NSS

Neutrosophic Sets and Systems, Vol. 33, 2020

University of New Mexico

Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making

Muhammad Saeed[1], Muhammad Saqlain[1*], Asad Mehmood[2], Khushbakht Naseer[3] and Sonia Yaqoob[4]

[1]Department of Mathematics, University of Management and Technology, Lahore, Pakistan; muhammad.saeed@umt.edu.pk

[1*]Department of Mathematics, Lahore Garrison University, Lahore, Pakistan; msaqlain@lgu.edu.pk

[2]Department of Mathematics, University of Management and Technology, Lahore, Pakistan; a.asadkhan.khi@gmail.com

[3]Department of Mathematics, University of Management and Technology, Lahore, Pakistan; khushbakht433@gmail.com

[4]Department of Mathematics, University of Management and Technology, Lahore, Pakistan; soniayaqoob1122@gmail.com

Abstract. A Similarity measure for Neutrosophic function performs a fundamental role in tackling the problems that include blurred and hazed information but is not able to handle the fuzziness and vagueness of the problems which have numerous information. The objective of this research paper is to generalize neutrosophic soft set to the multi-polar neutrosophic soft set (mNS set), aggregation operators and their properties on mNS sets. It also discusses the distance-based similarity measures that rely on between two mNS sets. It explains with the help of examples that the intended similarity measures of mNS sets are applicable in the field of medical diagnosis and decision-making problem for selection of lecturer in universities. Eventually, this proposed method is concluded as an algorithm in the application.

Keywords: mNS Set; Operators on mNS; Properties; Distance and Similarity Measure; Medical Diagnosis; Decision-Making

---

1. Introduction

Zadeh [1] introduced fuzzy sets as an additional classical conception of set. The theory of fuzzy set can be widely utilized in those domains where the information is deficient or incomplete like in bio-informatics fuzzy set logics, the members in a set are allowed to have a moderate assessment of membership, this is explained by the help of membership function admired in real unit interim [0,1]. Fuzzy sets have been derived from crisp sets, because the different aspects of functions of crisp sets are extraordinary occasions of degree functions of fuzzy sets if the later contains the values of fuzzy relations like 0 or 1 used in different areas like clustering (Bezdek [16]), linguistics (Cock [17]), and decision-making (Deli [18]) are significant theories of L-relations when the unit interval L is [0,1]. Further than intuitionistic fuzzy set was being proposed by Atanassov [2] which was an extension of Zadeh's conviction and it was in itself adjunct the classical conviction of a set. The Intuitionistic fuzzy set was only able to grasp insufficient particulars and facts, not unspecified particulars and facts. Neutrosophy was proposed by Florentin [19]. It is a limb of philosophy that scrutinizes the origination, essence, and range of neutralities along with interconnection with various ideational spectra. Neutrosophic set is a powerful general authorized substructure which deducts the theory of fuzzy sets and intuitionistic fuzzy set. The Soft set supposition is induction of fuzzy set supposition that was being presented by Molodtsov [3] to trade with unpredictability in a parametric model. He designated strains and troubles that were in mathematical representations and tackle the complications by proposing a soft set theory. Maji [4,23] expanded the soft set scheme into a fuzzy soft set and neutrosophic soft set theory. Feng [20] further explored decision-making that was supported by fuzzy soft sets.

Bipolar fuzzy sets and its connections were further introduced by Zhang [5], then Chen [6] proposed the notion of multi-polar fuzzy sets that was an abstraction of bipolar fuzzy sets. It was being manifested that bipolar fuzzy sets were isomorphic mathematical conceptions. Further than Wang and Liu [22] proposed decision-making on the multi-polar neutrosophic numbers. Deli [24] studied the neutrosophic soft multi-set theory with the multi distinct universe.

Chen [9,10] researched the notion of similarity measure of a vague set which was unsuccessful to retain in some scenarios. To conquer this problem Hong and Kim [21] proposed some altered calculations. Majumdar and Samanta [11,12,13] proposed uncertainty measures of soft sets and fuzzy sets. Kharal [14] proposed similarity measures of soft sets on set-theoretic functions. Li and Cheng [7] suggested the vision of new SMs allying in intuitionistic fuzzy sets. Deli [18,24] introduced decision-making methods on interval-valued neutrosophic soft set and neutrosophic soft multi-set theory. Huang [25] proposed TOPSIS for group decision-making problems. Furthermore, Anisseh [27] extend the TOPSIS model for group decision-making problem under multiple criteria.

Smarandache [28] generalized the soft set to hypersoft set by converting the function into a multi-decision function. Aggregate operators, similarity measure and a TOPSIS technique is introduced by [29-34] in his work. Application of fuzzy numbers in mobile selection in metros like Lahore is proposed by [35]. TOPSIS technique of MCDM can also be used for the prediction of games, and it's applied in FIFA 2018 by [36], prediction of games is a very complex topic and this game is also predicted by [37]. Abdel-Basset [38-41] has published a set of articles on Medical disease diagnosis based on neutrosophic environment.

1.1. *Motivation*

A huge number of articles is published in neutrosophic field, and as well as this theory is applied and extended in different branches such as Decision Making. The extension of neutrosophic environment to m-polar Nuetrosophic Soft set is totally new. Here, a set of questions arises that how mNS set can be represented? what is the purpose of m-polar structure? and how m-polar structures can be utilized in medical diagnosis and in decision making problems? From this point of view, mNS structure is good choice to get better results to the problems in decision making.

1.2. *Paper Presentation*

This article visualizes new concept m-polar Neutrosophic Soft Set as an extension of Neutrosophic Soft Set.

- Basic operators such as union, intersection on mNS set are introduced
- Properties related to operators on mNS structure
- Distance measure of different types are introduced between any two mNS sets
- A case study of two applications are concluded with an algorithm in the field of medical diagnosis and decision-making problem.

2. **Preliminaries**

This section studies some basic definitions related to this article.

2.1. *Neutrosophic Set*

Definition 2.1 [19] Let Z be a universal set. A neutrosophic set X is defined as :

$$X = \{z, (T_X(z), I_X(z), F_X(z)) : z \in Z\},$$

where,

$$T_X(z), I_X(x), F_X(z) \in [0, 1]$$
$$0 \leq T_X(z) + I_X(z) + F_X(z) \leq 3$$
$$\text{for all } z \in Z$$

2.2. *Multi-Polar Neutrosophic Set*

Definition 2.2 [22] An mN set on a universal set Z is a mapping

$$X = ((s_1 \circ T_X(z), s_2 \circ T_X(z), \cdots, s_m \circ T_X(z)), (s_1 \circ I_X(z), s_2 \circ I_X(z), \cdots, s_m \circ I_X(z)),$$
$$(s_1 \circ F_X(z), s_2 \circ F_X(z), \cdots, s_m \circ F_X(z))) : Z \to ([0, 1]^m, [0, 1]^m, [0, 1]^m)$$

where $i-th$ mapping is defined as

$$s_i \circ T_X : [0,1]^m \to [0,1]$$
$$s_i \circ I_X : [0,1]^m \to [0,1]$$
$$s_i \circ F_X : [0,1]^m \to [0,1]$$

and

$$0 \leq s_i \circ T_X(z) + s_i \circ I_X + s_i \circ F_X \leq 3$$
$$\text{for all } i = 1, 2, \cdots, m \text{ and } z \in Z$$

2.3. *Soft Set*

Definition 2.3 [3] Let Z be a universal set and E be the set of attributes of elements in Z. Take X to be a subset of E then a function F defined as

$$F : X \to P(Z),$$

then a pair (F, X) is called a soft set over Z such as

$$(F, X) = \{e, F(e) : e \in X,\ F(e) \in P(Z)\}$$

2.4. *Not Set*

Definition 2.4 [23] The Not Set of set of parameters $E = \{e_1, e_2, \cdots, e_q\}$, denoted by $\neg E$ is defined as

$$\neg E = \{\neg e_1, \neg e_2, \cdots, \neg e_q\}$$

where $\neg e_j$ means not e_j for all $j = 1, 2, \cdots, q$.

2.5. *Neutrosophic Soft Sets*

Definition 2.5 [23] A Neutrosophic Soft set (ω, X) over a universal set Z is a mapping from X to $P(Z)$ and defined as

$$(\omega, X) = \Omega_X = \{(e, (z, T_X(e)(z), I_X(e)(z), F_X(e)(z)) : z \in Z,\ e \in E)\}$$

where, $P(Z)$ denotes collection of all neutrosophic subsets of Z. Each of $T_X(e), I_X(e)$ and $F_X(e)$ is a mapping from Z to interval $[0,1]$ and

$$0 \leq T_X(e)(z) + I_X(e)(z) + F_X(e)(z) \leq 3$$
$$\text{for all } e \in E \text{ and } z \in Z$$

2.6. *Multi-Polar Nuetrosophic Soft set*

Definition 2.6 [24] Let Z be a universal set, E be a set of attributes and $X \subseteq E$.
Define $\omega : X \to m\mathrm{N}^Z$ where $m\mathrm{N}^Z$ is the collection of all mN subsets of set Z. Then (ω, X) is called an mNS set over Z as follows

$$\Omega_X = (\omega, X) = \{e, \omega_X(e) : e \in E, \omega_X(e) \in mN^Z\}$$

and $\omega_X(e)$ is an mN set denoted by,

$$\omega_X(e) = \{z, s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z) : z \in Z\}$$

and

$$0 \leq s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z) \leq 3$$
$$\text{for all } i = 1, 2, \cdots, m;\ e \in E \text{ and } z \in Z$$

3. Operations on mN Soft Sets

This section discusses some operators on mNS sets.

