Introduction to Plithogenic Hypersoft Subgroup

Sudipta Gayen[1], Florentin Smarandache[2], Sripati Jha[1], Manoranjan Kumar Singh[3], Said Broumi[4] and Ranjan Kumar[5,*]

[1] National Institute of Technology Jamshedpur, India; sudi23dipta@gmail.com

[2] University of New Mexico, USA

[3] Magadh University, Bodh Gaya, India

[4] Faculty of Science Ben MSik, University Hassan II, Morocco

[5] Jain Deemed to be University, Jayanagar, Bengaluru, India; ranjank.nit52@gmail.com

[*] Correspondence: ranjank.nit52@gmail.com

Abstract. In this article, some essential aspects of plithogenic hypersoft algebraic structures have been analyzed. Here the notions of plithogenic hypersoft subgroups i.e. plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup have been introduced and studied. For doing that we have redefined the notions of plithogenic crisp hypersoft set, plithogenic fuzzy hypersoft set, plithogenic intuitionistic fuzzy hypersoft set, and plithogenic neutrosophic hypersoft set and also given their graphical illustrations. Furthermore, by introducing function in different plithogenic hypersoft environments, some homomorphic properties of plithogenic hypersoft subgroups have been analyzed.

Keywords: Hypersoft set; Plithogenic set; Plithogenic hypersoft set; Plithogenic hypersoft subgroup

A LIST OF ABBREVIATIONS

US signifies universal set.

CS signifies crisp set.

FS signifies fuzzy set.

IFS signifies intuitionistic fuzzy set.

NS signifies neutrosophic set.

PS signifies plithogenic set.

SS signifies soft set.

HS signifies hypersoft set.

CHS signifies crisp hypersoft set.

FHS signifies fuzzy hypersoft set.

IFHS signifies intuitionistic fuzzy hypersoft set.

NHS signifies neutrosophic hypersoft set.

PHS signifies plithogenic hypersoft set.

PCHS signifies plithogenic crisp hypersoft set.

PFHS signifies plithogenic fuzzy hypersoft set.

PIFHS signifies plithogenic intuitionistic fuzzy hypersoft set.

PNHS signifies plithogenic neutrosophic hypersoft set.

CG signifies crisp group.

FSG signifies fuzzy subgroup.

IFSG signifies intuitionistic fuzzy subgroup.

NSG signifies neutrosophic subgroup.

DAF signifies degree of appurtenance function.

DCF signifies degree of contradiction function.

PSG signifies plithogenic subgroup.

PCHSG signifies plithogenic crsip hypersoft subgroup.

PFHSG signifies plithogenic fuzzy hypersoft subgroup.

PIFHSG signifies plithogenic intuitionistic fuzzy hypersoft subgroup.

PNHSG signifies plithogenic neutrosophic hypersoft subgroup.

DMP signifies decision making problem.

$\rho(U)$ signifies power set of U.

1. Introduction

FS [1] theory was first initiated by Zadeh to handle uncertain real-life situations more precisely than CSs. Gradually, some other set theories like IFS [2], NS [3], Pythagorean FS [4], PS [5], etc., have emerged. These sets are able to handle ambiguous situations more appropriately than FSs. NS theory was introduced by Smarandache which was generalizations of IFS and FS. He has also introduced neutrosophic probability, measure [6,7], psychology [8], pre-calculus and calculus [9], etc. Presently, NS theory is vastly used in various pure as well as applied fields. For instance, in medical diagnosis [10, 11], shortest path problem [12–20], DMP [21–26], transportation problem [27, 28], forecasting [29], mobile edge computing [30], abstract algebra [31], pattern recognition problem [32], image segmentation [33], internet of things [34], etc. Another set theory of profound importance is PS theory which is extensively used in handling various uncertain situations. This set theory is more general than CS, FS, IFS, and NS theory. Gradually, plithogenic probability and statistics [35], plithogenic logic [35], etc., have evolved which are generalizations of crisp probability, statistics, and logic. Smarandache has also introduced the notions of plithogenic number, plithogenic measure function, bipolar PS, tripolar PS, multipolar PS, complex PS, refined PS, etc. Presently, PS theory is extensively used in numerous research domains.

The notion of SS [36] theory is another fundamental set theory. Presently, SS theory has become one of the most popular branches in mathematics for its huge areas of applications in various research fields. For instance, nowadays in DMP [37], abstract algebra [38–40], etc., it is widely used. Again, there exist concepts like vague sets [41, 42], rough set [43], hard set [44], etc., which are well known for their vast applications in various domains. Gradually, based on SS theory the notions of fuzzy SS [45], intuitionistic SS [46], neutrosophic SS [47] theory, etc., have been introduced by various researchers. In fuzzy abstract algebra, the notions of FSG [48], IFSG [49], NSG [31], etc., have been developed and studied by different mathematicians. SS theory has opened some new windows of opportunities for researchers working not only in applied fields but also in pure fields. As a result, the notions of soft FSG [39], soft IFSG [50], soft NSG [51], etc., were introduced. Later on, Smarandache has proposed the concept of HS [52] theory which is a generalization of SS theory. Also, he has extended and introduced the concept of HS in the plithogenic environment and generalized that further. As a result, a new branch has emerged which can be a fruitful research field for its promising potentials. The following Table 1 contains some significant contributions in SS and PS theory by numerous researchers.

TABLE 1. Significance and influences of PS & SS theory in various fields.

Author & references	Year	Contributions in various fields
Majhi et al. [53]	2002	Applied SS theory in a DMP.
Feng et al. [54]	2010	Described an adjustable approach to fuzzy SS based DMP with some examples.
aman [55]	2011	Defined fuzzy soft aggregation operator which allows the construction of more efficient DMP.
Broumi et al. [56]	2014	Defined neutrosophic parameterized SS and neutrosophic parameterized aggregation operator and applied it in DMP.
Broumi et al. [57]	2014	Defined interval-valued neutrosophic parameterized SS a reduction method for it.
Deli et al. [58]	2014	Introduced neutrosophic soft multi-set theory and studied some of its properties.
Deli & Naim [59]	2015	Introduced intuitionistic fuzzy parameterized SS and studied some of its properties.
Smarandache [60]	2018	Introduced physical PS.
Smarandache [61]	2018	Studied aggregation plithogenic operators in physical fields.

continued ...

Author & references	Year	Contributions in various fields
Gayen et al. [62]	2019	Introduced the notions of plithogenic subgroups and studied some of their homomorphic properties.
Abdel-Basset et al. [63]	2019	Described a novel model for evaluation of hospital medical care systems based on PSs.
Abdel-Basset et al. [64]	2019	Described a novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management.
Abdel-Basset et al. [65]	2019	Proposed a hybrid plithogenic decision-making approach with quality function deployment.

This Chapter has been systematized as the following: In Section 2, literature reviews of FS, IFS, NS, FSG, IFSG, NSG, PS, PHS, etc., are mentioned. In Section 3, the concepts of PCHS, PFHS, PIFHS, and PNHS have been redefined in a different way and their graphical illustrations have been given. Also, the notions of PFHSG, PIFHSG, and PNHSG have been introduced and further the effects of homomorphism on those notions are studied. Finally, in Section 4, the conclusion is given mentioning some scopes of future researches.

2. **Literature Survey**

In this segment, some important notions like, FS, IFS, NS, FSG, IFSG, NSG, etc., have been discussed. We have also mentioned PS, SS, HS and some aspects of PHS. These notions will play vital roles in developing the concepts of PHSGs.

Definition 2.1. [1] Let U be a CS. A function $\sigma : U \to [0,1]$ is called a FS.

Definition 2.2. [2] Let U be a CS. An IFS γ of U is written as $\gamma = \{(m, t_\gamma(m), f_\gamma(m)) : m \in U\}$, where $t_\gamma(m)$ and $f_\gamma(m)$ are two FSs of U, which are called the degree of membership and non-membership of any $m \in U$. Here $\forall m \in U$, $t_\gamma(m)$ and $f_\gamma(m)$ satisfy the inequality $0 \le t_\gamma(m) + f_\gamma(m) \le 1$.

Definition 2.3. [3] Let U be a CS. A NS η of U is denoted as $\eta = \{(m, t_\eta(m), i_\eta(m), f_\eta(m)) : m \in U\}$, where $t_\eta(m), i_\eta(m), f_\eta(m) : U \to]^-0, 1^+[$ are the corresponding degree of truth, indeterminacy, and falsity of any $m \in U$. Here $\forall m \in U$ $t_\eta(m)$, $i_\eta(m)$ and $f_\eta(m)$ satisfy the inequality $^-0 \le t_\eta(m) + i_\eta(m) + f_\eta(m) \le 3^+$.

2.1. *Fuzzy, Intuitionistic fuzzy & Neutrosophic subgroup*

Definition 2.4. [48] A FS of a CG U is called as a FSG iff $\forall m, u \in U$, the conditions mentioned below are satisfied:

(i) $\alpha(mu) \ge min\{\alpha(m), \alpha(u)\}$

(ii) $\alpha(m^{-1}) \geq \alpha(m)$.

Definition 2.5. [49] An IFS $\gamma = \{(m, t_\gamma(m), f_\gamma(m)) : m \in U\}$ of a CG U is called an IFSG iff $\forall m, u \in U$,

(i) $t_\gamma(mu^{-1}) \geq min\{t_\gamma(m), t_\gamma(u)\}$

(ii) $f_\gamma(mu^{-1}) \leq \max\{f_\gamma(m), f_\gamma(u)\}$.

The set of all the IFSG of U will be denoted as IFSG(U).

Definition 2.6. [31] Let U be a CG and δ be a NS of U. δ is called a NSG of U iff the conditions mentioned below are satisfied:

(i) $\delta(mu) \geq \min\{\delta(m), \delta(u)\}$, i.e. $t_\delta(mu) \geq \min\{t_\delta(m), t_\delta(u)\}$, $i_\delta(mu) \geq \min\{i_\delta(m), i_\delta(u)\}$ and $f_\delta(mu) \leq \max\{f_\delta(m), f_\delta(u)\}$

(ii) $\delta(m^{-1}) \geq \delta(m)$ i.e. $t_\delta(m^{-1}) \geq t_\delta(u)$, $i_\delta(m^{-1}) \geq i_\delta(u)$ and $f_\delta(m^{-1}) \leq f_\delta(u)$.

