Linguistic Neutrosophic Topology

N. Gayathri[1] and M. Helen [2]

[1]Research scholar; Nirmala college for women; Coimbatore ;Tamil Nadu; India
[2]Associate professor; Nirmala college for women;Coimbatore; Tamil Nadu; India
* Correspondence: gayupadmagayu@gmail.com

Abstract. By utilizing linguistic neutrosophic sets and topological spaces, we construct and develop a new notion called "linguistic neutrosophic topological spaces", in this article. Many basic definitions, theorems and properties were defined with suitable examples.

Keywords: Linguistic Neutrosophic topology; Linguistic Neutrosophic open set; Linguistic Neutrosophic closed set; Linguistic Neutrosophic interior; Linguistic Neutrosophic closure; Linguistic Neutrosophic continuous function; Linguistic Neutrosophic neighborhood; Linguistic Neutrosophic derived sets; Linguistic Neutrosophic dense sets;

1. Introduction

Many investigators in business, science, economy and a variety of other branches deal with modeling unknown data on a regular basis. For these ambiguous and uncertainties, traditional techniques are not always successful. Lotfi A. Zadeh [16] instigated the idea of membership or truth value to the elements of collection of well-defined objects called, sets. These systems can handle a variety of inputs, including ambiguous, distorted, or inaccurate data. The idea of fuzzy topology was initially developed by Chang [2] in 1967. Many topological structures and generalizations have developed in time utilizing fuzzy sets. In addition to the degree of truth membership, Atanassov [1] paired non-membership value called false membership, which was the generalization of fuzzy sets, called intuitionistic fuzzy sets. In 1997, intuitionistic fuzzy topology was found by Coker [4]. Along with the two membership values, Smarandache [11] introduced the idea of indeterminacy membership function in 1999. Neutrosophic sets play an important part in many aspects like, decision making, medical diagnosis, etc., Wang and Smarandache [13] introduced the notion of interval valued neutrosophic sets.

Qualitative attributes can be easily expressed in linguistic terms, which was developed by Zadeh [17]. The idea of linguistic variables was applied in decision making by Herrara etc.,al [9] in 2000 and Herrara-Viedma, Vergegay [8] in 1996. Su [12] used linguistic preference information in group decision making. Chen, Liu, etc.,al [3] introduced linguistic intuitionistic fuzzy number(LIFN) in 2015. As LIFN lacks indeterminacy, Ye [15] in 2015, proposed the notion of single valued neutrosophic linguistic numbers and developed an extended TOPSIS model for MAGDM approach utilizing SVNLNs. An extended COPRAS model for MAGDM based on SVN 2-tuple neutrosophic environment was developed by Wei, Wu, etc.,al [14]. Fang, Zebo etc.,al [6] found linguistic neutrosophic numbers in 2017, with a concrete definition.This paper is categorized as follows: Section 2 deals with the basic definitions of LNNs. In Section 3, the idea of linguistic neutrosophic topology is introduced and some properties are discussed. Linguistic neutrosophic derived set is discussed in section 4. In last section, the notion of linguistic neutrosophic continuity and linguistic neutrosophic dense sets are defined and discussed with suitable examples.

2. **Preambles**

Definition 2.1. [11] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is
$T_A : S \to]0^-, 1^+[, I_A : S \to]0^-, 1^+[, F_A : S \to]0^-, 1^+[$
There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [11] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point S in S, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.
When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$.
When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 2.3. [6] Let $S = \{s_\theta | \theta = 0, 1, 2, ..., \tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable. For example, when $\tau = 6$, S can be expressed as, $S = \{$very bad, bad, fair, very fair, good, very good$\}$.

Su [12] extended the discrete linguistic term set S into a continuous term set $S = \{s_\theta | \theta \in [0, q]\}$, where, if $s_\theta \in S$, then we call s_θ the original term, otherwise it is called as a virtual term.

Definition 2.4. [6] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t+1$ and $\overline{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,

$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \leq \theta+\psi+\sigma \leq 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5. [6] Let $\alpha = (s_\theta, s_\psi, s_\sigma), \alpha_1 = (s_{\theta 1}, s_{\psi_1}, s_{\sigma 1}), \alpha_2 = (s_{\theta 2}, s_{\psi_2}, s_{\sigma 2})$ be three LSVNNs, then

(1) $\alpha^c = (s_\sigma, s_\psi, s_\theta)$;
(2) $\alpha_1 \cup \alpha_2 = (max(\theta_1, \theta_2), min(\psi_1, \psi_2), min(\sigma_1, \sigma_2))$;
(3) $\alpha_1 \cap \alpha_2 = (min(\theta_1, \theta_2), max(\psi_1, \psi_2), max(\sigma_1, \sigma_2))$;
(4) $\alpha_1 = \alpha_2$ iff $\theta_1 = \theta_2, \psi_1 = \psi_2, \sigma_1 = \sigma_2$;

Definition 2.6. Let $\alpha = (l_\theta, l_\psi, l_\sigma)$ be a LSVNN. The set of all labels is, $L = \{l_0, l_1, l_2, ..., l_t\}$. Then the unit linguistic neutrosophic set (1_{LN}) is defined as $1_{LN} = (l_t, l_0, l_0)$, which is the truth membership,and the zero linguistic neutrosophic set (0_{LN}) is defined as $0_{LN} = (l_0, l_t, l_t)$, which is the falsehood membership.

Example 2.7. For the linguistic neutrosophic set, $L = \{$very bad, bad, fair, very fair, good, very good$\}$, the set of all labels be, $L = \{l_0, l_1, l_2, l_3, l_4, l_5\}$.
Then the unit LNs is defined as $1_{LN} = (l_5, l_0, l_0)$, and the zero LNs is defined as $0_{LN} = (l_0, l_5, l_5)$.

3. Linguistic Neutrosophic Topology

In this chapter, we introduce the concepts of linguistic neutrosophic topological spaces.

Definition 3.1. For a linguistic neutrosophic topology π, the collection of linguistic neutrosophic sets should obey,

(1) $0_{LN}, 1_{LN} \in \pi$
(2) $K_1 \bigcap K_2 \in \pi$ for any $K_1, K_2 \in \pi$
(3) $\bigcup K_i \in \pi, \forall \{K_i : i \in J\} \subseteq \pi$

We call, the pair (S_{LN}, π_{LN}), a linguistic neutrosophic topological space.