3.1. *Multi-Polar Neutrosophic Soft subset*

Definition 3.1 Let Z be a universal set, X and Y are subsets of a set of attributes E. A set Ω_X is an mNS subset of Ψ_Y denoted by $\Omega_X \breve{\subseteq} \Psi_Y$ if

$$\text{(i) } X \subseteq Y$$
$$\text{(ii) } \omega_X(e) \subseteq \psi_Y(e) \text{ i.e.}$$

$$s_i \circ T_X(e)(z) \leq s_i \circ T_Y(e)(z),\ s_i \circ I_X(e)(z) \leq s_i \circ I_Y(e)(z) \text{ and } s_i \circ F_X(e)(z) \geq s_i \circ F_Y(e)(z)$$
$$\text{for all } i = 1, 2, \cdots, m;\ e \in X \text{ and } z \in Z$$

Example 3.1 Let $Z = \{z_1, z_2\}$ be a universal set and $E = \{e_1, e_2, e_3, \}$ be a set of attributes. $X = \{e_1, e_2\}, Y = \{e_1, e_2\} \subseteq E$.Let Ω_X and Ψ_Y be two 3−NS set defined as:

$$\Omega_X = \{e_1, (z_1, (0.7, 0.5, 0.5), (0.2, 0.2, 0.3), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.3, 0.2, 0.4), (0.3, 0.5, 0.8)),$$
$$e_2, (z_1, (0.3, 0.5, 0.6), (0.1, 0.4, 0.2), (0.7, 0.5, 0.3)), (z_2, (0.7, 0.2, 0.7), (0.3, 0.4, 0.4), (0.2, 0.7, 0.4))\}$$

$$\Psi_Y = \{e_1, (z_1, (0.8, 0.8, 0.7), (0.4, 0.5, 0.4), (0.3, 0.3, 0.2)), (z_2, (0.4, 0.7, 0.4), (0.6, 0.4, 0.5), (0.3, 0.3, 0.6)),$$
$$e_2, (z_1, (0.7, 0.8, 0.8), (0.3, 0.8, 0.2), (0.3, 0.4, 0.3)), (z_2, (0.8, 0.5, 0.8), (0.5, 0.8, 0.4), (0.2, 0.5, 0.3))\}$$

$$\text{this implies } \Omega_X \breve{\subseteq} \Psi_Y$$

3.2. *Equal Multi-Polar Neutrosophic Soft set*

Definition 3.2 Let Ω_X and Ψ_Y be two mNS set over a universal set Z, where X and Y are subsets of sets of attributes E. Then two mNS sets Ω_X and Ψ_Y are said to be equal denoted as $\Omega_X \breve{=} \Psi_Y$ if and only if $\Omega_X \breve{\subseteq} \Psi_Y$ and $\Psi_Y \breve{\subseteq} \Omega_X$

3.3. *Relative Null Multi-Polar Neutrosophic Soft set*

Definition 3.3 An mNS set over the universal set Z is said to be relative empty or relative null mNS set concering the set of attributes $X \subseteq E$, denoted by $\breve{\Phi}_X$ if

$$s_i \circ T_X(e)(z) = 0$$
$$s_i \circ I_X(e)(z) = 0$$
$$s_i \circ F_X(e)(z) = 1$$
for all $i = 1, 2, \cdots, m$; $e \in X$ and $z \in Z$

that is

$$\breve{\Phi}_X = \{e, (z, ((0, 0, \cdots, 0), (0, 0, \cdots, 0), (1, 1, \cdots, 1))) : z \in Z, e \in X\}$$

3.4. *Relative Whole Multi-Polar Neutrosophic Soft set*

Definition 3.4 An mNS set over the universal set Z is said to be relative whole mNS set concering the set of attributes $X \subseteq E$, denoted by $\breve{Z}_X$ if

$$s_i \circ T_X(e)(z) = 1$$
$$s_i \circ I_X(e)(z) = 1$$
$$s_i \circ F_X(e)(z) = 0$$
for all $i = 1, 2, \cdots, m$; $e \in X$ and $z \in Z$

that is

$$\breve{Z}_X = \{e, (z, ((1, 1, \cdots, 1), (1, 1, \cdots, 1), (0, 0, \cdots, 0))) : z \in Z, e \in X\}$$

3.5. *Absolute Multi-Polar Neutrosophic Soft set*

Definition 3.5 An mNS set over the universal set Z is said to be an absolute mNS set concering the set of attributes E, denoted by $\breve{Z}_E$ if

$$s_i \circ T_X(e)(z) = 1$$
$$s_i \circ I_X(e)(z) = 1$$
$$s_i \circ F_X(e)(z) = 0$$
for all $i = 1, 2, \cdots, m$; $e \in E$ and $z \in Z$

that is

$$\breve{Z}_E = \{e, (z, ((1, 1, \cdots, 1), (1, 1, \cdots, 1), (0, 0, \cdots, 0))) : z \in Z, e \in E\}$$

Example 3.2 Let $Z = \{z_1, z_2\}$ be universal set and $E = \{e_1, e_2, e_3\}$ is set of attributes, if $X = \{e_1, e_2\} \subseteq E$, A $3-$NS set Ω_X such that

$$\Omega_X = \{e_1, (z_1, (0, 0, 0), (0, 0, 0), (1, 1, 1)), (z_2, (0, 0, 0), (0, 0, 0), (1, 1, 1)),$$
$$e_2, (z_1, (0, 0, 0), (0, 0, 0), (1, 1, 1)), (z_2, (0, 0, 0), (0, 0, 0), (1, 1, 1))\} = \breve{\Phi}_X$$

then Ω_X is a relative null $3-$NS set $\breve{\Phi}_X$.

if $Y = \{e_1, e_3\} \subseteq E$, A $3-$NS set Ψ_Y such that

$$\Psi_Y = \{e_1, (z_1, (1, 1, 1), (1, 1, 1), (0, 0, 0)), (z_2, (1, 1, 1), (1, 1, 1), (0, 0, 0)),$$
$$e_3, (z_1, (1, 1, 1), (1, 1, 1), (0, 0, 0)), (z_2, (1, 1, 1), (1, 1, 1), (0, 0, 0))\} = \breve{Z}_Y$$

then Ψ_Y is a relative whole $3-$NS set $\breve{Z}_Y$.

if $W = E = \{e_1, e_2, e_3\}$, A $3-$NS set Λ_W such that

$$\begin{aligned}\Lambda_W = \{&e_1, (z_1, (1,1,1), (1,1,1), (0,0,0)), (z_2, (1,1,1), (1,1,1), (0,0,0)),\\ &e_2, (z_1, (1,1,1), (1,1,1), (0,0,0)), (z_2, (1,1,1), (1,1,1), (0,0,0)),\\ &e_3, (z_1, (1,1,1), (1,1,1), (0,0,0)), (z_2, (1,1,1), (1,1,1), (0,0,0))\} = \breve{Z}_E\end{aligned}$$

then Λ_W is an absolute $3-$NS set $\breve{Z}_E$.

Proposition 3.1 Let Z be a universal set, E a set of attributes, $X, Y, W \subseteq E$, If Ω_X, Ψ_Y and Λ_W are mNS sets over Z, then

(i) $\Omega_X \breve{\subseteq} \breve{Z}_X$

(ii) $\breve{\Phi}_X \breve{\subseteq} \Omega_X$

(iii) $\Omega_X \breve{\subseteq} \Omega_X$

(iv) $\Omega_X \breve{\subseteq} \Psi_Y$ and $\Psi_Y \breve{\subseteq} \Lambda_W$, then $\Omega_X \breve{\subseteq} \Lambda_W$

(v) $\Omega_X \breve{=} \Psi_Y$ and $\Psi_Y \breve{=} \Lambda_W$, then $\Omega_X \breve{=} \Lambda_W$

3.6. *Complement of mN Soft Set*

Definition 3.6 The complement of an mNS set Ω_X over a universal set Z with respect to the set of attributes $X \subseteq E$, denoted by $\Omega^c_X = (\omega^c, X)$ where $\omega^c : \neg X \to m\mathrm{NS}^Z$ is a mapping given as

$$\begin{aligned}\omega^c(\neg e) = \{z, ((s_i \circ T^c_X(\neg e)(z) = s_i \circ F_X(e)(z)), (s_i \circ I^c_X(\neg e)(z) = (1, 1, \cdots, 1) - s_i \circ I_X(e)(z)),\\ (s_i \circ F^c_X(\neg e)(z) = s_i \circ T_X(e)(z)))\}\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m,\ \neg e \in \neg X \text{ and } z \in Z$$

3.7. *Relative Complement of mN Soft Set*

Definition 3.7 The relative complement of an mNS set Ω_X over a universal set Z with respect to the set of attributes $X \subseteq E$, denoted by $\Omega^r_X = (\omega^r, X)$ where $\omega^r : X \to m\mathrm{NS}^Z$ is a mapping given as

$$\begin{aligned}\omega^r(e) = \{z, ((s_i \circ T^r_X(e)(z) = s_i \circ F_X(e)(z)), (s_i \circ I^r_X(e)(z) = (1, 1, \cdots, 1) - s_i \circ I_X(e)(z)),\\ (s_i \circ F^r_X(e)(z) = s_i \circ T_X(e)(z)))\}\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m;\ e \in X \text{ and } z \in Z$$

Example 3.3 Let $Z = \{z_1, z_2\}$ be universal set and $E = \{e_1, e_2, e_3\}$ is set of attributes, if $X = \{e_1, e_2\} \subseteq E$, A $3-$NS set Ω_X such that

$$\begin{aligned}\Omega_X = \{&e_1, (z_1, (0.4, 0.4, 0.6), (0.2, 0.4, 0.5), (0.5, 0.3, 0.7)), (z_2, (0.5, 0.7, 0.5), (0.6, 0.5, 0.3), (0.5, 0.7, 0.3)),\\ &e_2, (z_1, (0.7, 0.4, 0.6), (0.8, 0.6, 0.4), (0.2, 0.6, 0.7)), (z_2, (0.5, 0.7, 0.6), (0.8, 0.4, 0.7), (0.9, 0.3, 0.6))\}\end{aligned}$$

Then,

$$\begin{aligned}\Omega^c_X = \{&\neg e_1, (z_1, (0.5, 0.3, 0.7), (0.8, 0.6, 0.4), (0.4, 0.4, 0.6)), (z_2, (0.5, 0.7, 0.3), (0.4, 0.5, 0.7), (0.5, 0.7, 0.5)),\\ &\neg e_2, (z_1, (0.2, 0.6, 0.7), (0.2, 0.4, 0.6), (0.7, 0.4, 0.6)), (z_2, (0.9, 0.3, 0.6), (0.2, 0.6, 0.3), (0.5, 0.7, 0.6))\}\end{aligned}$$

$$\Omega^r_X = \{e_1, (z_1, (0.5, 0.3, 0.7), (0.8, 0.6, 0.4), (0.4, 0.4, 0.6)), (z_2, (0.5, 0.7, 0.3), (0.4, 0.5, 0.7), (0.5, 0.7, 0.5)),$$
$$e_2, (z_1, (0.2, 0.6, 0.7), (0.2, 0.4, 0.6), (0.7, 0.4, 0.6)), (z_2, (0.9, 0.3, 0.6), (0.2, 0.6, 0.3), (0.5, 0.7, 0.6))\}$$

