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Decision-Making Application Based on Aggregations of Complex Fuzzy Hypersoft Set and Development of Interval-Valued Complex Fuzzy Hypersoft Set

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Abstract. Hypersoft set, an extension of soft set, deals with disjoint attribute-valued sets corresponding to distinct attributes. In this study, the innovation of complex fuzzy hypersoft set (CFH-set) is conferred, which can tackle with uncertainties and vagueness that lie in the data by taking into account the amplitude and phase terms of the complex numbers at the same time. This model establishes a gluing framework of the fuzzy set and hypersoft set characterized in the complex plane. This structure is more flexible and useful as it consents a broad range of values for membership function by expanding them to the unit circle in a complex plane through the characterization of the fuzzy hypersoft set to consider the periodic nature of the information and the attributes can further be classified into attribute-values sets for vivid understanding. With the characterization of its some fundamental properties and operations, aggregations of complex fuzzy hypersoft set: matrix, cardinal set, cardinal matrix of cardinal set, aggregation operator/set and matrix of aggregation set, are conceptualized along with application in decision-making. Moreover, complex interval-valued fuzzy hypersoft set is developed and some of its fundamentals i.e. subset, equal sets, null set, absolute set etc. and theoretic operations i.e. compliment, union, intersection etc. are investigated.

Keywords: Complex fuzzy sets (CF-Sets), soft set, hypersoft set and complex fuzzy hypersoft set.

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1. Introduction

The concept of complex fuzzy set theory (CFS-Theory) [1] is an extension of fuzzy set theory (FS-Theory) [2], which uses complex-valued state for the membership of its elements. FS-Theory and CFS-Theory have some kind of complexities which restrain them to solve problem involving uncertainty professionally. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. It demands a mathematical tool free of all such impediments to tackle such issues. This scantiness is resolved with the development of soft set theory (SS-Theory) [7] which is a new parameterized family of subsets of the universe of discourse. The researchers [8]- [17] studied and investigated some elementary properties, operations, laws and hybrids of SS-Theory with applications in decision making. The gluing concept of NS-Theory and SS-Theory, is studied in [18] to make the NS-Theory adequate with parameterized tool. In many real life situations, distinct attributes are further partitioned in disjoint attribute-valued sets but existing SS-Theory is insufficient for dealing with such kind of attribute-valued sets. Hypersoft set theory (HS-Theory) [19] is developed to make the SST in line with attribute-valued sets to tackle real life scenarios. HS-Theory is an extension of SS-Theory as it transforms the single argument function into a multi-argument function. Certain elementary properties, aggregation operations, laws, relations and functions of HS-Theory, are investigated by [20]- [22] for proper understanding and further utilization in different fields. The applications of HS-Theory in decision making is studied by [23]- [27] and the intermingling study of HS-Theory with complex sets, convex and concave sets is studied by [28, 29]. Deli [30] characterized hybrid set structures under uncertainly parameterized hypersoft sets with theory and applications. Gayen et al. [31] analyzed some essential aspects of plithogenic hypersoft algebraic structures. They also investigated the notions and basic properties of plithogenic hypersoft subgroups ie plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup. Saeed et al. [32, 33] discussed decision making techniques for neutrosophic hypersoft mapping and complex multi-fuzzy hypersoft set. Rahman et al. [34–36] studied decision making applications based on neutrosophic parameterized hypersoft Set, fuzzy parameterized hypersoft set and rough hypersoft set. Ihsan et al. [37] investigated hypersoft expert set with application in decision making for the best selection of product.

1.1. *Motivation*

In order to address the limitation of fuzzy soft set for dealing with periodic nature of data, Thirunavukarasu et al. [38] developed the theory of complex fuzzy soft set and discussed its some fundamentals along with applications. Kumar et al. [39] extended the work of Thirunavukarasu et al. to complex intuitionistic fuzzy soft sets and calculated its distance measures and entropies. Selvachandran et al. [40] investigated interval-valued complex fuzzy soft set with application. Abd et al. [41] discussed the fundamentals, properties and application of complex generalised fuzzy soft sets. These existing models employed single set of attributes for dealing uncertainties under fuzzy set-like environments but there are many situations when each attribute is required to be further partitioned into its attribute-valued set.

These existing structures has limitation regarding the consideration of such attribute-valued sets. Inspiring from the above literature, the decision system of complex fuzzy hypersoft set is developed with the help of the characterization of its aggregation operations and fundamental theory of interval-valued complex fuzzy hypersoft set is investigated. The proposed structure complex fuzzy hypersoft set (CFH-set) and interval-valued complex hypersoft set (IV-CFHS) are more flexible and useful as they

(i) generalize the existing structures of complex fuzzy soft set.
(ii) permit a broad range of values for membership function by expanding them to the unit circle in a complex plane.
(iii) consider the periodic nature of the information through the phase-terms.
(iv) classify distinct attributes into corresponding attribute-values sets for vivid understanding.

1.2. *Organization of Paper*

The rest of the paper is organized as: section 2 reviews the notions of fuzzy set, soft set, complex fuzzy set and relevant definitions used in the proposed work. Section 3, presents the decision system of complex fuzzy hypersoft set based on its some decisive aggregation operations along with application in decision-making. Section 4, investigates the fundamental theory of interval-valued complex fuzzy hypersoft set. Lastly, paper is summarized with future directions.

2. **Preliminaries**

Here some existing fundamental concepts regarding fuzzy set, fuzzy soft set and fuzzy hypersoft set are presented along with their structures with complex fuzzy set from literature. Throughout the paper, $\mathbb{U}$, $P(\mathbb{U})$, $F(\mathbb{U})$, $C(\mathbb{U})$ and $C_h(\mathbb{U})$ will present universe of discourse, power set of $\mathbb{U}$, collection of fuzzy sets, collection of complex fuzzy sets on soft sets and collection of complex fuzzy sets on hypersoft sets respectively.

Definition 2.1. [2]
Suppose a universal set $\mathbb{U}$ and a *fuzzy set* $X \subseteq \mathbb{U}$. The set X will be written as $X = \{(x, \alpha_X(x))|x \in \mathbb{U}\}$ such that

$$\alpha_X : \mathbb{U} \to [0, 1]$$

where $\alpha_X(x)$ describes the membership percentage of $x \in X$.

Definition 2.2. [1]
A *complex fuzzy set* $\mathbb{C}_f$ is of the form

$$\mathbb{C}_f = \left\{(\epsilon, \mu_{\mathbb{C}_f}(\epsilon)) : \epsilon \in \mathbb{U}\right\} = \left\{(\epsilon, r_{\mathbb{C}_f}(\epsilon)e^{i\omega_{\mathbb{C}_f}(\epsilon)}) : \epsilon \in \mathbb{U}\right\}.$$

where $\mu_{\mathbb{C}_f}(\epsilon)$ is a membership function of $\mathbb{C}_f$ with $r_{\mathbb{C}_f}(\epsilon) \in [0,1]$ and $\omega_{\mathbb{C}_f}(\epsilon) \in (0, 2\pi]$ as amplitude and phase terms respectively and $i = \sqrt{-1}$.

Buckley [3] and Zhang et al. [4] presented fuzzy complex number in different way. However, according to [5]- [6], both amplitude and phase terms are captured by fuzzy sets.

Definition 2.3. [7]
A *soft set* $\mathfrak{S}$ over $\mathbb{U}$, is defined as

$$\mathfrak{S} = \{(\epsilon, f_{\mathfrak{S}}(\epsilon)) : \epsilon \in E_1\}$$

where $f_{\mathfrak{S}} : E_1 \to P(\mathbb{U})$. and $E_1 \subseteq E$ (set of parameters).

