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Subtraction operational aggregation operators of simplified neutrosophic numbers and their multi-attribute decision making approach

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Abstract: The simplified form of a neutrosophic set (NS) was introduced as the simplified NS (S-NS) containing an interval-valued NS (IV-NS) and a single-valued NS (SV-NS) when its truth, indeterminacy and falsity membership degrees are constrained in the real standard interval [0, 1] for the convenience of actual applications. Then, Ye presented subtraction operations of simplified neutrosophic numbers (S-NNs), containing the subtraction operations of interval-valued neutrosophic numbers (IV-NNs) and single-valued neutrosophic numbers (SV-NNs) in S-NN setting. However, the subtraction operations of S-NSs lack actual applications in current research. Since simplified neutrosophic aggregation operators are one of critical mathematical tools in decision making (DM) applications, they have been not investigated so far. Regarding the subtraction operations of S-NNs (SV-NNs and IV-NNs), this work proposes an IV-NN subtraction operational weighted arithmetic averaging (IV-NNSOWAA) operator and a SV-NN subtraction operational weighted arithmetic averaging (SV-NNSOWAA) operator as a necessary complement to existing aggregation operators of S-NNs to aggregate S-NNs (SV-NNs and IV-NNs). Then, a DM approach is developed by means of the SV-NNSOWAA and IV-NNSOWAA operators. Finally, an illustrative example is presented to indicate the applicability and effectiveness of the developed approach.

Keywords: Decision making; Simplified neutrosophic set; Subtraction operation; Subtraction operational aggregation operator

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1. Introduction

Neutrosophic set (NS) introduced by Smarandache [1] can depict indeterminate and inconsistent information, which is characterized independently by its truth, falsity and indeterminacy membership degrees in the real standard or non-standard interval $]^{-}0, 1^{+}[$. As a simplified form of NS, however, a simplified NS (S-NS) can be introduced when the truth, falsity and indeterminacy membership degrees are constrained in the real standard [0, 1] for the convenience of actual applications. Thus, Ye [2] introduced the S-NS composed of an interval-valued NS (IV-NS) [3] and a single-valued NS (SV-NS) [4], as the subclass of NS. S-NSs can depict the inconsistent and indeterminate information which exists in actual situations, while (interval-valued) intuitionistic fuzzy sets (IFSs) cannot do it. Therefore, S-NSs have received more and more attention in various fields. So far S-NSs (SV-NSs and IV-NSs) have been utilized in image processing [5], medical diagnosis [6], clustering analysis [7, 8], fault diagnosis [9-11], decision making (DM) [12-23] and so on.

Then, division and subtraction operations of (interval-valued) IFSs were presented in existing literature [26-28], and then the subtraction operational aggregation operators (SOAOs) of IFSs were developed for DM problems of clay-brick selection [29]. After that, division and subtraction operations of S-NSs [30], containing the division and subtraction operations of IV-NNs and SV-NNs, were proposed as the operational generalization of (interval-valued) IFSs. However, the subtraction operations of S-NSs lack actual applications in current research. Since simplified neutrosophic number (S-NN) aggregation operators are one of critical mathematical tools in DM applications, the SOAOs of S-NNs have been not investigated so far. Since S-NSs are the extension of (interval-valued) IFSs, the SOAOs of (interval-valued) IFSs can be also generalized to S-NSs to form the SOAOs of S-NSs as a necessary complement to existing aggregation operators of S-NNs. Hence, this paper presents an IV-NN subtraction operational weighted arithmetic averaging (IV-NNSOWAA) operator and a SV-NN subtraction operational weighted arithmetic averaging (SV-NNSOWAA) operator based on the S-NN (IV-NN and SV-NN) aggregation operators [18-20] and establishes their multi-attribute DM approach in S-NN setting.

For this study, the remainder of this paper is formed as the structure. Section 2 introduces some basic notion of S-NSs and operations of S-NNs (IV-NNs and SV-NNs). Section 3 proposes the SV-NNSOWAA and IV-NNSOWAA operators of S-NNs. A multi-attribute DM approach is developed by using the SV-NNSPOWAA or IV-NNSOWAA operator in Section 4. In Section 5, an illustrative example is provided to indicate the applicability and effectiveness of the developed approach. Some conclusions and future work are contained in Section 6.

2. Some basic notion of S-NSs and operations of S-NNs

By the truth, falsity and indeterminacy membership degrees constrained in the real standard interval [0, 1] for the convenience of actual applications, Ye [2] introduced the S-NS notion as a subclass of NS.

Definition 1 [2]. A S-NS N in a universal of discourse U is characterized by a truth-membership function $TM_N(u)$, a falsity-membership function $FM_N(u)$, and an indeterminacy-membership function $IM_N(u)$, where the values of the three functions $TM_N(u)$, $IM_N(u)$ and $FM_N(u)$ are three real single/interval values in the real standard interval [0, 1], such that $TM_N(u)$, $IM_N(u)$, $FM_N(u) \in [0, 1]$ and $0 \le TM_N(u) + IM_N(u) + FM_N(u) \le 3$ for SV-NS and then $TM_N(u)$, $IM_N(u)$, $FM_N(u) \subseteq [0, 1]$ and $0 \le \sup TM_N(u) + \sup IM_N(u) + \sup FM_N(u) \le 3$ for IV-NS. Thus, a S-NS N is denoted as the following mathematical symbol:

$$N = \{\langle u, TM_N(u), IM_N(u), FM_N(u)\rangle \mid u \in U\}.$$

For the simplified representation, the element $\langle u, TM_N(u), IM_N(u), FM_N(u)\rangle$ in the S-NS N is simply denoted as the S-NN $a = \langle TM_a, IM_a, FM_a\rangle$, including IV-NN and SV-NN.

Suppose that two S-NNs are $a = \langle TM_a, IM_a, FM_a\rangle$ and $b = \langle TM_b, IM_b, FM_b\rangle$, then there are the following relations [2]:

(i) $a^c = \langle FM_a, 1 - IM_a, TM_a\rangle$ for the complement of the SV-NN a and $a^c = \langle [\inf FM_a, \sup FM_a], [1 - \sup IM_a, 1 - \inf IM_a], [\inf TM_a, \sup TM_a]\rangle$ for the complement of the IV-NN a;

(ii) $a \subseteq b$ if and only if $TM_a \le TM_b$, $IM_a \ge IM_b$, and $FM_a \ge FM_b$ for the SV-NN a and $\inf TM_a \le \inf TM_b$, $\inf IM_a \ge \inf IM_b$, $\inf FM_a \ge \inf FM_b$ $\sup TM_a \le \sup TM_b$, $\sup IM_a \ge \sup IM_b$, and $\sup FM_a \ge \sup FM_b$ for the IV-NN a;

(iii) $a = b$ if and only if $a \subseteq b$ and $b \subseteq a$.

