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A Note on Neutrosophic Almost Bitopological Group

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Abstract. Because the concept of an almost topological group is relatively new, so, in this paper, we introduce the notion of the neutrosophic almost bitopological group. In this work, we define the definition of the neutrosophic almost continuous mapping and then we define the neutrosophic almost bitopological group. Also, we investigate some properties of the neutrosophic almost bitopological group.

Keywords: Neutrosophic Set; Neutrosophic Almost Continuous Mapping; Neutrosophic Bitopological Group; Neutrosophic Almost Bitopological Group.

A list of Abbreviations

NS - neutrosophic set.

NG - neutrosophic group.

NT - neutrosophic topology.

NTS - neutrosophic topological space.

NOS - neutrosophic open set.

NCoS - neutrosophic closed set.

NBTS - neutrosophic bitopological space.

NTG - neutrosophic topological group.

NSOS - neutrosophic semi open set.

NSCoS - neutrosophic semi closed set.

NROS - neutrosophic regularly open set.

NRCS - neutrosophic regularly closed set.

NABTG - neutrosophic almost bitopological group.

1. Introduction

The Fuzzy set (FS) concept was first introduced by Zadeh [29] in 1965. The concept of membership function and explained the idea of uncertainty defined with the help of FS. Atanassov [8] generalized the concept of fuzzy set theory (FST) and introduced the degree of non-membership and proposed intuitionistic fuzzy set theory (IFST). Azad [9] discussed the Fuzzy Semi-continuity (FSC), Fuzzy Almost Continuity (FAC), and Fuzzy Weakly Continuity (FWC). Chang [11] defined the concept of the fuzzy topology space (FTS) and Coker [12] introduced the Intuitionistic fuzzy topological space (IFTS). Kandil [15] and Kelly [16] discussed the fuzzy bitopological spaces and bitopological space. Rosenfeld [20] introduced the fuzzy groups and Foster [13] defined the fuzzy topological groups.

F. Smarandache [25, 26] was introduced as an independent component of the degree of uncertainty and discovered the neutrosophic set (NS). After the discovery of NS, many researchers have developed the neutrosophic set theory for various branches of Science and Technology. NS is used as an independent measure of uncertainty Membership and Non-Membership Function. FS is used to control uncertainty by using the membership function only. While NS is used to control uncertainty by using the truth membership function, indeterminacy membership function, and falsity membership function. Salama and Alblowi [21] introduced the concept of neutrosophic topological space (NTS). Salama et. al [22] studied closed sets and neutrosophic continuous functions. Imran et. al [14] discussed some types of neutrosophic topological groups in relation to neutrosophic alpha open sets. Abdel-Basset et. al [1] have applied neutrosophic set theory (NST) as a tool on group discussion making framework. Abdel-Basset et al [2] done the work in solving chain problems using a base-worst method based on a novel plithogenic model. Sumathi et. al [27, 28] studied the Fuzzy Neutrosophic Groups (FNG) and Topological Group Structure of Neutrosophic set. Mwchahary et. al [17] did their work in neutrosophic bitopological space. Abdel-Basset et. al [3] developed supplier selection with group decision-making under the type-2 neutrosophic number of TOPSIS technology. Abdel-Basset et. al [4, 5] studied the chain management practices of evaluation of the green supply and defined for achieving sustainable supplier selection of VIKOR method. Also, Abdel-Basset et. al [6, 7] developed hybrid multi-criteria decision-making for the sustainability assessment of bioenergy production technologies and employed an evaluation approach for sustainable renewable energy systems under uncertain environments.

In this paper, we try to study the neutrosophic almost bitopological group (NABTG) and some of their properties by using the definition of neutrosophic almost continuous mapping (NACM).

2. Preliminaries

2.1. Definition:[28]

A NS A on a universe of discourse X can be expressed as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \Gamma_A(x)\rangle : x \in X\}$, where $\mathcal{T}, \mathcal{I}, \Gamma : X \to]^-0, 1^+[$. Note that $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \Gamma_A(x) \leq 3$.

2.2. Definition:[28]

The complement of NS A is expressed as $A^c(x) = \{\langle x, \mathcal{T}_{A^c}(x) = \Gamma_A(x), \mathcal{I}_{A^c}(x) = 1 - \mathcal{I}_A(x), \Gamma_{A^c}(x) = \mathcal{T}_A(x)\rangle : x \in X\}$.