Theorem 2.1. *[66] Let g be a homomorphism of a CG U_1 into another CG U_2. Then preimage of an IFSG γ of U_2 i.e. $g^{-1}(\gamma)$ is an IFSG of U_1.*

Theorem 2.2. *[66] Let g be a surjective homomorphism of a CG U_1 to another CG U_2. Then the image of an IFSG γ of U_1 i.e. $g(\gamma)$ is an IFSG of U_2.*

Theorem 2.3. *[31] The homomorphic image of any NSG is a NSG.*

Theorem 2.4. *[31] The homomorphic preimage of any NSG is a NSG.*

Some more references in the domains of FSG, IFSG, NSG, etc., which can be helpful to various other researchers are [67–71].

2.2. Plithogenic set & Plithogenic hypersoft set

Definition 2.7. [5] Let U be a US and $P \subseteq U$. A PS is denoted as $P_s = (P, \psi, V_\psi, a, c)$, where ψ be an attribute, V_ψ is the respective range of attributes values, $a : P \times V_\psi \to [0, 1]^s$ is the DAF and $c : V_\psi \times V_\psi \to [0, 1]^t$ is the corresponding DCF. Here $s, t \in \{1, 2, 3\}$.

In Definition 2.7, for $s = 1$ and $t = 1$ a will become a FDAF and c will become a FDCF. In general, we consider only FDAF and FDCF. Also, $\forall (u_i, u_j) \in V_\psi \times V_\psi$, c satisfies $c(u_i, u_i) = 0$ and $c(u_i, u_j) = c(u_j, u_i)$.

Definition 2.8. [36] Let U be a US, V_A be a set of attribute values. Then the ordered pair (Γ, U) is called a SS over U, where $\Gamma : V_A \to \rho(U)$.

Definition 2.9. [52] Let U be a US. Let $r_1, r_2, ..., r_n$ be n attributes and corresponding attribute value sets are respectively $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then the ordered pair (Γ, V_ψ) is called a HS of U, where $\Gamma : V_\psi \to \rho(U)$.

Definition 2.10. [72] A US U_C is termed as a crisp US if $\forall u \in U_C$, u fully belongs to U_C i.e. membership of u is 1.

Definition 2.11. [72] A US U_F is termed as a fuzzy US if $\forall u \in U_F$, u partially belongs to U_F i.e. membership of u belonging to $[0, 1]$.

Definition 2.12. [72] A US U_{IF} is termed as an intuitionistic fuzzy US if $\forall u \in U_{IF}$, u partially belongs to U_{IF} and also partially does not belong to U_{IF} i.e. membership of u belonging to $[0, 1] \times [0, 1]$.

Definition 2.13. [72] A US U_N is termed as an neutrosophic US if $\forall u \in U_N$, u has truth belongingness, indeterminacy belongingness, and falsity belongingness to U_N i.e. membership of u belonging to $[0, 1] \times [0, 1] \times [0, 1]$.

Definition 2.14. [72] A US U_P over an attribute value set ψ is termed as a plithogenic US if $\forall u \in U_P$, u belongs to U_P with some degree on the basis of each attribute value. This degree can be crisp, fuzzy, intuitionistic fuzzy, or neutrosophic.

Definition 2.15. [52] Let U_C be a crisp US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \to \rho(U_C)$ is termed as a CHS over U_C.

Definition 2.16. [52] Let U_F be a fuzzy US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \to \rho(U_F)$ is called a FHS over U_F.

Definition 2.17. [52] Let U_{IF} be an intuitionistic fuzzy US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \to \rho(U_{IF})$ is called an IFHS over U_{IF}.

Definition 2.18. [52] Let U_N be a neutrosophic US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \to \rho(U_N)$ is called a NHS over U_N.

Definition 2.19. [52] Let U_P be a plithogenic US and $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes with attribute value sets respectively as $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$ for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Also, let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$. Then (Γ, V_ψ), where $\Gamma : V_\psi \to \rho(U_P)$ is called a PHS over U_P.

Further, depending on someones preferences PHS can be categorized as PCHS, PFHS, PIFHS, and PNHS. In [52], Smarandache has wonderfully introduced and illustrated these categories with proper examples.

In the next section, we have mentioned an equivalent statement of Definition 2.19 and described its categories in a different way. Also, we have given some graphical representations of PCHS, PFHS, PIFHS, and PNHS. Again, we have introduced functions in the environments of PFHS, PIFHS, and PNHS. Furthermore, we have introduced the notions of PFHSG, PIFHSG, and PNHSG and studied their homomorphic characteristics.

3. Proposed Notions

As an equivalent statement to Definition 2.19, we can conclude that $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_i : M \times D_i \to [0, 1]^s$ ($s = 1, 2$ or 3) such that $\forall (m, d) \in M \times D_i$, $a_i(m, d)$ represent the DAFs of m to the set M on the basis of the attribute value d. Then the pair (Γ, V_ψ) is called a PHS.

So, based on someones requirement one may choose $s = 1, 2$ or 3 and further, depending on these choices PHS can be categorized as PFHS, PIFHS, and PNHS. Also, by defining DAF as $a_i : M \times D_i \to \{0, 1\}$, the notion of PCHS can be introduced. The followings are those aforementioned notions:

Let $\psi = \{r_1, r_2, ..., r_n\}$ be a set of n attributes and corresponding attribute value sets are respectively $D_1, D_2, ..., D_n$ (where $D_i \cap D_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$). Let $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$.

Definition 3.1. The pair (Γ, V_ψ) is called a PCHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$ $\exists a_{C_i} : M \times D_i \to \{0, 1\}$ such that $\forall (m, d) \in M \times D_i$, $a_{C_i}(m, d) = 1$.

A set of all the PCHSs over a set U will be denoted as $\text{PCHS}(U)$.

Example 3.2. Let a balloon seller has a set $U = \{b_1, b_2, ..., b_{20}\}$ of a total of 20 balloons some which are of different size, color, and cost. Also, let for the aforementioned attributes corresponding attribute value sets are $D_1 = \{\text{small, medium, large}\}$, $D_2 = \{\text{red, orange, blue}\}$ and $D_3 = \{\text{small, medium, large}\}$. Let a person is willing to buy some balloons having the attributes as big, red and expensive. Lets assume (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ and $V_\psi = D_1 \times D_2 \times D_3$. Also, let $\Gamma(\text{big, red, expensive}) = \{b_3, b_{10}, b_{12}\}$.

Then corresponding PCHS will be Γ(big, red, expensive) = $\{b_3(1,1,1), b_{10}(1,1,1), b_{12}(1,1,1)\}$.
Its graphical representation is shown in Figure 1.

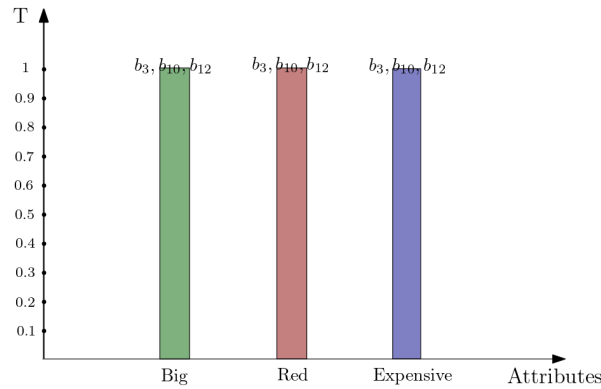FIGURE 1. PCHS according to Example 3.2

Definition 3.3. The pair (Γ, V_ψ) is called a PFHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_{F_i} : M \times D_i \to [0,1]$ such that $\forall (m,d) \in M \times D_i$, $a_{F_i}(m,d) \in [0,1]$.

A set of all the PFHSs over a set U will be denoted as PFHS(U).

Example 3.4. In Example 3.2 let corresponding PFHS is Γ(big, red, expensive) = $\{b_3(0.75, 0.3, 0.8), b_{10}(0.45, 0.57, 0.2), b_{12}(0.15, 0.57, 0.95)\}$. Its graphical representation is shown in Figure 2.
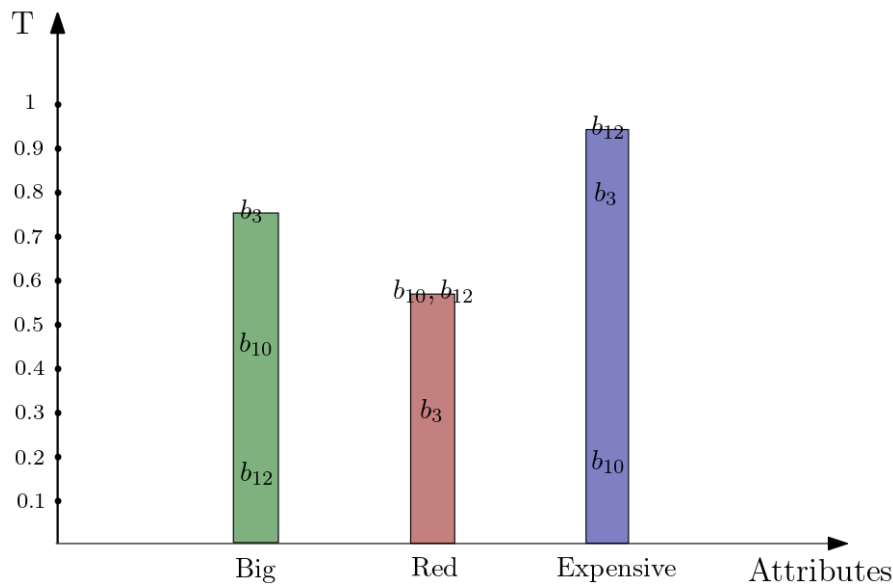

FIGURE 2. PFHS according to Example 3.4

Definition 3.5. The pair (Γ, V_ψ) is called a PIFHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_{IF_i} : M \times D_i \to [0, 1] \times [0, 1]$ such that $\forall (m, d) \in M \times D_i, a_{F_i}(m, d) \in [0, 1] \times [0, 1]$.

A set of all the PIFHSs over a set U will be denoted as PIFHS(U).

