees of freedom, and his Neutrosophic hypothesis is: [10]

$$NH_0 = b_N = 0$$

$$NH_1 = b_N \neq 0$$

The test index is:  $tt = \frac{b_N}{s_b}$

Where:

$$S_b = \frac{e_r}{\sqrt{\sum(t-\bar{t})^2}}$$

$$e_r = \sqrt{\frac{\sum(Y_t - \hat{Y}_t)^2}{n-2}}$$

In which:

$b_N$  :The coefficient of the Neutrosophic model we want to test its value.

$S_b$  : the error of Neutrosophic model Coefficient.

$\hat{Y}_t$  : Estimated values in the Neutrosophic model.

$Y_t$  : the Neutrosophic real values of the series.

Coefficient significance test ' $b_N$ ' in the previous example:

We had:

$$\hat{Y}_t = a_N + b_N t = [11.85, 12.59] + [9.147, 9.573] t$$

Let us calculate the test index value:

$$tt = \frac{b_N}{S_b}$$

To do this, we calculate first:

$$e_r = \sqrt{\frac{\sum(Y_t - \hat{Y}_t)^2}{n-2}}$$

$Y_t$	$\hat{Y}_t$	$(Y_t - \hat{Y}_t)^2$
[25,27]	[21, 22.16]	[16, 23.4]
30	[30.15, 31.73]	[0.02, 2.99]
45	[39.3, 41.3]	[13.7, 32.5]
[50,55]	[48.45, 50.87]	[2.4, 17.1]
[44,46]	[57.6, 60.44]	[184.96, 208.5]
60	[66.75, 70.01]	[45.6, 100.2]
80	[75.9, 79.58]	[0.18, 16.8]
[90,95]	[85.05, 89.15]	[24.5, 34.2]
100	[94.2, 98.72]	[1.64, 33.6]
[105,107]	[103.35, 108.29]	[1.67, 2.7]
<b>The sum</b>		<b>[290.7, 472]</b>

$$e_r = \sqrt{\frac{\sum(Y_t - \hat{Y}_t)^2}{n-2}} = \sqrt{\frac{[290.7, 472]}{8}} = \sqrt{[36.3, 59]} = [6.02, 7.68]$$

$$S_b = \frac{e_r}{\sqrt{\sum(t - \bar{t})^2}} = \frac{[6.02, 7.68]}{62.25} = [0.097, 0.123]$$

$$tt = \frac{b_N}{S_b} = \frac{[9.147, 9.573]}{[0.097, 0.123]} = [77.8, 94.3]$$

Let us discuss the significance of '  $b_N$  ', by comparing the test index '  $tt$  ' with the tabular value, at the significance level of  $\alpha = 0.05$  and the degree of freedom ( $n-2 = 8$ ), then:

$$tt(\alpha, n-2) = tt(0.05, 8) = 1.8595$$

We note that:

$$tt = [77.8, 94.3] > tt(\alpha, n-2) = 1.8595$$

Thus, we reject the primary hypothesis and take the alternative hypothesis. That is, the coefficient of the Neutrosophic linear model is significant. In addition, the model can be used to pave the series and prediction.

## 6. The logarithmic model of a Neutrosophic time series:

The general form of the logarithmic model of the Neutrosophic time series is:

$$\hat{Y}_t = a_N + b_N \ln t$$

Where:

$Y_t$  : The real values of the time series.

$\hat{Y}_t$  : The estimated values.

$a_N$  : Constant coefficient.

$b_N$  : Regression coefficient.

$T$  : Time.

We transform this model into a linear model by the following assumption:  $T = \ln t$

Then, the form becomes linear as follows:

$$\hat{Y}_t = a_N + b_N T$$

We compute  $a_N$ ,  $b_N$  by the least squares method [10], which is:

$$\text{(We used the second formula)} \quad \hat{b}_N = \frac{(\sum Y.T - \sum Y \sum T)/n}{(\sum T^2 - (\sum T)^2)/n}$$

$$\hat{a}_N = \bar{y} - b\bar{T}$$

We want pave the Neutrosophic time series "in the previous example" using the logarithmic model:

<b>t</b>	<b>T = lnt</b>	<b>Y<sub>t</sub></b>	<b>Y<sub>t</sub> . T</b>	<b>T<sup>2</sup></b>
1	0	[25,27]	0	0
2	0.69	30	20.7	0.48
3	1.10	45	49.5	1.21
4	1.39	[50,55]	[69.5 , 76.45]	1.92
5	1.61	[44,46]	[70.84 , 74.06]	2.59
6	1.79	60	107.4	3.21
7	1.95	80	156	3.79
8	2.08	[90,95]	[187.2 , 197.6]	4.32
9	2.20	100	220	4.83
10	2.30	[105,107]	[241.5 , 246.1]	5.30
<b>55</b>	<b>15.10</b>	<b>[629 , 645]</b>	<b>[1122.64 , 1147.81]</b>	<b>27.65</b>

$$\hat{b}_N = \frac{(\sum Y.T - \sum Y \sum T)/n}{(\sum T^2 - (\sum T)^2/n)} = \frac{([1122.64, 1147.81] - [629, 645](15.10))/10}{(27.65 - (15.10)^2)/10} = [41.8 , 42.9]$$

$$\hat{a}_N = \bar{y} - b\bar{T} = [62.9 , 64.5] - [41.8 , 42.9](1.51) = [-0.3 , -0.22]$$