Definition 2.4. [9]
A *fuzzy soft set* (FS-set) Γ_{E_1} on $\mathbb{U}$, is defined as

$$\Gamma_{E_1} = \{(\epsilon, \gamma_{E_1}(\epsilon)) : \epsilon \in E_1, \gamma_{E_1}(\epsilon) \in F(\mathbb{U})\}$$

where $\gamma_{E_1} : E_1 \to F(\mathbb{U})$ such that $\gamma_{E_1}(\epsilon) = \emptyset$ if $\epsilon \notin E_1$, and for all $\epsilon \in E_1$,

$$\gamma_{E_1}(\epsilon) = \left\{\mu_{\gamma_{E_1}(\epsilon)}(v)/v : v \in \mathbb{U}, \mu_{\gamma_{E_1}(\epsilon)}(v) \in [0,1]\right\}$$

is a fuzzy set over $\mathbb{U}$. Also γ_{E_1} is the approximate function of Γ_{E_1} and the value $\gamma_A(x)$ is a fuzzy set called ϵ-element of FS-set. Note that if $\gamma_{E_1}(\epsilon) = \emptyset$, then $(\epsilon, \gamma_{E_1}(\epsilon)) \notin \Gamma_{E_1}$.

Definition 2.5. [38]
A *complex fuzzy soft set* (CFS-set) χ_{E_1} over $\mathbb{U}$, is defined as

$$\chi_{E_1} = \{(\epsilon, \psi_{E_1}(\epsilon)) : \epsilon \in E_1, \psi_{E_1}(\epsilon) \in C(\mathbb{U})\}.$$

where $\psi_{E_1} : E_1 \to C(\mathbb{U})$ such that $\psi_{E_1}(\epsilon) = \emptyset$ if $\epsilon \notin E_1$ and it is complex fuzzy approximate function of CFS-set χ_{E_1} and its value $\psi_{E_1}(\epsilon)$ is called ϵ-member of CFS-set χ_{E_1} for all $\epsilon \in E_1$. Operations of CF-sets and CFS-sets were defined in [1] and [38] respectively.

Definition 2.6. [19]
The pair (H, G) is called a *hypersoft set* over $\mathbb{U}$, where G is the cartesian product of n disjoint sets $H_1, H_2, H_3, ..., H_n$ having attribute values of n distinct attributes $h_1, h_2, h_3, ..., h_n$ respectively and $H : G \to P(\mathbb{U})$.

Definition 2.7. [19]
A hypersoft set over a fuzzy universe of discourse is called *fuzzy hypersoft set.*

For more definitions and operations of hypersoft set, see [20]- [22]

2.1. *Complex Fuzzy Hypersoft Set*

The following subsections 2.1 and 2.2 are reviewed from [28].

Definition 2.8. Let $A_1, A_2, A_3, ..., A_n$ are disjoint sets having attribute values of n distinct attributes $a_1, a_2, a_3, ..., a_n$ respectively for $n \geq 1, G = A_1 \times A_2 \times A_3 \times ..... \times A_n$ and $\psi(\underline{x})$ be a CF-set over $\mathbb{U}$ for all $\underline{\epsilon} = (d_1, d_2, d_3, ..., d_n) \in G$. Then, *complex fuzzy hypersoft set* (CFH-set) χ_G over $\mathbb{U}$ is defined as

$$\chi_G = \{(\underline{\epsilon}, \psi(\underline{\epsilon})) : \underline{\epsilon} \in G, \psi(\underline{\epsilon}) \in C(\mathbb{U})\}$$

where

$$\psi : G \to C(\mathbb{U}), \qquad \psi(\underline{\epsilon}) = \emptyset \ if \ \underline{\epsilon} \notin G.$$

is a CF-approximate function of χ_G and its value $\psi(\underline{\epsilon})$ is called $\underline{\epsilon}$-member of CFH-set $\forall \underline{\epsilon} \in G$.

Example 2.9. Suppose a Department Promotion Committee (DPC) wants to observe(evaluate) the characteristics of some teachers by some defined indicators for departmental promotion. For this purpose, consider a set of teachers as a universe of discourse $\mathbb{U} = \{t_1, t_2, t_3, t_4\}$. The attributes of the teachers under consideration are the set $E = \{A_1, A_2, A_3\}$, where

$A_1 =$ Total experience in years $= \{3, < 10\} = \{e_{11}, e_{12}\}$

$A_2 =$ Total no. of publications$= \{10, 10 <\} = \{e_{21}, e_{22}\}$

$A_3 =$ Performance Evaluation Report (PER) remarks $= \{eligible, not\, eligible\} = \{e_{31}, e_{32}\}$

and

$$G = A_1 \times A_2 \times A_3 = \left\{ \begin{array}{l} (e_{11}, e_{21}, e_{31}), (e_{11}, e_{21}, e_{32}), (e_{11}, e_{22}, e_{31}), \\ (e_{11}, e_{22}, e_{32}), (e_{12}, e_{21}, e_{31}), (e_{12}, e_{21}, e_{32}), \\ (e_{12}, e_{22}, e_{31}), (e_{12}, e_{22}, e_{32}) \end{array} \right\} = \{e_1, e_2, e_3, ..., e_8\}$$

Complex fuzzy set $\psi_G(e_1), \psi_G(e_2), ..., \psi_G(e_8)$ are defined as,

$$\psi_G(e_1) = \left\{ \frac{0.4e^{i0.5\pi}}{t_1}, \frac{0.8e^{i0.6\pi}}{t_2}, \frac{0.8e^{i0.8\pi}}{t_3}, \frac{1.0e^{i0.75\pi}}{t_4} \right\},$$

$$\psi_G(e_2) = \left\{ \frac{0.6e^{i0.7\pi}}{t_1}, \frac{0.9e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_3) = \left\{ \frac{0.5e^{i0.6\pi}}{t_1}, \frac{0.8e^{i0.9\pi}}{t_2}, \frac{0.6e^{i0.9\pi}}{t_3}, \frac{0.65e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_4) = \left\{ \frac{0.3e^{i0.7\pi}}{t_1}, \frac{0.7e^{i0.9\pi}}{t_2}, \frac{0.5e^{i0.9\pi}}{t_3}, \frac{0.75e^{i0.65\pi}}{t_4} \right\},$$

$$\psi_G(e_5) = \left\{ \frac{0.2e^{i0.5\pi}}{t_1}, \frac{0.3e^{i0.8\pi}}{t_2}, \frac{0.8e^{i0.7\pi}}{t_3}, \frac{0.45e^{i0.65\pi}}{t_4} \right\},$$

$$\psi_G(e_6) = \left\{ \frac{0.5e^{i0.9\pi}}{t_1}, \frac{0.3e^{i0.9\pi}}{t_2}, \frac{0.7e^{i0.8\pi}}{t_3}, \frac{0.85e^{i0.95\pi}}{t_4} \right\},$$

$$\psi_G(e_7) = \left\{ \frac{0.6e^{i0.9\pi}}{t_1}, \frac{0.9e^{i0.6\pi}}{t_2}, \frac{0.5e^{i0.6\pi}}{t_3}, \frac{0.85e^{i0.75\pi}}{t_4} \right\},$$

and

$$\psi_G(e_8)=\left\{\frac{0.8e^{i0.9\pi}}{t_1},\frac{0.8e^{i0.8\pi}}{t_2},\frac{0.6e^{i0.8\pi}}{t_3},\frac{0.65e^{i0.85\pi}}{t_4}\right\}$$