2.3. Definition:[28]

Let X be non-empty set and $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \Gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mathcal{T}_B(x), \mathcal{I}_B(x), \Gamma_B(x)\rangle : x \in X\}$, are NSs. Then

(i) $A \Cap B = \{\langle x, min(\mathcal{T}_A(x), \mathcal{T}_B(x)), min(\mathcal{I}_A(x), \mathcal{I}_B(x)), max(\Gamma_A(x), \Gamma_B(x)), \rangle : x \in X\}$
(ii) $A \Cup B = \{\langle x, max(\mathcal{T}_A(x), \mathcal{T}_B(x)), max(\mathcal{I}_A(x), \mathcal{I}_B(x)), min(\Gamma_A(x), \Gamma_B(x)), \rangle : x \in X\}$
(iii) $A \leq B$ if for each $x \in X, \mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \leq \mathcal{I}_B(x), \Gamma_A(x) \geq \Gamma_B(x)$.

2.4. Definition:[28]

Let $(X, *)$ be a group and let A be a NG in X. Then A is said to be a NG in X if it satisfies the following conditions:

(i) $\mathcal{T}_A(xy) \geq \mathcal{T}_A(x) \Cap \mathcal{T}_A(y)$, $\mathcal{I}_A(xy) \geq \mathcal{I}_A(x) \Cap \mathcal{I}_A(y)$ and $\Gamma_A(xy) \leq \Gamma_A(x) \Cup \Gamma_A(y)$,
(ii) $\mathcal{T}_A(x^{-1}) \geq \mathcal{T}_A(x)$, $\mathcal{I}_A(x^{-1}) \geq \mathcal{I}_A(x)$, and $\Gamma_A(x^{-1}) \leq \Gamma_A(x)$.

2.5. Definition:[21]

Let X be a group and let $\mathbb{G}$ be NG in X and e be the identity of X. We define the NS $\mathbb{G}_e$ by

$$\mathbb{G}_e = \{x \in X : \mathcal{T}_{\mathbb{G}}(x) = \mathcal{T}_{\mathbb{G}}(e), \mathcal{I}_{\mathcal{G}}(x) = \mathcal{I}_{\mathcal{G}}(e), \Gamma_{\mathbb{G}}(x) = \Gamma_{\mathbb{G}}(e)\}.$$

We note for a NG $\mathbb{G}$ in a group X, for every $x \in X : \mathcal{T}_{\mathbb{G}}(x^{-1}) = \mathcal{T}_{\mathbb{G}}(x)$, $\mathcal{I}_{\mathbb{G}}(x^{-1}) = \mathcal{I}_{\mathbb{G}}(x)$ and $\Gamma_{\mathbb{G}}(x^{-1}) = \Gamma_{\mathbb{G}}(x)$. Also for the identity e of the group $X : \mathcal{T}_{\mathbb{G}}(e) \geq \mathcal{T}_{\mathbb{G}}(x), \mathcal{I}_{\mathbb{G}}(e) \geq \mathcal{I}_{\mathbb{G}}(x)$, and $\Gamma_{\mathbb{G}}(e) \leq \Gamma_{\mathbb{G}}(x)$.

2.6. Definition:[21]

Let X be a non-empty set and a NT on X is a family $\daleth_X$ of neutrosophic subsets of X satisfying the following axioms:

(i) $0_N, 1_N \in \daleth_X$
(ii) $G_1 \Cap G_2 \in \daleth_X$ for any $G_1, G_2 \in \daleth_X$

(iii) $\Cup G_i \in \daleth_X \ \forall \ \{G_i : i \in J\} \subseteq \daleth_X$

In this case, the pair $(X, \daleth_X)$ is called a neutrosophic topological space (NTS) and any NS in $\daleth_X$ is known as neuterosophic open set (NOS). The elements of $\daleth_X$ are called NOSs, a NS F is neutrosophic closed set (NCoS) if and only if it F^C is NOS.

2.7. **Definition:**[8]

It is known that $f : (X, \daleth_X) \to (Y, \daleth_Y)$ is neutrosophic continuous if the preimage of each neutrosophic open set in Y is neutrosophic open set in X.

2.8. **Definition:**[17]

Let $(X, \daleth_i^X)$ and $(X, \daleth_j^X)$ be the two neutrosophic topologies on X, then $(X, \daleth_i^X, \daleth_j^X)$ is said to be a neutrosophic bitopological space (NBTS). Throughout in this paper the indices i, j take the value $\in \{i, j\}$ and $i \neq j$.

