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NEUTROSOPHIC *d*-IDEAL OF NEUTROSOPHIC *d*-ALGEBRA

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Abstract: In this article, we introduce the concept of neutrosophic *d*-ideal of neutrosophic *d*-algebra. Also we have studied several properties of them. We also furnish some suitable examples.

Keywords: *Fuzzy Set*; *Intuitionistic Fuzzy Set*; *Neutrosophic Set*; *d-Algebra*; *d-Ideal*; *d-Sub-Algebra*.

1. Introduction:

The concept of *BCK* algebra and *BCI* algebra are introduced by Imai & Iseki [18]. Thereafter, Negger & Kim [23] introduced the *d*-algebra as a generalization of *BCK* algebra. Negger et al. [22] discussed the ideal theory in *d*-algebra. In the year 1965, Zadeh introduced the idea of fuzzy set [26]. Thereafter, Atanassov introduced the notion of intuitionistic fuzzy set [1], which is the natural generalization of fuzzy set. Later on, Jun et al. [20] applied the notion of intuitionistic fuzzy set on *d*-algebra. Afterwards, the notion of intuitionistic fuzzy *d*-ideal of *d*-algebra was introduced by Hasan [16] in 2017. Thereafter, the concept of intuitionistic fuzzy *d*-filter was introduced by Hasan [17] in 2020. The concept of neutrosophic set was introduced by Smarandache [24]. In this article, we procure the notion of neutrosophic *d*-algebra and neutrosophic *d*-ideal by extending the notion of intuitionistic fuzzy *d*-ideal of *d*-algebra.

Research gap: No investigation on neutrosophic *d*-algebra and neutrosophic *d*-ideal has been reported in the recent literature.

Motivation: To fill the research gap, we introduce the neutrosophic *d*-algebra and neutrosophic *d*-ideal.

The rest of the paper is designed as follows:

In section-2, we recall *d*-algebra, *d*-ideal, fuzzy *d*-algebra, fuzzy *d*-ideal, intuitionistic fuzzy *d*-algebra, intuitionistic fuzzy *d*-ideal. In section-3, we introduce the notion of neutrosophic *d*-algebra, neutrosophic *d*-ideal, and the proofs of some propositions, theorems on neutrosophic *d*-algebra, and neutrosophic *d*-ideal. In section-4, we give the conclusions of work done in this paper.

2. Preliminaries and Some Results:

Definition 2.1.[17] Assume that W be a non-empty set and 0 be a constant. Then, W with a binary operation $*$ is called a d-algebra if it satisfies the following three axioms:

(*i*) $c*c=0$, $\forall\, c\in W$

(*ii*) $0*c=0$, $\forall\, c\in W$

(*iii*) $c*d=0$ and $d*c=0\Rightarrow c=d$, $\forall\, c, d\in W$.

We will refer to $c*d$ by cd. And $c\leq d$ iff $cd=0$.

Definition 2.2.[17] A d-algebra W is called commutative if $c(cd)=d(dc)$, $\forall c, d\in W$, and $d(dc)$ is denoted by $(c\wedge d)$.

Definition 2.3.[17] A d-algebra W is called bounded if there exist $a\in W$ such that $c\leq a$ for all $c\in W$, i.e. $ca=0$, $\forall c\in W$.

Definition 2.4.[17] Let W be a d-algebra with binary operator $*$ and $A\subseteq W$. Then, A is said to be a d-sub-algebra of W, if $c, d\in A\Rightarrow cd\in A$.

Definition 2.5.[16] Let W be a d-algebra with binary operator $*$ and a constant 0. Then, $D\subseteq W$ is called a d-ideal of W if it satisfies the following:

(*i*) $a*b\in D$ and $b\in D\Rightarrow a\in D$;

(*ii*) $a\in D$ and $b\in W\Rightarrow a*b\in D$.

Definition 2.6.[15] Let $Y=\{(c,T_Y(c)):c\in W\}$ be a fuzzy set over a d-algebra W. Then, A is called a fuzzy d-algebra if $T_Y(cd)\geq \min\{T_Y(c), T_Y(d)\}$, for all $c, d\in W$.