Remark 3.2. Let (S_{LN}, π_{LN}) be a linguistic neutrosophic topological space (LNTS). Then, $(S_{LN}, \pi_{LN})^c$ is the dual LN topology, whose elements are $K^c{}_{LN}$ for $K_{LN} \in (S_{LN}, \pi_{LN})$. Any open set in π_{LN} is known as linguistic neutrosophic open set(LNOsS). Any closed set in π_{LN} is known as linguistic neutrosophic closed set(LNCS) iff it's complement is linguistic neutrosophic open set.

Example 3.3. Let U_{LN} be the universe of discourse $U_{LN} = \{u, v, w, z\}$ and $S_{LN} = \{u, v\}$ and the linguistic term set be, $L = \{$ very poor, poor, very bad, bad, fair, good, very good$\}$
Then L can be taken as, $L = \{l_0, l_1, l_2, l_3, l_4, l_5, l_6\}$.
Let $K_{LN} = \{\langle u, (l_5, l_3, l_4)\rangle, \langle v, (l_4, l_2, l_3)\rangle\}$,
That is, the element u's degree of appurtenance to the set K_{LN} is good(l_5)
the element u's degree of indeterminate-appurtenance to the set K_{LN} is bad(l_3)
the element u's degree of non-appurtenance to the set K_{LN} is fair(l_4).
And the element v's degree of appurtenance to the set K_{LN} is fair(l_4)
the element v's degree of indeterminate-appurtenance to the set K_{LN} is very bad(l_2)
the element v's degree of non-appurtenance to the set K_{LN} is bad(l_3).
Let, $H_{LN} = \{\langle u, (l_6, l_2, l_2)\rangle, \langle v, (l_6, l_1, l_0)\rangle\}$
That is, the element u's degree of appurtenance to the set H_{LN} is very good(l_6)
the element u's degree of indeterminate-appurtenance to the set H_{LN} is very bad(l_2)
the element u's degree of non-appurtenance to the set H_{LN} is very bad(l_2).
And the element v's degree of appurtenance to the set H_{LN} is very good(l_6)
the element v's degree of indeterminate-appurtenance to the set H_{LN} is poor(l_1)
the element v's degree of non-appurtenance to the set H_{LN} is very poor(l_0).
Similarly, let $M_{LN} = \{\langle u, (l_6, l_3, l_2)\rangle, \langle v, (l_6, l_2, l_0)\rangle\}$
That is, the element u's degree of appurtenance to the set M_{LN} is very good(l_6)
the element u's degree of indeterminate-appurtenance to the set M_{LN} is bad(l_3)
the element u's degree of non-appurtenance to the set M_{LN} is very bad(l_2).
And the element v's degree of appurtenance to the set M_{LN} is very good(l_6)
the element v's degree of indeterminate-appurtenance to the set M_{LN} is very bad(l_2)
the element v's degree of non-appurtenance to the set M_{LN} is very poor(l_0).

Then the collection $\pi_{LN} = \{0_{LN}, K_{LN}, H_{LN}, M_{LN}, K_{LN} \vee H_{LN}, 1_{LN}\}$ forms a LN topology on (S_{LN}, π_{LN}).

Definition 3.4. The linguistic neutrosophic closure and linguistic neutrosophic interior are given by,

(i) $LNint(K_{LN}) = \bigcup\{O_{LN}/O_{LN}$ is a $LNOSinS_{LN}$ where $O_{LN} \subseteq K_{LN}\}$ and it is the largest LN open subset of K_{LN}.

(ii) $LNcl(H_{LN}) = \bigcap\{J_{LN}/J_{LN}$ is a $LNCSinS_{LN}$ where $H_{LN} \subseteq J_{LN}\}$ and it is the smallest LN closed set containing H_{LN}.

Example 3.5. In example 3.3, $LNint(K_{LN}) = N_{LN}$ and $LNcl(K_{LN}) = 1_{LN}$

Theorem 3.6. *Let (S_{LN}, π_{LN}) be a LNTS and $K_{LN}, H_{LN} \in S_{LN}$. Then*

(i) $K_{LN} \in LNcl(K_{LN})$

(ii) K_{LN} *is LN closed if and only if* $K_{LN} = LNcl(K_{LN})$

(iii) $LNcl(\phi_{LN}) = \phi_{LN}$ *and* $LNcl(S_{LN}) = S_{LN}$.

(iv) $K_{LN} \subseteq H_{LN} \Rightarrow LNcl(K_{LN}) \subseteq LNcl(H_{LN})$

(v) $LNcl(K_{LN} \cup H_{LN}) = LNcl(K_{LN}) \cup LNcl(H_{LN})$

(vi) $LNcl(K_{LN} \cap H_{LN}) \subseteq LNcl(K_{LN}) \cap LNcl(H_{LN})$

(vii) $LNcl(LNcl(K_{LN})) = LNcl(K_{LN})$

Proof:

(i) From the definition, $K_{LN} \in LNcl(K_{LN})$

(ii) If K_{LN} is LN closed, then K_{LN} is the smallest LN closed set containing K_{LN}. So, $K_{LN} = LNcl(K_{LN})$.
Conversely, if $K_{LN} = LNcl(K_{LN})$, then K_{LN} is the smallest LN closed set containing K_{LN} and hence K_{LN} is LN closed.

(iii) If K_{LN} is LN closed, then $K_{LN} = LNcl(K_{LN})$. As ϕ_{LN} and S_{LN} are LN closed, $LNcl(\phi_{LN}) = \phi_{LN}$ and $LNcl(S_{LN}) = S_{LN}$.