For two S-NNs $a = \langle TM_a, IM_a, FM_a \rangle$ and $b = \langle TM_b, IM_b, FM_b \rangle$, their operational laws are introduced as follows [18, 20]:

(i) $a + b = \langle TM_a + TM_b - TM_a TM_b, IM_a IM_b, FM_a FM_b \rangle$ for SV-NNs and $a + b = \left\langle \begin{array}{l} [\inf TM_a + \inf TM_b - \inf TM_a \inf TM_b, \sup TM_a + \sup TM_b - \sup TM_a \sup TM_b], \\ [\inf IM_a \inf IM_b, \sup IM_a \sup IM_b], [\inf FM_a \inf FM_b, \sup FM_a \sup FM_b] \end{array} \right\rangle$ for IV-NNs;

(ii) $a \times b = \langle TM_a TM_b, IM_a + IM_b - IM_a IM_b, FM_a + FM_b - FM_a FM_b \rangle$ for SV-NNs and $a \times b = \left\langle \begin{array}{l} [\inf TM_a \inf TM_b, \sup TM_a \sup TM_b], \\ [\inf IM_a + \inf IM_b - \inf IM_a \inf IM_b, \sup IM_a + \sup IM_b - \sup IM_a \sup IM_b], \\ [\inf FM_a + \inf FM_b - \inf FM_a \inf FM_b, \sup FM_a + \sup FM_b - \sup FM_a \sup FM_b] \end{array} \right\rangle$ for IV-NNs;

(iii) $\rho a = \langle 1 - (1 - TM_a)^\rho, IM_a^\rho, FM_a^\rho \rangle$ for SV-NN and ρ > 0 and $\rho a = \left\langle \begin{array}{l} [1 - (1 - \inf TM_a)^\rho, 1 - (1 - \sup TM_a)^\rho], \\ [(\inf IM_a)^\rho, (\sup IM_a)^\rho], [(\inf FM_a)^\rho, (\sup FM_a)^\rho] \end{array} \right\rangle$ for IV-NN and ρ > 0;

(iv) $a^\rho = \langle TM_a^\rho, 1 - (1 - IM_a)^\rho, 1 - (1 - FM_a)^\rho \rangle$ for SV-NN and ρ > 0 and $a^\rho = \left\langle \begin{array}{l} [(\inf TM_a)^\rho, (\sup TM_a)^\rho], [1 - (1 - \inf IM_a)^\rho, 1 - (1 - \sup IM_a)^\rho], \\ [1 - (1 - \inf FM_a)^\rho, 1 - (1 - \sup FM_a)^\rho] \end{array} \right\rangle$ for IV-NN and ρ > 0.

For any S-NN $a = \langle TM_a, IM_a, FM_a \rangle$, its score functions can be introduced as follows [18]:

$$S(a) = (2 + TM_a - IM_a - FM_a)/3, \quad S(a) \in [0,1] \text{ for SV-NN,} \tag{1}$$

$$S(a) = (4 + \inf TM_a - \inf IM_a - \inf FM_a + \sup TM_a - \sup IM_a - \sup FM_a)/6, \quad S(a) \in [0,1] \text{ for IV-NN.} \tag{2}$$

Set $a_j = \langle TM_{a_j}, IM_{a_j}, FM_{a_j} \rangle$ (*j* = 1, 2, …, *n*) as a group of S-NNs. Then we can introduce the following SV-NN weighted arithmetic averaging (SV-NNWAA) and IV-NN weighted arithmetic averaging (IV-NNWAA) operators [18, 20]:

$$SV-NNWAA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j a_j = \left\langle 1 - \prod_{j=1}^{n} (1 - TM_{a_j})^{w_j}, \prod_{j=1}^{n} (IM_{a_j})^{w_j}, \prod_{j=1}^{n} (FM_{a_j})^{w_j} \right\rangle, \tag{3}$$

$$IV-NNWAA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j a_j = \left\langle \begin{array}{l} \left[1 - \prod_{j=1}^{n} (1 - \inf TM_{a_j})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \sup TM_{a_j})^{w_j} \right], \\ \left[\prod_{j=1}^{n} (\inf IM_{a_j})^{w_j}, \prod_{j=1}^{n} (\sup IM_{a_j})^{w_j} \right], \left[\prod_{j=1}^{n} (\inf FM_{a_j})^{w_j}, \prod_{j=1}^{n} (\sup FM_{a_j})^{w_j} \right] \end{array} \right\rangle, \tag{4}$$

where w_j (*j* = 1, 2, …, *n*) is the weight of a_j (*j* = 1, 2, …, *n*) for $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

3. SOAOs of S-NNs (SV-NNs and IV-NNs)

In this section, we present SOAOs based on the subtraction operation of S-NNs (SV-NNs and IV-NNs).

Definition 2 [30]. Set $a = \langle TM_a, IM_a, FM_a \rangle$ and $b = \langle TM_b, IM_b, FM_b \rangle$ as two S-NNs (SV-NNs and IV-NNs), then the subtraction operations of the S-NNs *a* and *b* are defined below:

$$a-b=\left\langle \frac{TM_a-TM_b}{1-TM_b}, \frac{IM_a}{IM_b}, \frac{FM_a}{FM_b} \right\rangle \text{ for SV-NNs,} \tag{5}$$

$$a-b=\left\langle \begin{array}{l} \left[\dfrac{\inf TM_a-\inf TM_b}{1-\inf TM_b}, \dfrac{\sup TM_a-\sup TM_b}{1-\sup TM_b}\right], \\ \left[\dfrac{\inf IM_a}{\inf IM_b}, \dfrac{\sup IM_a}{\sup IM_b}\right], \left[\dfrac{\inf FM_a}{\inf FM_b}, \dfrac{\sup FM_a}{\sup FM_b}\right] \end{array} \right\rangle \text{ for IV-NNs,} \tag{6}$$

which is valid under the conditions $a \ge b$, $TM_b \ne 1$, $IM_b \ne 0$, and $FM_b \ne 0$ for the SV-NNs a and b, and then $a \supseteq b$, $TM_b \ne [1, 1]$, $IM_b \ne [0, 0]$, and $FM_b \ne [0, 0]$ for the IV-NNs a and b.