2.9. **Definition:**[17]

Let $(X, \daleth_i^X, \daleth_j^X)$ be a NBTS. Then a set for $A = \big\{\langle x, \alpha_{ij}, \beta_{ij}, \gamma_{ij}\rangle : x \in X\big\}$, neutrosophic $(\daleth_i^X, \daleth_j^X)N \vdash$ interior of A is the union of all $(\daleth_i^X, \daleth_j^X)N \vdash$ open sets of X contained in A and can be defined as follows:
$(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})N \vdash Int(A) = \big\{\langle x, \Cup_{\daleth_i^X} \Cup_{\daleth_j^X} \alpha_{ij}, \Cap_{\daleth_i^X} \Cap_{\daleth_j^X} \beta_{ij}, \Cap_{\daleth_i^X} \Cap_{\daleth_j^X} \gamma_{ij}\rangle : x \in X\big\}$

2.10. **Definition:**[17]

Let $(X, \daleth_i^X, \daleth_j^X)$ be a NBTS. Then a set for $A = \big\{\langle x, \alpha_{ij}, \beta_{ij}, \gamma_{ij}\rangle : x \in X\big\}$, neutrosophic $(\daleth_i^X, \daleth_j^X)N \vdash$ closure of A is the intersection of all $(\daleth_i^X, \daleth_j^X)N \vdash$ closed sets of X contained in A and can be defined as follows:
$(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})N \vdash Cl(A) = \big\{\langle x, \Cap_{\daleth_i^X} \Cap_{\daleth_j^X} \alpha_{ij}, \Cup_{\daleth_i^X} \Cup_{\daleth_j^X} \beta_{ij}, \Cup_{\daleth_i^X} \Cup_{\daleth_j^X} \gamma_{ij}\rangle : x \in X\big\}$

2.11. **Definition:**[10]

Let X be a group and $\mathbb{G}$ be a NG on X. Let $\daleth_X$ be a NT on $\mathbb{G}$ then $(\mathbb{G}, \daleth_X)$ is called a neutrosophic topological group (NTG) if the following conditions are satisfied:

(i) The mapping $\lambda : (\mathbb{G}, \daleth_X) \times (\mathbb{G}, \daleth_X) \to (\mathbb{G}, \daleth_X)$ defined by $\lambda(g_1, g_2) = g_1 g_2$, for all $g_1, g_2 \in X$, is relatively neutrosophic continuous.
(ii) The mapping $\mu : (\mathbb{G}, \daleth_X) \to (\mathbb{G}, \daleth_X)$ defined by $\mu(g) = g^{-1}$, for all $g \in X$, is relatively neutrosophic continuous.

3. Main Works:

3.1. Definition:

Let $\mathcal{A}$ be a NS of a NBTS $(X, \daleth_i^X, \daleth_j^X)$, then $\mathcal{A}$ is called a neutrosophic semi-open set (NSOS) of X if there exists a $\mathcal{B} \in (X, \daleth_i^X, \daleth_j^X)$ such that $\mathcal{A} \subseteq (\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})N \vdash Cl(\mathcal{B})$, for each $i = j = 1, 2$.

3.2. Definition:

Let $\mathcal{A}$ be a NS of a NBTS $(X, \daleth_i^X, \daleth_j^X)$, then $\mathcal{A}$ is called a neutrosophic semi-closed set (NSCoS) of X if there exists a $\mathcal{B}^c \in (X, \daleth_i^X, \daleth_j^X)$ such that $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})N \vdash Int(\mathcal{B}) \subseteq \mathcal{A}$, for each $i = j = 1, 2$.

3.3. Definition:

A NS $\mathcal{A}$ of a NBTS $(X, \daleth_i^X, \daleth_j^X)$ is said to be a neutrosophic regularly open set (NROS), if $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})N \vdash Int\Big((\daleth_i^X, \daleth_j^X)N \vdash Cl(\mathcal{A})\Big) = \mathcal{A}$, for each $i = j = 1, 2$.

3.4. Definition:

A NS $\mathcal{A}$ of a NBTS $(X, \daleth_i^X, \daleth_j^X)$ is said to be a neutrosophic regularly closed set (NRCS), if $(\daleth_i^{\mathcal{G}}, \daleth_j^{\mathcal{G}})N \vdash Cl\Big((\daleth_i^X, \daleth_j^X)N \vdash Int(\mathcal{A})\Big) = \mathcal{A}$, for each $i = j = 1, 2$.

3.5. Definition:

A mapping $\phi : (X, \daleth_i^X, \daleth_j^X) \to (Y, \daleth_i^Y, \daleth_j^Y)$ is said to be a neutrosophic almost continuous mapping (NACM), if $\phi^{-1}(\mathcal{A}) \in (X, \daleth_i^X, \daleth_j^X)$ for each neutrosophic regularly closed set $\mathcal{A}$ of $(Y, \daleth_i^Y, \daleth_j^Y); i = j = 1, 2$.