Example 3.6. In Example 3.2 let corresponding PIFHS is

$$\Gamma(\text{big, red, expensive}) = \left\{ \begin{array}{l} b_3(0.87, 0.52, 0.66), b_{10}(0.6, 0.52, 0.2), b_{12}(0.33, 0.2, 0.83) \\ b_3(0.3, 0.4, 0.72), b_{10}(0.5, 0.19, 0.98), b_{12}(1, 0.72, 0.3) \end{array} \right\}.$$

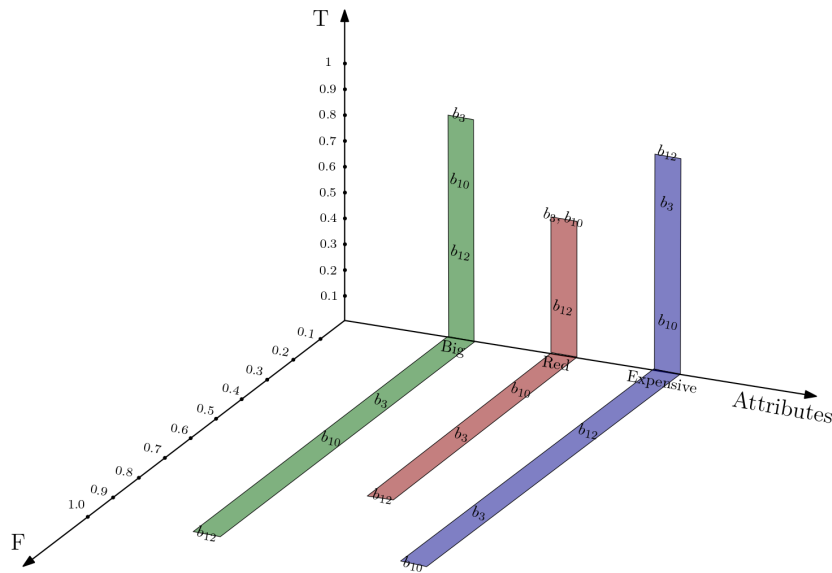Its graphical representation is shown in Figure 3.

FIGURE 3. PIFHS according to Example 3.6

Definition 3.7. The pair (Γ, V_ψ) is called a PNHS if $\forall M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2, ..., n\}$, $\exists a_{N_i} : M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$ such that $\forall (m, d) \in M \times D_i, \ a_{N_i}(m, d) \in [0, 1] \times [0, 1] \times [0, 1]$.

A set of all the PNHSs over a set U will be denoted as PNHS(U).

Example 3.8. In Example 3.2 let corresponding PNHS is

$$\Gamma(\text{big, red, expensive}) = \left\{ \begin{array}{l} b_3(0.87, 1, 0.66), b_{10}(0.61, 0.25, 0.2), b_{12}(0.32, 0.7, 0.83) \\ b_3(0.15, 0.72, 0.47), b_{10}(0.77, 0.4, 0.48), b_{12}(0.37, 0.18, 0.2) \\ b_3(0.76, 0.17, 0.29), b_{10}(0.5, 0.71, 0.98), b_{12}(1, 0.35, 0.67) \end{array} \right\}.$$

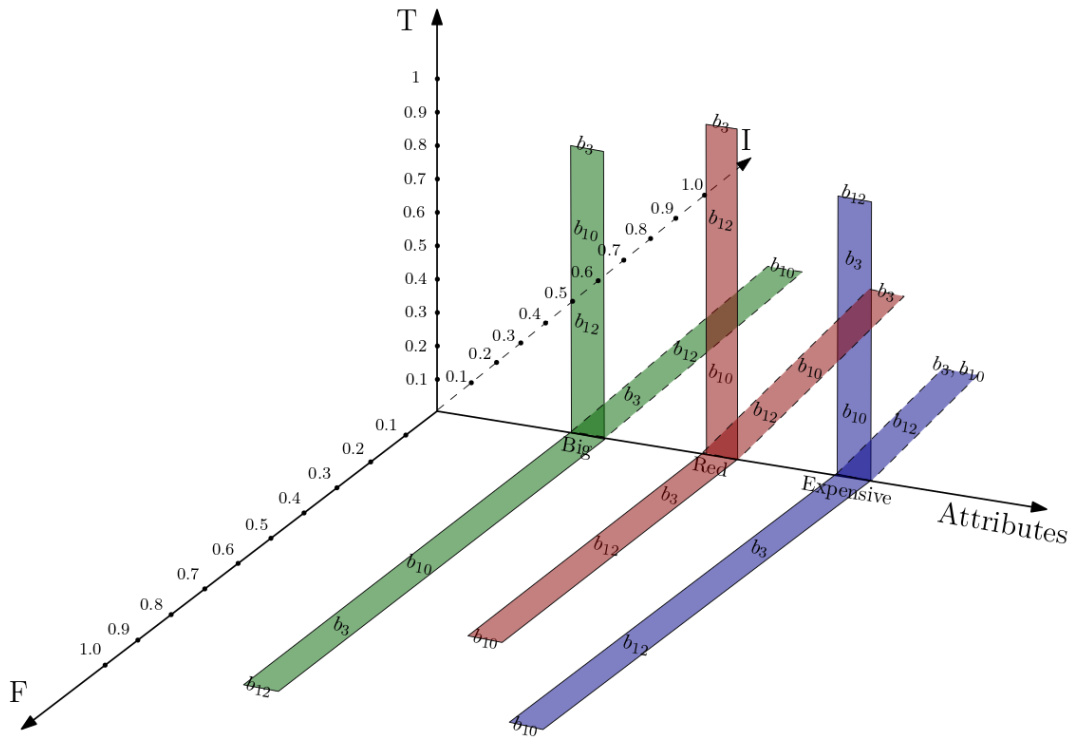Its graphical representation is shown in Figure 4.

FIGURE 4. PNHS according to Example 3.8

3.1. *Images & Preimages of PFHS, PIFHS & PNHS under a function*

Let U_1 and U_2 be two CSs and $\forall i,j \in \{1,2,...,n\}$, D_i and P_j are attribute value sets consisting of some attribute values. Again, let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are some functions. Then the followings can be defined:

Definition 3.9. Let $(\Gamma_1, V_\psi^1) \in \text{PFHS}(U_1)$ and $(\Gamma_2, V_\psi^2) \in \text{PFHS}(U_2)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$ and $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Also, let $\forall M \in \text{range}(\Gamma_1)$, $a_{F_i} : M \times D_i \to [0,1]$ are the corresponding FDAFs. Again, let $\forall N \in \text{range}(\Gamma_2)$, $b_{F_j} : N \times P_j \to [0,1]$ are the corresponding FDAFs. Then the images of (Γ_1, V_ψ^1) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PFHS over U_2 and they are denoted as $g_{ij}(\Gamma_1, V_\psi^1)$, where the corresponding FDAFs are defined as:

$$g_{ij}(a_{F_i})(n,p) = \begin{cases} \max a_{F_i}(m,d) \text{ if } (m,d) \in g_{ij}^{-1}(n,p) \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

The preimages of (Γ_2, V_ψ^2) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PFHSs over U_1, which are denoted as $g_{ij}^{-1}(\Gamma_2, V_\psi^2)$ and the corresponding FDAFs are defined as $g_{ij}^{-1}(b_{F_j})(m,d) = b_{F_j}(g_{ij}(m,d))$.

Definition 3.10. Let $(\Gamma_1, V_\psi^1) \in \text{PIFHS}(U_1)$ and $(\Gamma_2, V_\psi^2) \in \text{PIFHS}(U_2)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$ and $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Also, let $\forall M \in \text{range}(\Gamma_1)$, $a_{IF_i} :$

$M \times D_i \to [0,1] \times [0,1]$ with $a_{IF_i}(m,d) = \{((m,d), a_{IF_i}^T(m,d), a_{IF_i}^F(m,d)) : (m,d) \in M \times D_i\}$ are the corresponding IFDAFs. Again, let $\forall N \in \text{range}(\Gamma_2)$, $b_{IF_j} : N \times P_j \to [0,1] \times [0,1]$ with $b_{IF_j}(n,p) = \{((n,p), b_{IF_j}^T(n,p), b_{IF_j}^F(n,p)) : (n,p) \in N \times P_j\}$ are the corresponding IFDAFs. Then the images of (Γ_1, V_ψ^1) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PIFHS over U_2, which are denoted as $g_{ij}(\Gamma_1, V_\psi^1)$ and the corresponding IFDAFs are defined as: $g_{ij}(a_{IF_i})(n,p) = (g_{ij}(a_{IF_i}^T)(n,p), g_{ij}(a_{IF_i}^F)(n,p))$, where

$$g_{ij}(a_{IF_i}^T)(n,p) = \begin{cases} \max a_{IF_i}^T(m,d) \text{ if } (m,d) \in g_{ij}^{-1}(n,p) \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

and

$$g_{ij}(a_{IF_i}^F)(n,p) = \begin{cases} \min a_{IF_i}^F(m,d) \text{ if } (m,d) \in g_{ij}^{-1}(n,p) \\ 1 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

The preimages of (Γ_2, V_ψ^2) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PIFHSs over U_1, which are denoted as $g_{ij}^{-1}(\Gamma_2, V_\psi^2)$ and the corresponding IFDAFs are defined as $g_{ij}^{-1}(b_{IF_j})(m,d) = (g_{ij}^{-1}(b_{IF_j}^T)(m,d), g_{ij}^{-1}(b_{IF_j}^F)(m,d))$, where $g_{ij}^{-1}(b_{IF_j}^T)(m,d) = b_{IF_j}^T(g_{ij}(m,d))$ and $g_{ij}^{-1}(b_{IF_j}^F)(m,d) = b_{IF_j}^F(g_{ij}(m,d))$