Corresponding to the operational laws of S-NNs, we give the following theorem.

Theorem 1. Set $a = <TM_a, IM_a, FM_a>$ and $b = <TM_b, IM_b, FM_b>$ as two S-NNs and ρ > 0. Then, there are the following subtraction operational laws:

$$\rho(a-b)=\left\langle 1-\left(1-\frac{TM_a-TM_b}{1-TM_b}\right)^{\rho}, \left(\frac{IM_a}{IM_b}\right)^{\rho}, \left(\frac{FM_a}{FM_b}\right)^{\rho} \right\rangle, \text{if } a \ge b, TM_b \ne 1, IM_b, FM_b \ne 0 \text{ for SV-NNs,} \tag{7}$$

$$\rho(a-b)=\left\langle \begin{array}{l} \left[1-\left(1-\dfrac{\inf TM_a-\inf TM_b}{1-\inf TM_b}\right)^{\rho}, 1-\left(1-\dfrac{\sup TM_a-\sup TM_b}{1-\sup TM_b}\right)^{\rho}\right], \\ \left[\left(\dfrac{\inf IM_a}{\inf IM_b}\right)^{\rho}, \left(\dfrac{\sup IM_a}{\sup IM_b}\right)^{\rho}\right], \left[\left(\dfrac{\inf FM_a}{\inf FM_b}\right)^{\rho}, \left(\dfrac{\sup FM_a}{\sup FM_b}\right)^{\rho}\right] \end{array} \right\rangle, \text{if } a \supseteq b, TM_b \ne [1,1], IM_b, FM_b \ne [0,0]$$

for IV-NNs, (8)

$$(a-b)^{\lambda}=\left\langle \left(\frac{TM_a-TM_b}{1-TM_b}\right)^{\lambda}, 1-\left(1-\frac{IM_a}{IM_b}\right)^{\lambda}, 1-\left(1-\frac{FM_a}{FM_b}\right)^{\lambda} \right\rangle, \text{if } a \ge b, TM_b \ne 1, IM_b, FM_b \ne 0 \text{ for}$$

SV-NNs, (9)

$$(a-b)^{\rho}=\left\langle \begin{array}{l} \left[\left(\dfrac{\inf TM_a-\inf TM_b}{1-\inf TM_b}\right)^{\rho}, \left(\dfrac{\sup TM_a-\sup TM_b}{1-\sup TM_b}\right)^{\rho}\right], \\ \left[1-\left(1-\dfrac{\inf IM_a}{\inf IM_b}\right)^{\rho}, 1-\left(1-\dfrac{\sup IM_a}{\sup IM_b}\right)^{\rho}\right], \\ \left[1-\left(1-\dfrac{\inf FM_a}{\inf FM_b}\right)^{\rho}, 1-\left(1-\dfrac{\sup FM_a}{\sup FM_b}\right)^{\rho}\right] \end{array} \right\rangle, \text{if } a \supseteq b, TM_b \ne [1,1], IM_b, FM_b \ne [0,0]$$

for IV-NNs. (10)

Obviously, Eqs. (7)-(10) are true according to the operational laws of S-NNs.

Definition 3. Set $a_j = \left\langle TM_{a_j}, IM_{a_j}, FM_{a_j} \right\rangle$ and $b_j = \left\langle TM_{b_j}, IM_{b_j}, FM_{b_j} \right\rangle$ (j = 1, 2, …, n) as two groups of S-NNs and $c_j = a_j - b_j = \left\langle TM_{c_j}, IM_{c_j}, FM_{c_j} \right\rangle$ (j = 1, 2, …, n) as a group of subtraction operations between a_j and b_j. Based on Eqs. (3), (4) and (7)-(10), we can present the SV-NNSOWAA and IV-NNSOWAA operators:

$$\begin{array}{l} SV-NNSOWAA(c_1, c_2, ..., c_n)=\sum_{j=1}^{n} w_j c_j = \sum_{j=1}^{n} w_j (a_j-b_j) \\ = \left\langle 1-\prod_{j=1}^{n}(1-TM_{c_j})^{w_j}, \prod_{j=1}^{n}(IM_{c_j})^{w_j}, \prod_{j=1}^{n}(FM_{c_j})^{w_j} \right\rangle \end{array} \text{ for SV-NNs,} \tag{11}$$

$$IV-NNSOWAA(c_1,c_2,...,c_n)=\sum_{j=1}^{n} w_j c_j = \sum_{j=1}^{n} w_j (a_j - b_j)$$

$$= \left\langle \begin{array}{l} \left[1-\prod_{j=1}^{n}(1-\inf TM_{c_j})^{w_j}, 1-\prod_{j=1}^{n}(1-\sup TM_{c_j})^{w_j} \right], \\ \left[\prod_{j=1}^{n}(\inf IM_{c_j})^{w_j}, \prod_{j=1}^{n}(\sup IM_{c_j})^{w_j} \right], \left[\prod_{j=1}^{n}(\inf FM_{c_j})^{w_j}, \prod_{j=1}^{n}(\sup FM_{c_j})^{w_j} \right] \end{array} \right\rangle \quad \text{for IV-NNs, (12)}$$

where w_j (j = 1, 2, …, n) is the weight of $c_j = a_j - b_j$ (j = 1, 2, …, n) for $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, and the three elements in c_j for the SV-NNs a and b contain the following forms:

$$TM_{c_j} = \begin{cases} \dfrac{TM_{a_j} - TM_{b_j}}{1-TM_{b_j}} \in [0,1], \; if \; TM_{a_j} \ge TM_{b_j} \; and \; TM_{b_j} \ne 1 \\ 0, \; otherwise \end{cases}, \quad (13)$$

$$IM_{c_j} = \begin{cases} \dfrac{IM_{a_j}}{IM_{b_j}} \in [0,1], \; if \; IM_{a_j} \le IM_{b_j} \; and \; IM_{b_j} \ne 0 \\ 1, \; otherwise \end{cases}, \quad (14)$$

$$FM_{c_j} = \begin{cases} \dfrac{FM_{a_j}}{FM_{b_j}} \in [0,1], \; if \; FM_{a_j} \le FM_{b_j} \; and \; FM_{b_j} \ne 0 \\ 1, \; otherwise \end{cases}, \quad (15)$$