Proposition 3.2 Let Ω_X be an mNS set over a universal set Z. Then

(i) $(\Omega^c_X)^c = \Omega_X$

(ii) $(\Omega^r_X)^r = \Omega_X$

(iii) $\breve{Z}^c_X = \breve{\Phi}_X = \breve{Z}^r_X$

(iv) $\breve{\Phi}^c_X = \breve{Z}_X = \breve{\Phi}^r_X$

3.8. *Union of Two mN Soft Sets*

Definition 3.8 Let Z be a universal set and X and Y are subsets of the set of attributes E. The set Ω_X and Ψ_Y are two mNS sets. Let $W = X \cup Y$, then the union of Ω_X and Ψ_Y is an mNS set denoted by $\Omega_X \breve{\cup} \Psi_Y$ and defined as for all $e \in W$

$$\Omega_X \breve{\cup} \Psi_Y = \begin{cases} \omega_X(e), & e \in X \setminus Y; \\ \psi_Y(e), & e \in Y \setminus X; \\ \omega_X(e) \cup \psi_Y(e), & e \in X \cap Y. \end{cases}$$

where,

$$\omega_X(e) \cup \psi_Y(e) = (max(s_i \circ\ T_X(e)(z), s_i \circ\ T_Y(e)(z)), max(s_i \circ\ I_X(e)(z), s_i \circ\ I_Y(e)(z)),$$
$$min(s_i \circ\ F_X(e)(z), s_i \circ\ F_Y(e)(z)))$$
$$\text{for all } i = 1, 2, \cdots, m;\ e \in W \text{ and } z \in Z$$

3.9. *Intersection of Two mN Soft Sets*

Definition 3.9 Let Z be a universal set and X and Y are subsets of the set of attributes E. The set Ω_X and Ψ_Y are two mNS sets. Let $W = X \cap Y$, then the intersection of Ω_X and Ψ_Y is an mNS set denoted by $\Omega_X \breve{\cap} \Psi_Y$ and defined as for all $e \in W$

$$\Omega_X \breve{\cap} \Psi_Y = \omega_X(e) \cap \psi_Y(e)$$

where,

$$\omega_X(e) \cap \psi_Y(e) = (min(s_i \circ\ T_X(e)(z), s_i \circ\ T_Y(e)(z)), min(s_i \circ\ I_X(e)(z), s_i \circ\ I_Y(e)(z)),$$
$$max(s_i \circ\ F_X(e)(z), s_i \circ\ F_Y(e)(z)))$$
$$\text{for all } i = 1, 2, \cdots, m;\ e \in W \text{ and } z \in Z$$

3.10. *Restricted Union of Two mN Soft Sets*

Definition 3.10 Let Z be a universal set and X and Y are subsets of the set of attributes E. The set Ω_X and Ψ_Y are two mNS sets. Let $W = X \cap Y$, then the Restricted union of Ω_X and Ψ_Y is an mNS set denoted by $\Omega_X \cup_R \Psi_Y$ and defined as for all $e \in W$

$$\Omega_X \cup_R \Psi_Y = \omega_X(e) \cup \psi_Y(e)$$

3.11. *Extended Intersection of Two mN Soft Sets*

Definition 3.11 Let Z be a universal set and X and Y are subsets of the set of attributes E. The set Ω_X and Ψ_Y are two mNS sets. Let $W = X \cup Y$, then the Extended intersection of Ω_X and Ψ_Y is an mNS set denoted by $\Omega_X \cap_\varepsilon \Psi_Y$ and defined as for all $e \in W$

$$\Omega_X \cap_\varepsilon \Psi_Y = \begin{cases} \omega_X(e), & e \in X \setminus Y; \\ \psi_Y(e), & e \in Y \setminus X; \\ \omega_X(e) \cap \psi_Y(e), & e \in X \cap Y. \end{cases}$$

3.12. *OR-operator of Two mN Soft Sets*

Definition 3.12 Let Z be a universal set and X and Y are subsets of the set of attributes E. The set Ω_X and Ψ_Y are two mNS sets, then the OR-operator of Ω_X and Ψ_Y is an mNS set denoted by $\Omega_X \vee \Psi_Y$ and defined as $\Omega_X \vee \Psi_Y = \Lambda_{X\times Y}$ where

$$\lambda_{X\times Y}(x,y) = \omega_X(x) \cup \psi_Y(y)$$
$$\text{for all } (x,y) \in X \times Y$$

3.13. *AND-operator of Two mN Soft Sets*

Definition 3.13 Let Z be a universal set and X and Y are subsets of the set of attributes E. The set Ω_X and Ψ_Y are two mNS sets, then the AND-operator of Ω_X and Ψ_Y is an mNS set denoted by $\Omega_X \wedge \Psi_Y$ and defined as $\Omega_X \wedge \Psi_Y = \Lambda_{X\times Y}$ where

$$\lambda_{X\times Y}(x,y) = \omega_X(x) \cap \psi_Y(y)$$
$$\text{for all } (x,y) \in X \times Y$$

Example 3.4 Let $Z = \{z_1, z_2\}$ be a universal set and $E = \{e_1, e_2, e_3,\}$ be a set of attributes. Let $X = \{e_1, e_2\}, Y = \{e_2, e_3\} \subseteq E$. Let Ω_X and Ψ_Y be two $3-$N soft set defined as.

$$\begin{aligned}\Omega_X = \{&e_1, (z_1, (0.7, 0.5, 0.5), (0.4, 0.5, 0.4), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.6, 0.4, 0.5), (0.3, 0.5, 0.8)), \\ &e_2, (z_1, (0.3, 0.5, 0.6), (0.3, 0.8, 0.2), (0.7, 0.5, 0.3)), (z_2, (0.7, 0.2, 0.7), (0.5, 0.8, 0.4), (0.2, 0.7, 0.4))\}\end{aligned}$$

$$\begin{aligned}\Psi_Y = \{&e_2, (z_1, (0.3, 0.5, 0.7), (0.2, 0.6, 0.3), (0.3, 0.4, 0.6)), (z_2, (0.4, 0.6, 0.7), (0.3, 0.5, 0.7), (0.4, 0.7, 0.3)), \\ &e_3, (z_1, (0.4, 0.7, 0.4), (0.7, 0.5, 0.3), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.7, 0.4), (0.3, 0.8, 0.9), (0.2, 0.1, 0.6))\}\end{aligned}$$

Then,

$$\begin{aligned}\Omega_X \breve{\cup} \Psi_Y = \{&e_1, (z_1, (0.7, 0.5, 0.5), (0.4, 0.5, 0.4), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.6, 0.4, 0.5), (0.3, 0.5, 0.8)), \\ &e_2, (z_1, (0.3, 0.5, 0.7), (0.3, 0.8, 0.3), (0.3, 0.4, 0.3)), (z_2, (0.7, 0.6, 0.7), (0.5, 0.8, 0.7), (0.2, 0.7, 0.3)), \\ &e_3, (z_1, (0.4, 0.7, 0.4), (0.7, 0.5, 0.3), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.7, 0.4), (0.3, 0.8, 0.9), (0.2, 0.1, 0.6))\}\end{aligned}$$

$$\Omega_X \breve{\cap} \Psi_Y = \{e_2, (z_1, (0.3, 0.5, 0.6), (0.2, 0.6, 0.2), (0.7, 0.5, 0.6)), (z_2, (0.4, 0.2, 0.7), (0.3, 0.5, 0.4), (0.4, 0.7, 0.4))\}$$

$$\Omega_X \cup_R \Psi_Y = \{e_2, (z_1, (0.3, 0.5, 0.7), (0.3, 0.8, 0.3), (0.3, 0.4, 0.3)), (z_2, (0.7, 0.6, 0.7), (0.5, 0.8, 0.7), (0.2, 0.7, 0.3))\}$$

$$\Omega_X \cap_\varepsilon \Psi_Y = \{e_1, (z_1, (0.7, 0.5, 0.5), (0.4, 0.5, 0.4), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.4, 0.5), (0.6, 0.4, 0.5), (0.3, 0.5, 0.8)),$$
$$e_2, (z_1, (0.3, 0.5, 0.6), (0.2, 0.6, 0.2), (0.7, 0.5, 0.6)), (z_2, (0.4, 0.2, 0.7), (0.3, 0.5, 0.4), (0.4, 0.7, 0.4)),$$
$$e_3, (z_1, (0.4, 0.7, 0.4), (0.7, 0.5, 0.3), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.7, 0.4), (0.3, 0.8, 0.9), (0.2, 0.1, 0.6))\}$$

$$\Omega_X \vee \Psi_Y = \{(e_1, e_2), (z_1, (0.7, 0.5, 0.7), (0.4, 0.6, 0.4), (0.3, 0.4, 0.3)), (z_2, (0.4, 0.6, 0.7), (0.6, 0.5, 0.7), (0.3, 0.5, 0.3)),$$
$$(e_1, e_3), (z_1, (0.7, 0.7, 0.5), (0.7, 0.5, 0.4), (0.3, 0.5, 0.3)), (z_2, (0.3, 0.7, 0.5), (0.6, 0.8, 0.9), (0.2, 0.1, 0.6)),$$
$$(e_2, e_2), (z_1, (0.3, 0.5, 0.7), (0.3, 0.8, 0.3), (0.3, 0.4, 0.3)), (z_2, (0.7, 0.6, 0.7), (0.5, 0.8, 0.7), (0.2, 0.7, 0.3)),$$
$$(e_2, e_3), (z_1, (0.4, 0.7, 0.6), (0.7, 0.8, 0.3), (0.7, 0.5, 0.3)), (z_2, (0.7, 0.7, 0.7), (0.5, 0.8, 0.9), (0.2, 0.1, 0.4))\}$$

$$\Omega_X \wedge \Psi_Y = \{(e_1, e_2), (z_1, (0.3, 0.5, 0.5), (0.2, 0.5, 0.3), (0.3, 0.5, 0.6)), (z_2, (0.3, 0.4, 0.5), (0.3, 0.4, 0.5), (0.4, 0.7, 0.8)),$$
$$(e_1, e_3), (z_1, (0.4, 0.5, 0.4), (0.4, 0.5, 0.3), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.4, 0.4), (0.3, 0.4, 0.5), (0.3, 0.5, 0.8)),$$
$$(e_2, e_2), (z_1, (0.3, 0.5, 0.6), (0.2, 0.6, 0.2), (0.7, 0.5, 0.6)), (z_2, (0.4, 0.2, 0.7), (0.3, 0.5, 0.4), (0.4, 0.7, 0.4)),$$
$$(e_2, e_3), (z_1, (0.3, 0.5, 0.4), (0.3, 0.5, 0.2), (0.8, 0.5, 0.7)), (z_2, (0.2, 0.2, 0.4), (0.3, 0.8, 0.4), (0.4, 0.7, 0.6))\}$$