Definition 2.7.[15] An fuzzy set $Y=\{(c,T_Y(c),F_Y(c)): c\in W\}$ over a d-algebra W is called the fuzzy d-ideal if it satisfies the following inequalities:

(*i*) $T_Y(c)\geq \min\{T_Y(cd), T_Y(d)\}$;

(*iii*) $T_Y(cd)\geq T_Y(c)$, for all $c, d\in W$.

Definition 2.8.[14] Let $Y=\{(c,T_Y(c),F_Y(c)): c\in W\}$ be an intuitionistic fuzzy set over a d-algebra W. Then, A is called an intuitionistic fuzzy d-algebra if it satisfies the followings:

(*i*) $T_Y(cd)\geq \min\{T_Y(c), T_Y(d)\}$;

(*ii*) $F_Y(cd)\leq \max\{F_Y(c), F_Y(d)\}$;

where $c, d\in W$.

Proposition 2.1.[14] Every intuitionistic fuzzy d-algebra $Y=\{(c,T_Y(c),F_Y(c)): c\in W\}$ of W satisfies the following inequalities:

(*i*) $T_Y(0)\geq T_Y(c)$, for all $c\in W$;

(*ii*) $F_Y(0)\leq F_Y(c)$, for all $c\in W$.

Definition 2.9.[10] An intuitionistic fuzzy set $Y=\{(c,T_Y(c),F_Y(c)): c\in W\}$ over a d-algebra W is called the intuitionistic fuzzy d-ideal if it satisfies the following inequalities:

(*i*) $T_Y(c)\geq \min\{T_Y(cd), T_Y(d)\}$;

(*ii*) $F_Y(c)\leq \max\{F_Y(cd), F_Y(d)\}$;

(*iii*) $T_Y(cd)\geq T_Y(c)$;

(*iv*) $F_Y(cd)\geq F_Y(c)$; for all $c, d\in Y$.

Proposition 2.2.[10] Let Y={(c,$T_Y(c)$,$F_Y(c)$): $c\in W$} be an intuitionistic fuzzy d-ideal over a d-algebra W. Then, the following inequalities hold:

$T_Y(0)\geq T_Y(c)$, $F_Y(0)\leq F_Y(c)$, for all $c\in W$.

Definition 2.10.[18] A neutrosophic set over a universal set W is defined as follows:

H = {(y, $T_H(y)$, $I_H(y)$, $F_H(y)$): $y\in W$}, where $T_H(y)$, $I_H(y)$ and $F_H(y)$ ($\in]^{-}0,1^{+}[$) are the truth, indeterminacy and false membership value of y and $^{-}0 \leq T_H(y) + I_H(y) + F_H(y) \leq 3^{+}$.

Definition 2.11.[18] The neutrosophic whole set (1_N) and neutrosophic null set (0_N) over a universal set W is defined as follows:

(i) 1_N={(y,1,0,0): $y\in W$}.

(ii) 0_N={(y,0,0,1): $y\in W$}.

Definition 2.12.[18] Assume that H={(y, $T_H(y)$, $I_H(y)$, $F_H(y)$):$y\in W$} and K={(y, $T_K(y)$, $I_K(y)$, $F_K(y)$):$y\in W$} are any two neutrosophic sets over X. Then,

(i) $H\cup K$= {(y,$T_H(y)\vee T_K(y)$, $I_H(y)\wedge I_K(y)$, $F_H(y)\wedge F_K(y)$): $y\in W$};

(ii) $H\cap K$= {(y, $T_H(y)\wedge T_K(y)$, $I_H(y)\vee I_K(y)$, $F_H(y)\vee F_K(y)$): $y\in W$};

(iii) H^c= {(y, 1-$T_H(y)$, 1-$I_H(y)$, 1-$F_H(y)$): $y\in W$};

(iv) $H\subseteq K \Leftrightarrow T_H(y)\leq T_K(y)$, $I_H(y)\geq I_K(y)$, $F_H(y)\geq F_K(y)$, for each $y\in W$.

3. Neutrosophic d-Algebra and Neutrosophic d-Ideal:

Definition 3.1. Let Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$): $c\in W$} be an neutrosophic set over a d-algebra W. Then, A is called a neutrosophic d-algebra if it satisfies the followings:

(i) $T_Y(c*d)\geq \min\{T_Y(c), T_Y(d)\}$;

(ii) $I_Y(c*d)\leq \max\{I_Y(c), I_Y(d)\}$;

(iii) $F_Y(c*d)\leq \max\{F_Y(c), F_Y(d)\}$;

where c, $d\in W$.