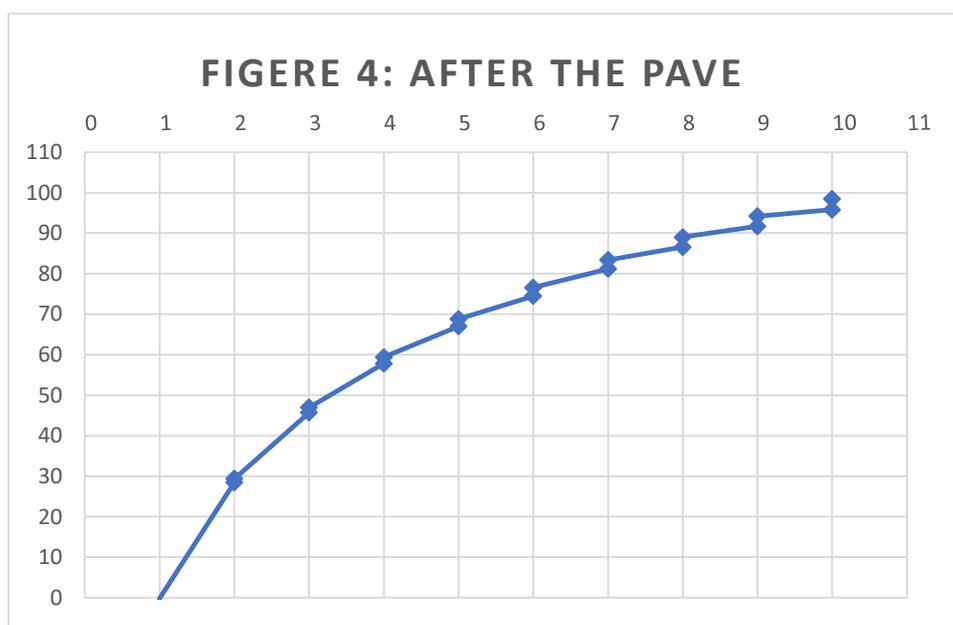
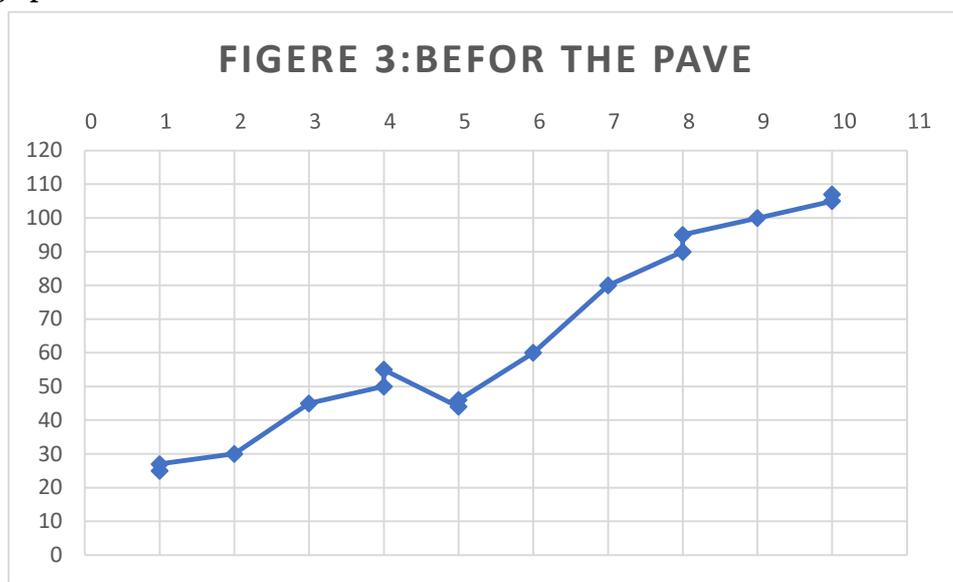
Subsequently:

$$\hat{Y}_t = a_N + b_N \ln t = [-0.3 , -0.22] + [41.8 , 42.9] \ln t$$

Now we calculate the estimated values for  $\hat{Y}_t$ :

<b>T = lnt</b>	<b>t</b>	<b><math>\hat{Y}_t</math></b>
0	1	[-0.3 , -0.22]
0.69	2	[28.5 , 29.4]
1.10	3	[45.7 , 46.97]
1.39	4	[57.8 , 59.4]
1.61	5	[66.998 , 68.8]
1.79	6	[74.5 , 76.57]
1.95	7	[81.2 , 83.4]
2.08	8	[86.6 , 89.01]
2.20	9	[91.7 , 94.2]
2.30	10	[95.8 , 98.5]

**6.1 The graph:**



We notice from figures (3) and (4) it is clear that the model represents the data well, and this appears from the approximation of the graphic line of the estimated values from the graphic line of the real values.

**7. Conclusions:**

We conclude that dealing with Neutrosophic time series provides us with a comprehensive and general study of the studied phenomenon, in which we do not neglect any data only because it is not specific. Working within the framework of classical logic is not sufficient now. Where we found that the indeterminacy actually affects the estimated values, and therefore the unspecified values we cannot be ignored and removed from the study framework in order to obtain the most accurate results as possible, thus drawing the future of the series and predicting it better and more accurately. In the near future, we look forward to applying the Neutrosophic logic to other models of time series, and to studying the seasonal, periodic and random components of the Neutrosophic time series.

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