(iv) When $K_{LN} \subseteq H_{LN}$, since $H_{LN} \subseteq LNcl(H_{LN})$ and $K_{LN} \subseteq LNcl(H_{LN})$. That is, $LNcl(H_{LN})$ is a LN closed set that contains K. But $LNcl(K_{LN})$ is the smallest LN closed set contains K. Thus, $LNcl(K_{LN}) \subseteq LNcl(H_{LN})$

(v) As $K_{LN} \subseteq K_{LN} \cap H_{LN}$ and $H_{LN} \subseteq K_{LN} \cap H_{LN}$, $LNcl(K_{LN}) \subseteq LNcl(K_{LN} \cap H_{LN})$ and $LNcl(H_{LN}) \subseteq LNcl(K_{LN} \cap H_{LN})$. Thus, $LNcl(K_{LN}) \cap LNcl(H_{LN}) \subseteq LNcl(K_{LN} \cap H_{LN})$. Since, $K_{LN} \cup H_{LN} \subseteq LNcl(K_{LN}) \cap LNcl(H_{LN})$, and since $LNcl(K_{LN} \cup H_{LN})$ is the smallest LN closed set containing $K_{LN} \cup H_{LN}, LNcl(K_{LN} \cup H_{LN}) \subseteq LNcl(K_{LN}) \cup LNcl(H_{LN})$.
Thus, $LNcl(K_{LN} \cup H_{LN}) = LNcl(K_{LN}) \cup LNcl(H_{LN})$.

(vi) Since $(K_{LN} \cap H_{LN}) \subseteq K_{LN}$ and $(K_{LN} \cap H_{LN}) \subseteq H_{LN}, LNcl(K_{LN} \cap H_{LN}) \subseteq LNcl(H_{LN}) \subseteq LNcl(H_{LN})$.

(vii) AS $LNcl(K_{LN})$ is a LN closed set, $LNcl(LNcl(K_{LN})) = LNcl(K_{LN})$.

Remark 3.7. If $LNint(K_{LN})$ is $LNcl(K_{LN})$ is a LNCS, then we have,

(i) $LNint(K_{LN}) = K_{LN}$ if and only if K_{LN} is LNOS in (S_{LN}, π_{LN}).

(ii) $LNcl(K_{LN}) = K_{LN}$ if and only if K_{LN} is LNCS in (S_{LN}, π_{LN}).

Theorem 3.8. *Let* (S_{LN}, π_{LN}) *be a LNTS and* $K_{LN} \in S_{LN}$. *Then*

(i) $S - LNint(K_{LN}) = LNint(S_{LN} - K_{LN})$

(ii) $S - LNcl(K_{LN}) = LNcl(S_{LN} - K_{LN})$

Proof: (i): Let $S \in S_{LN} - LNint(K_{LN}) \Rightarrow S \notin LNint(K_{LN})$. Thus, $G \nsubseteq K_{LN} \forall$ LN open set G containing S, (i.e) $C_{LN} \cap (S - K_{LN}) =\neq \phi_{LN}, \forall$ LN open set G. Hence, $S \in LNcl(S_{LN} - K_{LN})$ and $S_{LN} - LNint(K_{LN}) \subseteq LNcl(S_{LN} - K_{LN})$.
Conversely, if $S \in LNcl(S_{LN} - K_{LN})$, then $G_{LN} \cap (S_{LN} - K_{LN}) \neq \phi_{LN}$ for every LN open set containing S, (i.e) $G \nsubseteq A \forall$ LN open set G containing S. That is, $S \notin LNint(A) \Rightarrow S \in S - LNint(A)$. Then, $LNcl(S_{LN} - K_{LN}) \subseteq S_{LN} - LNint(K_{LN})$. Thus, $S_{LN} - LNint(K_{LN}) = LNint(S_{LN} - K_{LN})$
(ii) Proof is similar to (i).

Remark 3.9. On taking complements on both sides of $S_{LN} - LNint(K_{LN}) = LNint(S_{LN} - K_{LN})$ and $S_{LN} - LNcl(K_{LN}) = LNcl(S_{LN} - K_{LN})$, we have,
$LNint(K_{LN}) = S_{LN} - LNcl(S_{LN} - K_{LN})$ and $LNcl(K_{LN}) = S_{LN} - LNint(S_{LN} - K_{LN})$

Theorem 3.10. *Let* (S_{LN}, π_{LN}) *be a LNTS and* $K_{LN}, H_{LN} \in S_{LN}$. *Then*

(i) $LNint(K_{LN}) = K_{LN}$ *if and only if* K_{LN} *is LN open.*
(ii) $LNint(\phi_{LN}) = \phi_{LN}$ *and* $LNint(S_{LN}) = S_{LN}$.
(iii) $K_{LN} \subseteq H_{LN} \Rightarrow LNint(K_{LN}) \subseteq LNint(H_{LN})$
(iv) $LNint(K_{LN}) \cup LNint(H_{LN}) \subseteq LNint(K_{LN} \cup H_{LN})$
(iv) $LNint(K_{LN} \cap H_{LN}) = LNint(K_{LN}) \cap LNint(H_{LN})$
(vi) $LNint(LNint(K_{LN})) = LNint(K_{LN})$

Proof: (i): K_{LN} is LN open if and only if $S_{LN} - K_{LN}$ is LN closed, if and only if, $LNcl(S_{LN} - K_{LN}) = S_{LN} - K_{LN}$, if and only if, $S_{LN} - LNcl(K_{LN}) = K_{LN}$ iff $LNint(K_{LN}) = K_{LN}$ bT remark.
(ii): Since ϕ_{LN} and S_{LN} are LN open, $LNint(\phi_{LN}) = \phi_{LN}$ and $LNint(S_{LN}) = S_{LN}$
(iii): $K_{LN} \subseteq H_{LN} \Rightarrow S_{LN} - H_{LN} \subseteq S_{LN} - K_{LN}$. Thus, $LNcl(S_{LN} - H_{LN}) \subseteq LNcl(S_{LN} - K_{LN}), (i.e) S_{LN} - LNcl(S_{LN} - K_{LN}) \subseteq S_{LN} - LNcl(S_{LN} - H_{LN})$. Therefore, $LNint(K_{LN}) \subseteq LNint(H_{LN})$.

Definition 3.11. Let S_{LN} be a non-void set and $K_{LN} = \{\langle S, [T_{K_{LN}}, I_{K_{LN}}, F_{K_{LN}}]\rangle\}$ and $H_{LN} = \{\langle S, [T_{H_{LN}}, I_{H_{LN}}, F_{H_{LN}}]\rangle\}$ are LNSs in LNTS.