then CFH-set χ_G is written by,

$$\chi_G=\left\{\begin{array}{l}(e_1,\frac{0.4e^{i0.5\pi}}{t_1},\frac{0.8e^{i0.6\pi}}{t_2},\frac{0.8e^{i0.8\pi}}{t_3},\frac{1.0e^{i0.75\pi}}{t_4}),(e_2,\frac{0.6e^{i0.7\pi}}{t_1},\frac{0.9e^{i0.9\pi}}{t_2},\frac{0.7e^{i0.9\pi}}{t_3},\frac{0.75e^{i0.95\pi}}{t_4}),\\(e_3,\frac{0.5e^{i0.6\pi}}{t_1},\frac{0.8e^{i0.9\pi}}{t_2},\frac{0.6e^{i0.9\pi}}{t_3},\frac{0.65e^{i0.95\pi}}{t_4}),(e_4,\frac{0.3e^{i0.7\pi}}{t_1},\frac{0.7e^{i0.9\pi}}{t_2},\frac{0.5e^{i0.9\pi}}{t_3},\frac{0.75e^{i0.65\pi}}{t_4}),\\(e_5,\frac{0.2e^{i0.5\pi}}{t_1},\frac{0.3e^{i0.8\pi}}{t_2},\frac{0.8e^{i0.7\pi}}{t_3},\frac{0.45e^{i0.65\pi}}{t_4}),(e_6,\frac{0.5e^{i0.9\pi}}{t_1},\frac{0.3e^{i0.9\pi}}{t_2},\frac{0.7e^{i0.8\pi}}{t_3},\frac{0.85e^{i0.95\pi}}{t_4}),\\(e_7,\frac{0.6e^{i0.9\pi}}{t_1},\frac{0.9e^{i0.6\pi}}{t_2},\frac{0.5e^{i0.6\pi}}{t_3},\frac{0.85e^{i0.75\pi}}{t_4}),(e_8,\frac{0.8e^{i0.9\pi}}{t_1},\frac{0.8e^{i0.8\pi}}{t_2},\frac{0.6e^{i0.8\pi}}{t_3},\frac{0.65e^{i0.85\pi}}{t_4})\end{array}\right\}$$

Definition 2.10. Let $\chi_{G_1}=(\psi_1,G_1)$ and $\chi_{G_2}=(\psi_2,G_2)$ be two CFH-sets over the same $\mathbb{U}$. The set $\chi_{G_1}=(\psi_1,G_1)$ is said to be the *subset* of $\chi_{G_2}=(\psi_2,G_2)$, if

i. $G_1\subseteq G_2$

ii. $\forall\ \underline{x}\in G_1,\psi_1(\underline{x})\subseteq\psi_2(\underline{x})$ i.e. $r_{G_1}(\underline{x})\leq r_{G_2}(\underline{x})$ and $\omega_{G_1}(\underline{x})\leq\omega_{G_2}(\underline{x})$, where $r_{G_1}(\underline{x})$ and $\omega_{G_1}(\underline{x})$ are amplitude and phase terms of $\psi_1(\underline{x})$, whereas $r_{G_2}(\underline{x})$ and $\omega_{G_2}(\underline{x})$ are amplitude and phase terms of $\psi_2(\underline{x})$.

Definition 2.11. Two CFH-sets $\chi_{G_1}=(\psi_1,G_1)$ and $\chi_{G_2}=(\psi_2,G_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\psi_1,G_1)\subseteq(\psi_2,G_2)$

ii. $(\psi_2,G_2)\subseteq(\psi_1,G_1)$.

Definition 2.12. Let (ψ,G) be a CFH-set over $\mathbb{U}$.Then

i. (ψ,G) is called a *null CFH-set*, denoted by $(\psi,G)_\Phi$ if for all $\underline{x}\in G$, the amplitude and phase terms of the membership function are given by $r_G(\underline{x})=0$ and $\omega_G(\underline{x})=0\pi$ respectively.

ii. (ψ,G) is called a *absolute CFH-set*, denoted by $(\psi,G)_\Delta$ if for all $\underline{x}\in G$, the amplitude and phase terms of the membership function are given by $r_G(\underline{x})=1$ and $\omega_G(\underline{x})=2\pi$ respectively.

Definition 2.13. Let (ψ_1,G_1) and (ψ_2,G_2) are two CFH-sets over the same universe $\mathbb{U}$.Then

i. A CFH-set (ψ_1,G_1) is called a *homogeneous CFH-set*, denoted by $(\psi_1,G_1)_{Hom}$ if and only if $\psi_1(\underline{x})$ is a homogeneous CF-set for all $\underline{x}\in G_1$.

ii. A CFH-set (ψ_1,G_1) is called a *completely homogeneous CFH-set*, denoted by $(\psi_1,G_1)_{CHom}$ if and only if $\psi_1(\underline{x})$ is a homogeneous with $\psi_1(\underline{y})$ for all $\underline{x},\underline{y}\in G_1$.

iii. A CFH-set (ψ_1,G_1) is said to be a completely homogeneous CFH-set with (ψ_2,G_2) if and only if $\psi_1(\underline{x})$ is a homogeneous with $\psi_2(\underline{x})$ for all $\underline{x}\in G_1\cap G_2$.

2.2. *Set Theoretic Operations and Laws on CFH-Sets*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on CFH-sets.

Definition 2.14. The *complement* of CFH-set (ψ, G), denoted by $(\psi, G)^c$ is defined as

$$(\psi, G)^c = \{(\underline{x}, \psi^c(\underline{x})) : \underline{x} \in G, \psi^c(\underline{x}) \in C(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $\psi^c(\underline{x})$ are given by $r^c_G(\underline{x}) = 1 - r_G(\underline{x})$ and $\omega^c_G(\underline{x}) = 2\pi - \omega_G(\underline{x})$ respectively.

Proposition 2.15. *Let* (ψ, G) *be a CFH-set over* $\mathbb{U}$*. Then* $((\psi, G)^c)^c = (\psi, G)$.

Proof. Since $\psi(\underline{x}) \in C(\mathbb{U})$, therefore (ψ, G) can be written in terms of its amplitude and phase terms as

$$(\psi, G) = \left\{ \left(\underline{x}, r_G(\underline{x}) e^{i\omega_G(\underline{x})}\right) : \underline{x} \in G \right\} \tag{1}$$

Now

$$\psi^c(\underline{x}) = \left\{ \left(\underline{x}, r^c_G(\underline{x}) e^{i\omega^c_G(\underline{x})}\right) : \underline{x} \in G \right\}$$

$$\psi^c(\underline{x}) = \left\{ \left(\underline{x}, (1 - r_G(\underline{x})) e^{i(2\pi - \omega_G(\underline{x}))}\right) : \underline{x} \in G \right\}$$

$$((\psi, G)^c)^c = \left\{ \left(\underline{x}, (1 - r_G(\underline{x}))^c e^{i(2\pi - \omega_G(\underline{x})^c)}\right) : \underline{x} \in G \right\}$$

$$((\psi, G)^c)^c = \left\{ \left(\underline{x}, (1 - (1 - r_G(\underline{x}))) e^{i(2\pi - (2\pi - \omega_G(\underline{x})))}\right) : \underline{x} \in G \right\}$$

$$((\psi, G)^c)^c = \left\{ \left(\underline{x}, r_G(\underline{x}) e^{i\omega_G(\underline{x})}\right) : \underline{x} \in G \right\} \tag{2}$$

from equations (1) and (2), we have $((\psi, G)^c)^c = (\psi, G)$. □

Proposition 2.16. *Let* (ψ, G) *be a CFH-set over* $\mathbb{U}$*. Then*

i. $((\psi, G)_\Phi)^c = (\psi, G)_\Delta$
ii. $((\psi, G)_\Delta)^c = (\psi, G)_\Phi$

Definition 2.17. The *intersection* of two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) over the same universe $\mathbb{U}$, denoted by $(\psi_1, G_1) \cap (\psi_2, G_2)$, is the CFH-set (ψ_3, G_3), where $G_3 = G_1 \cap G_2$, and $\psi_3(\underline{x}) = \psi_1(\underline{x}) \cap \psi_2(\underline{x})$ for all $\underline{x} \in G_3$.