or the three elements in c_j for the IV-NNs a and b contain the following forms:

$$TM_{c_j} = \begin{cases} \left[\dfrac{\inf TM_{a_j} - \inf TM_{b_j}}{1-\inf TM_{b_j}}, \dfrac{\sup TM_{a_j} - \sup TM_{b_j}}{1-\sup TM_{b_j}} \right] \subseteq [0,1], \; if \; TM_{a_j} \supseteq TM_{b_j} \; and \; TM_{b_j} \ne [1,1] \\ [0,0], \; otherwise \end{cases}, \quad (16)$$

$$IM_{c_j} = \begin{cases} \left[\dfrac{\inf IM_{a_j}}{\inf IM_{b_j}}, \dfrac{\sup IM_{a_j}}{\sup IM_{b_j}} \right] \subseteq [0,1], \; if \; IM_{a_j} \subseteq IM_{b_j} \; and \; IM_{b_j} \ne [0,0] \\ [1,1], \; otherwise \end{cases}, \quad (17)$$

$$FM_{c_j} = \begin{cases} \left[\dfrac{\inf FM_{a_j}}{\inf FM_{b_j}}, \dfrac{\sup FM_{a_j}}{\sup FM_{b_j}} \right] \subseteq [0,1], \; if \; FM_{a_j} \subseteq FM_{b_j} \; and \; FM_{b_j} \ne [0,0] \\ [1,1], \; otherwise \end{cases}. \quad (18)$$

4. Multi-attribute DM approach corresponding to the SV-NNSOWAA and IV-NNSOWAA operators

Regarding the SV-NNSOWAA and IV-NNSOWAA operators, we can establish a multi-attribute DM approach to deal with the DM problem with S-NNs (SV-NNs and IV-NNs).

As for a multi-attribute DM problem in S-NN setting, suppose that $P = \{P_1, P_2, …, P_m\}$ is a set of alternatives and $R = \{r_1, r_2, …, r_n\}$ is a set of attributes. Then the suitability assessment of an alternative P_i (i = 1, 2, …, m) over an attribute r_j (j = 1, 2, …, n) is expressed by a S-NN $a_{ij} = \langle TM_{a_{ij}}, IM_{a_{ij}}, FM_{a_{ij}} \rangle$ (i = 1, 2, …, m; j = 1, 2, …, n), where $TM_{a_{ij}}$ indicates the degree that the alternative P_i is satisfactory to the attribute r_j, $IM_{a_{ij}}$ indicates the indeterminate degree that the alternative P_i is satisfactory and/or unsatisfactory to the attribute r_j, and $FM_{a_{ij}}$ indicates the degree that the alternative P_i is unsatisfactory to the attribute r_j. Thus, all the assessment values of S-NNs can be structured as a S-NN decision matrix $D = (a_{ij})_{m\times n}$. Then, the weight of each attribute r_j (j = 1, 2, …, n) is w_j (j = 1, 2, …, n) for $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Regarding the DM problem with S-NNs (SV-NNs or IV-NNs), the decision steps is indicated as follows:

Step 1. From the S-NN decision matrix $D = (a_{ij})_{m\times n}$, the j-th S-NN positive ideal solution can be determined by the SV-NN $a_j^+ = \langle TM_j^+, IM_j^+, FM_j^+ \rangle = \langle \max_i(TM_{ij}), \min_i(IM_{ij}), \min_i(FM_{ij}) \rangle$ or the IV-NN $a_j^+ = \langle TM_j^+, IM_j^+, FM_j^+ \rangle = \left\langle \begin{array}{l} \left[\max_i(\inf TM_{ij}), \max_i(\sup TM_{ij})\right], \\ \left[\min_i(\inf IM_{ij}), \min_i(\sup IM_{ij})\right], \\ \left[\min_i(\inf FM_{ij}), \min_i(\sup FM_{ij})\right] \end{array} \right\rangle$ (j = 1, 2, …, n; i = 1, 2, …, m), while the j-th S-NN negative ideal solution can be determined by the SV-NN $a_j^- = \langle TM_j^-, IM_j^-, FM_j^- \rangle = \langle \min_i(TM_{ij}), \max_i(IM_{ij}), \max_i(FM_{ij}) \rangle$ or the IV-NN

$$a_j^- = \langle TM_j^-, IM_j^-, FM_j^- \rangle = \left\langle \begin{array}{l} \left[\min_i(\inf TM_{ij}), \min_i(\sup TM_{ij})\right], \\ \left[\max_i(\inf IM_{ij}), \max_i(\sup IM_{ij})\right], \\ \left[\max_i(\inf FM_{ij}), \max_i(\sup FM_{ij})\right] \end{array} \right\rangle \quad (j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m).$$

Step 2. Two collective values d_i^+ and d_i^- (i = 1, 2, …, m) for each alternative P_i (i = 1, 2, …, m) can be obtained by the SV-NNSOWAA and IV-NNSOWAA operators:

$$\begin{array}{l} d_i^+ = SV-NNSOWAA(c_{i1}^+, c_{i2}^+, \ldots, c_{in}^+) = \sum_{j=1}^{n} w_j c_{ij}^+ = \sum_{j=1}^{n} w_j (a_j^+ - a_{ij}) \\ = \left\langle 1 - \prod_{j=1}^{n} (1 - TM_{c_{ij}^+})^{w_j}, \prod_{j=1}^{n} (IM_{c_{ij}^+})^{w_j}, \prod_{j=1}^{n} (FM_{c_{ij}^+})^{w_j} \right\rangle \end{array} \quad \text{for the SV-NN } a_{ij}, \ (19)$$

$$\begin{array}{l} d_i^+ = IV-NNSOWAA(c_{i1}^+, c_{i2}^+, \ldots, c_{in}^+) = \sum_{j=1}^{n} w_j c_{ij}^+ = \sum_{j=1}^{n} w_j (a_j^+ - a_{ij}) \\ = \left\langle \begin{array}{l} \left[1 - \prod_{j=1}^{n} (1 - \inf TM_{c_{ij}^+})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \sup TM_{c_{ij}^+})^{w_j}\right], \\ \left[\prod_{j=1}^{n} (\inf IM_{c_{ij}^+})^{w_j}, \prod_{j=1}^{n} (\sup IM_{c_{ij}^+})^{w_j}\right], \left[\prod_{j=1}^{n} (\inf FM_{c_{ij}^+})^{w_j}, \prod_{j=1}^{n} (\sup FM_{c_{ij}^+})^{w_j}\right] \end{array} \right\rangle \end{array} \quad \text{for the IV-NN } a_{ij}, \ (20)$$