(I) $K_{LN} \cup H_{LN}$ can be defined as
   (a) $K_{LN} \cup H_{LN} = \{\langle S, [T_{K_{LN}} \wedge T_{H_{LN}}, I_{K_{LN}} \wedge I_{H_{LN}}, F_{K_{LN}} \vee F_{H_{LN}}]\rangle\}$
(II) $K_{LN} \cap H_{LN}$ can be defined as
   (a) $K_{LN} \cap H_{LN} = \{\langle S, [T_{K_{LN}} \wedge T_{H_{LN}}, I_{K_{LN}} \wedge I_{H_{LN}}, F_{K_{LN}} \vee F_{H_{LN}}]\rangle\}$

(III) The complement of $K_{LN} = \{\langle S, [T_{K_{LN}}, I_{K_{LN}}, F_{K_{LN}}]\rangle\}$ is defined as,
  (a) ${K_{LN}}^c = \{\langle S, [1 - F_{K_{LN}}, I_{K_{LN}}, 1 - T_{K_{LN}}]\rangle\}$
  (b) $({K_{LN}}^c)^c = K_{LN}$
  (c) $(K_{LN} \cap H_{LN})^c = {K_{LN}}^c \cup {H_{LN}}^c$
  (d) $(K_{LN} \cup H_{LN})^c = {K_{LN}}^c \cap {H_{LN}}^c$

Theorem 3.12. *Let (S_{LN}, π_{LN}) be a LNTS. $S \in LNcl(K_{LN})$ iff $U_{LN} \cap K_{LN} \neq \phi_{LN}$ for every LN open set U_{LN} containing S, where $K_{LN} \subseteq S_{LN}$.*

Proof:

If U_{LN} is a LN open set and if $S \in LNcl(K_{LN})$, then$S_{LN} - U_{LN}$ is LN closed. If $K_{LN} \cap U_{LN} = \phi_{LN}$, then $K_{LN} \subseteq S_{LN} - U_{LN}$.
That is, $S_{LN} - U_{LN}$ is LN closed set containing K_{LN}. Therefore, $LNcl(K_{LN}) \subseteq S_{LN} - U_{LN}$, which is a contradiction, since $S \in LNcl(K_{LN})$ but $S \notin S_{LN} - U_{LN}$. Hence, $K_{LN} \cap U_{LN} \neq \phi_{LN}$, for every LN open set U_{LN} containing S.
Conversely, if $K_{LN} \cap U_{LN} \neq \phi_{LN}$, for every LN open set U_{LN} containing S and if $S \notin LNcl(K_{LN}), S \in S_{LN} - LNcl(K_{LN})$ which is LN open.
Hence, $(S_{LN} - LNcl(K_{LN})) \cap K_{LN} \neq \phi_{LN}$. But $K_{LN} \subseteq LNcl(K_{LN})$ and hence $S_{LN} - LNcl(K_{LN}) \subseteq S_{LN} - K_{LN}$, that implies $(S_{LN} - LNcl(K_{LN})) \cap K_{LN} \subseteq (S_{LN} - K_{LN}) \cap K_{LN}$.
Thus, $(S_{LN} - K_{LN}) \cap K_{LN} \neq \phi_{LN}$, which is a contradiction. Hence, $S \in LNcl(K_{LN})$.

Definition 3.13. Let (S_{LN}, π_{LN}) be a LNTS and $\pi_{LN} = \{0, S_{LN}\}$. Then, π is called the LN indiscrete topology over S.

Definition 3.14. Let π be the collection of all LN sets that can be defined over S_{LN}. Then, (S_{LN}, π_{LN}) is the called the LN discrete topology over S_{LN}.

Theorem 3.15. *Let $(S_{LN}, {\pi^1}_{LN})$ and $(S_{LN}, {\pi^2}_{LN})$ be two LNTSs, then $(S_{LN}, \pi^1 \cap {\pi^2}_{LN})$ is a LNTS on S_{LN}.*

Proof:

(1) clearly, 0_{LN} and $1_{LN} \in {\pi^1}_{LN} \cap {\pi^2}_{LN}$
(2) Let $F_i \in {\pi^1}_{LN} \cap {\pi^2}_{LN}$. Then, $F_i \in {\pi^1}_{LN}$ and $F_i \in {\pi^2}_{LN} \forall i \in I$.
Therefore, $\cup_{i \in I} F_i \in {\pi^1}_{LN}$ and $\cup_{i \in I} F_i \in {\pi^2}_{LN}$. Thus, $\bigcup_{i \in I} F_i \in {\pi^1}_{LN} \cap {\pi^2}_{LN}$.
(3) Let K_{LN} and $H_{LN} \in {\pi^1}_{LN} \cap {\pi^2}_{LN}$, which implies, $K_{LN}, H_{LN} \in {\pi^1}_{LN}$ and $K_{LN}, H_{LN} \in {\pi^2}_{LN}$. Since, $K_{LN} \cap H_{LN} \in {\pi^1}_{LN}$ and $K_{LN} \cap H_{LN} \in {\pi^2}_{LN}$, $K_{LN} \cap H_{LN} \in {\pi^1}_{LN} \cap {\pi^2}_{LN}$

Thus, $(S_{LN}, {\pi^1}_{LN} \bigcap {\pi^2}_{LN})$ is a LNTS on S_{LN}..

Remark 3.16. Union of two LNTSs may not be a LN topology over S_{LN}..

Example 3.17. Let the universe of discourse be $U = \{a, b, c\}$ and $S = \{a\}$. The set of all linguistic term is, $L = \{$ very salt(l_0), salt(l_1), very sour(l_2), sour(l_3), bitter(l_4), sweet(l_5), very sweet(l_6)$\}$.
And $\pi^1{}_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$ where $K_{LN} = \{\langle a, (l_6, l_3, l_3)\rangle\}$, where the element a's degree of appurtenance to the set K_{LN} is very sweet(l_6), the element a's degree of indeterminate-appurtenance to the set K_{LN} is sour(l_3), the element a's degree of non-appurtenance to the set K_{LN} is bitter(l_4).
Let $\pi^2{}_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$ where $H_{LN} = \{\langle a, (l_4, l_5, l_2)\rangle\}$, where the element a's degree of appurtenance to the set H_{LN} is bitter(l_4), the element a's degree of indeterminate-appurtenance to the set H_{LN} is sweet(l_5), the element a's degree of non-appurtenance to the set H_{LN} is very sour(l_2).
Let $\pi^1{}_{LN}$ and $\pi^2{}_{LN}$ be two LN topologies on S_{LN}.
Then, $\pi^1{}_{LN} \cup \pi^2{}_{LN} = \{0_{LN}, 1_{LN}, K_{LN}, H_{LN}\} = \{0_{LN}, 1_{LN}, \{\langle a, (l_6, l_3, l_3)\rangle\}, \{\langle a, (l_6, l_5, l_2)\rangle\}\}$.
Now, $K_{LN} \cup H_{LN} = \{\langle a, (l_6, l_5, l_2)\rangle\} \notin \pi^1{}_{LN} \cup \pi^2{}_{LN}$.
$K_{LN} \cap H_{LN} = \{\langle a, (l_4, l_3, l_3)\rangle\} \notin \pi^1{}_{LN} \cup \pi^2{}_{LN}$.
Therefore, union of any two linguistic neutrosophic topologies need not be a linguistic neutrosophic topology.