Example 3.1. Take $W=\{0,c,d,w\}$ with the following table

$\star$	0	c	d	w
0	0	0	0	0
c	c	0	0	c
d	d	d	0	0
w	w	w	d	0

Note that if we define

$T_Y(a)=\begin{cases}0.2 & if\, a=0,c\\ 0.02 & if\, a=d,w\end{cases}$, $I_Y(a)=\begin{cases}0.09 & if\, a=0,c\\ 0.8 & if\, a=d,w\end{cases}$ and $F_Y(a)=\begin{cases}0.05 & if\, a=0,c\\ 0.7 & if\, a=d,w\end{cases}$

So we can show easily that $Y=\{(c,T_Y(c),I_Y(c),F_Y(c)): c\in W\}$ is a neutrosophic d-algebra

Proposition 3.1. Every neutrosophic d-algebra Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$): $c\in W$} of W satisfies the following inequalities:

(i) $T_Y(0)\geq T_Y(c)$, for all $c\in W$;

(ii) $I_Y(0)\leq I_Y(c)$, for all $c\in W$;

(*iii*) $F_Y(0) \leq F_Y(c)$, for all $c \in W$.

Proof. Assume that $Y=\{(c,T_Y(c),I_Y(c),F_Y(c)): c\in W\}$ be a neutrosophic *d*-algebra of *W*. Let $c\in W$. Then

(*i*) $T_Y(0)= T_Y(c*c) \geq \min\{ T_Y(c), T_Y(c)\}= T_Y(c)$, (using definition 2.1. & 3.1.)

(*ii*) $I_Y(0)= I_Y(c*c) \leq \max\{ I_Y(c), I_Y(c)\}= I_Y(c)$, (using definition 2.1. & 3.1.)

(*iii*) $F_Y(0)= F_Y(c*c) \leq \max\{ F_Y(c), F_Y(c)\}= F_Y(c)$, (using definition 2.1. & 3.1.)

Theorem 3.1. Let $\{Y_i: i\in\Delta\}$ be the family of neutrosophic *d*-algebra of *W*. Then, $\bigcap_{i\in\Delta} Y_i$ is a neutrosophic *d*-algebra of *W*.

Proof. Assume that $\{Y_i: i\in\Delta\}$ be the family of neutrosophic *d*-algebra of *W*. Now, $\bigcap_{i\in\Delta} Y_i=\{(c,\wedge T_{Y_i}(c),\vee I_{Y_i}(c),\vee F_{Y_i}(c)): c\in W\}$. Let $c, d\in W$. Then,

(*i*) $\wedge T_{Y_i}(c*d) \geq \wedge \min\{T_{Y_i}(c), T_{Y_i}(d)\}= \min\{\wedge T_{Y_i}(c),\wedge T_{Y_i}(d)\}$

$\Rightarrow \wedge T_{Y_i}(c*d) \geq \min\{\wedge T_{Y_i}(c),\wedge T_{Y_i}(d)\}$;

(*ii*) $\vee I_{Y_i}(c*d) \leq \vee \max\{I_{Y_i}(c), I_{Y_i}(d)\}= \max\{\vee I_{Y_i}(c),\wedge I_{Y_i}(d)\}$

$\Rightarrow \vee I_{Y_i}(c*d) \leq \max\{\vee I_{Y_i}(c),\wedge I_{Y_i}(d)\}$;

(*iii*) $\vee F_{Y_i}(c*d) \leq \vee \max\{F_{Y_i}(c), F_{Y_i}(d)\}= \max\{\vee F_{Y_i}(c),\wedge F_{Y_i}(d)\}$

$\Rightarrow \vee F_{Y_i}(c*d) \leq \max\{\vee F_{Y_i}(c),\wedge F_{Y_i}(d)\}$;

Therefore, $\bigcap_{i\in\Delta} Y_i$ is also a neutrosophic *d*-algebra of *W*.

Theorem 3.3. If $Y=\{(c,T_Y(c),I_Y(c),F_Y(c)): c\in W\}$ is a neutrosophic *d*-algebra of *W*, then the sets $W_T=\{c\in W: T_Y(c)=T_Y(0)\}$, $W_I=\{c\in W: I_Y(c)=I_Y(0)\}$, and $W_F=\{c\in W: F_Y(c)=F_Y(0)\}$ are *d*-sub-algebras of *W*.