$$\begin{array}{l} d_i^- = SV-NNSOWAA(c_{i1}^-, c_{i2}^-, \ldots, c_{in}^-) = \sum_{j=1}^{n} w_j c_{ij}^- = \sum_{j=1}^{n} w_j (a_{ij} - a_j^-) \\ = \left\langle 1 - \prod_{j=1}^{n} (1 - TM_{c_{ij}^-})^{w_j}, \prod_{j=1}^{n} (IM_{c_{ij}^-})^{w_j}, \prod_{j=1}^{n} (FM_{c_{ij}^-})^{w_j} \right\rangle \end{array} \quad \text{for the SV-NN } a_{ij}, \ (21)$$

$$d_i^- = IV - NNSOWAA(c_{i1}^-, c_{i2}^-, ..., c_{in}^-) = \sum_{j=1}^{n} w_j c_{ij}^- = \sum_{j=1}^{n} w_j (a_{ij} - a_j^-)$$
$$= \left\langle \begin{array}{l} \left[1 - \prod_{j=1}^{n} (1 - \inf TM_{c_{ij}^-})^{w_j}, 1 - \prod_{j=1}^{n} (1 - \sup TM_{c_{ij}^-})^{w_j} \right], \\ \left[\prod_{j=1}^{n} (\inf IM_{c_{ij}^-})^{w_j}, \prod_{j=1}^{n} (\sup IM_{c_{ij}^-})^{w_j} \right], \left[\prod_{j=1}^{n} (\inf FM_{c_{ij}^-})^{w_j}, \prod_{j=1}^{n} (\sup FM_{c_{ij}^-})^{w_j} \right] \end{array} \right\rangle$$ for the IV-NN a_{ij}, (22)

where w_j (j = 1, 2, …, n) is the attribute weight for $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, and the components in the SV-NNs c_{ij}^+ and c_{ij}^- contain the following forms:

$$TM_{c_{ij}^+} = \begin{cases} \dfrac{TM_{a_j^+} - TM_{a_{ij}}}{1 - TM_{a_{ij}}}, if\ TM_{a_j^+} \geq TM_{a_{ij}}\ and\ TM_{a_{ij}} \neq 1 \\ 0, otherwise \end{cases},$$

$$IM_{c_{ij}^+} = \begin{cases} \dfrac{IM_{a_j^+}}{IM_{a_{ij}}}, if\ IM_{a_j^+} \leq IM_{a_{ij}}\ and\ IM_{a_{ij}} \neq 0 \\ 1, otherwise \end{cases},$$

$$FM_{c_{ij}^+} = \begin{cases} \dfrac{FM_{a_j^+}}{FM_{a_{ij}}}, if\ FM_{a_j^+} \leq FM_{a_{ij}}\ and\ FM_{a_{ij}} \neq 0 \\ 1, otherwise \end{cases},$$

$$TM_{c_{ij}^-} = \begin{cases} \dfrac{TM_{a_{ij}} - TM_{a_j^-}}{1 - TM_{a_j^-}}, if\ TM_{a_j^-} \leq TM_{a_{ij}}\ and\ TM_{a_j^-} \neq 1 \\ 0, otherwise \end{cases},$$

$$IM_{c_{ij}^-} = \begin{cases} \dfrac{IM_{a_{ij}}}{IM_{a_j^-}}, if\ IM_{a_j^-} \geq IM_{a_{ij}}\ and\ IM_{a_j^-} \neq 0 \\ 1, otherwise \end{cases},$$

$$FM_{c_{ij}^-} = \begin{cases} \dfrac{FM_{a_{ij}}}{FM_{a_j^-}}, if\ FM_{a_j^-} \geq FM_{a_{ij}}\ and\ FM_{a_j^-} \neq 0 \\ 1, otherwise \end{cases},$$

and the components in the IV-NNs c_{ij}^+ and c_{ij}^- contain the following forms:

$$TM_{c_{ij}^+} = \begin{cases} \left[\dfrac{\inf TM_{a_j^+} - \inf TM_{a_{ij}}}{1 - \inf TM_{a_{ij}}}, \dfrac{\sup TM_{a_j^+} - \sup TM_{a_{ij}}}{1 - \sup TM_{a_{ij}}} \right], if\ TM_{a_j^+} \supseteq TM_{a_{ij}}\ and\ TM_{a_{ij}} \neq [1,1] \\ [0,0], otherwise \end{cases},$$

$$IM_{c_{ij}^+} = \begin{cases} \left[\dfrac{\inf IM_{a_j^+}}{\inf IM_{a_{ij}}}, \dfrac{\sup IM_{a_j^+}}{\sup IM_{a_{ij}}} \right], if\ IM_{a_j^+} \subseteq IM_{a_{ij}}\ and\ IM_{a_{ij}} \neq [0,0] \\ [1,1], otherwise \end{cases},$$

$$FM_{c_{ij}^{+}}=\begin{cases}\left[\dfrac{\inf FM_{a_j^{+}}}{\inf FM_{a_{ij}}},\dfrac{\sup FM_{a_j^{+}}}{\sup FM_{a_{ij}}}\right], if\ FM_{a_j^{+}}\subseteq FM_{a_{ij}}\ and\ FM_{a_{ij}}\neq[0,0]\\ [1,1], otherwise\end{cases},$$

$$TM_{c_{ij}^{-}}=\begin{cases}\left[\dfrac{\inf TM_{a_{ij}}-\inf TM_{a_j^{-}}}{1-\inf TM_{a_j^{-}}},\dfrac{\sup TM_{a_{ij}}-\sup TM_{a_j^{-}}}{1-\sup TM_{a_j^{-}}}\right], if\ TM_{a_j^{-}}\subseteq TM_{a_{ij}}\ and\ TM_{a_j^{-}}\neq[1,1]\\ [0,0], otherwise\end{cases},$$

$$IM_{c_{ij}^{-}}=\begin{cases}\left[\dfrac{\inf IM_{a_{ij}}}{\inf IM_{a_j^{-}}},\dfrac{\sup IM_{a_{ij}}}{\sup IM_{a_j^{-}}}\right], if\ IM_{a_j^{-}}\supseteq IM_{a_{ij}}\ and\ IM_{a_j^{-}}\neq[0,0]\\ [1,1], otherwise\end{cases},$$

$$FM_{c_{ij}^{-}}=\begin{cases}\left[\dfrac{\inf FM_{a_{ij}}}{\inf FM_{a_j^{-}}},\dfrac{\sup FM_{a_{ij}}}{\sup FM_{a_j^{-}}}\right], if\ FM_{a_j^{-}}\supseteq FM_{a_{ij}}\ and\ FM_{a_j^{-}}\neq[0,0]\\ [1,1], otherwise\end{cases}.$$

Step 3. By Eq. (1) and (2), we calculate the score values of $S(d_i^{+})$ and $S(d_i^{-})$ (i = 1, 2, …, m).