Definition 3.18. Let (S_{LN}, π_{LN}) be a LNTS and U_{LN} be a LN set over S_{LN}.. Then any point S is a LN interior point of U_{LN}, if there exists a LN open set V_{LN} such that $S \in U_{LN} \subseteq V_{LN}$.

Definition 3.19. Let (S_{LN}, π_{LN}) be a LNTS and U_{LN} be a LN set over S_{LN}.. Then, V_{LN} is called a LN neighborhood if there exists a LN open set V_{LN} such that $S \in U_{LN} \subseteq V_{LN}$.

Theorem 3.20. *Let (S_{LN}, π_{LN}) be a LNTS, then*

(1) *each $s \in S$ has a neighborhood.*
(2) *If U_{LN} and V_{LN} are LN neighborhoods of some $x \in S_{LN}$, then $U_{LN} \cap V_{LN}$ is also a LN neighborhood of s.*
(3) *If U_{LN} is a LN neighborhood of S and $U_{LN} \cap V_{LN}$, then V_{LN} is also a LN neighborhood of $s \in S_{LN}$.*

Proof:
(1) : (2): Let U_{LN} and V_{LN} are LN neighborhoods of some $s \in S$, then there exists $U^1{}_{LN}$ and $V^1{}_{LN} \in \tau$ such that $S \in U^1{}_{LN} \subseteq U_{LN}$ and
$S \in V^1{}_{LN} \subseteq V_{LN}$
Now, $S \in U_{LN}$ and $S \in V_{LN}$ implies that $S \in U^1{}_{LN} \cap V^1{}_{LN}$ and $U^1{}_{LN} \cap V^1{}_{LN} \in \tau$. So w have $S \in U_{1LN} \cap V_{1LN} \subseteq U_{LN} \cap V_{LN}$.
Thus, $U_{LN} \bigcap V_{LN}$ is a LN neighborhood of s.
(3): Let U_{LN} is a LN neighborhood of s and $U_{LN} \bigcap V_{LN}$. By definition, there exists a LN open

set ${U^1}_{LN}$ such that $s \in {U^1}_{LN} \subseteq U_{LN} \subseteq V_{LN}$.
Then, $s \in U_{LN} \subseteq V_{LN}$.
Therefore, V_{LN} is also a LN neighborhood of $s \in S$.

Theorem 3.21. *Let (S_{LN}, π_{LN}) be a LNTS. For any LN open set K_{LN} over S, K_{LN} is a LN neighborhood of each point of $\cap_{i \in I} A_i$.*

Proof:

Let $K_{LN} \in \pi_{LN}$. For any $S \in \cap_{i \in I} K_{LNi}$, we have $S \in A_i \forall i \in I$. Thus, $S \in K_{LN}$ and hence K_{LN} is a LN neighborhood of S.

4. **Linguistic neutrosophic derived sets**

Definition 4.1. Let (S_{LN}, π_{LN}) be a LNTS and $K_{LN} \subseteq S_{LN}$. Let $s \in S_{LN}$. s is called as a LN limit point of K_{LN} if $E_{LN} \cap (K_{LN} - \{s\}) \neq \phi$ for every LN open set E_{LN} containing s. The collection of all LN limit points of K_{LN} is the LN derived set $(LND(K_{LN})) of K_{LN}$.

Theorem 4.2. *$LNcl(K_{LN}) = K_{LN} \cup LND(K_{LN})$ where $K_{LN} \subseteq S_{LN}$*

Proof:

If $s \in K_{LN} \cup LND(K_{LN})$, then $s \in K_{LN}$ or $S \in LND(K_{LN})$. If $s \in K_{LN}$, then $s \in LNcl(K_{LN})$. Therefore, let $s \notin K_{LN}$. That is, $s \in LND(K_{LN})$. Then, $\forall$ LN open set E_{LN} containing s, $E_{LN} \cap (K_{LN} - s) \neq \phi$. Since $s \notin K_{LN}, E_{LN} \cap K_{LN} \notin \phi$. Thus, $s \in LNcl(K_{LN})$. Hence, $K_{LN} \cup LND(K_{LN}) \subseteq LNcl(K_{LN})$. If $s \in LNcl(K_{LN})$ and $s \in K_{LN}$, then $s \in K_{LN} \cup LND(K_{LN})$. If $s \in LNcl(K_{LN})$ but $s \notin K_{LN}$, then $E_{LN} \cap K_{LN} \notin \phi$ for every LN open set E_{LN} containing s and hence $E_{LN} \cap (K_{LN} - s) \notin \phi$. Therefore, $s \in LND(K_{LN})$, (i.e) $s \in K_{LN} \cup LND(K_{LN})$. Thus, $LNcl(K_{LN}) \subseteq K_{LN} \cup LND(K_{LN})$. Therefore, $LNcl(K_{LN}) = K_{LN} \cup LND(K_{LN})$.

Theorem 4.3. *If the derived set of K_{LN} is a subset of K_{LN}, then K_{LN} is LN closed.*

Proof:

K_{LN} is LN closed if and only if $LNcl(K_{LN}) = K_{LN}$, iff $K_{LN} \cup LND(K_{LN}) = K_{LN}$, iff $LND(K_{LN}) \subseteq K_{LN}$.