4. Properties of mNS Set Operators

In this section, we define some properties of mNS set operators that satisfied among mNS sets. We also give proof of some of them, while others can also be proved. Let Ω_X, Ψ_Y and Λ_W be three mNS sets over universal set Z with respect to parameter set E where X, Y and W are subsets of E. The approximation functions of Ω_X, Ψ_Y and Λ_W are defined as

$$\omega_X(e) = \{(z, s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z)) : z \in Z,\ e \in X\}$$
$$\psi_Y(e) = \{(z, s_i \circ T_Y(e)(z), s_i \circ I_Y(e)(z), s_i \circ F_Y(e)(z)) : z \in Z,\ e \in Y\}$$
$$\lambda_W(e) = \{(z, s_i \circ T_W(e)(z), s_i \circ I_W(e)(z), s_i \circ F_W(e)(z)) : z \in Z,\ e \in W\}$$
$$\text{for all } i = 1, 2, \cdots, m$$

4.1. *Idempotent properties*

(i) $\Omega_X \breve{\cup} \Omega_X = \Omega_X = \Omega_X \cup_R \Omega_X$
(ii) $\Omega_X \breve{\cap} \Omega_X = \Omega_X = \Omega_X \cap_\varepsilon \Omega_X$

4.2. *Identity Properties*

(i) $\Omega_X \breve{\cup} \breve{\Phi}_X = \Omega_X = \Omega_X \cup_R \breve{\Phi}_X$
(ii) $\Omega_X \breve{\cap} \breve{Z}_X = \Omega_X = \Omega_X \cap_\varepsilon \breve{Z}_X$

4.3. *Domination Properties*

(i) $\Omega_X \breve{\cup} \breve{Z}_X = \breve{Z}_X = \Omega_X \cup_R \breve{Z}_X$
(ii) $\Omega_X \breve{\cap} \breve{\Phi}_X = \breve{\Phi}_X = \Omega_X \cap_\varepsilon \breve{\Phi}_X$

4.4. *Complementation Properties*

(i) $\breve{Z}^c_X = \breve{\Phi}_X = \breve{Z}^r_X$
(ii) $\breve{\Phi}^c_X = \breve{Z}_X = \breve{\Phi}^r_X$

4.5. *Double Complementation Property*

(i) $(\Omega_X^c)^c = \Omega_X = (\Omega_X^r)^r$

4.6. *Absorption Properties*

(i) $\Omega_X \breve{\cup} (\Omega_X \breve{\cap} \Psi_Y) = \Omega_X$

(ii) $\Omega_X \breve{\cap} (\Omega_X \breve{\cup} \Psi_Y) = \Omega_X$

(iii) $\Omega_X \cup_R (\Omega_X \cap_\varepsilon \Psi_Y) = \Omega_X$

(iv) $\Omega_X \cap_\varepsilon (\Omega_X \cup_R \Psi_Y) = \Omega_X$

Remark 4.1

(i) Union $\breve{\cup}$ and extended intersection $\cap_\varepsilon$ do not absorb over each other among mNS sets

(ii) Restricted Union $\cup_R$ and intersection $\breve{\cap}$ do not absorb over each other among mNS sets

4.7. *Commutative Properties*

(i) $\Omega_X \breve{\cup} \Psi_Y = \Psi_Y \breve{\cup} \Omega_X$

(ii) $\Omega_X \cup_R \Psi_Y = \Psi_Y \cup_R \Omega_X$

(iii) $\Omega_X \breve{\cap} \Psi_Y = \Psi_Y \breve{\cap} \Omega_X$

(iv) $\Omega_X \cap_\varepsilon \Psi_Y = \Psi_Y \cap_\varepsilon \Omega_X$

Remark 4.2

(i) OR-operator $\vee$ and AND-operator $\wedge$ do not commute among mNS sets

4.8. *Associative Properties*

(i) $\Omega_X \breve{\cup} (\Psi_Y \breve{\cup} \Lambda_W) = (\Omega_X \breve{\cup} \Psi_Y) \breve{\cup} \Lambda_W$

(ii) $\Omega_X \breve{\cap} (\Psi_Y \breve{\cap} \Lambda_W) = (\Omega_X \breve{\cap} \Psi_Y) \breve{\cap} \Lambda_W$

(iii) $\Omega_X \cup_R (\Psi_Y \cup_R \Lambda_W) = (\Omega_X \cup_R \Psi_Y) \cup_R \Lambda_W$

(iv) $\Omega_X \cap_\varepsilon (\Psi_Y \cap_\varepsilon \Lambda_W) = (\Omega_X \cap_\varepsilon \Psi_Y) \cap_\varepsilon \Lambda_W$

(v) $\Omega_X \vee (\Psi_Y \vee \Lambda_W) = (\Omega_X \vee \Psi_Y) \vee \Lambda_W$

(vi) $\Omega_X \wedge (\Psi_Y \wedge \Lambda_W) = (\Omega_X \wedge \Psi_Y) \wedge \Lambda_W$

Proof(i)

$$\begin{gathered}\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = max\{s_i \circ T_X(e)(z), max(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\},\\ max\{s_i \circ I_X(e)(z), max(s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z))\},\\ min\{s_i \circ F_X(e)(z), min(s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))\}\\ \textit{for all } i = 1, 2, \cdots, m;\ e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z \textit{ and } z \in Z\end{gathered}$$

$$\begin{gathered}\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)),\\ max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)), min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))\\ \textit{for all } i = 1, 2, \cdots, m;\ e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z \textit{ and } z \in Z\end{gathered}$$

$$\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = max\{max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), s_i \circ T_W(e)(z)\},$$
$$max\{max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), s_i \circ I_W(e)(z)\},$$
$$min\{min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), s_i \circ F_W(e)(z)\}$$
$$for\ all\ i = 1, 2, \cdots, m;\ e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cup \lambda_Y(e)) = (\omega_X(e) \cup \psi_Y(e)) \cup \lambda_Y(e)$$
$$for\ all\ i = 1, 2, \cdots, m;\ e \in X \cup (Y \cup Z) = (X \cup Y) \cup Z\ and\ z \in Z$$

$$\Rightarrow \Omega_X \breve{\cup} (\Psi_Y \breve{\cup} \Lambda_W) = (\Omega_X \breve{\cup} \Psi_Y) \breve{\cup} \Lambda_W$$

Proof(ii)

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = min\{s_i \circ T_X(e)(z), min(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\},$$
$$min\{s_i \circ I_X(e)(z), min(s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z))\},$$
$$max\{s_i \circ F_X(e)(z), max(s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))\}$$
$$for\ all\ i = 1, 2, \cdots, m;\ e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z)),$$
$$min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z)), max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z), s_i \circ F_W(e)(z))$$
$$for\ all\ i = 1, 2, \cdots, m;\ e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = min\{min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), s_i \circ T_W(e)(z)\},$$
$$min\{min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), s_i \circ I_W(e)(z)\},$$
$$max\{max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), s_i \circ F_W(e)(z)\}$$
$$for\ all\ i = 1, 2, \cdots, m;\ e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cap \lambda_Y(e)) = (\omega_X(e) \cup \psi_Y(e)) \cup \lambda_Y(e)$$
$$for\ all\ i = 1, 2, \cdots, m;\ e \in X \cap (Y \cap Z) = (X \cap Y) \cap Z\ and\ z \in Z$$

$$\Rightarrow \Omega_X \breve{\cap} (\Psi_Y \breve{\cap} \Lambda_W) = (\Omega_X \breve{\cap} \Psi_Y) \breve{\cap} \Lambda_W$$

Similarly, others associative properties also satisfy equality.