Step 4. The relative closeness degree of each alternative with respect to the S-NN ideal solution (i = 1, 2, …, m) is calculated by

$$C_i=\frac{S(d_i^{-})}{S(d_i^{-})+S(d_i^{+})}\quad \text{for } C_i\in[0, 1]. \tag{23}$$

Clearly, the larger value of C_i reveals that an alternative is closer to the ideal solution and farther from the negative ideal solution simultaneously. Therefore, all the alternatives can be ranked in the descending order according to the values of C_i (i = 1, 2, …, m). The alternative with the largest value is chosen as the best one.

Step 5. End.

5. Illustrative example

For convenient comparison, we consider the multi-attribute DM problem adapted from [12]. Some investment company needs to invest a sum of money into the best company. Then, the panel indicates four possible alternatives as their set P = {P_1, P_2, P_3, P_4}, where P_1, P_2, P_3, and P_4 are denoted as a car company, a food company, a computer company, and an arms company, respectively. To select the best company, they must be assessed by the three attributes: r_1 (risk), r_2 (growth) and r_3 (environmental impact), while their weight vector is specified by w = (0.35, 0.25, 0.4). The four alternatives are assessed over the three attributes by the suitable assessments, then their assessment values are represented by the form of S-NNs (SV-NNs and IV-NNs) and constructed as the SV-NN decision matrix:

$$D=(a_{ij})_{m\times n}=\begin{bmatrix}\langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.2,0.2,0.5\rangle\\ \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle\\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.3,0.2\rangle\\ \langle 0.7,0.1,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.4,0.3,0.2\rangle\end{bmatrix},$$

and the IV-NN decision matrix:

$$D=(a_{ij})_{m\times n}$$
$$=\begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.2,0.3],[0.2,0.3],[0.5,0.6]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.2,0.3]\rangle \\ \langle[0.3,0.4],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.3,0.4],[0.2,0.3]\rangle \\ \langle[0.7,0.8],[0.1,0.2],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.4,0.5],[0.3,0.4],[0.2,0.3]\rangle \end{bmatrix}.$$

On the one hand, the proposed DM approach can be applied in the DM problem with SV-NNs and depicted by the following decision steps:

Step 1. By $a_j^+=\langle TM_j^+, IM_j^+, FM_j^+\rangle=\left\langle \max_i(TM_{ij}), \min_i(IM_{ij}), \min_i(FM_{ij})\right\rangle$ (i = 1, 2, 3, 4; j = 1, 2, 3) for the SV-NN decision matrix $D=(a_{ij})_{m\times n}$, we can determine the SV-NN positive ideal solution (ideal alternative):

$$P^+=\{a_1^+,a_2^+,a_3^+\}=\{<0.7,0.1,0.1>,<0.6,0.1,0.2>,<0.5,0.2,0.2>\},$$

then by $a_j^-=\langle TM_j^-, IM_j^-, FM_j^-\rangle=\left\langle \min_i(TM_{ij}), \max_i(IM_{ij}), \max_i(FM_{ij})\right\rangle$ (j = 1, 2, …, n), we can determine the SV-NN negative ideal solution (non-ideal alternative):

$$P^-=\{a_1^-,a_2^-,a_3^-\}=\{<0.3,0.2,0.3>,<0.4,0.2,0.3>,<0.2,0.3,0.5>\}.$$

Step 2. By using Eqs. (19) and (21), we can obtain the two aggregated values d_i^+ and d_i^- (i = 1, 2, …, m) for each alternative P_i (i = 1, 2, …, m):

d_1^+ = <0.4126, 0.6598, 0.4264>, d_2^+ = <0.0958, 1.0000, 0.7846>, d_3^+ = <0.2970, 0.5610, 0.6152>, and d_4^+ = <0.0703, 0.8503, 1.0000>;

d_1^- = <0.0525, 0.8503, 1.0000>, d_2^- = <0.3844, 0.5610, 0.5435>, d_3^- = <0.2083, 1.0000, 0.6931>, and d_4^- = <0.4013, 0.6598, 0.4264>.

Step 3. By applying Eq. (1), we calculate the score values of $S(d_i^+)$ and $S(d_i^-)$ (i = 1, 2, 3, 4):

$S(d_1^+)$ = 0.4421, $S(d_2^+)$ = 0.1037, $S(d_3^+)$ = 0.3736, and $S(d_4^+)$ = 0.0733;

$S(d_1^-)$ = 0.0674, $S(d_2^-)$ = 0.4267, $S(d_3^-)$ = 0.1717, and $S(d_4^-)$ = 0.4384.

Step 4. By using Eq. (23), we calculate the relative closeness degrees of each alternative with respect to the SV-NN ideal solution:

C_1 = 0.1323, C_2 = 0.8044, C_3 = 0.3149, and C_4 = 0.8567.

Since the ranking order of the relative closeness degrees is $C_4 > C_2 > C_3 > C_1$, the ranking order of the four alternatives is $P_4 \succ P_2 \succ P_3 \succ P_1$. Hence, the best alternative is P_4.