Definition 3.11. Let $(\Gamma_1, V_\psi^1) \in \text{PNHS}(U_1)$ and $(\Gamma_2, V_\psi^2) \in \text{PNHS}(U_2)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$ and $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Also, let $\forall M \in \text{range}(\Gamma_1)$, $a_{N_i} : M \times D_i \to [0,1] \times [0,1] \times [0,1]$ with $a_{N_i}(m,d) = \{((m,d), a_{N_i}^T(m,d), a_{N_i}^I(m,d), a_{N_i}^F(m,d)) : (m,d) \in M \times D_i\}$ are the corresponding NDAFs. Again, let $\forall N \in \text{range}(\Gamma_2)$, $b_{N_j} : N \times P_j \to [0,1] \times [0,1] \times [0,1]$ with $b_{N_j}(n,p) = \{((n,p), b_{N_i}^T(n,p), b_{N_i}^I(n,p), b_{N_i}^F(n,p)) : (n,p) \in N \times P_i\}$ are the corresponding NDAFs. Then the images of (Γ_1, V_ψ^1) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PNHS over U_2, which are denoted as $g_{ij}(\Gamma_1, V_\psi^1)$ and the corresponding NDAFs are defined as: $g_{ij}(a_{N_i})(n,p) = (g_{ij}(a_{N_i}^T)(n,p), g_{ij}(a_{N_i}^I)(n,p), g_{ij}(a_{N_i}^F)(n,p))$, where

$$g_{ij}(a_{N_i}^T)(n,p) = \begin{cases} \max a_{N_i}^T(m,d) \text{ if } (m,d) \in g_{ij}^{-1}(n,p) \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases},$$

$$g_{ij}(a_{N_i}^I)(n,p) = \begin{cases} \max a_{N_i}^I(m,d) \text{ if} (m,d) \in g_{ij}^{-1}(n,p) \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases},$$

and

$$g_{ij}(a_{N_i}^F)(n,p) = \begin{cases} \min a_{N_i}^F(m,d) \text{ if } (m,d) \in g_{ij}^{-1}(n,p) \\ 1 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

The preimages of (Γ_2, V_ψ^2) under the functions $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are PNHS over U_1, which are denoted as $g_{ij}^{-1}(\Gamma_2, V_\psi^2)$ and the corresponding NDAFs are defined as
$g_{ij}^{-1}(b_{N_j})(m, d) = (g_{ij}^{-1}(b_{N_j}^T)(m, d), g_{ij}^{-1}(b_{N_j}^I)(m, d), g_{ij}^{-1}(b_{N_j}^F)(m, d))$, where
$g_{ij}^{-1}(b_{N_j}^T)(m, d) = b_{N_j}^T(g_{ij}(m, d))$, $g_{ij}^{-1}(b_{N_j}^I)(m, d) = b_{N_j}^I(g_{ij}(m, d))$ and
$g_{ij}^{-1}(b_{N_j}^F)(m, d) = b_{N_j}^F(g_{ij}(m, d))$.

In the next segment, we have defined plithogenic hypersoft subgroups in fuzzy, intuitionistic fuzzy, and neutrosophic environments. We have also, analyzed their homomorphic properties.

3.2. *Plithogenic Hypersoft Subgroup*

3.2.1. *Plithogenic Fuzzy Hypersoft Subgroup*

Definition 3.12. Let the pair (Γ, V_ψ) be a PFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PFHSG of U if and only if $\forall M \in$ range(Γ), $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{F_i} : M \times D_i \to [0, 1]$, the conditions mentioned below are satisfied:

(i) $a_{F_i}((m_1, d) \cdot (m_2, d')) \geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\}$ and
(ii) $a_{F_i}(m_1, d)^{-1} \geq a_{F_i}(m_1, d)$.

A set of all PFHSG of a CG U is denoted as PFHSG(U).

Example 3.13. Let $U = \{e, m, u, mu\}$ be the Kleins 4-group and $\psi = \{r_1, r_2\}$ is a set of two attributes and corresponding attribute value sets are respectively, $D_1 = \{1, i, -1, -i\}$ and $D_2 = \{1, w, w^2\}$, which are two cyclic groups. Let $V_\psi = D_1 \times D_2$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ such that the range of Γ i.e. R$(\Gamma) = \{\{e, m\}, \{e, u\}, \{e, mu\}\}$. Let for $M = \{e, m\}$, $a_{F_1} : M \times D_1 \to [0, 1]$ is defined in Table 2 and $a_{F_2} : M \times D_2 \to [0, 1]$ is defined in Table 3 respectively.

TABLE 2. Membership values of a_{F_1}

a_{F_1}	1	i	-1	$-i$
e	0.4	0.2	0.4	0.2
m	0.2	0.2	0.2	0.2

TABLE 3. Membership values of a_{F_2}

a_{F_2}	1	w	w^2
e	0.8	0.5	0.5
m	0.6	0.5	0.5

Let for $M = \{e, u\}$, $a_{F_1} : M \times D_1 \to [0, 1]$ is defined in Table 4 and $a_{F_2} : M \times D_2 \to [0, 1]$ is defined in Table 5 respectively.

TABLE 4. Membership values of a_{F_1} TABLE 5. Membership values of a_{F_2}

a_{F_1}	1	i	-1	$-i$
e	0.8	0.2	0.7	0.2
u	0.5	0.2	0.5	0.2

a_{F_2}	1	w	w^2
e	0.7	0.4	0.4
u	0.3	0.3	0.3

Let for $M = \{e, mu\}$, $a_{F_1} : M \times D_1 \to [0,1]$ is defined in Table 6 and $a_{F_2} : M \times D_2 \to [0,1]$ is defined in Table 7 respectively.

TABLE 6. Membership values of a_{F_2} TABLE 7. Membership values of a_{F_1}

a_{F_1}	1	i	-1	$-i$
e	0.9	0.2	0.4	0.2
mu	0.7	0.2	0.7	0.2

a_{F_2}	1	w	w^2
e	1	0.3	0.3
mu	0.2	0.2	0.2

Here, for any $M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2\}$, a_{F_i} satisfy Definition 3.12. Hence, $(\Gamma, V_\psi) \in$ PFHSG(U).

Proposition 3.1. *Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U), where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then for any $M \in \text{range}(\Gamma)$, $\forall (m, d) \in M \times D_i$ and $\forall a_{F_i} : M \times D_i \to [0, 1]$, the followings are satisfied:*

(i) $a_{F_i}(e, d_e^i) \geq a_{F_i}(m, d)$, *where e and d_e^i are the neutral elements of U and D_i.*
(ii) $a_{F_i}(m, d)^{-1} = a_{F_i}(m, d)$

Proof. (i) Let e and d_e^i be the neutral elements of U and D_i. Then $\forall (m, d) \in M \times D_i$,

$$a_{F_i}(e, d_e^i) = a_{F_i}((m, d) \cdot (m, d)^{-1}),$$
$$\geq \min\{a_{F_i}(m, d), a_{F_i}(m, d)^{-1}\} \text{ (by Definition 3.12)}$$
$$\geq \min\{a_{F_i}(m, d), a_{F_i}(m, d)\} \text{ (by Definition 3.12)}$$
$$\geq a_{F_i}(m, d)$$

(ii) Let U be a group and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.12,

$$a_{F_i}(m, d)^{-1} \geq a_{F_i}(m, d) \tag{3.1}$$

Again,

$$a_{F_i}(m, d) = a_{F_i}((m, d)^{-1})^{-1})$$
$$\geq a_{F_i}(m, d)^{-1} \tag{3.2}$$

Hence, from Equation 3.1 and Equation 3.2, $a_{F_i}(m, d)^{-1} = a_{F_i}(m, d)$. $\square$

Proposition 3.2. *Let the pair (Γ, V_ψ) be a PFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PFHSG of U if and only if $\forall M \in range(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{F_i} : M \times D_i \to [0, 1]$, $a_{F_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\}$.*

Proof. Let U be a CG and $(\Gamma, V_\psi) \in \text{PFHSG}(U)$. Then by Definition 3.12 and Proposition 3.1

$$a_{F_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')^{-1}\}$$
$$= \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\}$$

Conversely, let $a_{F_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\}$. Also, let e and d_e^i be the neutral elements of U and D_i. Then,

$$a_{F_i}(m, d)^{-1} = a_{F_i}((e, d_e^i) \cdot (m, d)^{-1})$$
$$\geq \min\{a_{F_i}(e, d_e^i), a_{F_i}(m, d)\}$$
$$= \min\{a_{F_i}((m, d) \cdot (m, d)^{-1}), a_{F_i}(m, d)\}$$
$$\geq \min\{a_{F_i}(m, d), a_{F_i}(m, d), a_{F_i}(m, d)\}$$
$$= a_{F_i}(m, d) \tag{3.3}$$

Now,

$$a_{F_i}((m_1, d) \cdot (m_2, d')) = a_{F_i}((m_1, d) \cdot ((m_2, d')^{-1})^{-1})$$
$$\geq \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')^{-1}\}$$
$$= \min\{a_{F_i}(m_1, d), a_{F_i}(m_2, d')\} \text{ (by Equation 3.3)} \tag{3.4}$$

Hence, by Equation 3.3 and Equation 3.4, $(\Gamma, V_\psi) \in \text{PFHSG}(U)$. $\square$

Proposition 3.3. *Intersection of two PFHSGs is also a PFHSG.*

Theorem 3.4. *The homomorphic image of a PFHSG is a PFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_1, V_\psi^1) \in \text{PFHSG}(U_1)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$. Again, let $\forall M \in \text{range}(\Gamma_1)$, $a_{F_i} : M \times D_i \to [0, 1]$ are the corresponding FDAFs.
Assuming $(n_1, p_1), (n_2, p_2) \in U_2 \times P_j$, if $g_{ij}^{-1}(n_1, p_1) = \phi$ and $g_{ij}^{-1}(n_2, p_2) = \phi$, then $g_{ij}(\Gamma_1, V_\psi^1) \in \text{PFHSG}(U_2)$.
Lets assume that $\exists (m_1, d_1), (m_2, d_2) \in U_1 \times D_i$ such that $g_{ij}(m_1, d_1) = (n_1, p_1)$ and