Definition 2.18. The *difference* between two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) is defined as

$$(\psi_1, G_1) \setminus (\psi_2, G_2) = (\psi_1, G_1) \cap (\psi_2, G_2)^c$$

Definition 2.19. The *union* of two CFH-sets (ψ_1, G_1) and (ψ_2, G_2) over the same universe $\mathbb{U}$, denoted by $(\psi_1, G_1) \cup (\psi_2, G_2)$, is the CFH-set (ψ_3, G_3), where $G_3 = G_1 \cup G_2$, and for all $\underline{x} \in G_3$,

$$\psi_3(\underline{x}) = \begin{cases} \psi_1(\underline{x}) & , if\ \underline{x} \in G_1 \backslash G_2 \\ \psi_2(\underline{x}) & , if\ \underline{x} \in G_2 \backslash G_1 \\ \psi_1(\underline{x}) \cup \psi_2(\underline{x}) & , if\ \underline{x} \in G_1 \cap G_2 \end{cases}$$

Proposition 2.20. *Let (ψ, G) be a CFH-set over $\mathbb{U}$.Then the following results hold true:*

i. $(\psi, G) \cup (\psi, G)_\Phi = (\psi, G)$
ii. $(\psi, G) \cup (\psi, G)_\Delta = (\psi, G)_\Delta$
iii. $(\psi, G) \cap (\psi, G)_\Phi = (\psi, G)_\Phi$
iv. $(\psi, G) \cap (\psi, G)_\Delta = (\psi, G)$
v. $(\psi, G)_\Phi \cup (\psi, G)_\Delta = (\psi, G)_\Delta$
vi. $(\psi, G)_\Phi \cap (\psi, G)_\Delta = (\psi, G)_\Phi$

Proposition 2.21. *Let (ψ_1, G_1), (ψ_2, G_2) and (ψ_3, G_3) are three CFH-sets over the same universe $\mathbb{U}$. Then the following commutative and associative laws hold true:*

i. $(\psi_1, G_1) \cap (\psi_2, G_2) = (\psi_2, G_2) \cap (\psi_1, G_1)$
ii. $(\psi_1, G_1) \cup (\psi_2, G_2) = (\psi_2, G_2) \cup (\psi_1, G_1)$
iii. $(\psi_1, G_1) \cap ((\psi_2, G_2) \cap (\psi_3, G_3)) = ((\psi_1, G_1) \cap (\psi_2, G_2)) \cap (\psi_3, G_3)$
iv. $(\psi_1, G_1) \cup ((\psi_2, G_2) \cup (\psi_3, G_3)) = ((\psi_1, G_1) \cup (\psi_2, G_2)) \cup (\psi_3, G_3)$

Proposition 2.22. *Let (ψ_1, G_1) and (ψ_2, G_2) are two CFH-sets over the same universe $\mathbb{U}$. Then the following De Moganss laws hold true:*

i. $((\psi_1, G_1) \cap (\psi_2, G_2))^c = (\psi_1, G_1)^c \cup (\psi_2, G_2)^c$
ii. $(\psi_1, G_1) \cup (\psi_2, G_2))^c = (\psi_1, G_1)^c \cap (\psi_2, G_2)^c$

3. Aggregation of Complex Fuzzy Hypersoft Set

In this section, we define an aggregation operator on complex fuzzy hypersoft set that produces an aggregate fuzzy set from a complex fuzzy hypersoft set and its cardinal set. The approximate functions of a complex fuzzy hypersoft set are fuzzy. Here G, E, χ_G and $C_H(\mathbb{U})$ will be in accordance with definition (2.8).

Definition 3.1. Let $\chi_G \in C_H(\mathbb{U})$. Assume that $\mathbb{U} = \{u_1, u_2, ..., u_m\}$ and $E = \{A_1, A_2, ..., A_n\}$ with
$A_1 = \{e_{11}, e_{12}, ..., e_{1n}\}, A_2 = \{e_{21}, e_{22}, ..., e_{2n}\}, ..., A_n = \{e_{n1}, e_{n2}, ..., e_{nn}\}$
and $G = A_1 \times A_2 \times ..... \times A_n = \{x_1, x_2, ..., x_n, ..., x_{n^n} = x_r\}$, each x_i is n-tuple element of G and $|G| = r = n^n$ then the χ_G can be presented as

χ_G	x_1	x_2	...	x_r
u_1	$\mu_{\psi_G(x_1)}(u_1)$	$\mu_{\psi_G(x_2)}(u_1)$	$\cdots$	$\mu_{\psi_G(x_n)}(u_1)$
u_2	$\mu_{\psi_G(x_1)}(u_2)$	$\mu_{\psi_G(x_2)}(u_2)$	$\cdots$	$\mu_{\psi_G(x_n)}(u_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\mu_{\psi_G(x_1)}(u_m)$	$\mu_{\psi_G(x_2)}(u_m)$	$\cdots$	$\mu_{\psi_G(x_n)}(u_m)$

Where $\mu_{\psi_G(x)}$ is the membership function of ψ_G. If $a_{ij} = \mu_{\psi_G(x_j)}(u_i)$, for $i = \mathbb{N}_1^m$ and $j = \mathbb{N}_1^r$ then CFH-set χ_G is uniquely characterized by a matrix,

$$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is called an $m \times r$ CFH-set matrix..

Definition 3.2. Let $\chi_G \in C_H(\mathbb{U})$. Then, the *cardinal set* of χ_G is defined as

$$\|\chi_G\| = \left\{\mu_{\|\chi_G\|}(\underline{x})/\underline{x} : \underline{x} \in G\right\},$$

where $\mu_{\|\chi_G\|} : G \to [0,1]$ is a membership function of $\|\chi_G\|$ with $\mu_{Card(\chi_G)(\underline{x})} = \frac{|\psi_G(\underline{x})|}{|U|}$.
Note that $\|C_H(\mathbb{U})\|$ is the collection of all cardinal sets of CFH-sets and $\|C_H(\mathbb{U})\| \subseteq F(G)$.

Definition 3.3. Let $\chi_G \in C_H(\mathbb{U})$ and $\|\chi_G\| \in \|C_H(\mathbb{U})\|$. Consider E as in definition (4.1) then $\|\chi_G\|$ can be presented as

G	x_1	x_2	$\cdots$	x_r
$\mu_{\|\chi_G\|}$	$\mu_{\|\chi_G\|}(x_1)$	$\mu_{\|\chi_G\|}(x_2)$	$\cdots$	$\mu_{\|\chi_G\|}(x_r)$

If $a_{1j} = \mu_{\|\chi_G\|(x_j)}$, for $j = \mathbb{N}_1^r$ then the cardinal set $\|\chi_G\|$ is represented by a matrix,

$$[a_{ij}]_{1\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix}$$

and is called *cardinal matrix* of $\|\chi_G\|$.

Definition 3.4. Let $\chi_G \in C_H(\mathbb{U})$ and $\|\chi_G\| \in \|C_H(\mathbb{U})\|$. Then *CFH-aggregation operator* is defined as

$$\overbrace{\chi_G} = A_{CFH}\left(\|\chi_G\|, \chi_G\right)$$

where

$$A_{CFH} : \|C_H(\mathbb{U})\| \times C_H(\mathbb{U}) \to F(U).$$

$\overbrace{\chi_G}$ is called the aggregate fuzzy set of CFH-set χ_G.
Its membership function is given as

$$\mu_{\overbrace{\chi_G}} : U \to [0,1]$$

with

$$\mu_{\overbrace{\chi_G}}(u) = \frac{1}{|G|} \sum_{\underline{x} \in G} \mu_{Card(\chi_G)}(\underline{x})\mu_{Card(\psi_G)}(u).$$

Definition 3.5. Let $\chi_G \in C_H(\mathbb{U})$ and $\overbrace{\chi_G}$ be its aggregate fuzzy set. Assume that $\mathbb{U} = \{u_1, u_2, ..., u_m\}$, then $\overbrace{\chi_G}$ can be presented as

χ_G	$\mu_{\overbrace{\chi_G}}$
u_1	$\mu_{\overbrace{\chi_G}}(u_1)$
u_2	$\mu_{\overbrace{\chi_G}}(u_2)$
$\vdots$	$\vdots$
u_m	$\mu_{\overbrace{\chi_G}}(u_m)$

If $a_{i1} = \mu_{\overbrace{\chi_G}}(u_i)$ for $i = \mathbb{N}_1^m$ then $\overbrace{\chi_G}$ is represented by the matrix,

$$[a_{i1}]_{m\times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

which is called *aggregate matrix* of $\overbrace{\chi_G}$ over $\mathbb{U}$.

3.1. *Applications of Complex Fuzzy Hypersoft Set*

In this section, an algorithm is presented to solve the problems in decision making by having under consideration the concept of aggregations defined in previous section. An example is demonstrated to explain the proposed algorithm.