Theorem 4.4. *If K_{LN} is a singleton subset of S_{LN}, then $LND(K_{LN}) = LNcl(K_{LN}) - K_{LN}$.*

Proof:

If $s \in LND(K_{LN})$, then for every LN open set E_{LN} containing , $E_{LN} \cap (K_{LN} - s) \neq \phi$. Then $s \notin K_{LN}$. Suppose if $s \in K_{LN}$, then $K_{LN} = \{s\}$, and $E_{LN} \cap (K_{LN} - s) = \phi$. It is true that, $LND(K_{LN}) \subseteq LNcl(K_{LN})$. Then, $s \in LNcl(K_{LN})$ but $s \notin K_{LN}$, when $s \in LND(K_{LN})$. Thus, $LND(K_{LN}) \subseteq LNcl(K_{LN}) - K_{LN}$. Thus, $s \in LNcl(K_{LN}) - K_{LN}, s \in LNcl(K_{LN})$

but $s \notin K_{LN}$. Thus, $E_{LN} \cap K_{LN} \neq \phi$ for every LN open set E_{LN} containing s, (i.e) $E_{LN} \cap (K_{LN} - s) \neq \phi$ for every LN open set E_{LN} containing s. Thus, $s \in LND(K_{LN})$. Thus, $LNcl(K_{LN}) - K_{LN} \subseteq LND(K_{LN})$. Hence, $LND(K_{LN}) = LNcl(K_{LN}) - K_{LN}$, if K_{LN} is a singleton set.

Definition 4.5. (1) Linguistic Neutrosophic semi-closed set if $LNint(LNcl(K_{LN})) \subseteq K_{LN}$

(2) Linguistic Neutrosophic semi-open set if $K_{LN} \subseteq LNcl(LNint(K_{LN}))$
(3) Linguistic Neutrosophic semi-pre closed if $LNint(LNcl(LNint(K_{LN}))_{\subseteq} K_{LN}$
(4) Linguistic Neutrosophic semi-pre open if $K_{LN} \subseteq LNcl(LNint(LNcl(K_{LN})))$
(5) Linguistic Neutrosophic pre-closed if $LNcl(LNint(K_{LN}) \subseteq K_{LN}$ -doubt
(6) Linguistic Neutrosophic pre-open if $K_{LN} \subseteq LNint(LNcl(K_{LN}))$
(7) Linguistic Neutrosophic regular closed if $K_{LN} = LNint(LNcl(K_{LN}))$
(8) Linguistic Neutrosophic regular open if $K_{LN} = LNcl(LNint(K_{LN}))$

5. Linguistic Neutrosophic continuity

Definition 5.1. Define the image and pre-image of linguistic neutrosophic sets. Let S_{LN} and T_{LN} be two non-void sets and $f : S_{LN} \to T_{LN}$ be a function, then

(i) If $E_{LN} = \{\langle S, [T_{E_{LN}}(S), I_{E_{LN}}(S), F_{E_{LN}}(S)]\rangle\}$ is a LN set in T_{LN}, then the pre image of E_{LN} under f is denoted by, $f^{-1}(E_{LN})$ is defined by,
$f^{-1}(E_{LN}) = \{\langle S, [f^{-1}(T_{E_{LN}}(S)), f^{-1}(I_{E_{LN}}(S)), f^{-1}(F_{E_{LN}}(S))]\rangle\}$

(ii) If $F_{LN} = \{\langle S, [T_{F_{LN}}(S), I_{F_{LN}}(S), F_{_{LN}}F(S)]\rangle; S \in S_{LN}\}$ is a LN set in S_{LN}, then the image of F_{LN} under f is denoted bT,
$f(F_{LN}) = \{\langle T, [f(T_{F_{LN}}(T)), f(I_{F_{LN}}(T)), f(F_{F_{LN}}(T))]\rangle; T \in T_{LN}\}$

Definition 5.2. A function $f : S_{LN} \to T_{LN}$ is called a linguistic neutrosophic continuous function if the inverse image of every linguistic neutrosophic open set F_{LN} is linguistic neutrosophic open in S_{LN}.

Example 5.3. Let the universe of discourse be $U_{LN} = \{a, b, c, d, x, y, z, w\}$ and $S_1 = \{a, b, c\}$ and $S_2 = \{x, y, z\}$. The set of all linguistic term is, $L = \{$ very salt(l_0), salt(l_1), very sour(l_2), sour(l_3), bitter(l_4), sweet(l_5), very sweet(l_6)$\}$. Define linguistic neutrosophic sets K_{LN} and H_{LN} as $K_{LN} = \{s_1, (a, \langle l_0, l_6, l_0\rangle), (b, \langle l_4, l_0, l_2\rangle), (c, \langle l_2, l_3, l_1\rangle)\}$, where the element a's degree of appurtenance to the set K_{LN} is very sweet(l_0), the element a's degree of indeterminate-appurtenance to the set K_{LN} is very sweet(l_6), the element a's degree of non-appurtenance to the set K_{LN} is very salt(l_0).

Similarly, b's degree of appurtenance to the set K_{LN} is bitter(l_4), b's degree of indeterminate-appurtenance to the set K_{LN} is very salt(l_0), b's degree of non-appurtenance to the set K_{LN} is very sour(l_2).
And, c's degree of appurtenance to the set K_{LN} is very sour(l_2), c's degree of indeterminate-appurtenance to the set K_{LN} is sour(l_3), c's degree of non-appurtenance to the set K_{LN} is very salt(l_1).
Also, let $H_{LN} = \{s_2, (x, \langle l_6, l_0, l_0\rangle), (y, \langle l_0, l_4, l_2\rangle), (z, \langle l_3, l_2, l_1\rangle)\}$. Then, $\pi_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$ are linguistic neutrosophic topologies on S_1, S_2 respectively. Let $g : (S_1, \pi_{LN}) \to (S_2, \eta_{LN})$ be defined by $g(a) = b, g(b) = a, g(c) = c$. Then, g is linguistic neutrosophic continuous function.