$g_{ij}(m_2, d_2) = (n_2, p_2)$. Then

$$g_{ij}(a_{F_i})(n_1, p_1) \cdot (n_2, p_2)^{-1} = \max_{(n_1,p_1) \cdot (n_2,p_2)^{-1} = g_{ij}(m,d)} a_{F_i}(m, d)$$

$$\geq a_{F_i}(m_1, d_1) \cdot (m_2, d_2)^{-1}$$

$$\geq \min\{a_{F_i}(m_1, d_1), a_{F_i}(m_2, d_2)\} \text{ (as } (\Gamma_1, V_\psi^1) \in \text{PFHSG}(U_1))$$

$$\geq \min\{\max_{(n_1,p_1)=g_{ij}(m_1,d_1)} a_{F_i}(m_1, d_1), \max_{(n_2,p_2)=g_{ij}(m_2,d_2)} a_{F_i}(m_2, d_2)\}$$

$$\geq \min\{g_{ij}(a_{F_i})(n_1, p_1), g_{ij}(a_{F_i})(n_2, p_2)\}$$

Hence, $g_{ij}(\Gamma_1, V_\psi^1) \in \text{PFHSG}(U_2)$. $\square$

Theorem 3.5. *The homomorphic preimage of a PFHSG is a PFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_2, V_\psi^2) \in \text{PFHSG}(U_2)$, $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Again, $\forall N \in \text{range}(\Gamma_2)$, $b_{F_j} : N \times P_j \to [0, 1]$ are the corresponding FDAFs. Lets assume $(m_1, d_1), (m_2, d_2) \in U_1 \times D_i$. As g_{ij} is a homomorphism the followings can be concluded:

$$g_{ij}^{-1}(b_{F_i})(m_1, d_1) \cdot (m_2, d_2)^{-1}$$

$$= b_{F_i}(g_{ij}((m_1, d_1) \cdot (m_2, d_2)^{-1}))$$

$$= b_{F_i}(g_{ij}(m_1, d_1) \cdot g_{ij}(m_2, d_2)^{-1}) \text{ (As } g_{ij} \text{ is a homomorphism)}$$

$$\geq \min\{b_{F_i}(g_{ij}(m_1, d_1)), b_{F_i}(g_{ij}(m_2, d_2))\} \text{ (As } (\Gamma_2, V_\psi^2) \in \text{PFHSG}(U_2))$$

$$\geq \min\{g_{ij}^{-1}(b_{F_i})(m_1, d_1), g_{ij}^{-1}(b_{F_i})(m_2, d_2)\}$$

Then $g_{ij}^{-1}(\Gamma_2, V_\psi^2) \in \text{PFHSG}(U_1)$. $\square$

3.2.2. *Plithogenic Intuitionistic Fuzzy Hypersoft Subgroup*

Definition 3.14. Let the pair (Γ, V_ψ) be a PIFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PIFHSG of U if and only if $\forall M \in \text{range}(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{IF_i} : M \times D_i \to [0, 1] \times [0, 1]$ with $a_{IF_i}(m, d) = \{((m, d), a_{IF_i}^T(m, d), a_{IF_i}^F(m, d)) : (m, d) \in M \times D_i\}$, the subsequent conditions are fulfilled:

   (i) $a_{IF_i}^T((m_1, d) \cdot (m_2, d')) \geq \min\{a_{IF_i}^T(m_1, d), a_{IF_i}^T(m_2, d')\}$
   (ii) $a_{IF_i}^T(m_1, d)^{-1} \geq a_{IF_i}^T(m_1, d)$
   (iii) $a_{IF_i}^F((m_1, d) \cdot (m_2, d')) \leq \max\{a_{IF_i}^F(m_1, d), a_{IF_i}^F(m_2, d')\}$
   (iv) $a_{IF_i}^F(m_1, d)^{-1} \leq a_{IF_i}^F(m_1, d)$

  A set of all PIFHSG of a CG U is denoted as PIFHSG(U).

Example 3.15. Let $U = S_3$ be a CG and $\psi = \{r_1, r_2\}$ is a set of two attributes and corresponding attribute value sets are respectively, $D_1 = A_3$ and $D_2 = S_2$, which are respectively an alternating group of order 3 and a symmetric group of order 2. Let $V_\psi = D_1 \times D_2$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ such that the range of Γ i.e. $\mathrm{R}(\Gamma) = \{\{(1), (13)\}, \{(1), (23)\}\}$. Let for $M = \{(1), (13)\}$, $a_{IF_1} : M \times D_1 \to [0, 1] \times [0, 1]$ is defined in Table 8–9 and $a_{IF_2} : M \times D_2 \to [0, 1] \times [0, 1]$ is defined in Table 10–11 respectively.

TABLE 8. Membership values of a_{IF_1}

$a_{IF_1}^T$	(1)	(123)	(132)
(1)	0.4	0.5	0.5
(13)	0.2	0.2	0.2

TABLE 9. Non-membership values of a_{IF_1}

$a_{IF_1}^F$	(1)	(123)	(132)
(1)	0.4	0.7	0.7
(13)	0.8	0.8	0.8

TABLE 10. Membership values of a_{IF_2}

$a_{IF_2}^T$	(1)	(12)
(1)	0.8	0.4
(13)	0.3	0.3

TABLE 11. Non-membership values of a_{IF_2}

$a_{IF_2}^F$	(1)	(12)
(1)	0.4	0.8
(13)	0.9	0.9

Let for $M = \{(1), (23)\}$ $a_{IF_1} : M \times D_1 \to [0, 1] \times [0, 1]$ is defined in Table 12–13 and $a_{IF_2} : M \times D_2 \to [0, 1] \times [0, 1]$ is defined in Table 14–15 respectively.

TABLE 12. Membership values of a_{IF_1}

$a_{IF_1}^T$	(1)	(123)	(132)
(1)	0.6	0.4	0.4
(23)	0.5	0.4	0.4

TABLE 13. Non-membership values of a_{IF_1}

$a_{IF_1}^F$	(1)	(123)	(132)
(1)	0.4	0.7	0.7
(23)	0.6	0.7	0.7

TABLE 14. Membership values of a_{IF_2}

$a_{IF_2}^T$	(1)	(12)
(1)	0.7	0.6
(23)	0.7	0.6

TABLE 15. Non-membership values of a_{IF_2}

$a_{IF_2}^F$	(1)	(12)
(1)	0.5	0.9
(23)	0.8	0.9

Here, for any $M \in \mathrm{range}(\Gamma)$ and $\forall i \in \{1, 2\}$, a_{IF_i} satisfy Definition 3.14. Hence, $(\Gamma, V_\psi) \in$ PIFHSG(U).

Gayen et al., Introduction to Plithogenic Hypersoft Subgroup

Proposition 3.6. *Let U be a CG and $(\Gamma, V_\psi) \in PIFHSG(U)$, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then for any $M \in range(\Gamma)$ and $\forall(m, d_i) \in M \times D_i$ and $\forall a_{IF_i} : M \times D_i \to [0, 1] \times [0, 1]$ with $a_{IF_i}(m, d) = \{((m, d), a^T_{IF_i}(m, d), a^F_{IF_i}(m, d)) : (m, d) \in M \times D_i\}$, the subsequent conditions are satisfied:*

(i) $a^T_{IF_i}(e, d^i_e) \geq a^T_{IF_i}(m, d)$, *where e and d^i_e are the neutral elements of U and D_i.*
(ii) $a^T_{IF_i}(m, d)^{-1} = a^T_{IF_i}(m, d)$
(iii) $a^F_{IF_i}(e, d^i_e) \leq a^F_{IF_i}(m, d)$, *where e and d^i_e are the neutral elements of U and D_i.*
(iv) $a^F_{IF_i}(m, d)^{-1} = a^F_{IF_i}(m, d)$

Proof. Here, (i) and (ii) can be easily proved using Proposition 3.1.

(iii) Let e and d^i_e be the neutral elements of U and D_i. Then $\forall(m, d) \in M \times D_i$,

$$
\begin{aligned}
a^F_{IF_i}(e, d^i_e) &= a^F_{IF_i}((m, d) \cdot (m, d)^{-1}) \\
&\leq \max\{a^F_{IF_i}(m, d), a^F_{IF_i}(m, d)^{-1}\} \text{ (by Definition 3.14)} \\
&\leq \max\{a^F_{IF_i}(m, d), a^F_{IF_i}(m, d)\} \text{ (by Definition 3.14)} \\
&\leq a^F_{IF_i}(m, d)
\end{aligned}
$$

(iv) Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.14,

$$a^F_{IF_i}(m, d)^{-1} \leq a^F_{IF_i}(m, d) \tag{3.5}$$

Again,

$$
\begin{aligned}
a^F_{IF_i}(m, d) &= a^F_{IF_i}((m, d)^{-1})^{-1} \\
&\leq a^F_{IF_i}(m, d)^{-1}
\end{aligned} \tag{3.6}
$$

Hence, by Equation 3.5 and Equation 3.6, $a^F_{IF_i}(m, d)^{-1} = a^F_{IF_i}(m, d)$. $\square$

Proposition 3.7. *Let the pair (Γ, V_ψ) be a PIFHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PIFHSG of U if and only if $\forall M \in range(\Gamma)$, $\forall(m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{IF_i} : M \times D_i \to [0, 1] \times [0, 1]$ with $a_{IF_i}(m, d) = \{((m, d), a^T_{IF_i}(m, d), a^F_{IF_i}(m, d)) : (m, d) \in M \times D_i\}$, the subsequent conditions are fulfilled:*

(i) $a^T_{IF_i}((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a^T_{IF_i}(m_1, d), a^T_{IF_i}(m_2, d')\}$ *and*
(ii) $a^F_{IF_i}((m_1, d) \cdot (m_2, d')^{-1}) \leq \max\{a^F_{IF_i}(m_1, d), a^F_{IF_i}(m_2, d')\}$

Proof. Here, (i) can be proved using Proposition 3.2.