It is necessary to determine an aggregate fuzzy set of CFH-set for choosing the best option (parameter) from the given set (set of choices/alternatives). Following algorithm may help in making appropriate decision.

Step 1: Determine a CFH-set χ_G over $\mathbb{U}$,

Step 2: Determine $\| \chi_G \|$ for amplitude term and phase term separately,

Step 3: Find $\overbrace{\chi_G}$ for amplitude term and phase term separately,

Step 4: Find the best option by max modulus of $\mu_{\overbrace{\chi_G}}(u)$

Example 3.6. Suppose a business man wants to buy a share from share market. There are four same kind of share which form the set , $\mathbb{U} = \{s_1, s_2, s_3, s_4\}$. The expert committee consider a set of attributes , $E = \{e_1, e_2, e_3\}$. For i = 1, 2, 3, 4, the attributes e_i stand for current trend of company performance, particular companys stock price for last one year, and Home country inflation rate, respectively. Corresponding to each attribute, the sets of attribute values are: $A_1 = \{e_{11}, e_{12}\}$; $A_2 = \{e_{21}\}$ and $A_3 = \{e_{31}, e_{32}\}$. Then the set $G = A_1 \times A_2 \times A_3 = \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$ where each ϵ_i is a 3-tuple. Complex fuzzy sets $\psi_G(\epsilon_1), \psi_G(\epsilon_2), \psi_G(\epsilon_3), \psi_G(\epsilon_4)$ are defined as,

$$\psi_G(\epsilon_1) = \left\{ \frac{0.4e^{i0.5\pi}}{s_1}, \frac{0.8e^{i0.6\pi}}{s_2}, \frac{0.8e^{i0.8\pi}}{s_3}, \frac{1.0e^{i0.75\pi}}{s_4} \right\},$$

$$\psi_G(\epsilon_2) = \left\{ \frac{0.3e^{i0.7\pi}}{s_1}, \frac{0.6e^{i0.8\pi}}{s_2}, \frac{0.5e^{i0.2\pi}}{s_3}, \frac{1.0e^{i0.85\pi}}{s_4} \right\},$$

$$\psi_G(\epsilon_3) = \left\{ \frac{0.6e^{i0.7\pi}}{s_1}, \frac{0.9e^{i0.9\pi}}{s_2}, \frac{0.7e^{i0.95\pi}}{s_3}, \frac{0.75e^{i0.95\pi}}{s_4} \right\},$$

and

$$\psi_G(\epsilon_4) = \left\{ \frac{0.5e^{i0.6\pi}}{s_1}, \frac{0.7e^{i0.8\pi}}{s_2}, \frac{0.6e^{i0.85\pi}}{s_3}, \frac{0.75e^{i0.85\pi}}{s_4} \right\},$$

Step 1: CFH-set χ_G is written as,

$$\chi_G = \left\{ \begin{array}{l} \left(\epsilon_1, \frac{0.4e^{i0.5\pi}}{s_1}, \frac{0.8e^{i0.6\pi}}{s_2}, \frac{0.8e^{i0.8\pi}}{s_3}, \frac{1.0e^{i0.75\pi}}{s_4}\right), \left(\epsilon_2, \frac{0.3e^{i0.7\pi}}{s_1}, \frac{0.6e^{i0.8\pi}}{s_2}, \frac{0.5e^{i0.2\pi}}{s_3}, \frac{1.0e^{i0.85\pi}}{s_4}\right), \\ \left(\epsilon_3, \frac{0.6e^{i0.7\pi}}{s_1}, \frac{0.9e^{i0.9\pi}}{s_2}, \frac{0.7e^{i0.95\pi}}{s_3}, \frac{0.75e^{i0.95\pi}}{s_4}\right), \left(\epsilon_4, \frac{0.5e^{i0.6\pi}}{s_1}, \frac{0.7e^{i0.8\pi}}{s_2}, \frac{0.6e^{i0.85\pi}}{s_3}, \frac{0.75e^{i0.85\pi}}{s_4}\right) \end{array} \right\}$$

Step 2: The cardinal is computed as,

$$\parallel \chi_G \parallel (Amplitude\ Term) = \{0.75/\epsilon_1, 0.6/\epsilon_2, 0.74/\epsilon_3, 0.64/\epsilon_4\}$$

$$\parallel \chi_G \parallel (Phase\ Term) = \{0.66/\epsilon_1, 0.64/\epsilon_2, 0.87/\epsilon_3, 0.78/\epsilon_4\}$$

Step 3: The set $\widehat{\chi_G}$ can be determined as,

$$\widehat{\chi_G}(Apmlitude\ Term) = \frac{1}{4} \begin{bmatrix} 0.4 & 0.3 & 0.6 & 0.5 \\ 0.8 & 0.6 & 0.9 & 0.7 \\ 0.8 & 0.5 & 0.7 & 0.6 \\ 1.0 & 1.0 & 0.75 & 0.75 \end{bmatrix} \begin{bmatrix} 0.75 \\ 0.6 \\ 0.74 \\ 0.64 \end{bmatrix} = \begin{bmatrix} 0.3110 \\ 0.5185 \\ 0.4505 \\ 0.5963 \end{bmatrix}$$

$$\widehat{\chi_G}(Phase\ Term) = \frac{1}{4} \begin{bmatrix} 0.5 & 0.7 & 0.7 & 0.6 \\ 0.6 & 0.8 & 0.9 & 0.8 \\ 0.8 & 0.2 & 0.95 & 0.85 \\ 0.75 & 0.85 & 0.95 & 0.85 \end{bmatrix} \begin{bmatrix} 0.66 \\ 0.64 \\ 0.87 \\ 0.78 \end{bmatrix} = \begin{bmatrix} 0.4638 \\ 0.5788 \\ 0.5364 \\ 0.6321 \end{bmatrix}$$

$$\widehat{\chi_G} = \left\{0.3110e^{i0.4638\pi}/s_1, 0.5185e^{i0.5788\pi}/s_2, 0.4505e^{i0.5364\pi}/s_3, 0.5963e^{i0.6321\pi}/s_4\right\}$$

Consider the modulus value of $Max(\mu_{\widehat{\chi_G}}) = \{0.31098/s_1, 0.5185/s_2, 0.4504/s_3, 0.5963/s_4\} = 0.5963/s_4$ This means that the 4th share s_4 may be recommended for suitable investment.

4. Interval-Valued Complex Fuzzy Hypersoft Set(IV-CFHS)

In this section, the basic theory of interval-valued complex fuzzy hypersoft set is developed.

Definition 4.1. Let $W_1, W_2, W_3, ..., W_n$ are disjoint sets having attribute values of n distinct attributes $w_1, w_2, w_3, ..., w_n$ respectively for $n \geq 1, W = W_1 \times W_2 \times W_3 \times ..... \times W_n$ and $\Psi(\underline{\omega})$ be a IV-CFS over $\mathbb{U}$ for all $\underline{\omega} = (b_1, b_2, b_3, ..., b_n) \in W$. Then, *interval-valued complex fuzzy hypersoft set* (IV-CFHS) $\Omega_W = (\Psi, W)$ over $\mathbb{U}$ is defined as

$$\Omega_W = \{(\underline{\omega}, \Psi(\underline{\omega})) : \underline{\omega} \in W, \Psi(\underline{\omega}) \in C_{IV}(\mathbb{U})\}$$

where

$$\Psi : W \to C_{IV}(\mathbb{U}), \qquad \Psi(\underline{\omega}) = \emptyset \ if \ \underline{\omega} \notin W.$$

is a IV-CF approximate function of Ω_W and $\Psi(\underline{\omega}) = (\overleftarrow{\Psi}(\underline{\omega}), \overrightarrow{\Psi}(\underline{\omega}))$. $\overleftarrow{\Psi}(\underline{\omega}) = \overleftarrow{r} e^{i\overleftarrow{\theta}}$ and $\overrightarrow{\Psi}(\underline{\omega}) = \overrightarrow{r} e^{i\overrightarrow{\theta}}$ are lower and upper bounds of the membership function of Ω_W respectively and its value $\Psi(\underline{\omega})$ is called $\underline{\omega}$-member of IV-CFHS $\forall \underline{\omega} \in W$.