Example 3.1. Let $X = \{x, y\}$ and $Y = \{p, q\}$
$P = \{\langle\frac{0.6,0.4,0.4}{a}\rangle, \langle\frac{0.7,0.2,0.3}{b}\rangle\}$, $Q = \{\langle\frac{0.2,0.3,0.7}{a}\rangle, \langle\frac{0.2,0.1,0.7}{b}\rangle\}$, $R = \{\langle\frac{0.6,0.3,0.3}{a}\rangle, \langle\frac{0.7,0.1,0.2}{b}\rangle\}$,
$S = \{\langle\frac{0.1,0.8,0.7}{a}\rangle, \langle\frac{0.1,0.8,0.7}{b}\rangle\}$.
Then $\daleth_{N_1} = \{0_{N_X}, 1_{N_X}, P\}$, $\daleth_{N_2} = \{0_{N_X}, 1_{N_X}, Q\}$, $\daleth_{N_3} = \{0_{N_X}, 1_{N_X}, R\}$, $\daleth_{N_4} = \{0_{N_X}, 1_{N_X}, S\}$ are neutrosophic topological spaces. Then
$(\daleth_{N_1}, \daleth_{N_2}) \vdash$ open sets $= \{0_{N_X}, 1_{N_X}, P, Q\}$, $(\daleth_{N_1}, \daleth_{N_2}) \vdash$ closed sets $= \{0_{N_X}, 1_{N_X}, P^C, Q^C\}$,
$(\daleth_{N_3}, \daleth_{N_4}) \vdash$ open sets $= \{0_{N_X}, 1_{N_X}, R, S\}$, $(\daleth_{N_1}, \daleth_{N_2}) \vdash$ closed sets $= \{0_{N_X}, 1_{N_X}, R^C, S^C\}$.
Let $f : (X, \daleth_{N_1}, \daleth_{N_2}) \to (Y, \daleth_{N_1}, \daleth_{N_2})$ define by $f(x) = p$ and $f(y) = q$. Then we have $(\daleth_{N_3}, \daleth_{N_4}) \vdash Cl(R^C)$ is closed set.
Now, R^C is $(\daleth_{N_3}, \daleth_{N_4}) \vdash$ closed set in Y and $f^{-1}(R^C) \subseteq P$, where P is $(\daleth_{N_1}, \daleth_{N_2}) \vdash$ open set in X. Also, $Cl(f^{-1}(R^C)) \subseteq P$.
Therefore, f is neutrosophic almost continuous mapping.

3.6. **Definition:**

Let $\mathbb{G}$ be a NG on a group X. Let $\daleth_i^{\mathbb{G}}$ be a NT on $\mathbb{G}$ for each $i = 1, 2$; then $(\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}})$ is said to be a neutrosophic almost bitopological group (NABTG) if the following conditions are satisfied:

(i) A mapping $\lambda : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \times (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ defined by $\lambda(x, y) = xy$, for all $x, y \in X$, is neutrosophic almost i-continuous, for each $i = 1, 2$.

(ii) A mapping $\mu : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ defined by $\mu(x) = x^{-1}$, for all $x \in X$, is neutrosophic almost i-continuous, for each $i = 1, 2$.

Example 3.2. Let $\mathbb{G} = \{a, e\}$, where e is the identity element of $\mathbb{G}$. Then $\mathbb{G}$ is a group. Let $P = \{\langle \frac{0.6,0.4,0.4}{a} \rangle, \langle \frac{0.7,0.2,0.3}{b} \rangle\}$, $Q = \{\langle \frac{0.2,0.3,0.7}{a} \rangle, \langle \frac{0.2,0.1,0.7}{b} \rangle\}$, $R = \{\langle \frac{0.6,0.3,0.3}{a} \rangle, \langle \frac{0.7,0.1,0.2}{b} \rangle\}$, $S = \{\langle \frac{0.1,0.8,0.7}{a} \rangle, \langle \frac{0.1,0.8,0.7}{b} \rangle\}$.
Then, it is clear that the mapping $\mu : (a, b) \to ab$ of $(\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}}) \times (\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}})$ in to $(\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}})$ is neutrosophic almost continuous and $\lambda : a \to a^{-1}$ of $(\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}})$ in to $(\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}})$ is neutrosophic almost continuous.
Hence, $(\mathbb{G}, \daleth_1^{\mathbb{G}}, \daleth_2^{\mathbb{G}})$ is neutrosophic almost topological group.

Remark 3.1. $(\mathbb{G}, \daleth_i^{\mathbb{G}})$ is a NABTG for each $i = 1, 2$; if following conditions hold good:

(i) for $g_1, g_2 \in \mathbb{G}$ and for every $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS $\mathcal{U}$ containing g_1, g_2 in $\mathbb{G}$, $\exists\ \daleth_i^{\mathbb{G}}-$ neutrosophic open, $i = 1, 2$ nbds $\mathscr{P}$ and $\mathscr{Q}$ of g_1 and g_2 respectively in $\mathbb{G}$ so that $\mathscr{P} * \mathscr{Q} \subseteq \mathcal{U}$ and

(ii) for $g \in \mathbb{G}$ and for every $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS $\mathscr{Q}$ in $\mathbb{G}$ containing g^{-1}, $\exists\ \daleth_i^{\mathbb{G}}-$ neutrosophic open, $i = 1, 2$ nbd $\mathscr{P}$ of g in $\mathbb{G}$ such that $\mathscr{P}^{-1} \subseteq \mathscr{Q}$.