(ii) Let U be a CG and $(\Gamma, V_\psi) \in$ PFHSG(U). Then by Definition 3.14 and Proposition 3.6

$$
\begin{aligned}
a^F_{IF_i}((m_1, d) \cdot (m_2, d')^{-1}) &\leq \max\{a^F_{IF_i}(m_1, d), a^F_{IF_i}(m_2, d')^{-1}\} \\
&\leq \max\{a^F_{IF_i}(m_1, d), a^F_{IF_i}(m_2, d')\}
\end{aligned}
$$

Conversely, let $a_{IF_i}^F((m_1, d) \cdot (m_2, d')^{-1}) \leq \max\{a_{IF_i}^F(m_1, d), a_{IF_i}^F(m_2, d')\}$. Also, let e and d_e^i be the neutral elements of U and D_i. Then

$$\begin{aligned}
a_{IF_i}^F(m, d)^{-1} &= a_{IF_i}^F((e, d_e^i) \cdot (m, d)^{-1}) \\
&\leq \max\{a_{IF_i}^F(e, d_e^i), a_{IF_i}^F(m, d)\} \\
&\leq \max\{a_{IF_i}^F((m, d) \cdot (m, d)^{-1}), a_{IF_i}^F(m, d)\} \\
&\leq \max\{a_{IF_i}^F(m, d), a_{IF_i}^F(m, d), a_{IF_i}^F(m, d)\} \\
&= a_{IF_i}^F(m, d) \quad\quad\quad (3.7)
\end{aligned}$$

Now,

$$\begin{aligned}
a_{IF_i}^F((m_1, d) \cdot (m_2, d')) &= a_{IF_i}^F((m_1, d) \cdot ((m_2, d')^{-1})^{-1}) \\
&\leq \max\{a_{IF_i}^F(m_1, d), a_{IF_i}^F(m_2, d')^{-1}\} \\
&= \max\{a_{IF_i}^F(m_1, d), a_{IF_i}^F(m_2, d')\} \text{ (by Equation 3.7)} \quad (3.8)
\end{aligned}$$

Hence, by Equation 3.7 and Equation 3.8, $(\Gamma, V_\psi) \in \text{PFHSG}(U)$. $\square$

Proposition 3.8. *Intersection of two PIFHSGs is also a PIFHSG.*

Theorem 3.9. *The homomorphic image of a PIFHSG is a PIFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_1, V_\psi^1) \in \text{PIFHSG}(U_1)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$. Again, let $\forall M \in \text{range}(\Gamma_1)$ and $a_{IF_i} : M \times D_i \to [0, 1] \times [0, 1]$ with $a_{IF_i}(m, d) = \{((m, d), a_{IF_i}^T(m, d), a_{IF_i}^F(m, d)) : (m, d) \in M \times D_i\}$ are the corresponding IFDAFs. Assuming $(n_1, p_1), (n_2, p_2) \in U_2 \times P_j$, if $g_{ij}^{-1}(n_1, p_1) = \phi$ and $g_{ij}^{-1}(n_2, p_2) = \phi$, then $g_{ij}(\Gamma_1, V_\psi^1) \in \text{PIFHSG}(U_2)$. Lets assume that $\exists (m_1, d_1), (m_2, d_2) \in U_1 \times D_i$ such that $g_{ij}(m_1, d_1) = (n_1, p_1)$ and $g_{ij}(m_2, d_2) = (n_2, p_2)$. Then by Theorem 3.4

$$g_{ij}(a_{IF_i}^T)(n_1, p_1) \cdot (n_2, p_2)^{-1} \geq \min\{g_{ij}(a_{IF_i}^T)(n_1, p_1), g_{ij}(a_{IF_i}^T)(n_2, p_2)\}.$$

Again,

$$\begin{aligned}
g_{ij}(a_{IF_i}^F)(n_1, p_1) \cdot (n_2, p_2)^{-1} &= \min_{(n_1, p_1) \cdot (n_2, p_2)^{-1} = g_{ij}(m, d)} a_{IF_i}^F(m, d) \\
&\leq a_{IF_i}^F(m_1, d_1) \cdot (m_2, d_2)^{-1} \\
&\leq \max\{a_{IF_i}^F(m_1, d_1), a_{IF_i}^F(m_2, d_2)\} \text{ (as } (\Gamma_1, V_\psi^1) \in PIFHSG(U_1)) \\
&\leq \max\{\min_{(n_1, p_1) = g_{ij}(m_1, d_1)} a_{IF_i}^F(m_1, d_1), \min_{(n_2, p_2) = g_{ij}(m_2, d_2)} a_{IF_i}^F(m_2, d_2)\} \\
&\leq \max\{g_{ij}(a_{IF_i}^F)(n_1, p_1), g_{ij}(a_{IF_i}^F)(n_2, p_2)\} \text{ (by Definition 3.10)}
\end{aligned}$$

Hence, $g_{ij}(\Gamma_1, V_\psi^1) \in$ PIFHSG(U_2). $\square$

Theorem 3.10. *The homomorphic preimage of a PIFHSG is a PIFHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_2, V_\psi^2) \in$ PIFHS(U_2), where $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Again, let $\forall N \in$ range(Γ_2), $b_{IF_j} :$ $N \times P_j \to [0, 1] \times [0, 1]$ with $b_{IF_j}(n, p) = \{((n, p), b_{IF_j}^T(n, p), b_{IF_j}^F(n, p)) : (n, p) \in N \times P_j\}$ are the corresponding IFDAFs. Lets assume$(m_1, d_1), (m_2, d_2) \in U_1 \times D_i$. Since, g_{ij} is a homomorphism, by Theorem 3.5

$$g_{ij}^{-1}(b_{IF_j}^T)(m_1, d_1) \cdot (m_2, d_2)^{-1} \geq \min\{g_{ij}^{-1}(b_{IF_j}^T)(m_1, d_1), g_{ij}^{-1}(b_{IF_j}^T)(m_2, d_2)\}.$$

Again,

$$
\begin{aligned}
g_{ij}^{-1}(b_{IF_j}^F)(m_1, d_1) \cdot (m_2, d_2)^{-1} &= b_{IF_j}^F(g_{ij}((m_1, d_1) \cdot (m_2, d_2)^{-1})) \\
&= b_{IF_j}^F(g_{ij}(m_1, d_1) \cdot g_{ij}(m_2, d_2)^{-1}) \text{ (As } g_{ij} \text{ is a homomorphism)} \\
&\leq \max\{b_{IF_j}^F(g_{ij}(m_1, d_1)), b_{IF_j}^F(g_{ij}(m_2, d_2))\} \\
&\qquad\qquad\qquad\qquad (\text{As } (\Gamma_2, V_\psi^2) \in PIFHSG(U_2)) \\
&\leq \max\{g_{ij}^{-1}(b_{IF_j}^F)(m_1, d_1), g_{ij}^{-1}(b_{IF_j}^F)(m_2, d_2)\}
\end{aligned}
$$

Hence, $g_{ij}^{-1}(\Gamma_2, V_\psi^2) \in$ PIFHSG(U_1). $\square$

3.2.3. *Plithogenic Neutrosophic Hypersoft Subgroup*

Definition 3.16. content.Let the pair (Γ, V_ψ) be a PNHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PNHSG of U if and only if $\forall M \in$ range(Γ), $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{N_i} : M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$, with $a_{N_i}(m, d) = \{((m, d), a_{N_i}^T(m, d), a_{N_i}^I(m, d), a_{N_i}^F(m, d)) : (m, d) \in M \times D_i\}$, the subsequent conditions are fulfilled:

(i) $a_{N_i}^T((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{N_i}^T(m_1, d), a_{N_i}^T(m_2, d')\}$
(ii) $a_{N_i}^T(m_1, d)^{-1} \geq a_{N_i}^T(m_1, d)$
(iii) $a_{N_i}^I((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{N_i}^I(m_1, d), a_{N_i}^I(m_2, d')\}$
(iv) $a_{N_i}^I(m_1, d)^{-1} \geq a_{N_i}^I(m_1, d)$
(v) $a_{N_i}^F((m_1, d) \cdot (m_2, d')^{-1}) \leq \max\{a_{N_i}^F(m_1, d), a_{N_i}^F(m_2, d')\}$
(vi) $a_{N_i}^F(m_1, d)^{-1} \leq a_{N_i}^F(m_1, d)$

A set of all PNHSG of a CG U is denoted as PNHSG(U).

Example 3.17. Let $D_6 = \{e, m, u, mu, um, mum\}$ be a dihedral group of order 6 and $\psi = \{r_1, r_2\}$ is a set of two attributes and corresponding attribute value sets are respectively, $D_1 = \{1, w, w^2\}$ and $D_2 = A_3$, which are respectively a cyclic group of order 3 and an alternating group of order 3. Let $V_\psi = D_1 \times D_2$ and (Γ, V_ψ) be a HS over U, where $\Gamma : V_\psi \to \rho(U)$ such that the range of Γ i.e. $R(\Gamma) = \{\{e, mu, um\}, \{e, mum\}\}$.

Let for $M = \{e, mu\}$, $a_{N_1} : M \times D_1 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 16–18 and $a_{N_2} : M \times D_2 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 19–21 respectively.

TABLE 16. Truth values of a_{N_1}

$a_{N_1}^T$	1	w	w^2
e	0.7	0.5	0.5
mu	0.3	0.3	0.3
um	0.3	0.3	0.3

TABLE 17. Indeterminacy values of a_{N_1}

$a_{N_1}^I$	1	w	w^2
e	0.8	0.4	0.4
mu	0.5	0.4	0.4
um	0.5	0.4	0.4

TABLE 18. Falsity values of a_{N_1}

$a_{N_1}^F$	1	w	w^2
e	0.3	0.5	0.5
mu	0.7	0.7	0.7
um	0.7	0.7	0.7

TABLE 19. Truth values of a_{N_2}

$a_{N_2}^T$	(1)	(123)	(132)
e	0.7	0.2	0.2
mu	0.1	0.1	0.1
um	0.1	0.1	0.1

TABLE 20. Indeterminacy values of a_{N_2}

$a_{N_2}^I$	(1)	(123)	(132)
e	0.8	0.5	0.5
mu	0.8	0.5	0.5
um	0.8	0.5	0.5

TABLE 21. Falsity values of a_{N_2}

$a_{N_2}^F$	(1)	(123)	(132)
e	0.3	0.8	0.8
mu	0.9	0.9	0.9
um	0.9	0.9	0.9

Let for $M = \{e, mum\}$, $a_{N_1} : M \times D_1 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 22–24 and $a_{N_2} : M \times D_2 \to [0,1] \times [0,1] \times [0,1]$ is defined in Table 25–27 respectively.

TABLE 22. Truth values of a_{N_1}

$a_{N_1}^T$	1	w	w^2
e	0.8	0.4	0.4
mum	0.2	0.2	0.2

TABLE 23. Indeterminacy values of a_{N_1}

$a_{N_1}^I$	1	w	w^2
e	0.8	0.6	0.6
mum	0.7	0.6	0.6

TABLE 24. Falsity values of a_{N_1}

$a_{N_1}^F$	1	w	w^2
e	0.2	0.6	0.6
mum	0.8	0.8	0.8

TABLE 25. Truth values of a_{N_2}

$a_{N_2}^T$	(1)	(123)	(132)
e	0.9	0.8	0.8
mum	0.9	0.8	0.8

TABLE 26. Indeterminacy values of a_{N_2}

$a_{N_2}^I$	(1)	(123)	(132)
e	0.5	0.2	0.2
mum	0.1	0.1	0.1

TABLE 27. Falsity values of a_{N_2}

$a_{N_2}^F$	(1)	(123)	(132)
e	0.1	0.2	0.2
mum	0.1	0.2	0.2

Here, for any $M \in \text{range}(\Gamma)$ and $\forall i \in \{1, 2\}$, a_{N_i} satisfy Definition 3.16. Hence, $(\Gamma, V_\psi) \in$ PNHSG(U).