Example 4.2. Considering example 2.9 with $W = \{e_1, e_2, e_3, ..., e_8\}$, IV-Complex fuzzy sets $\Psi_W(e_1), \Psi_W(e_2), ..., \Psi_W(e_8)$ are defined as,

$$\Psi_W(e_1) = \left\{ \frac{[0.4, 0.5]e^{i[0.5,06]\pi}}{t_1}, \frac{[0.7, 0.8]e^{i[0.5,0.6]\pi}}{t_2}, \frac{[0.6, 0.7]e^{i[0.7,0.8]\pi}}{t_3}, \frac{[0.3, 0.4]e^{i[0.65,0.75]\pi}}{t_4} \right\},$$

$$\Psi_W(e_2) = \left\{ \frac{[0.5, 0.6]e^{i[0.6,0.7]\pi}}{t_1}, \frac{[0.8, 0.9]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.6, 0.7]e^{i[0.8,0.9]\pi}}{t_3}, \frac{[0.65, 0.75]e^{i[0.85,0.95]\pi}}{t_4} \right\},$$

$$\Psi_W(e_3) = \left\{ \frac{[0.4, 0.5]e^{i[0.5,0.6]\pi}}{t_1}, \frac{[0.7, 0.8]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.5, 0.6]e^{i[0.8,0.9]\pi}}{t_3}, \frac{[0.55, 0.65]e^{i[0.85,0.95]\pi}}{t_4} \right\},$$

$$\Psi_W(e_4) = \left\{ \frac{[0.2, 0.3]e^{i[0.6,0.7]\pi}}{t_1}, \frac{[0.6, 0.7]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.4, 0.5]e^{i[0.8,0.9]\pi}}{t_3}, \frac{[0.65, 0.75]e^{i[0.55,0.65]\pi}}{t_4} \right\},$$

$$\Psi_W(e_5) = \left\{ \frac{[0.1, 0.2]e^{i[0.4,0.5]\pi}}{t_1}, \frac{[0.2, 0.3]e^{i[0.7,0.8]\pi}}{t_2}, \frac{[0.7, 0.8]e^{i[0.6,0.7]\pi}}{t_3}, \frac{[0.35, 0.45]e^{i[0.55,0.65]\pi}}{t_4} \right\},$$

$$\Psi_W(e_6) = \left\{ \frac{[0.4, 0.5]e^{i[0.8,0.9]\pi}}{t_1}, \frac{[0.2, 0.3]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.6, 0.7]e^{i[0.7,0.8]\pi}}{t_3}, \frac{[0.75, 0.85]e^{i[0.85,0.95]\pi}}{t_4} \right\},$$

$$\Psi_W(e_7) = \left\{ \frac{[0.5, 0.6]e^{i[0.8,0.9]\pi}}{t_1}, \frac{[0.8, 0.9]e^{i[0.5,0.6]\pi}}{t_2}, \frac{[0.4, 0.5]e^{i[0.5,0.6]\pi}}{t_3}, \frac{[0.75, 0.85]e^{i[0.65,0.75]\pi}}{t_4} \right\},$$

and

$$\Psi_W(e_8) = \left\{ \frac{[0.7, 0.8]e^{i[0.8,0.9]\pi}}{t_1}, \frac{[0.7, 0.8]e^{i[0.7,0.8]\pi}}{t_2}, \frac{[0.5, 0.6]e^{i[0.7,0.8]\pi}}{t_3}, \frac{[0.55, 0.65]e^{i[0.75,0.85]\pi}}{t_4} \right\}$$

then IV-CFHS Ω_W is written by,

$$\Omega_W = \left\{ \begin{array}{l} (e_1, \frac{[0.4,0.5]e^{i[0.5,06]\pi}}{t_1}, \frac{[0.7,0.8]e^{i[0.5,0.6]\pi}}{t_2}, \frac{[0.6,0.7]e^{i[0.7,0.8]\pi}}{t_3}, \frac{[0.3,0.4]e^{i[0.65,0.75]\pi}}{t_4}), \\ (e_2, \frac{[0.5,0.6]e^{i[0.6,0.7]\pi}}{t_1}, \frac{[0.8,0.9]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.6,0.7]e^{i[0.8,0.9]\pi}}{t_3}, \frac{[0.65,0.75]e^{i[0.85,0.95]\pi}}{t_4}), \\ (e_3, \frac{[0.4,0.5]e^{i[0.5,0.6]\pi}}{t_1}, \frac{[0.7,0.8]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.5,0.6]e^{i[0.8,0.9]\pi}}{t_3}, \frac{[0.55,0.65]e^{i[0.85,0.95]\pi}}{t_4}), \\ (e_4, \frac{[0.2,0.3]e^{i[0.6,0.7]\pi}}{t_1}, \frac{[0.6,0.7]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.4,0.5]e^{i[0.8,0.9]\pi}}{t_3}, \frac{[0.65,0.75]e^{i[0.55,0.65]\pi}}{t_4}), \\ (e_5, \frac{[0.1,0.2]e^{i[0.4,0.5]\pi}}{t_1}, \frac{[0.2,0.3]e^{i[0.7,0.8]\pi}}{t_2}, \frac{[0.7,0.8]e^{i[0.6,0.7]\pi}}{t_3}, \frac{[0.35,0.45]e^{i[0.55,0.65]\pi}}{t_4}), \\ (e_6, \frac{[0.4,0.5]e^{i[0.8,0.9]\pi}}{t_1}, \frac{[0.2,0.3]e^{i[0.8,0.9]\pi}}{t_2}, \frac{[0.6,0.7]e^{i[0.7,0.8]\pi}}{t_3}, \frac{[0.75,0.85]e^{i[0.85,0.95]\pi}}{t_4}), \\ (e_7, \frac{[0.5,0.6]e^{i[0.8,0.9]\pi}}{t_1}, \frac{[0.8,0.9]e^{i[0.5,0.6]\pi}}{t_2}, \frac{[0.4,0.5]e^{i[0.5,0.6]\pi}}{t_3}, \frac{[0.75,0.85]e^{i[0.65,0.75]\pi}}{t_4}), \\ (e_8, \frac{[0.7,0.8]e^{i[0.8,0.9]\pi}}{t_1}, \frac{[0.7,0.8]e^{i[0.7,0.8]\pi}}{t_2}, \frac{[0.5,0.6]e^{i[0.7,0.8]\pi}}{t_3}, \frac{[0.55,0.65]e^{i[0.75,0.85]\pi}}{t_4}) \end{array} \right\}$$

Definition 4.3. Let $\Omega_{W_1} = (\Psi_1, W_1)$ and $\Omega_{W_2} = (\Psi_2, W_2)$ be two IV-CFHS over the same $\mathbb{U}$. The set $\Omega_{W_1} = (\Psi_1, W_1)$ is said to be the *subset* of $\Omega_{W_2} = (\Psi_2, W_2)$, if

i. $W_1 \subseteq W_2$

ii. $\forall\ \underline{x} \in W_1, \Psi_1(\underline{x}) \subseteq \Psi_2(\underline{x})$ implies $\overleftarrow{\Psi}_1(\underline{x}) \subseteq \overleftarrow{\Psi}_2(\underline{x}), \overrightarrow{\Psi}_1(\underline{x}) \subseteq \overrightarrow{\Psi}_2(\underline{x})$ i.e. $\overleftarrow{r}_{W_1}(\underline{x}) \leq \overleftarrow{r}_{W_2}(\underline{x}), \overrightarrow{r}_{W_1}(\underline{x}) \leq \overrightarrow{r}_{W_2}(\underline{x}), \overleftarrow{\theta}_{W_1}(\underline{x}) \leq \overleftarrow{\theta}_{W_2}(\underline{x})$ and $\overrightarrow{\theta}_{W_1}(\underline{x}) \leq \overrightarrow{\theta}_{W_2}(\underline{x})$, where

$\overleftarrow{r}_{W_1}(\underline{x})$ and $\overleftarrow{\theta}_{W_1}(\underline{x})$ are amplitude and phase terms of $\overleftarrow{\Psi}_1(\underline{x})$,
$\overrightarrow{r}_{W_1}(\underline{x})$ and $\overrightarrow{\theta}_{W_1}(\underline{x})$ are amplitude and phase terms of $\overrightarrow{\Psi}_1(\underline{x})$,
$\overleftarrow{r}_{W_2}(\underline{x})$ and $\overleftarrow{\theta}_{W_2}(\underline{x})$ are amplitude and phase terms of $\overleftarrow{\Psi}_2(\underline{x})$, and
$\overrightarrow{r}_{W_2}(\underline{x})$ and $\overrightarrow{\theta}_{W_2}(\underline{x})$ are amplitude and phase terms of $\overrightarrow{\Psi}_2(\underline{x})$.