Theorem 5.4. *A function $f : S_{LN} \to T_{LN}$ is linguistic neutrosophic continuous if and only if the pre image of every linguistic neutrosophic closed set in T_{LN} is linguistic neutrosophic closed in S_{LN}.*

Proof:

Let f be a LN continuous function and E_{LN} be a LN closed set in T_{LN}, (i.e) $T_{LN} - E_{LN}$ is LN open in T_{LN}. $f^{-1}(T_{LN} - E_{LN})$ is a LN open set in S_{LN}, as f is LN continuous function. Thus, $S_{LN} - f^{-1}(E_{LN})$ is LN open set in S_{LN}. That is, $f^{-1}(E_{LN})$ is LN closed set in S_{LN}.
Conversely, let the inverse image of each LN closed set be LN closed. Let F_{LN} be a LN open set in T_{LN}, (i.e) $T_{LN} - F_{LN}$ is LN closed. Then, $S_{LN} - f^{-1}(F_{LN})$ is LN closed set in S_{LN}, which implies, $f^{-1}(F_{LN})$ is LN open set in S_{LN}. Thus, f is LN continuous function on S_{LN}.

Theorem 5.5. *A function $f : S_{LN} \to T_{LN}$ is LN continuous if and only if $f(LNcl(K_{LN})) \subseteq LNcl(f(K_{LN}))$ for each subset K_{LN} of S_{LN}.*

Proof:

Let f be LN continuous function. If $K_{LN} \subseteq S_{LN}$, then $f(K_{LN}) \subseteq T_{LN}$. As f is LN continuous and $LNcl(f(K_{LN}))$ is LN closed in T_{LN}, $f^{-1}(LNcl(f(K_{LN})))$ is LN closed set in S_{LN}. Since, $f(K_{LN}) \subseteq LNcl(f(K_{LN})), K_{LN} \subseteq f^{-1}(LNcl(f(K_{LN})))$, which implies, $f^{-1}(LNcl(f(K_{LN})))$ is the smallest LN closed set that contains K_{LN}. But, $LNcl(K_{LN})$ is the smallest LN closed set that contains K_{LN}. Hence, $LNcl(K_{LN}) \subseteq f^{-1}(LNcl(f(K_{LN})))$, (i.e) $f(LNcl(K_{LN})) \subseteq LNcl(f(K_{LN}))$. Conversely, let $f(LNcl(H_{LN})) \subseteq LNcl(f(H_{LN}))$. If H_{LN} is LN closed in T_{LN}, $f(LNcl(f^{-1}(H_{LN}))) \subseteq LNcl(H_{LN})$. Thus, $LNcl(f^{-1}(H_{LN})) \subseteq f^{-1}((LNcl(H_{LN}))) = f^{-1}((H_{LN}))$. But, $f^{-1}((H_{LN})) \subseteq LNcl(f^{-1}((H_{LN})))$, that implies, $LNcl(f^{-1}((H_{LN}))) = f^{-1}((H_{LN})) \Rightarrow f^{-1}((H_{LN}))$ is LN closed set in S_{LN} for each LN closed set H_{LN} in T_{LN}. Therefore, f is LN continuous.

Example 5.6. In example(5.3), g is a linguistic neutrosophic continuous function. Let $K_{LN} = \{s_1, \langle a, (l_0, l_6, l_0)\rangle, \langle b, (l_4, l_0, l_2)\rangle, \langle c, (l_2, l_3, l_1)\rangle\} \subseteq (S_1, \pi_{LN})$. Then, $g(LNcl(K_{LN})) =$

$\{s_1, \langle a, (l_6, l_6, l_6)\rangle, \langle b, (l_0, l_4, l_2)\rangle, \langle c, (l_3, l_5, l_4)\rangle\}$.
But, $LNcl(g(K_{LN})) = LNcl(H_{LN}) = B^c \neq \{s_1, \langle a, (l_6, l_6, l_6)\rangle, \langle b, (l_0, l_4, l_2)\rangle, \langle c, (l_3, l_5, l_4)\rangle\}$, even though g is linguistic neutrosophic continuous function. Thus, equality is not necessarily holds when g is linguistic neutrosophic continuous function.

Theorem 5.7. *A function $f : S_{LN} \to T_{LN}$ is LN continuous if and only if $LNcl(f^{-1}(E_{LN})) \subseteq f^{-1}(LNcl(E_{LN}))$ for each subset E_{LN} of T_{LN}.*

Proof:

If f is LN continuous and $E_{LN} \subseteq T_{LN}$, then $LNcl(E_{LN})$ is LN closed in T_{LN} and hence $f^{-1}(LNcl(E_{LN}))$ is LN closed in S_{LN}. Thus, $LNcl(f^{-1}(LNcl(E_{LN}))) = f^{-1}(LNcl(E_{LN}))$. Since, $E_{LN} \subseteq LNcl(E_{LN}), f^{-1}(E_{LN}) \subseteq f^{-1}(LNcl(E_{LN}))$. Therefore, $LNcl(f^{-1}(E_{LN})) \subseteq LNcl(f^{-1}(LNcl(E_{LN}))) = f^{-1}(LNcl(E_{LN}))$, (i.e) $LNcl(f^{-1}(E_{LN})) \subseteq f^{-1}(LNcl(E_{LN}))$. Conversely, let $LNcl(f^{-1}(E_{LN}))f^{-1}(LNcl(E_{LN}))$ for all E_{LN} of T_{LN}. If E_{LN} is LN closed, then $LNcl(E_{LN}) = E_{LN}$. By assumption, $LNcl(f^{-1}(E_{LN})) \subseteq f^{-1}(LNcl(E_{LN}))$. Thus, $LNcl(f^{-1}(E_{LN})) \subseteq f^{-1}(E_{LN})$. But, $f^{-1}(E_{LN}) \subseteq LNcl(f^{-1}(E_{LN}))$. Thus, $LNcl(f^{-1}(E_{LN})) = f^{-1}(E_{LN})$, (i.e) $f^{-1}(E_{LN})$ is LN closed in S_{LN} for every LN closed set E_{LN} in T_{LN}. Hence, f is LN continuous.