Proof. Assume that $Y=\{(c,T_Y(c),I_Y(c),F_Y(c)): c\in W\}$ be a neutrosophic *d*-algebra of *W*. Given $W_T=\{c\in W: T_Y(c)=T_Y(0)\}$, $W_I=\{c\in W: I_Y(c)=I_Y(0)\}$, and $W_F=\{c\in W: F_Y(c)=F_Y(0)\}$. Let $c, d\in W_T$. Therefore, $T_Y(c)=T_Y(0)$, $T_Y(d)=T_Y(0)$. Now by definition 2.1, $T_Y(c*d) \geq \min\{T_Y(c), T_Y(d)\}= \min\{T_Y(0), T_Y(0)\}=T_Y(0)$, i.e. $T_Y(c*d) \geq T_Y(0)$. Again from proposition 3.1, it is clear that $T_Y(0) \leq T_Y(c*d)$. Therefore $T_Y(c*d)= T_Y(0)$. This implies that $c*d\in W_T$. Hence $c, d\in W_T \Rightarrow c*d\in W_T$. Therefore the set $W_T=\{c\in W: T_Y(c)=T_Y(0)\}$ is a *d*-sub-algebra of *W*.

Similarly we can easily show that $W_I=\{c\in W: I_Y(c)=I_Y(0)\}$ and $W_F=\{c\in W: F_Y(c)=F_Y(0)\}$ are *d*-sub-algebras of *W*.

Definition 3.2. Assume that $Y = \{(c,T_Y(c),I_Y(c),F_Y(c)): c\in W\}$ be a neutrosophic set over *W*. Then, the sets $W(T_Y,\alpha)=\{c\in W:T_Y(c)\geq\alpha\}$, $W(I_Y,\alpha)=\{c\in W:I_Y(c)\leq\alpha\}$, $W(F_Y,\alpha)=\{c\in W:F_Y(c)\leq\alpha\}$ are respectively called *T*-level α-cut, *I*-level α-cut, *F*-level α-cut of *Y*.

Theorem 3.4. Assume that $Y=\{(c,T_Y(c),I_Y(c),F_Y(c)):c\in W\}$ be a neutrosophic *d*-algebra of *W*. Then, for any $\alpha\in[0,1]$, the *T*-level α-cut, *I*-level α-cut, *F*-level α-cut of *Y* are *d*-sub-algebra of *W*.

Proof. Assume that $Y=\{(c,T_Y(c),I_Y(c),F_Y(c)):c\in W\}$ be a neutrosophic *d*-algebra of *W*. Then, *T*-level α-cut of $Y= W(T_Y,\alpha)=\{c\in W:T_Y(c)\geq\alpha\}$, *I*-level α-cut of $Y= W(I_Y,\alpha)=\{c\in W:I_Y(c)\leq\alpha\}$, and *F*-level α-cut of $Y= W(F_Y,\alpha)=\{c\in W:F_Y(c)\leq\alpha\}$.

Let $c, d\in W(T_Y,\alpha)$. Therefore, $T_Y(c)\geq\alpha$, $T_Y(d)\geq\alpha$. Now $T_Y(c*d) \geq \min\{ T_Y(c), T_Y(d)\} \geq \min\{ \alpha, \alpha\}\geq \alpha$. This implies, $c*d \in W(T_Y,\alpha)$. Hence, $c, d \in W(T_Y,\alpha) \Rightarrow c*d \in W(T_Y,\alpha)$. Therefore, $W(T_Y,\alpha)$ i.e. *T*-level α-cut of *Y* is a *d*-sub-algebra of *W*.

Similarly, we can easily show that *I*-level α-cut, *F*-level α-cut of *Y* are *d*-sub-algebra of *W*.

Definition 3.3. An neutrosophic set Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$):c∈W} over a d-algebra W is called a neutrosophic d-ideal if it satisfies the following inequalities:

(i) $T_Y(c)\geq\min\{T_Y(cd), T_Y(d)\}$;

(ii) $F_Y(c)\leq\max\{F_Y(cd), F_Y(d)\}$;

(iii) $I_Y(c)\leq\max\{I_Y(cd), I_Y(d)\}$;

(iv) $T_Y(cd)\geq T_Y(c)$;

(v) $F_Y(cd)\geq F_Y(c)$;

(vi) $I_Y(cd)\geq I_Y(c)$, for all c, $d\in Y$.