Theorem 3.1. *Let $(\mathbb{G}, \daleth_i^{\mathbb{G}})$, $i = 1, 2$; be a NABTG and let $a \in \mathbb{G}$ be any element of $\mathbb{G}$. Then*

(i) $\pi_a : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$*:* $\pi_a(x) = ax$*, for all* $x \in \mathbb{G}$*, is neutrosophic almost i-continuous, for each* $i = 1, 2$*.*

(ii) $\sigma_a : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$*:* $\sigma_a(x) = xa$*, for all* $x \in \mathbb{G}$*, is neutrosophic almost i-continuous, for each* $i = 1, 2$*.*

Proof.

(i) Let $p \in \mathbb{G}$ and let $\mathscr{W}$ be a $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1, 2$; containing ap in $\mathbb{G}$. By Definition 3.6, $\exists\ \daleth_i^{\mathbb{G}}-$ neutrosophic open, $i = 1, 2$ neighborhoods $\mathcal{U}, \mathcal{V}$ of ap in $\mathbb{G}$ so that $\mathcal{U}\mathcal{V} \subseteq \mathscr{W}$. Especially, $a\mathcal{V} \subseteq \mathscr{W}$ that is $\pi_a(\mathcal{V}) \subseteq \mathscr{W}$. This shows that π_a is NACM at p and therefore π_a is NACM.

(ii) Suppose $p \in \mathbb{G}$ and $\mathscr{W} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$, containing pa. Then $\exists\ \daleth_i^{\mathbb{G}}-$ NOSs, $i = 1, 2$; $p \in \mathcal{U}$ and $a \in \mathcal{V}$ in $\mathbb{G}$ so that $\mathcal{U}\mathcal{V} \subseteq \mathscr{W}$. This shows $\mathcal{U}a \subseteq \mathscr{W}$, i.e., $\sigma_a(\mathcal{U}) \subseteq \mathscr{W}$. This implies σ_a is NACM at p. As arbitrary element p is in $\mathbb{G}$, σ_a is NACM.

Theorem 3.2. *Let $\mathbb{G}$ be any $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1,2$; in a NABTG $(\mathbb{G}, \daleth_i^{\mathbb{G}})$. The following conditions hold good:*

(1) $a\mathcal{U} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$, for all $a \in \mathbb{G}$.
(2) $\mathcal{U}a \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$, for all $a \in \mathbb{G}$.
(3) $\mathcal{U}^{-1} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$.

Proof.

(1) First, we have to prove that $a\mathcal{U} \in \daleth_i^{\mathbb{G}}, i = 1,2$. Let $p \in a\mathcal{U}$. Then from Definition 3.6 of NABTGs, $\exists\ \daleth_i^{\mathbb{G}}-$ neutrosophic open neighborhoods, $i = 1,2$; $a^{-1} \in \mathscr{W}_1$ and $p \in \mathscr{W}_2$ in $\mathbb{G}$ so that $\mathscr{W}_1\mathscr{W}_2 \subseteq \mathcal{U}$. Especially, $a^{-1}\mathscr{W}_2 \subseteq \mathcal{U}$. i.e., equivalently $\mathscr{W}_2 \subseteq a\mathcal{U}$. This indicates that $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{U})$ and thus, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{U}) = a\mathcal{U}$. i.e., $a\mathcal{U} \in \daleth_i^{\mathbb{G}}, i = 1,2$. Consequently, $a\mathcal{U} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big)$.

Now, we have to prove that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big) \subseteq a\mathcal{U}$. Since $\mathcal{U}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1,2$; $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}) \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS$(\mathbb{G})$. From theorem 3.1, $\pi_{a^{-1}} : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ is NACM, $i = 1,2$ and therefore, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$. Thus, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U}) \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$. i.e., $a^{-1}(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$. Since $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big)$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1,2$; it follows that $a^{-1}(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})\Big) = \mathcal{U}$. i.e., $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big) \subseteq a\mathcal{U}$. Thus, $a\mathcal{U} = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U})\Big)$. This shows that $a\mathcal{U} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$.

(2) Following the same steps as in part (1) above, then we can prove $\mathcal{U}a \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$, for all $a \in \mathbb{G}$.