Proposition 3.11. *Let the pair (Γ, V_ψ) be a PNHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PNHSG of U if and only if $\forall M \in \text{range}(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{N_i} : M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$, with $a_{N_i}(m, d) = \{((m, d), a_{N_i}^T(m, d), a_{N_i}^I(m, d), a_{N_i}^F(m, d)) : (m, d) \in M \times D_i\}$. Then the subsequent conditions are satisfied:*

(i) *$a_{N_i}^T(e, d) \geq a_{N_i}^T(m, d)$, where e is the neutral element of U.*
(ii) *$a_{N_i}^T(m, d)^{-1} = a_{N_i}^T(m, d)$*
(iii) *$a_{N_i}^I(e, d) \geq a_{N_i}^I(m, d)$, where e is the neutral element of U.*
(iv) *$a_{N_i}^I(m, d)^{-1} = a_{N_i}^I(m, d)$*
(v) *$a_{N_i}^F(e, d) \leq a_{N_i}^F(m, d)$, where e is the neutral element of U.*
(vi) *$a_{N_i}^F(m, d)^{-1} = a_{N_i}^F(m, d)$*

Proof. This can be proved using Proposition 3.1 and Proposition 3.6. $\square$

Proposition 3.12. *Let the pair (Γ, V_ψ) be a PNHS of a CG U, where $V_\psi = D_1 \times D_2 \times \cdots \times D_n$ and $\forall i \in \{1, 2, ..., n\}$, D_i are CGs. Then (Γ, V_ψ) is called a PNHSG of U if and only if $\forall M \in \text{range}(\Gamma)$, $\forall (m_1, d), (m_2, d') \in M \times D_i$ and $\forall a_{N_i} : M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$, with $a_{N_i}(m, d) = \{((m, d), a_{N_i}^T(m, d), a_{N_i}^I(m, d), a_{N_i}^F(m, d)) : (m, d) \in M \times D_i\}$. Then the subsequent conditions are fulfilled:*

(i) *$a_{N_i}^T((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{N_i}^T(m_1, d), a_{N_i}^T(m_2, d')\}$*
(ii) *$a_{N_i}^I((m_1, d) \cdot (m_2, d')^{-1}) \geq \min\{a_{N_i}^I(m_1, d), a_{N_i}^I(m_2, d')\}$*
(iii) *$a_{N_i}^F((m_1, d) \cdot (m_2, d')^{-1}) \leq \max\{a_{N_i}^F(m_1, d), a_{N_i}^F(m_2, d')\}$*

Proof. This can be proved using Proposition 3.2 and Proposition 3.7. □

Proposition 3.13. *Intersection of two PNHSGs is also a PNHSG.*

Theorem 3.14. *The homomorphic image of a PNHSG is a PNHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\}$, D_i and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_1, V_\psi^1) \in \text{PNHSG}(U_1)$, where $V_\psi^1 = D_1 \times D_2 \times \cdots \times D_n$. Again, let $\forall M \in \text{range}(\Gamma_1)$, $a_{N_i} :$ $M \times D_i \to [0, 1] \times [0, 1] \times [0, 1]$ with $a_{N_i}(m, d) = \{((m, d), a_{N_i}^T(m, d), a_{N_i}^I(m, d), a_{N_i}^F(m, d)) :$ $(m, d) \in M \times D_i\}$ are the corresponding NDAFs.

Assuming $(n_1, p_1), (n_2, p_2) \in U_2 \times P_j$, if $g_{ij}^{-1}(n_1, p_1) = \phi$ and $g_{ij}^{-1}(n_2, p_2) = \phi$ then $g_{ij}(\Gamma_1, V_\psi^1) \in$ $\text{NHSG}(U_2)$. Lets assume that $\exists (m_1, d_1), (m_2, d_2) \in U_1 \times D_i$ such that $g_{ij}(m_1, d_1) = (n_1, p_1)$ and $g_{ij}(m_2, d_2) = (n_2, p_2)$. Then by Theorem 3.4 and Theorem 3.9, we can prove the followings:

$$g_{ij}(a_{N_i}^T)(n_1, p_1) \cdot (n_2, p_2)^{-1} \geq \min\{g_{ij}(a_{N_i}^T)(n_1, p_1), g_{ij}(a_{N_i}^T)(n_2, p_2)\},$$

$$g_{ij}(a_{N_i}^I)(n_1, p_1) \cdot (n_2, p_2)^{-1} \geq \min\{g_{ij}(a_{N_i}^I)(n_1, p_1), g_{ij}(a_{N_i}^I)(n_2, p_2)\},$$

and

$$g_{ij}(a_{N_i}^F)(n_1, p_1) \cdot (n_2, p_2)^{-1} \leq \max\{g_{ij}(a_{N_i}^F)(n_1, p_1), g_{ij}(a_{N_i}^F)(n_2, p_2)\}.$$

Hence, $g_{ij}(\Gamma_1, V_\psi^1) \in \text{PNHSG}(U_2)$. □

Theorem 3.15. *The homomorphic preimage of a PNHSG is a PNHSG.*

Proof. Let U_1 and U_2 be two CGs and $\forall i, j \in \{1, 2, ..., n\} D_i$ and P_j are attribute value sets consisting of some attribute values and let $g_{ij} : U_1 \times D_i \to U_2 \times P_j$ are homomorphisms. Also, let $(\Gamma_2, V_\psi^2) \in \text{PNHSG}(U_2)$, where $V_\psi^2 = P_1 \times P_2 \times \cdots \times P_n$. Again, let $\forall N \in \text{range}(\Gamma_2), b_{N_j} :$ $N \times P_j \to [0, 1] \times [0, 1] \times [0, 1]$ with $b_{N_j}(n, p) = \{((n, p), b_{N_j}^T(n, p), b_{N_j}^I(n, p), b_{N_j}^F(n, p)) : (n, p) \in$ $N \times P_j\}$ are the corresponding IFDAFs. Lets assume $(m_1, d_1), (m_2, d_2) \in U_1 \times D_i$. Since g_{ij} is a homomorphism by Theorem 3.5 and Theorem 3.10, the followings can be proved:

$$g_{ij}^{-1}(b_{N_j}^T)(m_1, d_1) \cdot (m_2, d_2)^{-1} \geq \min\{g_{ij}^{-1}(b_{N_j}^T)(m_1, d_1), g_{ij}^{-1}(b_{N_j}^T)(m_2, d_2)\},$$

$$g_{ij}^{-1}(b_{N_j}^I)(m_1, d_1) \cdot (m_2, d_2)^{-1} \geq \min\{g_{ij}^{-1}(b_{N_j}^I)(m_1, d_1), g_{ij}^{-1}(b_{N_j}^I)(m_2, d_2)\},$$

and

$$g_{ij}^{-1}(b_{N_j}^F)(m_1, d_1) \cdot (m_2, d_2)^{-1} \leq \max\{g_{ij}^{-1}(b_{N_j}^F)(m_1, d_1), g_{ij}^{-1}(b_{N_j}^F)(m_2, d_2)\}.$$

Hence, $g_{ij}^{-1}(\Gamma_2, V_\psi^2) \in \text{PNHSG}(U_1)$. □

4. **Conclusions**

Hypersoft set theory is more general than soft set theory and it has a huge area of applications. That is why we have adopted and implemented it in plithogenic environment so that we can introduce various algebraic structures. Because of this, the notions of plithogenic hypersoft subgroups have become general than fuzzy, intuitionistic fuzzy, neutrosophic subgroups, and plithogenic subgroups. Again, we have introduced functions in different plithogenic hypersoft environments. Hence, homomorphism can be introduced and its effects on these newly defined plithogenic hypersoft subgroups can be studied. In the future, to extend this study one may introduce general T-norm and T-conorm and further generalize plithogenic hypersoft subgroups. Also, one may extend these notions by introducing different normal versions of plithogenic hypersoft subgroups and by studying the effects of homomorphism on them.

References

1. L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–358, 1965.
2. K. T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1):87–96, 1986.
3. F. Smarandache. A unifying field in logics: neutrosophic logic. In *Philosophy*, pages 1–141. American Research Press, 1999.
4. R. R. Yager. Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting*, pages 57–61. IEEE, 2013.
5. F. Smarandache. *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited.* Infinite Study, 2018.
6. F. Smarandache. *An introduction to the neutrosophic probability applied in quantum physics.* Infinite Study, 2000.
7. F. Smarandache. *Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability.* Infinite Study, 2013.
8. F. Smarandache. *Neutropsychic personality: A mathematical approach to psychology.* Infinite Study, 2018.
9. F. Smarandache and H. E. Khalid. *Neutrosophic precalculus and neutrosophic calculus.* Infinite Study, 2015.
10. S. Broumi, I. Deli, and F. Smarandache. N-valued interval neutrosophic sets and their application in medical diagnosis. *Critical Review, Center for Mathematics of Uncertainty, Creighton University, Omaha, NE, USA*, 10:45–69, 2015.
11. Gaurav, M. Kumar, K. Bhutani, and S. Aggarwal. Hybrid model for medical diagnosis using neutrosophic cognitive maps with genetic algorithms. In *2015 IEEE International Conference on Fuzzy Systems*, pages 1–7. IEEE, 2015.
12. R. Kumar, A. Dey, F. Smarandache, and S. Broumi. *Chapter 6: A study of neutrosophic shortest path problem*, Neutrosophic Graph Theory and Algorithms, pages 144–175. IGI-Global, 2019.
13. R. Kumar, S. A. Edalatpanah, S. Jha, S. Broumi, R. Singh, and A. Dey. A multi objective programming approach to solve integer valued neutrosophic shortest path problems. *Neutrosophic Sets and Systems*, 24:134–149, 2019.
14. R. Kumar, S. A. Edalatpanah, S. Jha, and R. Singh. A novel approach to solve gaussian valued neutrosophic shortest path problems. *International Journal of Engineering and Advanced Technology*, 8(3):347–353, 2019.
15. R. Kumar, S. A. Edalatpanah, S. Jha, S. Gayen, and R. Singh. Shortest path problems using fuzzy weighted arc length. *International Journal of Innovative Technology and Exploring Engineering*, 8(6):724–731, 2019.