Example 3.2. Take $W = \{0, c, d, w\}$ with the following table

⋆	0	c	d
0	0	0	0
c	d	0	d
d	c	c	0

Note that if we define

$$T_Y(a) = \begin{cases} 0.9 & if a = 0 \\ 0.01 & if a = c, d \end{cases}, \; I_Y(a) = \begin{cases} 0.1 & if a = 0 \\ 0.5 & if a = \mathrm{c}, d \end{cases} \text{ and } F_Y(a) = \begin{cases} 0.2 & if a = 0 \\ 0.6 & if a = c, d \end{cases}$$

Then $Y = \{(c, T_Y(c), I_Y(c), F_Y(c)) \colon c \in W\}$ is a neutrosophic d-ideal of d-algebra

Proposition 3.2. If Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$): $c\in W$} is a neutrosophic d-ideal of W, then $T_Y(0)\geq T_Y(c)$, $I_Y(0)\leq I_Y(c)$, $F_Y(0)\leq F_Y(c)$, for all $c\in W$.

Proof. Assume that Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$): $c\in W$} is a neutrosophic d-ideal of W, and c be any arbitrary element of W. Since $T_Y(c*c)\geq T_Y(c)$, so $T_Y(0)\geq T_Y(c)$. Similarly, since $I_Y(c*c)\leq I_Y(c)$, so $I_Y(0)\leq I_Y(c)$. Again, since $F_Y(c*c)\leq F_Y(c)$, so $F_Y(0)\leq F_Y(c)$.

Theorem 3.6. Assume that Y={(x,$T_Y(x)$,$I_Y(x)$,$F_Y(x)$):$x\in W$} is a neutrosophic d-ideal of W. If $x*y \leq z$, then $T_Y(x)\geq \min\{T_Y(y), T_Y(z)\}$, $I_Y(x)\leq \max\{I_Y(y), I_Y(z)\}$, $F_Y(x)\leq \max\{F_Y(y), F_Y(z)\}$.

Proof. Assume that Y={(x,$T_Y(x)$,$I_Y(x)$,$F_Y(x)$):$x\in W$} be an neutrosophic d-ideal of W. Let x, y, z be any three element of W such that $x*y \leq z$. Then by definition 2.1, $(x*y)*z=0$.

Now, $T_Y(x)\geq \min\{T_Y(x*y), T_Y(y)\}\geq \min\{\min\{T_Y((x*y)*z), T_Y(z)\}, T_Y(y)\}= \min\{\min\{T_Y(0),T_Y(z)\}, T_Y(y)\}\geq \min\{T_Y(z), T_Y(y)\}$. Therefore, $T_Y(x)\geq \min\{T_Y(y), T_Y(z)\}$.

Now, $I_Y(x)\leq \max\{I_Y(x*y), I_Y(y)\}\leq \max\{\max\{I_Y((x*y)*z), I_Y(z)\}, I_Y(y)\}= \max\{\max\{I_Y(0),I_Y(z)\}, I_Y(y)\}\leq \max\{I_Y(z), I_Y(y)\}$. Therefore, $I_Y(x)\leq \max\{I_Y(y), I_Y(z)\}$.

Again, $F_Y(x)\leq \max\{F_Y(x*y), F_Y(y)\}\leq \max\{\max\{F_Y((x*y)*z), F_Y(z)\}, F_Y(y)\}= \max\{\max\{F_Y(0),F_Y(z)\}, F_Y(y)\}\leq \max\{F_Y(z), F_Y(y)\}$. Therefore, $F_Y(x)\leq \max\{F_Y(y), F_Y(z)\}$.

Theorem 3.7. Assume that Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$): $c\in W$} is a neutrosophic d-ideal of W. If $x \leq z$, then $T_Y(x)\geq T_Y(z)$, $I_Y(x)\leq I_Y(z)$, $F_Y(x)\leq F_Y(z)$.

Proof. Assume that Y={(c,$T_Y(c)$,$I_Y(c)$,$F_Y(c)$): $c\in W$} is a neutrosophic d-ideal of W. Also let x, z be any two element of W such that $x \leq z$. Then by the definition 2.1, $x*z=0$.