Definition 4.4. Two IV-CFHS $\Omega_{W_1} = (\Psi_1, W_1)$ and $\Omega_{W_2} = (\Psi_2, W_2)$ over the same $\mathbb{U}$, are said to be *equal* if

i. $(\Psi_1, W_1) \subseteq (\Psi_2, W_2)$

ii. $(\Psi_2, W_2) \subseteq (\Psi_1, W_1)$.

Definition 4.5. Let (Ψ, W) be a IV-CFHS over $\mathbb{U}$.Then

i. (Ψ, W) is called a *null IV-CFHS*, denoted by $(\Psi, W)_\Phi$ if for all $\underline{x} \in W$, the amplitude and phase terms of the membership function are given by $\overleftarrow{r}_W(\underline{x}) = \overrightarrow{r}_W(\underline{x}) = 0$ and $\overleftarrow{\theta}_W(\underline{x}) = \overrightarrow{\theta}_W(\underline{x}) = 0\pi$ respectively.

ii. (Ψ, W) is called a *absolute IV-CFHS*, denoted by $(\Psi, W)_\Delta$ if for all $\underline{x} \in W$, the amplitude and phase terms of the membership function are given by $\overleftarrow{r}_W(\underline{x}) = \overrightarrow{r}_W(\underline{x}) = 1$ and $\overleftarrow{\theta}_W(\underline{x}) = \overrightarrow{\theta}_W(\underline{x}) = 2\pi$ respectively.

Definition 4.6. Let (Ψ_1, W_1) and (Ψ_2, W_2) are two CFH-sets over the same universe $\mathbb{U}$.Then

i. A IV-CFHS (Ψ_1, W_1) is called a *homogeneous IV-CFHS*, denoted by $(\Psi_1, W_1)_{Hom}$ if and only if $\Psi_1(\underline{x})$ is a homogeneous CF-set for all $\underline{x} \in W_1$.

ii. A IV-CFHS (Ψ_1, W_1) is called a *completely homogeneous IV-CFHS*, denoted by $(\Psi_1, W_1)_{CHom}$ if and only if $\Psi_1(\underline{x})$ is a homogeneous with $\Psi_1(\underline{y})$ for all $\underline{x}, \underline{y} \in W_1$.

iii. A IV-CFHS (Ψ_1, W_1) is said to be a completely homogeneous IV-CFHS with (Ψ_2, W_2) if and only if $\Psi_1(\underline{x})$ is a homogeneous with $\Psi_2(\underline{x})$ for all $\underline{x} \in W_1 \prod W_2$.

4.1. *Set Theoretic Operations and Laws on IV-CFHS*

Here some basic set theoretic operations (i.e.complement, union and intersection) and laws (commutative laws, associative laws etc.) are discussed on IV-CFHS.

Definition 4.7. The *complement* of IV-CFHS (Ψ, W), denoted by $(\Psi, W)^c$ is defined as

$$(\Psi, W)^c = \{(\underline{x}, (\Psi(\underline{x}))^c) : \underline{x} \in W, (\Psi(\underline{x}))^c \in C_{IV}(\mathbb{U})\}$$

such that the amplitude and phase terms of the membership function $(\Psi(\underline{x}))^c$ are given by
$(\overleftarrow{r}_W(\underline{x}))^c = 1 - \overleftarrow{r}_W(\underline{x})$
$(\overrightarrow{r}_W(\underline{x}))^c = 1 - \overrightarrow{r}_W(\underline{x})$
and
$(\overleftarrow{\theta}_W(\underline{x}))^c = 2\pi - \overleftarrow{\theta}_W(\underline{x})$,
$(\overrightarrow{\theta}_W(\underline{x}))^c = 2\pi - \overrightarrow{\theta}_W(\underline{x})$ respectively.

Proposition 4.8. *Let* (Ψ, W) *be a IV-CFHS over* $\mathbb{U}$*.Then* $((\Psi, W)^c)^c = (\Psi, W)$.

Proof. Since $\Psi(\underline{x}) \in C_{IV}(\mathbb{U})$, therefore (Ψ, W) can be written in terms of its amplitude and phase terms as

$$(\Psi, W) = \left\{\left(\underline{x}, \left(\overleftarrow{r}_W(\underline{x})e^{i\overleftarrow{\theta}_W(\underline{x})}, \overrightarrow{r}_W(\underline{x})e^{i\overrightarrow{\theta}_G(\underline{x})}\right)\right) : \underline{x} \in W\right\} \tag{3}$$

Now

$$(\Psi, W)^c(\underline{x}) = \left\{\left(\underline{x}, \left((\overleftarrow{r}_W(\underline{x}))^c e^{i(\overleftarrow{\theta}_W(\underline{x}))^c}, (\overrightarrow{r}_W(\underline{x}))^c e^{i(\overrightarrow{\theta}_G(\underline{x}))^c}\right)\right) : \underline{x} \in W\right\}$$

$$(\Psi, W)^c(\underline{x}) = \left\{\left(\underline{x}, \left((1 - \overleftarrow{r}_W(\underline{x}))e^{i(2\pi - \overleftarrow{\theta}_W(\underline{x}))}, (1 - \overrightarrow{r}_W(\underline{x}))e^{i(2\pi - \overrightarrow{\theta}_G(\underline{x}))}\right)\right) : \underline{x} \in W\right\}$$

$$((\psi, G)^c)^c = \left\{\left(\underline{x}, \left((1 - \overleftarrow{r}_W(\underline{x}))^c e^{i(2\pi - \overleftarrow{\theta}_W(\underline{x}))^c}, (1 - \overrightarrow{r}_W(\underline{x}))^c e^{i(2\pi - \overrightarrow{\theta}_G(\underline{x}))^c}\right)\right) : \underline{x} \in W\right\}$$

$$((\Psi, W)^c)^c = \left\{\left(\underline{x}, \left((1 - (1 - \overleftarrow{r}_W(\underline{x})))e^{i(2\pi - (2\pi - \overleftarrow{\theta}_W(\underline{x})))}, (1 - (1 - \overrightarrow{r}_W(\underline{x})))e^{i(2\pi - (2\pi - \overrightarrow{\theta}_G(\underline{x})))}\right)\right) : \underline{x} \in W\right\}$$

$$((\Psi, W)^c)^c = \left\{\left(\underline{x}, \left(\overleftarrow{r}_W(\underline{x})e^{i\overleftarrow{\theta}_W(\underline{x})}, \overrightarrow{r}_W(\underline{x})e^{i\overrightarrow{\theta}_G(\underline{x})}\right)\right) : \underline{x} \in W\right\} \tag{4}$$

from equations (3) and (4), we have $((\Psi, W)^c)^c = (\Psi, W)$. □

Proposition 4.9. *Let* (Ψ, W) *be a IV-CFHS over* $\mathbb{U}$*.Then*

i. $((\Psi, W)_\Phi)^c = (\Psi, W)_\Delta$
ii. $((\Psi, W)_\Delta)^c = (\Psi, W)_\Phi$