4.9. *Distributive Properties*

(i) $\Omega_X \breve{\cup} (\Psi_Y \breve{\cap} \Lambda_W) = (\Omega_X \breve{\cup} \Psi_Y) \breve{\cap} (\Omega_X \breve{\cup} \Lambda_W)$
(ii) $\Omega_X \breve{\cap} (\Psi_Y \breve{\cup} \Lambda_W) = (\Omega_X \breve{\cap} \Psi_Y) \breve{\cup} (\Omega_X \breve{\cap} \Lambda_W)$
(iii) $\Omega_X \cup_R (\Psi_Y \cap_\varepsilon \Lambda_W) = (\Omega_X \cup_R \Psi_Y) \cap_\varepsilon (\Omega_X \cup_R \Lambda_W)$
(iv) $\Omega_X \cap_\varepsilon (\Psi_Y \cup_R \Lambda_W) = (\Omega_X \cap_\varepsilon \Psi_Y) \cup_R (\Omega_X \cap_\varepsilon \Lambda_W)$
(v) $\Omega_X \cup_R (\Psi_Y \breve{\cap} \Lambda_W) = (\Omega_X \cup_R \Psi_Y) \breve{\cap} (\Omega_X \cup_R \Lambda_W)$
(vi) $\Omega_X \breve{\cap} (\Psi_Y \cup_R \Lambda_W) = (\Omega_X \breve{\cap} \Psi_Y) \cup_R (\Omega_X \breve{\cap} \Lambda_W)$

Proof(i)

$$\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cap \lambda_W(e)) = max\{s_i \circ T_X(e)(z), min(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\},$$
$$max\{s_i \circ I_X(e)(z), min(s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z))\},$$
$$min\{s_i \circ T_X(e)(z), max(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\}$$

$$for\ all\ i = 1,2,\cdots,m;\ e \in X \cup (Y \cap W) = (X \cup Y) \cap (X \cup W)\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cap \lambda_W(e) =$$
$$min\{max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), max(s_i \circ T_X(e)(z), s_i \circ T_W(e)(z))\},$$
$$min\{max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), max(s_i \circ I_X(e)(z), s_i \circ I_W(e)(z))\},$$
$$max\{min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), min(s_i \circ F_X(e)(z), s_i \circ F_W(e)(z))\}$$
$$for\ all\ i = 1,2,\cdots,m;\ e \in X \cup (Y \cap W) = (X \cup Y) \cap (X \cup W)\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cup (\psi_Y(e) \cap \lambda_W(e)) = (\omega_X(e) \cup \psi_Y(e)) \cap (\omega_X(e) \cup \lambda_W(e))$$
$$for\ all\ i = 1,2,\cdots,m;\ e \in X \cup (Y \cap W) = (X \cup Y) \cap (X \cup W)\ and\ z \in Z$$

$$\Rightarrow \Omega_X \breve{\cup} (\Psi_Y \breve{\cap} \Lambda_W) = (\Omega_X \breve{\cup} \Psi_Y) \breve{\cap} (\Omega_X \breve{\cup} \Lambda_W)$$

Proof(ii)

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cup \lambda_W(e)) = min\{s_i \circ T_X(e)(z), max(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\},$$
$$min\{s_i \circ I_X(e)(z), max(s_i \circ I_Y(e)(z), s_i \circ I_W(e)(z))\},$$
$$max\{s_i \circ T_X(e)(z), min(s_i \circ T_Y(e)(z), s_i \circ T_W(e)(z))\}$$
$$for\ all\ i = 1,2,\cdots,m;\ e \in X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cup \lambda_W(e) =$$
$$max\{min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)), min(s_i \circ T_X(e)(z), s_i \circ T_W(e)(z))\},$$
$$max\{min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), min(s_i \circ I_X(e)(z), s_i \circ I_W(e)(z))\},$$
$$min\{max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)), max(s_i \circ F_X(e)(z), s_i \circ F_W(e)(z))\}$$
$$for\ all\ i = 1,2,\cdots,m;\ e \in X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)\ and\ z \in Z$$

$$\Rightarrow \omega_X(e) \cap (\psi_Y(e) \cup \lambda_W(e)) = (\omega_X(e) \cap \psi_Y(e)) \cup (\omega_X(e) \cap \lambda_W(e))$$
$$for\ all\ i = 1,2,\cdots,m;\ e \in X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)\ and\ z \in Z$$

$$\Rightarrow \Omega_X \breve{\cap} (\Psi_Y \breve{\cup} \Lambda_W) = (\Omega_X \breve{\cap} \Psi_Y) \breve{\cup} (\Omega_X \breve{\cap} \Lambda_W)$$

Similarly, others distributive properties also satisfy equality.

Remark 4.3

(i) Union $\breve{\cup}$ and extended intersection $\cap_\varepsilon$ do not distribute over each other among mNS sets

(ii) OR-operator $\vee$ and AND-operator $\wedge$ do not distribute over each other among mNS sets

(iii) Restricted union $\cup_R$ distribute over union $\breve{\cup}$ but converse does not hold true

(iv) Intersection $\breve{\cap}$ distribute over extended intersection $\cap_\varepsilon$ but converse does not hold true

Counter-Example 4.1

Let $\Omega_X = \{e_2, (z_1, (0.5, 0.6), (0.3, 0.2), (0.9, 0.7)), (z_2, (0.3, 0.6), (0.4, 0.7), (0.5, 0.8))\}$;
$\Psi_Y = \{e_1, (z_1(0.3, 0.2), (0.4, 0.6), (0.1, 0.8)), (z_2, (0.6, 0.4), (0.6, 0.8), (0.8, 0.8))\}$ and
$\Lambda_W = \{e_2, (z_1, (0.7, 0.2), (0.3, 0.5), (0.2, 0.1)), (z_2, (0.6, 0.5), (0.3, 0.6), (0.5, 0.4))\}$ be three $2-$NS sets over the universal set $Z = \{z_1, z_2\}$ with respect to set of attributes $E = \{e_1, e_2\}$, then

$$\Omega_X \cap_\varepsilon (\Psi_Y \breve{\cup} \Lambda_W) = \{e_1, (z_1(0.3, 0.2), (0.4, 0.6), (0.1, 0.8)), (z_2, (0.6, 0.4), (0.6, 0.8), (0.8, 0.8)),$$
$$e_2, (z_1, (0.5, 0.2), (0.3, 0.2), (0.9, 0.7)), (z_2, (0.3, 0.5), (0.3, 0.6), (0.5, 0.8))\}$$

and

$$(\Omega_X \cap_\varepsilon \Psi_Y) \breve{\cup} (\Omega_X \cap_\varepsilon \Lambda_W) = \{e_1, (z_1(0.3, 0.2), (0.4, 0.6), (0.1, 0.8)), (z_2, (0.6, 0.4), (0.6, 0.8), (0.8, 0.8)),$$
$$e_2, (z_1, (0.5, 0.6), (0.3, 0.2), (0.9, 0.7)), (z_2, (0.3, 0.6), (0.4, 0.7), (0.5, 0.8))\}$$

Hence, $\Omega_X \cap_\varepsilon (\Psi_Y \breve{\cup} \Lambda_W) \neq (\Omega_X \cap_\varepsilon \Psi_Y) \breve{\cup} (\Omega_X \cap_\varepsilon \Lambda_W)$

4.10. *De Morgan's Properties*

(i) $(\Omega_X \cup_R \Psi_Y)^r = \Omega_X^r \breve{\cap} \Psi_Y^r$
(ii) $(\Omega_X \breve{\cap} \Psi_Y)^r = \Omega_X^r \cup_R \Psi_Y^r$
(iii) $(\Omega_X \wedge \Psi_Y)^r = \Omega_X^r \vee (\Psi_Y^r$
(iv) $(\Omega_X \vee \Psi_Y)^r = \Omega_X^r \wedge \Psi_Y^r$
(v) $(\Omega_X \breve{\cup} \Psi_Y)^r = \Omega_X^r \cap_\varepsilon \Psi_Y^r$
(vi) $(\Omega_X \cap_\varepsilon \Psi_Y)^r = \Omega_X^r \breve{\cup} \Psi_Y^r$
(vii) $(\Omega_X \cup_R \Psi_Y)^c = \Omega_X^c \breve{\cap} \Psi_Y^c$
(viii) $(\Omega_X \breve{\cap} \Psi_Y)^c = \Omega_X^c \cup_R \Psi_Y^c$
(ix) $(\Omega_X \wedge \Psi_Y)^c = \Omega_X^c \vee \Psi_Y^c$
(x) $(\Omega_X \vee \Psi_Y)^c = \Omega_X^c \wedge \Psi_Y^c$
(xi) $(\Omega_X \breve{\cup} \Psi_Y)^c = \Omega_X^c \cap_\varepsilon \Psi_Y^c$
(xii) $(\Omega_X \cap_\varepsilon \Psi_Y)^c = \Omega_X^c \breve{\cup} \Psi_Y^c$

Proof(i)

$$\Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = [max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)),$$
$$max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z))]^r$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)),$$
$$(1, 1, \cdots, 1) - max(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z))$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = min(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)),$$
$$min\{(1, 1, \cdots, 1) - s_i \circ I_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z))\}, max(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z))$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = [s_i \circ F_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_X(e)(z), s_i \circ T_X(e)(z)]$$
$$\cap [s_i \circ F_Y(e)(z)), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z)), s_i \circ T_Y(e)(z))]$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = [s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z)]^r$$
$$\cap [s_i \circ T_Y(e)(z)), s_i \circ I_Y(e)(z)), s_i \circ F_Y(e)(z))]^r$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cup \psi_Y(e))^r = \omega_X^r(e) \cap \psi_Y^r(e)$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\Omega_X \cup_R \Psi_Y)^r = (\Omega_X)^r \breve{\cap} (\Psi_Y)^r$$

Proof(ii)

$$\Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = [min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z)),$$
$$min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z))]^r$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)),$$
$$(1, 1, \cdots, 1) - min(s_i \circ I_X(e)(z), s_i \circ I_Y(e)(z)), min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z))$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = max(s_i \circ F_X(e)(z), s_i \circ F_Y(e)(z)),$$
$$max\{(1, 1, \cdots, 1) - s_i \circ I_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z))\}, min(s_i \circ T_X(e)(z), s_i \circ T_Y(e)(z))$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = [s_i \circ F_X(e)(z), (1, 1, \cdots, 1) - s_i \circ I_X(e)(z), s_i \circ T_X(e)(z)]$$
$$\cup [s_i \circ F_Y(e)(z)), (1, 1, \cdots, 1) - s_i \circ I_Y(e)(z)), s_i \circ T_Y(e)(z))]$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = [s_i \circ T_X(e)(z), s_i \circ I_X(e)(z), s_i \circ F_X(e)(z)]^r$$
$$\cup [s_i \circ T_Y(e)(z)), s_i \circ I_Y(e)(z)), s_i \circ F_Y(e)(z))]^r$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\omega_X(e) \cap \psi_Y(e))^r = \omega_X^r(e) \cup \psi_Y^r(e)$$
$$\textit{for all } i = 1, 2, \cdots, m;\ e \in X \cap Y \textit{ and } z \in Z$$

$$\Rightarrow (\Omega_X \breve{\cap} \Psi_Y)^r = (\Omega_X)^r \cup_R (\Psi_Y)^r$$

Similarly, all the other De Morgan's properties can be proved in the same way.