16. R. Kumar, S. A. Edaltpanah, S. Jha, S. Broumi, and A. Dey. Neutrosophic shortest path problem. *Neutrosophic Sets and Systems*, 23:5–15, 2018.

17. R. Kumar, S. Jha, and R. Singh. Shortest path problem in network with type-2 triangular fuzzy arc length. *Journal of Applied Research on Industrial Engineering*, 4(1):1–7, 2017.

18. R. Kumar, S. Jha, and R. Singh. A different appraoch for solving the shortest path problem under mixed fuzzy environment. *International Journal of Fuzzy System Applications*, 9(2):132–161, 2020.

19. S. Broumi, A. Bakal, M. Talea, F. Smarandache, and L. Vladareanu. Applying dijkstra algorithm for solving neutrosophic shortest path problem. In *2016 International Conference on Advanced Mechatronic Systems*, pages 412–416. IEEE, 2016.

20. S. Broumi, A. Dey, M. Talea, A. Bakali, F. Smarandache, D. Nagarajan, M. Lathamaheswari, and R. Kumar. Shortest path problem using bellman algorithm under neutrosophic environment. *Complex & Intelligent Systems*, pages 1–8, 2019.

21. M. Abdel-Basset, A. Atef, and F. Smarandache. A hybrid neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research*, 57:216–227, 2019.

22. M. Abdel-Basset, M. Mohamed, Y. Zhou, and I. M. Hezam. Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent and Fuzzy Systems*, 33(6):4055–4066, 2017.

23. P. Majumdar. Neutrosophic sets and its applications to decision making. In *Computational Intelligence for Big Data Analysis*, pages 97–115. Springer, 2015.

24. J. Ye. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *Journal of Intelligent & Fuzzy Systems*, 26(1):165–172, 2014.

25. H. Zhang, J. Wang, and X. Chen. Interval neutrosophic sets and their application in multicriteria decision making problems. *The Scientific World Journal*, 2014:1–16, 2014.

26. S. Broumi, M. Talea, F. Smarandache, and A. Bakali. Decision-making method based on the interval valued neutrosophic graph. In *2016 Future Technologies Conference*, pages 44–50. IEEE, 2016.

27. R. Kumar, S. A. Edalatpanah, S. Jha, and R. Singh. A pythagorean fuzzy approach to the transportation problem. *Complex & Intelligent Systems*, 5(2):255–263, 2019.

28. J. Pratihar, R. Kumar, A. Dey, and S. Broumi. *Chapter 7: Transportation problem in neutrosophic environment*, Neutrosophic Graph Theory and Algorithms, pages 180–212. IGI-Global, 2019.

29. M. Abdel-Basset, M. Mohamed, and F. Smarandache. A aefined approach for forecasting based on neutrosophic time series. *Symmetry*, 11(4):457, 2019.

30. M. Abdel-Basset, G. Manogaran, and M. Mohamed. A neutrosophic theory based security approach for fog and mobile-edge computing. *Computer Networks*, 157:122–132, 2019.

31. V. Çetkin and H. Aygün. An approach to neutrosophic subgroup and its fundamental properties. *Journal of Intelligent & Fuzzy Systems*, 29(5):1941–1947, 2015.

32. I. K. Vlachos and G. D. Sergiadis. Intuitionistic fuzzy information-applications to pattern recognition. *Pattern Recognition Letters*, 28(2):197–206, 2007.

33. Y. Guo and H. D. Cheng. New neutrosophic approach to image segmentation. *Pattern Recognition*, 42(5):587–595, 2009.

34. M. Abdel-Basset, N. A. Nabeeh, H. A. El-Ghareeb, and A. Aboelfetouh. Utilising neutrosophic theory to solve transition difficulties of iot-based enterprises. *Enterprise Information Systems*, pages 1–21, 2019.

35. F. Smarandache. *Plithogeny, plithogenic set, logic, probability, and statistics*. Infinite Study, 2017.

36. D. Molodtsov. Soft set theory–first results. *Computers & Mathematics with Applications*, 37(4-5):19–31, 1999.

37. P. K. Maji, R. Biswas, and A. R. Roy. Soft set theory. *Computers & Mathematics with Applications*, 45(4-5):555–562, 2003.

38. H. Aktaş and N. Çağman. Soft sets and soft groups. *Information sciences*, 177(13):2726–2735, 2007.

39. A. Aygünolu and H. Aygün. Introduction to fuzzy soft groups. *Computers & Mathematics with Applications*, 58(6):1279–1286, 2009.

40. M. Shabir, M. Ali, M. Naz, and F. Smarandache. Soft neutrosophic group. *Neutrosophic Sets and Systems*, 1:13–25, 2013.

41. H. Bustince and P. Burillo. Vague sets are intuitionistic fuzzy sets. *Fuzzy sets and systems*, 79(3):403–405, 1996.

42. D. H. Hong and C. H. Choi. Multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy sets and systems*, 114(1):103–113, 2000.

43. M. Kryszkiewicz. Rough set approach to incomplete information systems. *Information sciences*, 112(1-4):39–49, 1998.

44. Z. Pawlak. Hard and soft sets. In *Rough sets, fuzzy sets and knowledge discovery*, pages 130–135. Springer, 1994.

45. A. R. Roy and P. K. Maji. A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics*, 203(2):412–418, 2007.

46. P. K. Maji. More on intuitionistic fuzzy soft sets. In *International Workshop on Rough Sets, Fuzzy Sets, Data Mining, and Granular-Soft Computing*, pages 231–240. Springer, 2009.

47. P. K. Maji. *Neutrosophic soft set*. Infinite Study, 2013.

48. A. Rosenfeld. Fuzzy groups. *Journal of Mathematical Analysis and Applications*, 35(3):512–517, 1971.

49. R. Biswas. Intuitionistic fuzzy subgroups. In *Mathematical Forum*, volume 10, pages 37–46, 1989.

50. F. Karaaslan, K. Kaygisiz, and N. Çağman. On intuitionistic fuzzy soft groups. *Journal of New Results in Science*, 2(3):72–86, 2013.

51. T. Bera and N. K. Mahapatra. Introduction to neutrosophic soft groups. *Neutrosophic Sets and Systems*, 13:118–127, 2016.

52. F. Smarandache. *Extension of soft set to hypersoft set, and then to plithogenic hypersoft set*. Infinite Study, 2018.

53. P. K. Maji, A. R. Roy, and R. Biswas. An application of soft sets in a decision making problem. *Computers & Mathematics with Applications*, 44(8-9):1077–1083, 2002.

54. F. Feng, Y. B. Jun, X. Liu, and L. Li. An adjustable approach to fuzzy soft set based decision making. *Journal of Computational and Applied Mathematics*, 234(1):10–20, 2010.

55. N. Çağman, S. Enginoglu, and F. Citak. Fuzzy soft set theory and its applications. *Iranian Journal of Fuzzy Systems*, 8(3):137–147, 2011.

56. S. Broumi, I. Deli, and F. Smarandache. Neutrosophic parametrized soft set theory and its decision making. *International Frontier Science Letters*, 1(1):1–11, 2014.

57. S. Broumi, I. Deli, and F. Smarandache. Interval valued neutrosophic parameterized soft set theory and its decision making. *Journal of New Results in Science*, 3(7):58–71, 2014.

58. I. Deli, S. Broumi, and M. Ali. Neutrosophic soft multi-set theory and its decision making. *Neutrosophic Sets and Systems*, 5:65–76, 2014.

59. I. Deli and N. Çağman. Intuitionistic fuzzy parameterized soft set theory and its decision making. *Applied Soft Computing*, 28:109–113, 2015.

60. F. Smarandache. Physical plithogenic set. In *APS Meeting Abstracts*, 2018.

61. F. Smarandache. Aggregation plithogenic operators in physical fields. *Bulletin of the American Physical Society*, 63(13), 2018.

62. S. Gayen, F. Smarandache, S. Jha, M. K. Singh, S. Broumi, and R. Kumar. *Chapter 8: Introduction to plithogenic subgroup*, Neutrosophic Graph Theory and Algoritm, pages 209–233. IGI-Global, October 2019.

63. M. Abdel-Basset, M. El-hoseny, A. Gamal, and F. Smarandache. A novel model for evaluation hospital medical care systems based on plithogenic sets. *Artificial intelligence in medicine*, 100:1–8, 2019.

64. M. Abdel-Basset and R. Mohamed. A novel plithogenic topsis-critic model for sustainable supply chain risk management. *Journal of Cleaner Production*, 247, 2019.

65. M. Abdel-Basset, R. Mohamed, A. N. H. Zaied, and F. Smarandache. A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. *Symmetry*, 11(7):903, 2019.

66. P. K. Sharma. Homomorphism of intuitionistic fuzzy groups. In *International Mathematical Forum*, volume 6, pages 3169–3178, 2011.

67. S. Gayen, S. Jha, and M. Singh. On direct product of a fuzzy subgroup with an anti-fuzzy subgroup. *International Journal of Recent Technology and Engineering*, 8:1105–1111, 2019.

68. P. S. Das. Fuzzy groups and level subgroups. *Journal of Mathematical Analysis and Applications*, 84(1):264–269, 1981.

69. S. Gayen, F. Smarandache, S. Jha, and R. Kumar. *Chapter 10: Interval-valued neutrosophic subgroup based on interval-valued triple t-norm*, Neutrosophic Sets in Decision Analysis and Operations Research, pages 215–243. IGI-Global, December 2019.

70. R. Biswas. Rosenfeld's fuzzy subgroups with interval-valued membership functions. *Fuzzy Sets and Systems*, 63(1):87–90, 1994.

71. C. Y. Xu. Homomorphism of intuitionistic fuzzy groups. In *2007 International Conference on Machine Learning and Cybernetics*, volume 2, pages 1178–1183. IEEE, 2007.

72. S. Rana, M. Qayyum, M. Saeed, and F. Smarandache. Plithogenic fuzzy whole hypersoft set: construction of operators and their application in frequency matrix multi attribute decision making technique. *Neutrosophic Sets and Systems*, 28(1):34–50, 2019.