On the other hand, the proposed DM approach can be also applied in the DM problem with IV-NNs and depicted by the following decision steps:

Step 1. By $a_j^+=\langle TM_j^+, IM_j^+, FM_j^+\rangle=\left\langle \begin{matrix} \left[\max_i(\inf TM_{ij}), \max_i(\sup TM_{ij})\right], \\ \left[\min_i(\inf IM_{ij}), \min_i(\sup IM_{ij})\right], \\ \left[\min_i(\inf FM_{ij}), \min_i(\sup FM_{ij})\right] \end{matrix}\right\rangle$ (i = 1, 2, 3, 4; j = 1, 2, 3) for the IV-NN decision matrix $D=(a_{ij})_{m\times n}$, we can determine the IV-NN positive ideal solution (ideal alternative):

$$P^+=\{a_1^+,a_2^+,a_3^+\}=\left\langle \begin{matrix} <[0.7,0.8],[0.1,0.2],[0.1,0.2]>, \\ <[0.6,0.7],[0.1,0.2],[0.2,0.3]>, \\ <[0.5,0.6],[0.2,0.3],[0.2,0.3]> \end{matrix}\right\rangle.$$

By $a_j^- = \langle TM_j^-, IM_j^-, FM_j^- \rangle = \left\langle \begin{array}{l} \left[\min\limits_i(\inf TM_{ij}), \min\limits_i(\sup TM_{ij})\right], \\ \left[\max\limits_i(\inf IM_{ij}), \max\limits_i(\sup IM_{ij})\right], \\ \left[\max\limits_i(\inf FM_{ij}), \max\limits_i(\sup FM_{ij})\right] \end{array} \right\rangle$ (j = 1, 2, …, n), we can determine the IV-NN negative ideal solution (non-ideal alternative):

$$P^- = \{a_1^-, a_2^-, a_3^-\} = \left\langle \begin{array}{l} <[0.3,0.4],[0.2,0.3],[0.3,0.4]>, \\ <[0.4,0.5],[0.2,0.3],[0.3,0.4]>, \\ <[0.2,0.3],[0.3,0.4],[0.5,0.6]> \end{array} \right\rangle.$$

Step 2. By using Eqs. (20) and (22), we can obtain the two aggregated values d_i^+ and d_i^- (i = 1, 2, …, m) for each alternative P_i (i = 1, 2, …, m):

d_1^+ = <[0.4126, 0.4894], [0.6598, 0.7841], [0.4264, 0.5533]>, d_2^+ = <[0.0958, 0.1323], [1.0000, 1.0000], [0.7846, 0.8677]>, d_3^+ = <[0.2970, 0.3665], [0.5610, 0.6988], [0.6152, 0.7301]>, and d_4^+ = <[0.0703, 0.0854], [0.8503, 0.8913], [1.0000, 1.0000]>;

d_1^- = <[0.0525, 0.0618], [0.8503, 0.8913], [1.0000, 1.0000]>, d_2^- = <[0.3844, 0.4480], [0.5610, 0.6988], [0.5435, 0.6377]>, d_3^- = <[0.2083, 0.2439], [1.0000, 1.0000], [0.6931, 0.7579]>, and d_4^- = <[0.4013, 0.4763], [0.6598, 0.7841], [0.4264, 0.5533]>.

Step 3. By applying Eq. (2), we calculate the score values of $S(d_i^+)$ and $S(d_i^-)$ (i = 1, 2, 3, 4):

$S(d_1^+)$ = 0.4131, $S(d_2^+)$ = 0.0960, $S(d_3^+)$ = 0.3431, and $S(d_4^+)$ = 0.0690;

$S(d_1^-)$ = 0.0621, $S(d_2^-)$ = 0.3986, $S(d_3^-)$ = 0.1669, and $S(d_4^-)$ = 0.4090.

Step 4. By using Eq. (23), we calculate the relative closeness degrees of each alternative with respect to the SV-NN ideal solution:

C_1 = 0.1307, C_2 = 0.8059, C_3 = 0.3272, and C_4 = 0.8556.

Since the ranking order of the relative closeness degrees is $C_4 > C_2 > C_3 > C_1$, the ranking order of the four alternatives is $P_4 \succ P_2 \succ P_3 \succ P_1$. Hence, the best alternative is P_4.

By the comparison of the above decision results with the decision results obtained in [12], both the ranking order of the four alternatives and the best one above are the same as in [12].

Table 1. Decision results of multi-attribute DM approaches regarding various weighted aggregation operators of SV-NNs and IV-NNs

Aggregation operator	Score value	Relative closeness degree	Ranking order
SV-NNWSOAA	$S(d_1^+)$ = 0.4421, $S(d_2^+)$ = 0.1037, $S(d_3^+)$ = 0.3736, $S(d_4^+)$ = 0.0733; $S(d_1^-)$ = 0.0674, $S(d_2^-)$ = 0.4267, $S(d_3^-)$ = 0.1717, $S(d_4^-)$ = 0.4384	C_1 = 0.1323, C_2 = 0.8044, C_3 = 0.3149, C_4 = 0.8567	$P_4 \succ P_2 \succ P_3 \succ P_1$
IV-NNWSOAA	$S(d_1^+)$ = 0.4131, $S(d_2^+)$ = 0.0960, $S(d_3^+)$ = 0.3431, $S(d_4^+)$ = 0.0690; $S(d_1^-)$ = 0.0621, $S(d_2^-)$ = 0.3986, $S(d_3^-)$ = 0.1669, $S(d_4^-)$ = 0.4090	C_1 = 0.1307, C_2 = 0.8059, C_3 = 0.3272, C_4 = 0.8556	$P_4 \succ P_2 \succ P_3 \succ P_1$
SV-NNWAA [18, 20]	$S(d_1)$ = 0.5611, $S(d_2)$ = 0.6891, $S(d_3)$ = 0.6194, $S(d_4)$ = 0.6901	/	$P_4 \succ P_2 \succ P_3 \succ P_1$
IV-NNWAA [18, 20]	$S(d_1)$ = 0.5407, $S(d_2)$ = 0.6696, $S(d_3)$ = 0.5993, $S(d_4)$ = 0.6712	/	$P_4 \succ P_2 \succ P_3 \succ P_1$

To demonstrate the effectiveness and rationality of the proposed DM approach in this paper, we compare it with existing DM approaches based on the SV-NNWAA and IV-NNWAA operators [18, 20]. By directly using the SV-NNWAA operator of Eq. (3) and IV-NNWAA operator of Eq. (4) and the score function of Eqs. (1) and (2), we can obtain all the aggregated values of $d_i = SV - NNWAA(a_{i1}, a_{i2}, a_{i3}, a_{i4})$ and $d_i = IV - NNWAA(a_{i1}, a_{i2}, a_{i3}, a_{i4})$, and then the score values of $S(d_i)$ and decision results for each alternative P_i (i = 1, 2, …, m) are tabulated in Table 1.