4.11. *Exclusion and Contradiction Properties*

The Exclusion and Contradiction Properties among mNS set do not hold, we show it by a counter-example

(i) $\Omega_X \breve{\cup} \Omega_X^r \neq \breve{Z}_X \neq \Omega_X \cup_R \Omega_X^r$
(ii) $\Omega_X \breve{\cup} \Omega_X^c \neq \breve{Z}_X \neq \Omega_X \cup_R \Omega_X^c$
(iii) $\Omega_X \breve{\cap} \Omega_X^r \neq \breve{\Phi}_X \neq \Omega_X \cap_\varepsilon \Omega_X^r$
(iv) $\Omega_X \breve{\cap} \Omega_X^c \neq \breve{\Phi}_X \neq \Omega_X \cap_\varepsilon \Omega_X^c$

Counter-Example 4.2

Let $\Omega_X = \{e_1, (z_1, (0.5, 0.6), (0.3, 0.2), (0.9, 0.3)), (z_2, (0.3, 0.6), (0.4, 0.7), (0.5, 0.8))\}$ be a $2-$NS set over universal set $Z = \{z_1, z_2\}$ with respect to the set of attributes $X \subseteq E$, relative complement of $2-$NS set Ω_X will be

$$\Omega^r_X = \{e_1, (z_1, (0.9, 0.3), (0.7, 0.8), (0.5, 0.6)), (z_2, (0.5, 0.8), (0.6, 0.3), (0.3, 0.6))\}, \text{ then}$$
$$\Omega_X \breve{\cup} \Omega^r_X = \{e_1, (z_1, (0.9, 0.6), (0.7, 0.8), (0.5, 0.3)), (z_2, (0.5, 0.8), (0.6, 0.7), (0.3, 0.6))\} \neq \breve{Z}_X$$
and
$$\Omega_X \breve{\cap} \Omega^r_X = \{e_1, (z_1, (0.5, 0.3), (0.3, 0.2), (0.9, 0.6)), (z_2, (0.3, 0.6), (0.4, 0.3), (0.5, 0.8))\} \neq \breve{\Phi}_X$$

Similarly, others can also be proved by counter-example.

5. **Distances and Similarity Measure**

In this section we define distances and similarity measure formulas for mNS set as follows:

5.1. *Distances*

Definition 4.1 Let $Z = \{z_1, z_2, ..., z_n\}$ be a universal set, $E = \{e_1, e_2, ..., e_q\}$ be a set of attributes and $X, Y \in E$. Let Ω_X, Ψ_Y are two mNS sets over Z with their mN approximate mapping

$$\omega_X(e_j) = \{(z, s_i \circ T_X(e_j)(z_k), s_i \circ I_X(e_j)(z_k), s_i \circ F_X(e_j)(z_k))\}$$
$$\psi_Y(e_j) = \{(z, s_i \circ T_Y(e_j)(z_k), s_i \circ I_Y(e_j)(z_k), s_i \circ F_Y(e_j)(z_k))\}$$
$$\text{for all } i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, q \text{ and } k = 1, 2, \cdots, n$$

respectively, then the distance measure between Ω_X and Ψ_Y is defined as

(1) Hamming distance:

$$d_H(\Omega_X, \Psi_Y) = \frac{1}{3mq}\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}(|s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)| + |s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)| + |s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|)\} \quad (1)$$

(2) Normalized Hamming distance:

$$d_{NH}(\Omega_X, \Psi_Y) = \frac{1}{3mqn}\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}(|s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)| + |s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)| + |s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|)\} \quad (2)$$

(3) Euclidean distance:

$$d_E(\Omega_X, \Psi_Y) = \{\frac{1}{3mq}\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}((s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k))^2 + (s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k))^2 + (s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k))^2)\}^{\frac{1}{2}} \quad (3)$$

(4) Normalized Euclidean distance:

$$d_{NE}(\Omega_X,\Psi_Y)=\{\frac{1}{3mqn}\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}((s_i\circ\ T_X(e_j)(z_k)-s_i\circ\ T_Y(e_j)(z_k))^2 \\ +(s_i\circ\ I_X(e_j)(z_k)-s_i\circ\ I_Y(e_j)(z_k))^2 \\ +(s_i\circ\ F_X(e_j)(z_k)-s_i\circ\ F_Y(e_j)(z_k))^2)\}^{\frac{1}{2}} \tag{4}$$

Theorem 5.1 The distance measures between Ω_X and Ψ_Y satisfy the following inequality

(1) $d_H(\Omega_X,\Psi_Y)\le n$

(2) $d_{NH}(\Omega_X,\Psi_Y)\le 1$

(3) $d_E(\Omega_X,\Psi_Y)\le\sqrt{n}$

(4) $d_{NE}(\Omega_X,\Psi_Y)\le 1$

Theorem 5.2

The distance mappings d_H,d_{NH},d_E and d_{NE} are defined from $m\mathrm{N}^Z\to R^+$ are metric

Proof

Let $\Omega_X=(\omega,X),\Psi_Y=(\psi,Y)$ and $\Lambda_W=(\lambda,W)$ be three mNS sets over Z, then

(1) $d_H(\Omega_X,\Psi_Y)\ge 0$

(2) Suppose $d_H(\Omega_X,\Psi_Y)=0$

$$\Longleftrightarrow\frac{1}{3mqn}\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}(|s_i\circ\ T_X(e_j)(z_k)-p_i\circ\ T_Y(e_j)(z_k)| \\ +|s_i\circ\ I_X(e_j)(z_k)-p_i\circ\ I_Y(e_j)(z_k)| \\ +|s_i\circ\ F_X(e_j)(z_k)-p_i\circ\ F_Y(e_j)(z_k)|)\}=0$$

$$\text{for all } i=1,2,\cdots,m;\ j=1,2,\cdots,q \text{ and } k=1,2,\cdots,n$$

$$\Longleftrightarrow|s_i\circ\ T_X(e_j)(z_k)-s_i\circ\ T_Y(e_j)(z_k)| \\ +|s_i\circ\ I_X(e_j)(z_k)-s_i\circ\ I_Y(e_j)(z_k)| \\ +|s_i\circ\ F_X(e_j)(z_k)-s_i\circ\ F_Y(e_j)(z_k)|=0$$

$$\text{for all } i=1,2,\cdots,m;\ j=1,2,\cdots,q \text{ and } k=1,2,\cdots,n$$

$$\Longleftrightarrow|s_i\circ\ T_X(e_j)(z_k)-s_i\circ\ T_Y(e_j)(z_k)|=0 \\ +|s_i\circ\ I_X(e_j)(z_k)-s_i\circ\ I_Y(e_j)(z_k)|=0 \\ +|s_i\circ\ F_X(e_j)(z_k)-s_i\circ\ F_Y(e_j)(z_k)|=0$$

$$\text{for all } i=1,2,\cdots,m;\ j=1,2,\cdots,q \text{ and } k=1,2,\cdots,n$$

$$\begin{aligned}\iff s_i \circ\ T_X(e_j)(z_k) &= s_i \circ\ T_Y(e_j)(z_k),\\ s_i \circ\ I_X(e_j)(z_k) &= s_i \circ\ I_Y(e_j)(z_k),\\ s_i \circ\ F_X(e_j)(z_k) &= s_i \circ\ F_Y(e_j)(z_k)\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, q \text{ and } k = 1, 2, \cdots, n$$

$$\iff \Omega_X = \Psi_Y$$

(3) $d_H(\Omega_X, \Psi_Y) = d_H(\Psi_Y, \Omega_X)$

(4) For any three mNS sets Ω_X, Ψ_Y and Λ_W

$$\begin{aligned}&|s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)|\\ +&|s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)|\\ +&|s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, q \text{ and } k = 1, 2, \cdots, n$$

$$\begin{aligned}&= |s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_W(e_j)(z_k) + s_i \circ\ T_W(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)|\\ &+|s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_W(e_j)(z_k) + s_i \circ\ I_W M(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)|\\ &+|s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_W(e_j)(z_k) + s_i \circ\ F_W(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, q \text{ and } k = 1, 2, \cdots, n$$

$$\begin{aligned}&\leq |s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_W(e_j)(z_k)| + |s_i \circ\ T_W(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)|\\ &+|s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_W(e_j)(z_k)| + |s_i \circ\ I_W(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)|\\ &+|s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_W(e_j)(z_k)| + |s_i \circ\ F_W(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, q \text{ and } k = 1, 2, \cdots, n$$

$$\begin{aligned}&= \{|s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_W(e_j)(z_k)| + |s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_W(e_j)(z_k)|\\ &+|s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_W(e_j)(z_k)|\} + \{|s_i \circ\ T_W(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)|\\ &+|s_i \circ\ I_W(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)| + |s_i \circ\ F_W(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|\}\end{aligned}$$

$$\text{for all } i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, q \text{ and } k = 1, 2, \cdots, n$$

Thus,

$$d_H(\Omega_X, \Psi_Y) \leq d_H(\Omega_X, \Lambda_W) + d_H(\Lambda_W, \Psi_Y)$$

5.2. *Similarity Measure*

Definition 5.2 [16] The SM of Ω_X and Ψ_Y is defined as

$$S(\Omega_X, \Psi_Y) = \frac{1}{1 + d(\Omega_X, \Psi_Y)} \tag{5}$$

where $d(\Omega_X, \Psi_Y)$ is any of the above distance.

5.3. *Similarity of two mN Soft Set*

Definition 5.3 [16] The two mN soft sets Ω_X and Ψ_Y are γ similar if and only if

$$S(\Omega_X, \Psi_Y) \geq \gamma, \text{ i.e.,}$$

$$\Omega_X \approx^{\gamma} \Psi_Y \Leftrightarrow S(\Omega_X, \Psi_Y) \geq \gamma, \gamma \in (0,1) \tag{6}$$

Ω_X and Ψ_Y are significantly similar if $S(\Omega_X, \Psi_Y) \geq 0.5$

Theorem 5.3

The SM of Ω_X and Ψ_Y over Z satisfies the following.

(1) $0 \leq S(\Omega_X, \Psi_Y) \leq 1$

(2) $S(\Omega_X, \Psi_Y) = S(\Psi_Y, \Omega_X)$

(3) $S(\Omega_X, \Psi_Y) = 1 \Leftrightarrow \Omega_X = \Psi_Y$

6. Application of SM for mN Soft Set

In this section, we utilize similarity measure for mNS set in two different real-life applications like as in medical diagnosis and decision-making for selection of a lecturer for university.