(3) Let $x \in \mathcal{U}^{-1}$, then $\exists\ \daleth_i^{\mathbb{G}}-$ NOS, $i = 1,2$; $x \in \mathscr{W}$ in $\mathbb{G}$ so that $\mathscr{W}^{-1} \subseteq \mathcal{U}$ $\Rightarrow \mathscr{W} \subseteq \mathcal{U}^{-1}$. Therefore $\mathcal{U}^{-1}$ has interior point x. Thus, $\mathcal{U}^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1,2$ i.e., $\mathcal{U}^{-1} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1})\Big)$. Now we have to prove that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1})\Big) \subseteq \mathcal{U}^{-1}$. Since $\mathcal{U}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1,2$; $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1,2$ and hence $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$ in $\mathbb{G}$. Therefore, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1})\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$ $\Rightarrow (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1})\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})\Big)^{-1} = \mathcal{U}^{-1}$. Thus, $\mathcal{U}^{-1} = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1})\Big)$. This shows that $\mathcal{U}^{-1} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS$(\mathbb{G})$.

Corollary 3.1. *Let $\mathcal{Q}$ be any $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1,2$; in a NABTG. Then*

(1) $a\mathcal{Q} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ *NRCS*$(\mathbb{G})$*, for all* $a \in \mathbb{G}$*.*

(2) $\mathcal{Q}^{-1} \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ *NRCS*$(\mathbb{G})$*.*

Theorem 3.3. *Let* $\mathcal{U}$ *be any* $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ *NROS,* $i = j = 1, 2$*; in a NABTG. Then*

(1) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}a) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$*, for each* $a \in \mathbb{G}$*.*

(2) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})a$*, for each* $a \in \mathbb{G}$*.*

(3) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$*.*

Proof.

(1) Taking $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}a)$ and consider $q = pa^{-1}$. Let $q \in \mathcal{W}$ be $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$ in $\mathbb{G}$. Then $\exists$ $\daleth_i^{\mathbb{G}}-$ NOSs, $i = 1, 2$; $a^{-1} \in \mathcal{V}_1$ and $p \in \mathcal{V}_2$ in $\mathbb{G}$, so that $\mathcal{V}_1\mathcal{V}_2 \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big)$. By assumption, there is $g \in \mathcal{U}a \Cap \mathcal{V}_2 \Rightarrow ga^{-1} \in \mathcal{U} \Cap \mathcal{V}_1\mathcal{V}_2 \subseteq \mathcal{U} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big) \Rightarrow \mathcal{U} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big) \neq 0_N \Rightarrow \mathcal{U} \Cap \Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big) \neq 0_N$.

Since $\mathcal{U}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$; $\mathcal{U} \Cap \mathcal{W} \neq 0_N$. i.e., $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})a$.

Conversely, let $q \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})a$. Then $q = pa$ for some $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$. To prove $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})a \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}a)$.

Let $pa \in \mathcal{W}$ be an $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$ in $\mathbb{G}$. Then $\exists$ $\daleth_i^{\mathbb{G}}-$ NOSs, $i = 1, 2$; $a \in \mathcal{V}_1$ in $\mathbb{G}$ and $p \in \mathcal{V}_2$ in $\mathbb{G}$, so that $\mathcal{V}_1\mathcal{V}_2 \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big)$. Since $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$, $\mathcal{U} \Cap \mathcal{V}_2 \neq 0_N$. There is $g \in \mathcal{U} \Cap \mathcal{V}_2$. This gives $ag \in (\mathcal{U}a) \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big) \Rightarrow (\mathcal{U}a) \Cap \Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{W})\Big) \neq 0_N$. From Theorem 3.2, $\mathcal{U}a$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$ and thus $(\mathcal{U}a) \Cap \mathcal{W} \neq 0_N$, so $q \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}a)$.

Therefore, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}a) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})a$.

(2) Following the same steps as in part (1) above, then we can prove $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{U}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$.

(3) Since $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1, 2$; $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1, 2$ in $\mathbb{G}$. So, $\mathcal{U}^{-1} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$ this implies $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$. Next, $q \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$. Then $q = p^{-1}$ for some $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})$. Let $q \in \mathcal{V}_2$ be any $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$ in $\mathbb{G}$. Then $\exists$ $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$; $\mathcal{V}_1$ in $\mathbb{G}$ so that $p \in \mathcal{V}_1$ with $\mathcal{V}_1^{-1} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big\{(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{V}_2)\big\}$. Also, there is $g \in \mathcal{U} \Cap \mathcal{V}_1$ which implies $g^{-1} \in \mathcal{U}^{-1} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big\{(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{V}_2)\big\}$. i.e., $\in \mathcal{U}^{-1} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big\{(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{V}_2)\big\} \neq 0_N \Rightarrow \mathcal{U}^{-1} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{V}_2) \neq 0_N \Rightarrow \mathcal{U}^{-1} \Cap \mathcal{V}_2 \neq 0_N$, since $\mathcal{U}^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$. Therefore, $q \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1})$.