Theorem 5.8. *A function $f : S_{LN} \to T_{LN}$ is LN continuous if and only if $f^{-1}(LNint(E_{LN})) \subseteq LNint(f^{-1}(E_{LN}))$ for each subset E_{LN} of T_{LN}.*

Proof:

Let f be LN continuous function and $E \subseteq T_{LN}$. Then, $f^{-1}(LNint(E_{LN}))$ is LN open in S_{LN}. That means, $f^{-1}(LNint(E_{LN})) = LNint(f^{-1}(LNint(E_{LN})))$. As $LNint(E_{LN}) \subseteq E_{LN}$, implies $f^{-1}(LNint(E_{LN})) \subseteq f^{-1}(E_{LN})$. Thus, $LNint(f^{-1}(LNint(E_{LN}))) \subseteq LNint(f^{-1}(E_{LN}))$. Therefore,$f^{-1}(LNint(E_{LN})) \subseteq LNint(f^{-1}(E_{LN}))$. Conversely, let $f^{-1}(LNint(E_{LN})) \subseteq LNint(f^{-1}(E_{LN}))$, for each subset E_{LN} of T_{LN}. If E_{LN} is LN open, then $f^{-1}(E_{LN}) \subseteq LNint(f^{-1}(E_{LN}))$. But, $LNint(f^{-1}(E_{LN})) \subseteq f^{-1}(E_{LN})$. Thus, $f^{-1}(E_{LN}) = LNint(f^{-1}(E_{LN}))$. Hence, f is LN continuous.

Example 5.9. In example(5.3), $H_{LN} = \{s_2, \langle x, (l_6, l_0, l_0)\rangle, \langle y, (l_0, l_4, l_2)\rangle, \langle z, (l_3, l_2, l_1)\rangle\}$. Then, $g^{-1}(LNcl(H_{LN})) = g^{-1}({H_{LN}}^c) = \{s_2, \langle z, (l_0, l_6, l_0)\rangle, \langle y, (l_4, l_4, l_6)\rangle, \langle z, (l_2, l_5, l_3)\rangle\}$. And $LNcl(g^{-1}(H_{LN})) = {K_{LN}}^c$. Thus, $g^{-1}(LNcl(E_{LN})) \subseteq LNcl(g^{-1}(E_{LN}))$. Similarly, $g^{-1}(LNint(E_{LN})) \subseteq LNint(g^{-1}(E_{LN}))$. Even if g is LN continuous, equality does not hold in theorems (5.7) and (5.8).

Definition 5.10. Any subset of a LN topological space (S_{LN}, π_{LN}) is a LN dense set if $LNcl(K_{LN}) = S_{LN})$.

Theorem 5.11. *Let $f : S_{LN} \to T_{LN}$ be an onto function and linguistic neutrosophic continuous function. If U_{LN} is LN dense in S_{LN}, then $f(U_{LN})$ is LN dense in T_{LN}.*

Proof:

As U_{LN} is LN dense in S_{LN}, $f(LNcl(U_{LN})) = f(S_{LN}) = T_{LN}$, since f is onto. Also, $f(LNcl(U_{LN})) \subseteq LNcl(U_{LN})$, as f is LN continuous. Thus, $T_{LN} = LNcl(f(U_{LN}))$. But $LNcl(f(U_{LN})) \subseteq T_{LN}$. Hence,$LNcl(f(U_{LN})) = T_{LN}$, which implies, $f(U_{LN})$ is LN dense set.

Conclusion

We have introduced a new type of topology called linguistic neutrosophic topology and it was established with apt examples. Moreover, the basic properties of linguistic neutrosophic were discussed. In addition to this, the ideas of linguistic neutrosophic continuity and linguistic neutrosophic neighborhood were introduced and established. Linguistic neutrosophic derived sets and linguistic neutrosophic dense sets were talked through.

References

[1] Atanassov, K.T. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 1986; 20, pp. 87–96.

[2] Chang, C.L. Fuzzy topological spaces, J Math.Anal.Appl. 1968; 24, pp. 182–190.

[3] Chen, Z.C.; Liu, P.H.; Pei, Z. An approach to multiple attribute group decision making based on linguistic intuitionistic fuzzy numbers. Int. J. Comput. Intell. Syst. 2015, 8, 747–760.

[4] Coker, D. An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 1997; 88, pp. 81-89.

[5] Fan, Feng, and Hu. (2019). Linguistic Neutrosophic Numbers Einstein Operator and Its Application in Decision Making. Mathematics, 7(5), 389. doi:10.3390/math7050389

[6] Fang, Zebo and Te, Jun. Multiple Attribute Group Decision-Making Method Based on Linguistic Neutrosophic Numbers.Symmetry,9(7), 2017. 111; https://doi.org/10.3390/sTm9070111.

[7] Garg H and Nancy. Linguistic single-valued neutrosophic power aggregation operators and their applications to group decision-making problems. IEEE/CAA J. Autom. Sinica, vol. 7, no. 2, pp. 546–558, Mar. 2020.

[8] Herrera, F.; Herrera-Viedma, E.; Verdegay, L. A model of consensus in group decision making under linguistic assessments. Fuzzy Sets Syst. 1996, 79, 73–87.

[9] Herrera F.; Herrera-Viedma E. Linguistic decision analysis . Steps for solving decision problems under linguistic information. Fuzzy Sets and Systems, 2000, 115(1): 67–82.

[10] Munkres, James R. Topology: a First Course. Englewood Cliffs, N.J.: Prentice-Hall, 1974.

[11] Smarandache F. A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic. American Research Press, Rehoboth, 1999.

[12] Su Z S. Deviation measures of linguistic preference relations in group decision making. Omega, 2005, 33(3):249–254.

[13] Wang H, Smarandache F, Zhang T Q, Sunderraman R. Interval neutrosophic sets and logic: Theory and applications in computing. Hexis, Phoenis, AZ, 2005.

[14] Wei, G., Wu, J., Guo, Y., Wang, J., and Wei, C. (2021). An extended COPRAS model for multiple attribute group decision making based on single-valued neutrosophic 2-tuple linguistic environment. Technological and Economic Development of Economy, 27(2), 353-368. https://doi.org/10.3846/tede.2021.14057.

[15] Ye, J. An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. J. Intell. Fuzzy Syst. 2015, 28, 247–255.

[16] Zadeh, L.A. Fuzzy Sets. Information and Control, 1965; 8,pp. 338–353.

[17] Zadeh, L.A. The concept of a linguistic variable and its application to approximate reasoning Part I. Inf. Sci. 1975, 8, 199–249.

Received: May 29, 2021. Accepted: October 1, 2021