6.1. *Case Study I*

We use the notion of Similarity Measure to analyze whether the patient has dengue fever or not. An algorithm is given as follows

6.1.1. *Algorithm*

Step 1: Construct set of parameters $E = \{e_1, e_2, \cdots, e_q\}$ as all symptoms of a disease.

Step 2: Construct an mN soft set Ω_X of disease by a medical expert.

Step 3: Construct an mN soft set Ψ_Y by the medical report of the patient.

Step 4: Compute the distance between Ω_X and Ψ_Y by using the distance formula

$$\begin{aligned} d_H(\Omega_X, \Psi_Y) = \frac{1}{3mq}\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}(&|s_i \circ\ T_X(e_j)(z_k) - s_i \circ\ T_Y(e_j)(z_k)| \\ &+|s_i \circ\ I_X(e_j)(z_k) - s_i \circ\ I_Y(e_j)(z_k)| \\ &+|s_i \circ\ F_X(e_j)(z_k) - s_i \circ\ F_Y(e_j)(z_k)|)\} \end{aligned}$$

Step 5: Calculate similarity measure between Ω_X and Ψ_Y using formula

$$S(\Omega_X, \Psi_Y) = \frac{1}{1 + d(\Omega_X, \Psi_Y)}$$

Step 6: Analyze the result using similarity.

6.1.2. *Problem Formulation and Assumptions*

The proposed algorithm can be utilized in medical diagnosis problems, here we are giving one numerical example of solution for such medical diagnosis problem in the light of mathematics. This proposed algorithm can be applied for any medical disease diagnosis problems. We consider dengue fever disease as an medical diagnosis problem, wether a considered patient has dengue fever or not, since many of the symptoms of dengue fever are matched with other diseases such as malaria. For specification of disease we applied similarity measure on mNS structure to get insured and accurate results. The m-polar structure gives us data of m medical experts evaluation for particular disease.

6.1.3. *Application of Algorithm*

Now we consider a universal set $Z = \{z_1 = \text{dengue fever}, z_2 = \text{not dengue fever}\}$
We consider set of parameters $E = \{e_1 = \text{High Fever}, e_2 = \text{Bleeding}, e_3 = \text{Severe Pain}\}$ as some of the symptoms of dengue fever disease, where these parameters can be described as,
The patient may have "High Fever" may also suffering from irritability and headache
"Bleeding" from gums or under the skin or from nose
"Severe Pain" in joints or in muscles
Let $X, Y \subseteq E$. Then we construct an $3-$NS set Ω_X with the help of 3 medical expert (doctor) as follows:

Ω_X	z_1	z_2
e_1	(0.69,0.52,0.61),(0.37,0.44,0.23),(0.46,0.37,0.29)	(0.54,0.63,0.55),(0.48,0.44,0.26),(0.63,0.47,0.59)
e_2	(0.43,0.66,0.62),(0.48,0.45,0.53),(0.47,0.52,0.36)	(0.17,0.23,0.29),(0.37,0.41,0.47),(0.53,0.59,0.61)
e_3	(0.34,0.47,0.27),(0.46,0.48,0.37),(0.75,0.58,0.69)	(0.58,0.53,0.55),(0.37,0.35,0.32),(0.65,0.63,0.59)

Table 1: $3-$NS set by 3 medical expert (doctor)

Then we construct a $3-$NS set Ψ_Y by a medical report of the patient as follows:

Ψ_Y	z_1	z_2
e_1	(0.63,0.57,0.54),(0.47,0.46,0.32),(0.62,0.75,0.67)	(0.45,0.71,0.50),(0.50,0.43,0.26),(0.61,0.50,0.47)
e_2	(0.47,0.59,0.69),(0.53,0.50,0.60),(0.43,0.58,0.32)	(0.15,0.25,0.25),(0.32,0.40,0.43),(0.53,0.60,0.60)
e_3	(0.27,0.38,0.24),(0.58,0.37,0.47),(0.65,0.69,0.70)	(0.47,0.46,0.64),(0.44,0.40,0.30),(0.61,0.60,0.68)

Table 2: $3-$NS set by a medical report of a patient

Computing distances between Ω_X and Ψ_Y and the results are

$$d_H(\Omega_X, \Psi_Y) = 0.1381$$
$$d_{NH}(\Omega_X, \Psi_Y) = 0.0690$$
$$d_E(\Omega_X, \Psi_Y) = 0.0195$$
$$d_{NE}(\Omega_X, \Psi_Y) = 0.0097$$

Using Eculidean distance to calculate similarity measure of Ω_X and Ψ_Y and result is as follows

$$S(\Omega_X, \Psi_Y) = 0.98 \geq 0.5$$

Since $S(\Omega_X, \Psi_Y)$ is greater than 0.5, i.e. the similarity measure of two $3-$NS sets is significantly similar, this implies a patient is suffering from dengue fever.

6.2. *Case Study II*

Here, we generate an example of selecting lecturer for university after seeing candidate's interview reports. An algorithm is given as follows;

6.2.1. *Algorithm*

Step 1: Construct a set of attribute of selection purpose as $E = \{e_1, e_2, \cdots, e_q\}$
Step 2: Construct an mNS set Ω_X as the requirements of a firm concluded by decision-making team.
Step 3: Construct t mNS sets Ψ_Y^h by the help of evaluation of different alternatives given by decision-making team, where $h = 1, 2, \cdots, t$
Step 4: Compute the distance between Ω_X and Ψ_Y^h by using the distance formula

$$d_E(\Omega_X, \Psi_Y) = \{\frac{1}{3mq}\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}((s_i \circ \ T_X(e_j)(z_k) - s_i \circ \ T_Y(e_j)(z_k))^2$$
$$+(s_i \circ \ I_X(e_j)(z_k) - s_i \circ \ I_Y(e_j)(z_k))^2$$
$$+(s_i \circ \ F_X(e_j)(z_k) - s_i \circ \ F_Y(e_j)(z_k))^2)\}^{\frac{1}{2}}$$

Step 5: Calculate the similarity measure between Ω_X and Ψ_Y^h using formula

$$S(\Omega_X, \Psi_Y^h) = \frac{1}{1 + d(\Omega_X, \Psi_Y^h)}$$

Step 6: Analyze the result using similarity that which alternative is more suitable to be select as a lecturer.

6.2.2. *Problem Formulation and Assumption*

A Similarity measure of mNS sets can help in decision-making problem of selection of best alternative corresponds to attribute of selection purpose. It can be applicable in problems of group decision making where a group of people gives their own evaluation to all alternatives corresponds to attribute and wants an alternative to be selected which fulfills or near to the evaluation that was given individually by them. We consider one example of such type of problems that is department of mathematics wants to hire a new lecturer for university. The new lecturer must well aware of both Pure and Applied Mathematics. They made a decision-making team of three-person for the selection of a lecturer. All member of team first evaluate the requirements of a department for selecting the purpose of a new lecturer individually, then they took interviews and demo classes of four applicants who are willing to be a lecturer in that university and made reports of every applicant for decision-making process.

6.2.3. *Application of Algorithm*

Consider a universal set $Z = \{z_1 = \text{Pure Math}, z_2 = \text{Applied Math}\}$ and set of attributes for the selection purpose as $E = \{e_1 = \text{Teaching Techniques}, e_2 = \text{Research Work}, e_3 = \text{Expertise}\}$. Let $X = Y \subseteq E$, then we construct a 3−NS set Ω_X as requirements of a department.

Ω_X	z_1	z_2
e_1	(0.82,0.55,0.63),(0.55,0.46,0.28),(0.43,0.38,0.60)	(0.50,0.62,0.52),(0.93,0.57,0.80),(0.66,0.48,0.52)
e_2	(0.43,0.68,0.86),(0.47,0.67,0.56),(0.42,0.51,0.33)	(0.77,0.54,0.82),(0.75,0.54,0.72),(0.53,0.54,0.69)
e_3	(0.73,0.48,0.53),(0.87,0.43,0.77),(0.76,0.53,0.62)	(0.64,0.48,0.59),(0.32,0.58,0.22),(0.94,0.64,0.62)

Table 3: 3−NS set of requirement of a department

Now we will construct four 3−NS sets $\Psi_Y^1, \Psi_Y^2, \Psi_Y^3$ and Ψ_Y^4 of different applicants A_1 A_2 A_3 and A_4 respectively with the help of reports made by decision-making team.

Ψ_Y^1	z_1	z_2
e_1	(0.13,0.15,0.22),(0.89,0.78,0.83),(0.77,0.82,0.91)	(0.79,0.84,0.93),(0.36,0.18,0.26),(0.21,0.24,0.16)
e_2	(0.07,0.23,0.32),(0.12,0.18,0.20),(0.74,0.79,0.88)	(0.23,0.13,0.22),(0.31,0.25,0.43),(0.19,0.22,0.27)
e_3	(0.23,0.12,0.17),(0.25,0.16,0.22),(0.14,0.16,0.18)	(0.10,0.13,0.11),(0.91,0.84,0.69),(0.31,0.30,0.28)

Table 4: 3−NS set a report of Applicant 1 (A_1)

Ψ_Y^2	z_1	z_2
e_1	(0.16,0.20,0.27),(0.83,0.87,0.89),(0.70,0.75,0.86)	(0.88,0.81,0.90),(0.40,0.20,0.26),(0.22,0.27,0.17)
e_2	(0.13,0.21,0.24),(0.18,0.20,0.20),(0.70,0.84,0.90)	(0.15,0.16,0.25),(0.32,0.33,0.43),(0.22,0.25,0.30)
e_3	(0.20,0.16,0.27),(0.29,0.17,0.26),(0.14,0.15,0.12)	(0.16,0.17,0.14),(0.85,0.84,0.70),(0.30,0.30,0.30)

Table 5: 3−NS set a report of Applicant 2 (A_2)

Ψ_Y^3	z_1	z_2
e_1	(0.76,0.59,0.56),(0.47,0.52,0.33),(0.52,0.45,0.67)	(0.45,0.70,0.56),(0.90,0.50,0.85),(0.61,0.42,0.47)
e_2	(0.47,0.59,0.89),(0.53,0.60,0.60),(0.46,0.58,0.30)	(0.70,0.59,0.76),(0.70,0.50,0.68),(0.56,0.60,0.62)
e_3	(0.77,0.40,0.48),(0.83,0.37,0.74),(0.66,0.62,0.70)	(0.72,0.45,0.64),(0.39,0.50,0.31),(0.85,0.60,0.68)