Hence $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{U})^{-1}$.

Theorem 3.4. *Let $\mathcal{Q}$ be $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ neutrosophic regularly closed, $i=j=1,2$ subset in a NABTG $\mathbb{G}$. Then the following statements are satisfied:*

(1) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{Q}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})$, *for all* $a \in \mathbb{G}$.

(2) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q}a) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})a$, *for all* $a \in \mathbb{G}$.

(3) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})^{-1}$.

Proof.

(1) Since $\mathcal{Q}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i=j=1,2$ in $\mathbb{G}$. Consequently, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{Q})$.

Conversely, let q be an arbitrary element of $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{Q})$. Assume that $q = ap$ for some $p \in \mathcal{Q}$. By assumption, this shows $a\mathcal{Q}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i=1,2$; and that is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{Q})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i=j=1,2$ in $\mathbb{G}$. Suppose $a \in \mathcal{U}$ and $p \in \mathcal{V}$ be $\daleth_i^{\mathbb{G}}-$ NOSs, $i=1,2$ in $\mathbb{G}$, so that $\mathcal{U}\mathcal{V} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{Q})$. Then $a\mathcal{V} \subseteq a\mathcal{Q}$, which it follows that $a\mathcal{V} \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})$. Thus, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathcal{Q}) \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})$. Hence the statement follows.

(2) Following the same steps as in part (1) above, then we can prove $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q}a) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})a$.

(3) Since $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i=j=1,2$; so, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NOSs, $i=1,2$ in $\mathbb{G}$. Therefore, $\mathcal{Q}^{-1} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})^{-1}$ implies that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})^{-1}$. Next, let q be an arbitrary element of $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})^{-1}$. Then $q = p^{-1}$ for some $p \in (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})$. Let $q \in \mathcal{V}$ be $\daleth_i^{\mathbb{G}}-$ NOS, $i=1,2$ in $\mathbb{G}$. Then $\exists\ \daleth_i^{\mathbb{G}}-$ NOS, $i=1,2$; $\mathcal{U}$ is in $\mathbb{G}$ so that $p \in \mathcal{U}$ with $\mathcal{U}^{-1} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{V})\Big)$. Also, there is $g \in \mathcal{Q} \Cap \mathcal{U}$ which implies $g^{-1} \in \mathcal{Q}^{-1} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{V})\Big)$. That is $\mathcal{Q}^{-1} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{V})\Big) \neq 0_N \Rightarrow \mathcal{Q}^{-1} \Cap (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{V}) \neq 0_N \Rightarrow \mathcal{Q}^{-1} \Cap \mathcal{V} \neq 0_N$, since $\mathcal{Q}^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i=1,2$. Hence $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})^{-1}$.

Theorem 3.5. *Let $\mathcal{A}$ be any $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, $i=1,2$ in a NABTG $\mathbb{G}$. Then*

(1) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{A}) \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$, *for all* $a \in \mathbb{G}$.

(2) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}a) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})a$, *for all* $a \in \mathbb{G}$.

(3) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$.

Proof.

(1) As $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i=j=1,2$. From Theorem 3.1, $\pi_{a^{-1}} : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ is NACM, for each $i=1,2$.

So, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$. Hence $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{A}) \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$.

(2) As $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1,2$. From Theorem 3.1, $\sigma_{a^{-1}} : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ is NACM, for each $i = 1,2$. So, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})a$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$. Thus, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}a) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})a$.

(3) Since $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, $i = j = 1,2$; so, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1,2$ and hence $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$. Conserquently, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$.

Theorem 3.6. *Let $\mathcal{A}$ be both $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ neutrosophic semi open and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ neutrosophic semi closed subset of a NABTG, $i = j = 1,2$. Then the following statements hold:*

(1) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$, for all $a \in \mathbb{G}$.

(2) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}a) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})a$, for all $a \in \mathbb{G}$.

(3) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$.

Proof.

(1) Since $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1,2$; from which it follows that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{A}) \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$. Further, $\daleth_i^{\mathbb{G}}-$ neutrosophic semi-openness of $\mathcal{A}$, $i = 1,2$ implies $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})\Big) \Rightarrow a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})\Big)$. As $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSCoS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS in $\mathbb{G}$, $i = j = 1,2$. From Theorem 3.5, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})\Big) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{A})$. Hence $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(a\mathcal{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$.

(2) Following the same steps as in part (1) above, then we can prove $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}a) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})a$.

(3) By assumption, this shows $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1,2$ and therefore $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$. Consequently, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$. Next, as $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, $i = j = 1,2$; $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})\Big) \Rightarrow (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1} = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})\Big)^{-1}$. Also, as $\mathcal{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSCoS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1,2$. From Theorem 3.3, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1} = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{A})^{-1}\Big) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}^{-1})$. This shows that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathcal{A})^{-1}$.