In Table 1, all the ranking orders of the four alternatives given by the multi-attribute DM approaches based on the SV-NNWSOAA, IV-NNWSOAA, SV-NNWAA, and IV-NNWAA operators are identical, and then the best choices indicate the same alternative P_4, which show the effectiveness of the proposed approach. Clearly, the DM results obtained by the SV-NNWSOAA and IV-NNWSOAA operators reveals stronger identification than the DM results obtained by existing SV-NNWAA and IV-NNWAA operators [18, 19] because the values of the relative closeness degrees show bigger difference than the score values in existing approaches [18, 19]. Therefore, the DM method proposed in this paper is reasonable and provides an effective DM way for decision-makers.

6. Conclusion

Regarding existing subtraction operations of S-NNs (SV-NNs and IV-NNs), this paper firstly presented the SV-NNSOWAA and IV-NNSOWAA operators for S-NNs as a necessary complement to existing aggregation operators of S-NNs. Next, we developed a multi-attribute DM approach based on the SV-NNWSOAA and IV-NNSOWAA operators for the first time. Finally, an illustrative example was presented to demonstrate the applicability and effectiveness of the developed approach. However, the main advantage of the proposed DM approach is that the DM results in this study reveals stronger identification than the DM results of existing DM approaches. In the future work, the developed approach will be further extended to other fields, such as image processing and clustering analysis.

References

[1] Smarandache, F. Neutrosophy: Neutrosophic probability, set, and logic, American Research Press, Rehoboth, USA, 1998.

[2] Ye, J. Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, Artificial Intelligence in Medicine, 63(3) (2015), 171–179.

[3] Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Interval neutrosophic sets and logic: Theory and applications in computing, Hexis, Phoenix, AZ, 2005.

[4] Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets, Multispace Multistructure, 4 (2010), 410–413.

[5] Guo, Y.; Sengur, A.; Ye, J. A novel image thresholding algorithm based on neutrosophic similarity score, Measurement, 58 (2014), 175-186.

[6] Fu, J.; Ye, J. Simplified neutrosophic exponential similarity measures for the initial evaluation/diagnosis of benign prostatic hyperplasia symptoms, Symmetry, 9(8) (2017), 154; P. 12, doi:10.3390/sym9070154.

[7] Ye, J. Single valued neutrosophic minimum spanning tree and its clustering method. Journal of Intelligent Systems, 23(3) (2014), 311-324.

[8] Ye, J. Single-valued neutrosophic clustering algorithms based on similarity measures. Journal of Classification, 34 (2017), 148-162.

[9] Kong, L.; Wu,Y.; Ye, J. Misfire fault diagnosis method of gasoline engines using the cosine similarity measure of neutrosophic numbers. Neutrosophic Sets and Systems, 8 (2015), 43-46.

[10] Ye, J. Single valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. Soft Computing, 21(3) (2017), 817-825.

[11] Ye, J. Fault diagnoses of hydraulic turbine using the dimension root similarity measure of single-valued neutrosophic sets. Intelligent Automation & Soft Computing, 24(1) (2018), 1-8.

[12] Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42(4) (2013), 386-394.
[13] Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent & Fuzzy Systems, 26(5) (2014), 2459–2466.
[14] Liu, P.D.; Wang, Y.M. Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Computing and Applications, 25(7-8) (2014), 2001-2010.
[15] Liu, P.D.; Chu, Y.C.; Li, Y.W.; Chen, Y.B. Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making. Journal of Intelligent & Fuzzy Systems, 16(2), (2014), 242–255.
[16] Majumdar, P.; Samant, S.K. On similarity and entropy of neutrosophic sets, Journal of Intelligent & Fuzzy Systems, 26(3) (2014), 1245–1252.
[17] Peng, J.J.; Wang, J.Q.; Zhang, H.Y.; Chen, X.H. An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets, Applied Soft Computing, 25 (2014), 336–346.
[18] Zhang, H.Y.; Wang, J.Q.; Chen, X.H. Interval neutrosophic sets and their application in multicriteria decision making problems, The Science World Journal, 2014, Article ID 645953, 15 pages.
[19] Ye, J. Multiple attribute decision-making method based on the possibility degree ranking method and ordered weighted aggregation operators of interval neutrosophic numbers, Journal of Intelligent & Fuzzy Systems, 28(3) (2015), 1307–1317.
[20] Peng, J.J.; Wang, J.Q.; Wang, J.; Zhang, H.Y.; Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science, 47(10) (2016), 2342-2358.
[21] Wu, X.H.; Wang, J.Q.; Peng, J.J.; Chen, X.H. Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. International Journal of Fuzzy Systems, 18(6) (2016), 1104–1116.
[22] Pramanik, S.; Dalapati, S.; Alam, S.; Smarandache, F.; Roy, T.K. NS-cross entropy based MAGDM under single valued neutrosophic set environment. Information, 9(2) (2018), 37. doi:10.3390/info9020037.
[23] Wu, H.; Yuan, Y.; Wei, L.; Pei, L. On entropy, similarity measure and cross-entropy of single-valued neutrosophic sets and their application in multi-attribute decision making. Soft Computing, 22(22), (2018), 7367–7376.
[24] Yang, H.; Wang, X.; Qin, K. New similarity and entropy measures of interval neutrosophic sets with applications in multi-attribute decision-making. Symmetry, 11 (2019), 370; doi:10.3390/sym11030370.
[25] Abdel-Basset, M.; Gamal, A.; Manogaran, G.; Long, H.V. A novel group decision making model based on neutrosophic sets for heart disease diagnosis. Multimedia Tools & Applications, 2019, https://doi.org/10.1007/s11042-019-07742-7.
[26] Atanassov, K.T.; Riecan, B. On two operations over intuitionistic fuzzy sets, Journal of Applied Mathematics, Statistics and Informatics, 2(2) (2006), 145–148.
[27] Atanassov, K.T. Remark on operations "subtraction" over intuitionistic fuzzy sets. Notes on Intuitionistic Fuzzy Sets, 15(3) (2009), 20–24.
[28] Chen, T.Y. Remarks on the subtraction and division operations over intuitionistic fuzzy sets and interval-valued fuzzy sets. International Journal of Fuzzy Systems, 2007, 9(3), 169–172.
[29] Lu, Z.; Ye, J. Decision-making method for clay-brick selection based on subtraction operational aggregation operators of intuitionistic fuzzy values. The Open Cybernetics and Systemics Journal, 10 (2016), 283-291.
[30] Ye, J. Subtraction and division operations of simplified neutrosophic sets, Information, 8(2) (2017), 51, page 7; doi:10.3390/info8020051.

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