Now, $T_Y(x)\geq \min\{T_Y(x*z),T_Y(z)\}= \min\{T_Y(0),T_Y(z)\}, T_Y(z)\}=T_Y(z)$. Therefore, $T_Y(x)\geq T_Y(z)$.

Now, $I_Y(x)\leq \max\{I_Y(x*z),I_Y(z)\}= \max\{I_Y(0),I_Y(z)\}, I_Y(z)\}=I_Y(z)$. Therefore, $I_Y(x)\leq I_Y(z)$.

Now, $F_Y(x) \leq \max\{F_Y(x*z), F_Y(z)\} = \max\{F_Y(0), F_Y(z)\}, F_Y(z)\} = F_Y(z)$. Therefore, $F_Y(x) \leq F_Y(z)$.

Theorem 3.10. If $\{D_i: i \in \Delta\}$ be the collection of neutrosophic d-ideals of d-algebra W, then $\bigcap_{i\in\Delta} D_i$ is also a neutrosophic d-ideal of d-algebra W.

Proof. Assume that $\{D_i: i \in \Delta\}$ be the collection of neutrosophic d-ideals of d-algebra W. We have $\bigcap_{i\in\Delta} D_i = \{(c, \wedge T_{Y_i}(c), \vee I_{Y_i}(c), \vee F_{Y_i}(c)): c \in W\}$.

Now $\wedge T_{Y_i}(c) \geq \wedge\{\min\{T_{Y_i}(c*d), T_{Y_i}(d)\}\} \geq \min\{\wedge T_{Y_i}(c*d), \wedge T_{Y_i}(d)\}$,

$\vee I_{Y_i}(c) \leq \vee\{\max\{I_{Y_i}(c*d), I_{Y_i}(d)\}\} \leq \max\{\vee I_{Y_i}(c*d), \vee I_{Y_i}(d)\}$,

and $\vee I_{Y_i}(c) \leq \vee\{\max\{I_{Y_i}(c*d), I_{Y_i}(d)\}\} \leq \max\{\vee I_{Y_i}(c*d), \vee I_{Y_i}(d)\}$.

Since $T_{Y_i}(c*d) \geq T_{Y_i}(c)$, $I_{Y_i}(c*d) \leq I_{Y_i}(c)$, $I_{Y_i}(c*d) \leq I_{Y_i}(c)$, for all i, we have $\wedge T_{Y_i}(c*d) \geq \wedge T_{Y_i}(c)$, $\vee I_{Y_i}(c*d) \leq \vee I_{Y_i}(c)$, $\vee I_{Y_i}(c*d) \leq \vee I_{Y_i}(c)$, for all i. Hence $\bigcap_{i\in\Delta} D_i = \{(c, \wedge T_{Y_i}(c), \vee I_{Y_i}(c), \vee F_{Y_i}(c)): c \in W\}$ is a neutrosophic d-ideal of W.

Theorem 3.11. A neutrosophic set $Y=\{(c, T_Y(c), F_Y(c), I_Y(c)): c \in W\}$ is neutrosophic d-ideal of d-algebra W if and only if the corresponding fuzzy set $\{(c, T_Y(c)): c \in W\}$, $\{(c, 1\text{-}I_Y(c)): c \in W\}$, $\{(c, 1\text{-}F_Y(c)): c \in W\}$ are fuzzy d-ideal of W.

Proof. Assume that $Y=\{(c, T_Y(c), F_Y(c), I_Y(c)): c \in W\}$ be a neutrosophic d-ideal of W. Therefore for all $c,d \in W$, $T_Y(c) \geq \min\{T_Y(cd), T_Y(d)\}$; $T_Y(cd) \geq T_Y(c)$; $I_Y(c) \leq \max\{I_Y(cd), I_Y(d)\}$; $I_Y(cd) \leq I_Y(c)$; $F_Y(c) \leq \max\{F_Y(cd), F_Y(d)\}$; $F_Y(cd) \leq F_Y(c)$.

Since for all c, $d \in W$, $T_Y(c) \geq \min\{T_Y(cd), T_Y(d)\}$; $T_Y(cd) \geq T_Y(c)$, so the fuzzy set $\{(c, T_Y(c)): c \in W\}$ is a fuzzy d-ideal of W.