Theorem 3.7. *From Theorem 3.6, the following statements hold:*

(1) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathscr{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$*, for all* $a \in \mathbb{G}$*.*

(2) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}a) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})a$*, for all* $a \in \mathbb{G}$*.*

(3) $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})^{-1}$*.*

Proof.

(1) As $\mathscr{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSCoS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1, 2$. From Theorem 3.1, $\pi_{a^{-1}} : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ is NACM, for each $i = 1, 2$. So, $\pi_{a^{-1}}^{-1}\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})\Big) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$. Thus, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathscr{A})$. Next, by hypothesis, it follows $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big) \Rightarrow a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big)$. As $\mathscr{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1, 2$. From Theorem 3.4, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big)$ $\supseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathscr{A})$. That is, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathscr{A}) \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$. Therefore, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(a\mathscr{A}) = a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$. Hence proved.

(2) As $\mathscr{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSCoS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1, 2$. From Theorem 3.1, $\sigma_{a^{-1}} : (\mathbb{G}, \daleth_i^{\mathbb{G}}) \to (\mathbb{G}, \daleth_i^{\mathbb{G}})$ is NACM, for each $i = 1, 2$. So, $\sigma_{a^{-1}}^{-1}\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})\Big) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})a$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$. Thus, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})a \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}a)$. Next, by hypothesis, it shows that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big) \Rightarrow (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})a = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big)a$. Since $\mathscr{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1, 2$. From Theorem 3.4, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big)a = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})a\Big)$ $\supseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}a)$. That is, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}a) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})a$. Therefore, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}a) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})a$. Hence proved.

(3) From hypothesis, this shows that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NROS, $i = j = 1, 2$ and therefore $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})^{-1}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1, 2$. Consequently, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}^{-1}) \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})^{-1}$. Next, as $\mathscr{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSCoS, $i = j = 1, 2$; $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big)$ $\Rightarrow (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})^{-1} = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\Big)^{-1}$. Also, as $\mathscr{A}$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NSOS, and $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})$ is $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ NRCS, $i = j = 1, 2$. From Theorem 3.4, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})^{-1} = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})^{-1}\Big)$. $\subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}^{-1})$. This proves that $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A}^{-1}) = (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathscr{A})^{-1}$.

Theorem 3.8. *Let $\mathscr{A}$ be $\daleth_i^{\mathbb{G}}-$ NOS in a NABTG $\mathbb{G}, i = 1,2$. Then* $a\mathscr{A} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\big)\Big)$, *for each* $a \in \mathbb{G}$.

Proof.

Since $\mathscr{A}$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1,2$; so $\mathscr{A} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\big) \Rightarrow a\mathscr{A} \subseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\big)$. From Theorem 3.2, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\big)$ is $\daleth_i^{\mathbb{G}}-$ NOS, $i = 1,2$; $\big($in fact,$(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}}) -$ NROS$, i = j = 1,2\big)$.
Hence, $a\mathscr{A} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl(\mathscr{A})\big)\Big)$.

Theorem 3.9. *Let $\mathcal{Q}$ be any $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})-$ Neutrosophic closed subset in a NABTG $\mathbb{G}, i = j = 1,2$. Then* $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})\big)\Big) \subseteq a\mathcal{Q}$, *for each* $a \in \mathbb{G}$.

Proof.

Since $\mathcal{Q}$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$; so $\mathcal{Q} \supseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})\big) \Rightarrow a\mathcal{Q} \supseteq a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})\big)$. From Theorem 3.2, $a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})\big)$ is $\daleth_i^{\mathbb{G}}-$ NCoS, $i = 1,2$; $\big($in fact,$(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}}) -$ NRCS$, i = j = 1,2\big)$. Therefore, $a\mathcal{Q} \subseteq (\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})\big)\Big)$.
Hence, $(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\Big(a(\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Cl\big((\daleth_i^{\mathbb{G}}, \daleth_j^{\mathbb{G}})N \vdash Int(\mathcal{Q})\big)\Big) \subseteq a\mathcal{Q}$.

4. Conclusion:

In this paper, we have studied the neutrosophic almost bitopological group. We defined the definition of the neutrosophic regularly open (closed) set and proved some of their properties. After defining the definition of the neutrosophic regularly open set we have defined the definition of the neutrosophic almost continuous mapping with an example and we proved some properties of the neutrosophic almost continuous mapping. Finally, by using the definition of the neutrosophic almost continuous mapping, we defined the neutrosophic almost bitopological group and cited an example. Also, we have proved some of their properties. In future, we try to study the Neutrosophic Almost Ideal Bitopological Group. We hope that this work shall bring some new ideas in the development of the neutrosophic almost bitopological group.

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