Table 6: 3−NS set a report of Applicant 3 (A_3)

Ψ_Y^4	z_1	z_2
e_1	(0.40,0.32,0.33),(0.30,0.29,0.40),(0.24,0.67,0.83)	(0.28,0.45,0.73),(0.64,0.86,0.49),(0.48,0.70,0.80)
e_2	(0.63,0.40,0.59),(0.69,0.44,0.30),(0.70,0.70,0.60)	(0.55,0.31,0.60),(0.52,0.76,0.58),(0.72,0.79,0.90)
e_3	(0.49,0.28,0.34),(0.58,0.63,0.50),(0.60,0.69,0.79)	(0.47,0.65,0.80),(0.56,0.37,0.47),(0.61,0.76,0.80)

Table 7: 3−NS set a report of Applicant 4 (A_4)

The Euclidean distance between Ω_X and Ψ_Y^h is calculated as

$$d_E(\Omega_X, \Psi_Y^1) = 0.3798$$
$$d_E(\Omega_X, \Psi_Y^2) = 0.3610$$

$$d_E(\Omega_X, \Psi_Y^3) = 0.0076$$
$$d_E(\Omega_X, \Psi_Y^4) = 0.1087$$

The Similarity Measure of Ω_X and Ψ_Y^h is calculated as

$$S(\Omega_X, \Psi_Y^1) = 0.72$$
$$S(\Omega_X, \Psi_Y^2) = 0.73$$
$$S(\Omega_X, \Psi_Y^3) = 0.99$$
$$S(\Omega_X, \Psi_Y^4) = 0.90$$

Since, similarity measure of $S(\Omega_X, \Psi_Y^3) = 0.99$ is greater than 0.5 and greater than all others, the two $3-$NS sets are significantly similar, which shows applicant 3 (A_3) is more suitable and he also fulfills requirements of the university.

7. Conclusion

The existing multi-polar information is not completely defined by using the existing methods. now multi-polar neutrosophic models explain the things in a better way to solve the undetermined data having multi-polar information and having the vast applications in different fields. similarity measures based on distance play an important role to solve the problems that have indeterminacy. In this paper, we defined some basic operations and their properties on mN soft sets. Moreover, we have defined the distance-based similarity measure of multi-polar neutrosophic soft sets. We have used the concept of distance-based similarity measures in medical diagnosis and decision-making of the selection of lecturers for university with along algorithms. Moreover, we defined some basic operations and their properties on mN soft sets. In the future, Group MCDM problems can be solved using different methods of MCDM (TOPSIS, VIKOR, etc).

References

1. L. A. Zadeh, *Fuzzy sets*, Inf. and Cont. **8**, No. 3 (1965) 338-353
2. K. T. Atanassov *Intuitionistic fuzzy sets*, Fuzzy sets and Syst. **20**, No. 1 (1986) 87-96
3. D. A. Molodtsov, *Soft set theory-first results*, Comput. Math Appl **37**, No. 4-5 (1999) 19-31.
4. P. K. Maji, R. Biswas and A. R. Roy *Soft set theory*, Comput. Math Appl **45**, No. 37 (2003) 555-562.
5. W. R. Zhang, *Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multi-agent decision analysis*, Proc. of IEEE Conf. (1994) 305-309.
6. J. Chen, S. Li, S. Ma and X. Wang, *mpolar fuzzy sets: an extension of bipolar fuzzy sets*, Sci. World J. 2014, Article Id 416530 (2014) 8.
7. D.F. Li and C.T. Cheng, *New similarity measure of intuitionistic fuzzy set and application to pattern recognition*, Pattern Recognit. Lett. **23**, No. 1-3 (2002) 221-225
8. D. Chen, E. C. C. Tsang and D.S. Yeung, *Some notes on the parameterization reduction of soft sets*, Sci. World J. 2014, Article Id 416530 (2014) 8.
9. S.M. Chen, M.S. Yeh and P.Y. Hsiao, *A comparison of similarity measures of fuzzy values*, Fuzzy sets Syst. **72**, No. 1 (1995) 79-89.
10. S.M. Chen; *Measures of similarity between vague sets*, Fuzzy sets Syst. **74**, No. 2 (1995) 217-223.

11. P. Majumdar and S.K. Samanta, *Generalized fuzzy soft sets*, Comput Math Appl. **59**, No. 4 (2010) 1425-1432.
12. P. Majumdar and S.K. Samanta, *Similarity measure of soft sets*, New Math. Nat. Comput 4, No. 1 (2008) 1-12.
13. P.Majumdar and S. K. Samanta, *On distance based similarity measure between intuitionistic fuzzy soft sets*, Anusandhan, **12**, No. 22 (2010) 41-50.
14. A. Kharal, *Distance and similarity measures for soft set*, New Math Nat. Comput. **6**, No. 3 (2010) 321-334
15. M. Akram and N. Waseem *Similarity Measure for new Hybrid Models: mF sets and mF Soft Sets*, Punj. Uni. J. Math. **51**, No. 6 (2019) 115-130
16. J. C. Bezdek, R. Ehrlich and W. Full *FCM: The fuzzy c-means clustering algorithm*, Comput. Geosci. **10**, No. 2-3 (1984) 191-203
17. M. D. Cock, U. Bodenhofer and E. Kerre *Modelling linguistic expressions using fuzzy relations*, Proceed. Conference, (2000) 353-360
18. I. Deli, *Interval-valued neutrosophic soft sets and its decision making*, Int. J. Mach. Learn. Cyber. **8**, No. 2 (2017) 665-676
19. F. Smarandache, *Neutrosophic Set - A Generalization of the Intuitionistic Fuzzy Set*, Int. J. Pure Appl. Math. 24, No. 3 (2004)
20. F. Feng, Y. B. Jun, X. Liu and L. L, *An adjustable approach to fuzzy soft set based decision making*, J. Comput. Appl. Math. 234, No. 1 (2010) 10-20
21. D. H. Hong and C. Kim, *A note on similarity measures between vague sets and between elements*, Inf. Sci. **115**, No. 1-4 (1999) 83-96
22. P. Liu, L. Zhang, J. Q. Wang and X. Liu, *Multi-valued neutrosopohic number Bonferroni mean operators with their applications in multiple attribute group decision-making*, Int. J. Inf. Tech. Decis. Mak. **15**, No. 5 (2016) 1181-1210
23. P. K. Maji, *Neutrosophic soft set*, Ann. Fuzzy Math. Inf. **5**, No. 1 (2013) 157-168
24. I. Deli, S. Broumi and M. Ali, *Neutrosophic soft multiset theory and its Decison Making*, Neutro. Sets Syst. **5**, (2014) 65-76
25. Y. S. Huang and W. H. Li, *A study on aggregation of TOPSIS ideal solutions for group decision-making*, Group Decis. Negot. **21**, No. 4 (2012) 461-473
26. T. Kaya and C. Kahraman, *Multi criteria decision making in energy planning using a modified fuzzy TOPSIS methodology*, Expert Syst. Appl. 38, No. 6 (2011) 6577–6585
27. M. Anisseh, F. Piri, M. R. Shahraki and F. Aghamohamadi, *Fuzzy extension of TOPSIS model for group decision making under multiple criteria* Artifi. Intell. Review, **38**, No. 4 (2011) 325-338
28. F. Smarandache, *Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set*, Neutro. Sets Syst. **22**, (2018) 168-170
29. M. Saqlain, M. Saeed, M. R. Ahmad and F. Smarandache, *Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application*, Neutro. Sets Syst. **27**, (2019) 131-137
30. M. Saqlain, M. Sana, M. Saeed and F. Smarandache, *Aggregate Operators of Neutrosophic Hypersoft Set*, Neutro. Sets Syst. **32**, (2020) 294-306.
31. M. Saqlain, M. Sana, M. Saeed and B. Said, *Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets*, Neutro. Set Syst. **32**, (2020) 317-329.
32. M. Zulqarnain, F. Dayan and M. Saeed, *Topsis analysis for the prediction of diabetes based on general characteristics of humans*, Inter. J. Pharm. Sci. R. **9**, No. 7 (2018) 2932-2939
33. M. Saeed, Z. Anam, T. Kanwal, I. Saba, F. Memoona and M. F. Tabassum, *Generalization of TOPSIS from Soft Set to Fuzzy Soft Sets in Decision Making Problem*, Sci. Inq. Rev. **1**, No. 1 (2017) 11-18

34. M. Saeed, *Comparative study of airport evaluation problem by using fuzzy soft expert set and AHP technique*, Sci. Intern. **28**, No. 3 (2016) 2439-2443
35. M. Saqlain, N. Jafar and A. Riffat, *Smart phone selection by consumers' in Pakistan: FMCGDM fuzzy multiple criteria group decision making approach*, G. Uni. J. Res. **34**, No. 1 (2019) 27-31
36. M. Riaz, M. Saqlain and M. Saeed, *Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis*, P. Uni. J. Math. **51**, No. 8 (2019) 111-126
37. M. Saqlain, N. Jafar, R. Hamid and A. Shahzad, *Prediction of Cricket World Cup 2019 by TOPSIS Technique of MCDM-A Mathematical Analysis*, Int. J. Sci. Eng. R. **10**, No. 2 (2019) 789-792
38. M. Abdel-Basset, M. Mohamed, M. Elhoseny, F. Chiclana and A. E. N. H. Zaied, *Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases* Artifi. Intell. in Med. **101** (2019) 101735.
39. M. Abdel-Basset, M. El-hoseny, A. Gamal and F. Smarandache, *A novel model for evaluation Hospital medical care systems based on plithogenic sets*, Artifi. Intell. in Med. **100** (2019) 101710.
40. M. Abdel-Basset, G. Manogaran, A. Gamal and V. Chang, *A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT* IEEE IoT J. (2019)
41. M. Abdel-Basset, A. Gamal, G. Manogaran and H. V Long, *A novel group decision making model based on neutrosophic sets for heart disease diagnosis*, Multi. Tools App. (2019) 1-26.

Received: 9, February, 2020 / Accepted: 13, April, 2020