Now, for all c, $d \in W$,

$I_Y(c) \leq \max\{I_Y(cd), I_Y(d)\} \Rightarrow 1\text{-}I_Y(c) \geq \min\{1\text{-}I_Y(cd), 1\text{-}I_Y(d)\}$;

$I_Y(cd) \leq I_Y(c) \Rightarrow 1\text{-}I_Y(cd) \geq 1\text{-}I_Y(c)$;

Therefore, the fuzzy set $\{(c, 1\text{-}I_Y(c)): c \in W\}$ is a fuzzy d-ideal of W.

Again, for all c, $d \in W$,

$F_Y(c) \leq \max\{F_Y(cd), F_Y(d)\} \Rightarrow 1\text{-}F_Y(c) \geq \min\{1\text{-}F_Y(cd), 1\text{-}F_Y(d)\}$;

$F_Y(cd) \leq F_Y(c) \Rightarrow 1\text{-}F_Y(cd) \geq 1\text{-}F_Y(c)$;

Therefore, the fuzzy set $\{(c, 1\text{-}F_Y(c)): c \in W\}$ is a fuzzy d-ideal of W.

Hence for an neutrosophic d-ideal $Y=\{(c, T_Y(c), F_Y(c), I_Y(c)): c \in W\}$ of W, the corresponding fuzzy sets $\{(c, T_Y(c)): c \in W\}$, $\{(c, 1\text{-}I_Y(c)): c \in W\}$, $\{(c, 1\text{-}F_Y(c)): c \in W\}$ are fuzzy d-ideal of W.

Theorem 3.12. If a neutrosophic set $Y=\{(c, T_Y(c), F_Y(c), I_Y(c)): c \in W\}$ is neutrosophic d-ideal of d-algebra W, then the sets $W(T_Y)=\{c \in W: T_Y(c)=T_Y(0)\}$, $W(I_Y)=\{c \in W: I_Y(c)=I_Y(0)\}$, and $W(F_Y)=\{c \in W: F_Y(c)=F_Y(0)\}$ are d-ideal of W.

Proof. Assume that $Y=\{(c, T_Y(c), F_Y(c), I_Y(c)): c \in W\}$ be a neutrosophic d-ideal of a d-algebra W.

Let $a*b \in W(T_Y)$ and $b \in W(T_Y)$. Therefore, $T_Y(a*b)=T_Y(0)$ and $T_Y(b)=T_Y(0)$. Since Y is a neutrosophic d-ideal of a d-algebra W, so $T_Y(a) \geq \min\{T_Y(a*b), T_Y(b)\} = \min\{T_Y(0), T_Y(0)\} = T_Y(0)$. This implies that $T_Y(a) \geq T_Y(0)$. Again by proposition 3.2, it is clear that $T_Y(0) \geq T_Y(a)$. Hence $T_Y(a)=T_Y(0)$, i.e. $a \in W(T_Y)$. Therefore, $a*b \in W(T_Y)$ and $b \in W(T_Y) \Rightarrow a \in W(T_Y)$.

Again let $a \in W(T_Y)$ and $b \in W$. Therefore, $T_Y(a)=T_Y(0)$. Since Y is a neutrosophic d-ideal of a d-algebra W, so $T_Y(a*b) \geq T_Y(a)=T_Y(0)$. This implies that $T_Y(a*b) \geq T_Y(0)$. From proposition 3.2, it is clear that $T_Y(0) \geq$

$T_Y(a*b)$. Hence $T_Y(a*b)=T_Y(0)$, i.e. $a*b\in W(T_Y)$. Therefore, $a\in W(T_Y)$ and $b\in W \Rightarrow a*b\in W(T_Y)$. Hence the set $W(T_Y)=\{c\in W: T_Y(c)=T_Y(0)\}$ is a d-ideal of W.

Similarly we can show that, the sets $W(I_Y)=\{c\in W: I_Y(c)=I_Y(0)\}$, and $W(F_Y)=\{c\in W: F_Y(c)=F_Y(0)\}$ are d-ideal of W.

5. Conclusions:

In this article, we introduce the notion of neutrosophic d-ideals of d-algebra. Further we have investigated different properties and study some relations on neutrosophic d-algebra. By defining neutrosophic d-algebra, neutrosophic d-ideals, we prove some propositions, theorems on neutrosophic d-algebra and d-ideal.

In the future, we hope that many new notions namely neutrosophic d-filter, neutrosophic d-topology can be introduce based on these notions of neutrosophic d-algebra.

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