Definition 4.10. The *intersection* of two IV-CFHS (Ψ_1, W_1) and (Ψ_2, W_2) over the same universe $\mathbb{U}$, denoted by $(\Psi_1, W_1) \prod (\Psi_2, W_2)$, is the IV-CFHS (Ψ_3, W_3), where $W_3 = W_1 \coprod W_2$, and for all $\underline{x} \in W_3$,

$$\overleftarrow{\Psi}_3(\underline{x}) = \begin{cases} \overleftarrow{r}_{W_1}(\underline{x}) e^{i \overleftarrow{\theta}_{W_1}(\underline{x})} & , if\ \underline{x} \in W_1 \backslash W_2 \\ \overleftarrow{r}_{W_2}(\underline{x}) e^{i \overleftarrow{\theta}_{W_2}(\underline{x})} & , if\ \underline{x} \in W_2 \backslash W_1 \\ min[\overleftarrow{r}_{W_1}(\underline{x}), \overleftarrow{r}_{W_2}(\underline{x})] e^{i\, min[\overleftarrow{\theta}_{W_1}(\underline{x}), \overleftarrow{\theta}_{W_2}(\underline{x})]} & , if\ \underline{x} \in W_1 \prod W_2 \end{cases}$$

and

$$\overrightarrow{\Psi}_3(\underline{x}) = \begin{cases} \overrightarrow{r}_{W_1}(\underline{x}) e^{i \overrightarrow{\theta}_{W_1}(\underline{x})} & , if\ \underline{x} \in W_1 \backslash W_2 \\ \overrightarrow{r}_{W_2}(\underline{x}) e^{i \overrightarrow{\theta}_{W_2}(\underline{x})} & , if\ \underline{x} \in W_2 \backslash W_1 \\ min[\overrightarrow{r}_{W_1}(\underline{x}), \overrightarrow{r}_{W_2}(\underline{x})] e^{i\, min[\overrightarrow{\theta}_{W_1}(\underline{x}), \overrightarrow{\theta}_{W_2}(\underline{x})]} & , if\ \underline{x} \in W_1 \prod W_2 \end{cases}$$

Definition 4.11. The *difference* between two IV-CFHS (Ψ_1, W_1) and (Ψ_2, W_2) is defined as

$$(\Psi_1, W_1) \setminus (\Psi_2, W_2) = (\Psi_1, W_1) \prod (\Psi_2, W_2)^c$$

Definition 4.12. The *union* of two IV-CFHS (Ψ_1, W_1) and (Ψ_2, W_2) over the same universe $\mathbb{U}$, denoted by $(\Psi_1, W_1) \coprod (\Psi_2, W_2)$, is the IV-CFHS (Ψ_3, W_3), where $W_3 = W_1 \coprod W_2$, and for all $\underline{x} \in W_3$,

$$\overleftarrow{\Psi}_3(\underline{x}) = \begin{cases} \overleftarrow{r}_{W_1}(\underline{x}) e^{i \overleftarrow{\theta}_{W_1}(\underline{x})} & , if\ \underline{x} \in W_1 \backslash W_2 \\ \overleftarrow{r}_{W_2}(\underline{x}) e^{i \overleftarrow{\theta}_{W_2}(\underline{x})} & , if\ \underline{x} \in W_2 \backslash W_1 \\ max[\overleftarrow{r}_{W_1}(\underline{x}), \overleftarrow{r}_{W_2}(\underline{x})] e^{i\, max[\overleftarrow{\theta}_{W_1}(\underline{x}), \overleftarrow{\theta}_{W_2}(\underline{x})]} & , if\ \underline{x} \in W_1 \prod W_2 \end{cases}$$

and

$$\overrightarrow{\Psi}_3(\underline{x}) = \begin{cases} \overrightarrow{r}_{W_1}(\underline{x}) e^{i \overrightarrow{\theta}_{W_1}(\underline{x})} & , if\ \underline{x} \in W_1 \backslash W_2 \\ \overrightarrow{r}_{W_2}(\underline{x}) e^{i \overrightarrow{\theta}_{W_2}(\underline{x})} & , if\ \underline{x} \in W_2 \backslash W_1 \\ max[\overrightarrow{r}_{W_1}(\underline{x}), \overrightarrow{r}_{W_2}(\underline{x})] e^{i\, max[\overrightarrow{\theta}_{W_1}(\underline{x}), \overrightarrow{\theta}_{W_2}(\underline{x})]} & , if\ \underline{x} \in W_1 \prod W_2 \end{cases}$$

Proposition 4.13. *Let* (Ψ, W) *be a IV-CFHS over* $\mathbb{U}$.*Then the following results hold true:*

i. $(\Psi, W) \coprod (\Psi, W)_\Phi = (\Psi, W)$
ii. $(\Psi, W) \coprod (\Psi, W)_\Delta = (\Psi, W)_\Delta$
iii. $(\Psi, W) \prod (\Psi, W)_\Phi = (\Psi, W)_\Phi$
iv. $(\Psi, W) \prod (\Psi, W)_\Delta = (\Psi, W)$
v. $(\Psi, W)_\Phi \coprod (\Psi, W)_\Delta = (\Psi, W)_\Delta$
vi. $(\Psi, W)_\Phi \prod (\Psi, W)_\Delta = (\Psi, W)_\Phi$

Proposition 4.14. *Let* (Ψ_1, W_1), (Ψ_2, W_2) *and* (Ψ_3, W_3) *are three CFH-sets over the same universe* $\mathbb{U}$. *Then the following commutative and associative laws hold true:*

i. $(\Psi_1, W_1) \prod (\Psi_2, W_2) = (\Psi_2, W_2) \prod (\Psi_1, W_1)$
ii. $(\Psi_1, W_1) \coprod (\Psi_2, W_2) = (\Psi_2, W_2) \coprod (\Psi_1, W_1)$
iii. $(\Psi_1, W_1) \prod ((\Psi_2, W_2) \prod (\Psi_3, W_3)) = ((\Psi_1, W_1) \prod (\Psi_2, W_2)) \prod (\Psi_3, W_3)$

iv. $(\Psi_1, W_1) \coprod ((\Psi_2, W_2) \coprod (\Psi_3, W_3)) = ((\Psi_1, W_1) \coprod (\Psi_2, W_2)) \coprod (\Psi_3, W_3)$

Proposition 4.15. *Let* (Ψ_1, W_1) *and* (Ψ_2, W_2) *are two CFH-sets over the same universe* $\mathbb{U}$. *Then the following De Morganss laws hold true:*

i. $((\Psi_1, W_1) \prod (\Psi_2, W_2))^c = (\Psi_1, W_1)^c \coprod (\Psi_2, W_2)^c$
ii. $(\Psi_1, W_1) \coprod (\Psi_2, W_2))^c = (\Psi_1, W_1)^c \prod (\Psi_2, W_2)^c$

Conclusion

In this work, the complex fuzzy hypersoft sets (CFH-sets) are developed along with some fundamentals, theoretic set operations and aggregations. Also a method is proposed to solve decision making problems and demonstrated with a commerce-based application. Moreover, the rudiments of interval-valued fuzzy hypersoft set (IV-CFHS) are characterized with suitable examples. CFH-sets and IV-CFHS generalize the existing structures of complex fuzzy soft set, permit a broad range of values for membership function by expanding them to the unit circle in a complex plane, consider the periodic nature of the information through the phase-terms and classify distinct attributes into corresponding attribute-values sets for vivid understanding. Further work may include:

(i) the extension of proposed work to the development of:
- complex intuitionistic fuzzy hypersoft set,
- complex neutrosophic hypersoft set,
- interval-valued complex intuitionistic fuzzy hypersoft set,
- interval-valued complex neutrosophic hypersoft set,

(ii) the application of proposed work in multi-criteria decision-making,
(iii) the determination of similarity measures and entropies for proposed structures,
(iv) the parameterization of proposed structures with fuzzy, intuitionistic fuzzy and neutrosophic settings,
(v) the characterization of proposed structures under multi-decisive environment,
(vi) the introduction of refinement in the proposed structures for sub-membership grades.

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